Robust Observer synthesis for Bilinear Parameter Varying system

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***Abstract*— In the present paper, sufficient conditions for the synthesis of robust Unknown Input Observers (UIOs) are proposed for a class of nonlinear systems, both in continuous and discrete time. The considered class is general enough to contain bilinear systems as well as Linear Parameter-Varying (LPV) systems with no parameter variation on the output matrix. The proposed conditions are numerically tractable, and are expressed in terms of Linear Matrix Inequalities (LMIs) or Linear Matrix Equalities (LMEs). Furthermore, the gain synthesis problem is shown to be formulated as a convex optimisation one, directly enabling the minimization of the influence of noisy measurements and model uncertainty. Simulations on energy systems are provided to illustrate the proposed methodologies.**

I. INTRODUCTION

In all engineering domains, one of the most popular methods to reduce the costs associated to physical sensors while maintaining robustness and performances of a control system is the utilisation of observers. An observer is able to estimate parameters that are difficult to measure, or to ensure the redundancy of sensors. Since the 1960s, several types of observers have been proposed in the literature, and have considered broad classes of dynamical systems (see for instance [1], [2]). When an unknown input is acting on a dynamical system, the specific class of Unknown Input Observers (UIOs) has been investigated. Their goal is to reconstruct some variables of interests for any unknown input with a given structure. Due to the practical relevance of the problems the UIOs address , there are several contributions to this field in the literature (e.g., [3])

While the early works on UIOs were initially focused on linear systems [4], significant research effort has been dedicated to their design for several classes of nonlinear systems, namely LPV systems [5], [6], discrete Fuzzy systems [7] or bilinear continuous Fuzzy systems, with single input [8], [9] or multiple inputs [10]. Furthermore, a new approach as the interval UIOs have been recently developed, as shown in [11] for systems with non-noisy outputs and in [12], [13] for systems with noisy outputs. It must to be noticed that the aforementioned classes of systems are amenable to the use of convex approaches for both control and observation (see [14]). Additionally, during the last decade several researchers have been interested in the topic of observer design for bilinear systems with unknown input. For example, in [15], [16], the authors have considered the design of UIOs for Discrete Time Uncertain Takagi-Sugeno Fuzzy System when no noise is taken into account.

However, few results are nowadays available for the class of bilinear parameter-varying (BLPV) dynamical systems, despite the fact that several applications can be modeled as such (e.g. power converters, as DC-DC or AC-DC ones). Moreover, measurement noise and model uncertainty are rarely taken into account, with some notable exception as [12], [13] and [10]. In [12] and [13], the interval observer is first designed, and a bound on the error is given a posteriori, with the consequence that the sensitivity to noise is not optimized. To the best of our knowledge, the synthesis of an observer gain that minimizes the impact of noise and uncertainty for either LPV or bilinear systems has not been conducted.

By proposing a robust UIO (i.e., an UIO whose sensitivity to noise and model uncertainty is minimized) for BLPV system, the present paper contributes in filling this gap. Indeed, we consider a large class of nonlinear systems (including bilinear, switched bilinear [17] and LPV with no parameter variation on the output matrix ones), and establish numerically tractable sufficient conditions for robust UIO synthesis The proposed conditions are expressed in terms of Linear Matrix Inequalities (LMIs) or Linear Matrix Equalities (LMEs). Furthermore, the gain synthesis problem is shown to be formulated as a convex optimisation one, thus enabling the minimization of the influence of noisy measurements and model uncertainty. The considered models are close to the one discussed in [8] and [10], with the notable difference that we consider a bilinear term between the control input and the unknown input. Since no assumptions are made on the variation of the parameter p, the proposed methodology also applies to switched (linear and bilinear) systems.

The suggested approach fits several application domains, as the class of systems that is investigated describes a large variety of physical devices, e.g., power converters or photovoltaic systems.

The remaining of this paper is organized as follows. In Section II, the models and the structures of the observers under consideration are introduced, along with the relevant convergence definitions. Sufficient conditions for the synthesis of robust UIOs in terms of LMIs and LMEs are presented in Section III. In Section IV, examples in the electrical domain are provided. Lastly, Section V offers some concluding comments.

Notation:

- For a vector or a matrix v, v' denotes its transpose.

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- $\mathbb{R}^+$ denotes the set of non-negative real numbers.
- We define for a matrix A, $He(A) := A + A'$. For a matrix M, $\lambda_{min}(M)$ and $\lambda_{max}(M)$ denote its smallest and largest eigenvalues, respectively. Given a set of matrices with identical number of lines $A_1, ..., A_N$, we denote $(A_1, ..., A_N)$ as their concatenation.
- For two positive definite matrices (resp positive semi-definite) P and Q, we write $P \succ Q$ if $P-Q$ is positive definite (resp $P \succeq Q$ if $P-Q$ is positive semidefinite).
- For a finite set $\mathcal{S}$, we use $|\mathcal{S}|$ to denotes its cardinal.
- Given $p \in \mathbb{N}$, the set Λ_p denotes the unit simplex,

$$\Lambda_p := \left\{ \lambda \in \mathbb{R}^p_{\geq 0} : \sum_{i=1}^{p} \lambda_i = 1 \right\}.$$

For a set of matrices $R_i \in \mathbb{R}^{n\times m}$, $i = 1, \dots p$, $Cov\{R_i\}_{i\in\{1\dots p\}}$ denotes its closed convex hull:

$$\underset{i\in\{1\dots p\}}{Cov\{R_i\}} = \left\{ Z \in \mathbb{R}^{n\times m} : \exists \lambda \in \Lambda_p, \sum_{i=1}^{p} \lambda_i R_i = Z \right\}.$$

II. Modeling of systems and observers

A. Dynamical Systems

In this work, we consider LPV bilinear systems in the following form:

$$\begin{cases} \dot{x}(t) = & A(p)x + B(p)u(t) \\ & + \sum_{i=1}^{m} u_i G_i(p)x(t) + D(p)v(t) \\ & + \sum_{i=1}^{m} u_i F_i(p)v(t) + S(p) + \Omega(p)\omega(t), \\ y(t) = & Cx(t) + \Delta\delta(t), \end{cases} \tag{1}$$

$$\begin{cases} x_{k+1} = & A(p)x_k + B(p)u_k \\ & + \sum_{i=1}^{m} u_{k,i} G_i(p)x_k + D(p)v_k \\ & + \sum_{i=1}^{m} u_{k,i} F_i(p)v_k + S(p) + \Omega(p)\omega_k, \\ y_k = & Cx_k + \Delta\delta_k, \end{cases} \tag{2}$$

where the model in (1) describes the continuous-time (CT) case, while the one in (2) the discrete-time (DT) case.

In this setting, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{U} \subset \mathbb{R}^m$ is the known input, with $u_{k,i}$ the value of the i^{th} component of the input at time k, $v \in \mathbb{R}^r$ is the unknown input, $y \in \mathbb{R}^q$ is the output, while $\omega \in \mathbb{R}^{n_\omega}$ and $\delta \in \mathbb{R}^{q_\delta}$ are respectively an unknown disturbance and a noise. In the rest of this work we will furthermore assume that the noise is differentiable and therefore that $\dot{\delta}$ is well defined. The matrices $A(p), B(p), G_i(p), D(p), F_i(p), S(p), \Omega(p), C, \Delta$ are of appropriate dimensions, and $p \in \mathcal{P} \subset \mathbb{R}^p$ denotes a known scheduling parameter.

B. Unknown Input Observers

With respect to the models in (1) and (2), we consider the following UIOs, respectively:

$$\begin{cases} \dot{z}(t) = N(p)z(t) + L(p)y(t) + J(p)u(t) \\ \qquad + \sum_{i=1}^{m} u_i(t) H_i(p)y(t) + MS(p), \\ \hat{x}(t) = z(t) - Ey(t), \end{cases} \tag{3}$$

$$\begin{cases} z_{k+1} = N(p)z_k + L(p)y_k + J(p)u_k \\ \qquad + \sum_{i=1}^{m} u_{k,i} H_i(p)y_k + MS(p), \\ \hat{x}_k = z_k - Ey_k. \end{cases} \tag{4}$$

We stress that the structure of the suggested UIOs is a direct extension of classical UIOs (see [18]).

In the present paper, we refer to the following definitions:

Definition 1 (CT Exponential observer): A CT observer $\hat{x}(t)$ is an exponential observer of $x(t)$ with convergence rate $\alpha > 0$ if there exists M such that

$$||x(t) - \hat{x}(t)|| \leq M||x(0) - \hat{x}(0)||e^{-\alpha t}.$$

Definition 2 (DT Exponential observer): A DT observer $\hat{x}_k$ is an exponential observer of x_k with convergence rate $\alpha \in]0, 1[$ if there exists M such that

$$||x_k - \hat{x}_k|| \leq M||x_0 - \hat{x}_0||\alpha^k.$$

Definition 3 (CT Robust observer): An CT observer $\hat{x}(t)$ is a robust observer, with respect to perturbation ν, of $x(t)$ with L_2 gain γ if

$$\int_0^\infty (x(s) - \hat{x}(s))'(x(s) - \hat{x}(s)) - \gamma^2 \nu(s)'\nu(s) ds < \infty.$$

Definition 4 (DT Robust observer): A DT observer $\hat{x}_k$ is a robust observer, with respect to perturbation ν, of x_k with L_2 gain γ if

$$\sum_{l=0}^{\infty} (x_l - \hat{x}_l)'(x_l - \hat{x}_l) - \gamma^2 \nu_l'\nu_l < \infty.$$

C. Assumptions

We describe here some assumptions on the matrices of the considered systems that will be used in the sequel of the paper.

First, with respect to the matrices in (1), (2), (3) and (4), let us name the matrix M as $M = I_n + EC$. Therefore, for the system matrices $\forall p \in \mathcal{P}$, $\forall i = 1, ...m$, we consider the following:

Constraints on equality:

i : $MA(p) - N(p)M - L(p)C = 0$;
ii : $MG_i(p) = H_i(p)C$;
iii : $MB(p) = J(p)$;
iv : $M(D(p) + \sum_{i=1}^{m} u_i M F_i(p)) = 0$.

Then, for the parameter dependent matrices $\forall p \in \mathcal{P}$, $\forall i = 1, ...m$, we consider the following:

Assumptions on convexity:

v : $A(p) \in Cov\{A_j\}_{j\in\mathbb{I}_f}$, $B(p) \in Cov\{B_j\}_{j\in\mathbb{I}_f}$,
$G_i(p) \in Cov\{G_{ij}\}_{j\in\mathbb{I}_f}$, $F_i(p) \in Cov\{F_{ij}\}_{j\in\mathbb{I}_f}$,
$D(p) \in Cov\{D_j\}_{j\in\mathbb{I}_f}$, $\Omega(p) \in Cov\{\Omega_j\}_{j\in\mathbb{I}_f}$,

where $\mathbb{I}_f \subset \mathbb{N}$ is a finite set of indexes.

III. Observer synthesis

In this Section we provide the main results of the present paper. First, robust observers are designed in case of noise both for CT and DT systems. Then, the particular noiseless case is investigated, and the proposed observers are shown to exponentially converge to the dynamical systems.

Preliminary computations of the observation error defined as $e = x - \hat{x}$ in CT or $e_k = x_k - \hat{x}_k$ in DT follows.

1) CT - Observation error:

Since $M = I_n + EC$, it follows that

$$\dot{e} = \dot{x} + EC\dot{x} - \dot{z} + E\Delta\dot{\delta} = M\dot{x} - \dot{z} + E\Delta\dot{\delta},$$

which can be rewritten as

$$\begin{aligned}\dot{e} = M\Big(& A(p)x + B(p)u + D(p)v \\ & + \textstyle\sum_{i=1}^{m} u_i\Big(G_i(p)x + F_i(p)v\Big) + S(p) + \Omega(p)\omega\Big) \\ & -\Big(N(p)z + L(p)Cx + L(p)\Delta\delta + J(p)u \\ & + \textstyle\sum_{i=1}^{m} u_i H_i(p)y + MS(p)\Big) + E\Delta\dot{\delta}.\end{aligned}$$

By rearranging the terms, it results

$$\begin{aligned}\dot{e} = {} & N(p)e + \big(MA(p) - N(p)M - L(p)C\big)x \\ & + MD(p)v + \big(MB(p) - J(p)\big)u \\ & + \textstyle\sum_{i=1}^{m} u_i\Big(\big(MG_i(p) - H_i(p)C\big)x + MF_i(p)v\Big) \\ & + M\Omega(p)\omega + E\Delta\dot{\delta} \\ & - \big(L(p) + NE + \textstyle\sum_{i=1}^{m} u_i H_i(p)\big)\Delta\delta.\end{aligned}$$

Then, under Assumptions (i), (ii), (iii) and (iv), it is possible to write

$$\dot{e}(t) = N(p)e(t) + R(p, u(t))\nu(t) \tag{5}$$

where

$$R(p,u) = \left(M\Omega(p), -(L(p) + N(p)E + \sum_{i=1}^{m} u_i H_i(p))\Delta, E\Delta\right), \tag{6}$$

such that $R(p,u) \in \mathbb{R}^{n\times(2q_\delta+n_\omega)}$ and $\nu(t) = (w(t)', \delta(t)', \dot{\delta}(t)')'$. Note that due to its structure the UIO is sensitive to measurement noises as well as the measurement noise derivative.

2) DT - Observation error:

Similarly to the CT case, by the definition of the error as $e_k = x_k - \hat{x}_k$ and since $M = I_n + EC$, it results

$$\begin{aligned} e_{k+1} &= x_{k+1} + ECx_{k+1} - z_{k+1} + E\Delta\delta_{k+1} \\ &= Mx_{k+1} - z_{k+1} + E\Delta\delta_{k+1}\end{aligned}$$

and therefore

$$\begin{aligned} e_{k+1} = {} & N(p)e_k \\ & + \big(MA(p) - N(p)M - L(p)C\big)x_k \\ & + MD(p)v_k + \big(MB(p) - J(p)\big)u_k \\ & + \textstyle\sum_{i=1}^{m} u_{k,i}\big(MG_i(p) - H_i(p)C\big)x_k \\ & + M\Omega(p)\omega + E\Delta\delta_{k+1} \\ & - \big(L(p) + NE + \textstyle\sum_{i=1}^{m} u_{k,i}H_i(p)\big)\Delta\delta_k.\end{aligned}$$

Likewise for (5) in the CT case, under Assumptions (i), (ii), (iii) and (iv) it is possible to write

$$e_{k+1} = N(p)e_k + R(p, u_k)\nu_k \tag{7}$$

where $R(p,u_k)$ is defined similarly to $R(p,u)$ in (6) for the CT case, and $\nu_k = (w_k', \delta_k', \delta_{k+1}')'$.

A. Observer design as convex optimisation problem

The aim of this section is to show how UIO synthesis with robustness properties can be reformulated as a convex optimization problem. For this problem, the constraints described as LMEs will ensure that the unknown input has no impact on the observation error, while the ones described as LMIs will ensure convergence of the observation error. Furthermore, the objective function to minimize is defined as the sensitivity of the UIO to both noises and uncertainties.

Let us consider the convexity assumptions in (v). Together with the equality assumptions in (i), (ii), (iii) and (iv), it follows that the observer gains can be expressed as convex combination of matrices to be determined (i.e. $N(p) \in Cov\{N_j\}_{j\in\mathbb{I}_f}$, $L(p) \in Cov\{L_j\}_{j\in\mathbb{I}_f}$,$H_i(p) \in Cov\{H_{ij}\}_{j\in\mathbb{I}_f}, R(p,u) \in Cov\{R_j(u)\}_{j\in\mathbb{I}_f}$).

Hence, using the aforementioned convexity property of the matrices of the UIO, the complexity of its gain synthesis problem is drastically reduced. In the sequel, we give sufficient conditions to verify (i), (ii), (iii) and (iv) under the assumption that (v) is verified.

1) CT - Convergence of the observer: First let us define the following matrices inequalities:

$$\mathbb{P}_j^c(u) = \begin{pmatrix} He\big((P+SC)A_j - W_jC\big) + \varepsilon I & \Pi_j^c(u) \\ \star & -\rho I_{2q_\delta+n_\omega} \end{pmatrix} \prec 0 \tag{8}$$

with $\Pi_j^c(u) = [(P+SC)\Omega_j,\ (W_j - \sum_i u_i V_{ij})\Delta,\ S\Delta]$ and, additionally,

$$(P+SC)(D_j + \sum_{i=1}^{m} u_i F_{ij}) = 0, \tag{9}$$

$$(P+SC)B_j = O_j, \tag{10}$$

$$(P+SC)G_{ij} = V_{ij}C, \tag{11}$$

and the following matrices:

$$\begin{aligned} & Q = P^{-1},\ E = QS,\ M = I + EC \\ & J_j = QO_j,\ H_{ij} = QV_{ij}, \\ & N_j = MA_j - QW_jC,\ L_j = QW_j - N_jE. \end{aligned} \tag{12}$$

Theorem 1: If there exist a P positive definite matrix, S, W_j, V_{ij}, O_j matrices of appropriate dimension and scalars $\rho, \varepsilon > 0$, such that $\forall\, j \in \mathbb{I}_f, i = 1, ...m$ and $\forall u \in \mathbb{U}$ a (8),(9), (10), (11) are verified then, it is possible to synthesize an observer of the form (3) with gain (12) that has a L_2 gain bounded by $\gamma = \sqrt{\frac{\rho}{\varepsilon}}$.

Proof:

From (9), $(P+SC)(D_j + \sum_{i=1}^{m} u_i F_{ij}) = 0$. Since P is invertible, from $(P+SC) = P(I+EC) = PM$, one has $M(D_j + \sum_{i=1}^{m} u_i F_{ij}) = 0$.

Furthermore since $D(p) + \sum_{i=1}^{m} u_i F(p)_i = \sum_{j\in I_f} \lambda_j \big(D_j + \sum_{i=1}^{m} u_i F_{ij}\big)$ for some (measurable) $\lambda_j \in \Lambda_{|I_f|}$. Therefore by convexity, (iv) holds.

From (10), $P(I + EC)B_j = O_j = PJ_j$ so from the fact that P is invertible one has $MB_j = J_j$. Since $B(p) = \sum_{j\in I_f} \lambda_j B_j\big)$ for some (measurable) $\lambda_j \in \Lambda_{|I_f|}$. So by convexity, (iii) holds with $J(p) = \sum_{j\in I_f} \lambda_j J_j$.

From (11) $P(I + EC)G_{ij} = V_{ij}C = PH_{ij}C$. By assumption on $G_i(p) = \sum_{j\in I_f} \lambda_j G_{ij}$ for some (measurable) $\lambda_j \in \Lambda_{|I_f|}$ and from the fact that P is invertible, by convexity, (ii) holds with $H_i(p) = \sum_{j\in I_f} \lambda_j H_{ij}$.

By definition of L_j and N_j, one has $L_jC = QW_jC - N_jEC$, so

$$N_jM + L_jC = N_j + N_jEC + L_jC,$$

$$N_jM + L_jC = MA_j - QW_jC + N_jEC + QW_jC - N_iEC.$$

Therefore $MA_j - N_jM - L_jC = 0$. By assumption $A(p) = \sum_{j\in I_f} \lambda_j A_j$ for some (measurable) $\lambda_j \in \Lambda_{|I_f|}$. So by convexity, (i) holds with $L(p) = \sum_{j\in I_f} \lambda_j L_j$ and $N(p) = \sum_{j\in I_f} \lambda_j N_j$.

Considering $V(e) = e'Pe$, one has

$$\dot{V}(e) = (N(p)e + R(p,u)\nu)'Pe + e'P(N(p)e + R(p,u)\nu).$$

Remarking that by definition of $N_j = MA_j - QW_jC$ so

$$PN_j = PMA_j - W_jC = PA_j + PECA_j - W_jC.$$

By definition of $E = QS$ it follows that

$$PN_j = PA_j + SCA_j - W_jC.$$

By denoting

$$PR_j(u) := (P + SC, -W_j - \sum_{i=1}^{m} u_i V_{ij}, S)$$

and remarking that $P + SC = PM$, $W_j + \sum_{i=1}^{m} u_i V_{ij} = PL_j + PN_jE + \sum_{i=1}^{m} u_i PH_{ij}$ and $PE = S$, one has $PR_j(u) = \Pi_j^c(u)$. From (8) and convexity property, one has $\dot{V}(e) + \varepsilon e'e - \rho\nu'\nu = \dot{V}(e) + \varepsilon(e'e - \gamma^2\nu'\nu) \leq 0$. Since V is positive definite integrating, the previous expression leads to

$$\int_0^\infty e(s)'e(s) - \gamma^2\nu(s)'\nu(s)ds \leq \frac{V(e(0))}{\varepsilon}.$$

Therefore the observation error has an L_2 gain bounded by γ. ■

Claim 1: The previous theorem can be restated as a convex optimization problem targeting to optimise the observer gains. The optimisation variables would be the P positive definite matrix, the matrices S, W_j, V_{ij}, O_j, and the positive scalar ρ and $\varepsilon > 0$. Therefore, by solving the optimization problem

$$\min_{(8)-(11)} \rho - \varepsilon \tag{13}$$

it is possible to reduce the value of $\sqrt{\frac{\rho}{\varepsilon}}$, which is an upper bound on the L_2 gain of the observer in (3).

Remark 1: By minimizing $\gamma^2 = \frac{\rho}{\varepsilon}$, one can minimize the impact of both the noise and the model uncertainty on the UIO. While γ^2 is not convex, one can replace $\frac{\rho}{\varepsilon}$ by $\rho - \varepsilon$ if the targeted goal is to have a value for ε thath is maximum while the value for ρ is as small as possible. According to [19], in order to find the optimal value of γ it is also possible to solve a series of convex optimization problems.

2) DT - Convergence of the observer: First let us define the matrix

$$\mathbb{P}_j^d(u) = \begin{pmatrix} P & PA_j + SCA_j - W_jC & \Pi_j^d(u) \\ \star & P - \varepsilon I_n & 0_{n,2q_\delta+n_\omega} \\ \star & \star & \rho I_{2q_\delta+n_\omega} \end{pmatrix} \succ 0 \tag{14}$$

Theorem 2: If there exist a P positive definite matrix, matrices of appropriate dimension S, W_j, V_{ij}, O_j and scalars $\rho, \varepsilon > 0$ such that $\forall\, j \in \mathbb{I}_f, i = 1, ...m$ and $\forall u \in \mathbb{U}$ (14),(9), (10), (11) are verified then, it is possible to synthesize an observer of the form (4) with gains (12) that has a L_2 gain bounded by $\gamma = \sqrt{\frac{\rho}{\varepsilon}}$.
The proof is *mutatis mutandis* identical to the previous one (this is also true for the subsequent proof).

Note that Claim 1 applies also to Theorem 2.

Remark 2: Alternatively, the results in Theorem 1 (or Theorem 2) hold also when considering specific controllers that verify (8)-(11) , thus relaxing the conditions requiring the necessity to hold $\forall\ u \in \mathbb{U}$. Note that in this case the classic rank condition for UIOs, which is $rank(CD_j) = rank(D_j)$ (i.e. $(P + SC)D_j = 0$)), is no longer required in general, but is replaced by the condition $(P + SC)(D_j + \sum_{i=1}^{m} u_i F_{ij}) = 0$, for some specifically chosen inputs.

Remark 3: In Theorems 1 and 2, the convex optimization problem is dependent on u. To check the proposed conditions for all possible u, an infinite set of LMIs and LMEs is not tractable. In the case that $u \in \mathbb{U}$ is bounded, it is possible to consider a convex embedding where $\forall\ u \in \mathbb{U}$, $u = \sum_{s\in\mathbb{S}} \lambda_s \bar{u}_s$, where $\mathbb{S}$ is finite and $(u_s)_{s\in\mathbb{S}}$ is a finite set of vertices of a convex polytope containing $\mathbb{U}$. In order to ensure $\mathbb{P}_j^c(u) \prec 0, \forall u \in \mathbb{U}$, one can check a finite amount of LMIs on $\mathbb{P}_j^c(u) \prec 0, \forall u \in (u_s)_{s\in\mathbb{S}}$.
In the case where $V_{ij}\Delta = 0$, $(P + SC)D_j = 0$, $(P + SC)F_{ij} = 0$, the dependency of (8) (resp. (14)) vanishes and the observer (3) (resp. (4)) converges for any control input.

B. Observer synthesis: Noiseless case

Let us now focus on the noiseless case, thus assume $\Delta = 0$ and $\Omega(p) = 0$. We show that it is possible to derive conditions for the synthesis of exponential UIO. First consider the following set of LMIs :

$$He\Big(\big(P + SC\big)A_j - W_jC\Big) + \alpha^2 P \prec 0. \tag{15}$$

Theorem 3: Given a positive scalar α, if there exist a P positive definite matrix and matrices of appropriate dimensions S, W_j, O_j, V_{ij} such that $\forall\, j \in \mathbb{I}_f; i = 1...m, \forall u \in \mathbb{U}$ the following relations are verified: (15), (9), (10), (11).

Then, it is possible to synthesize an observer of the form (3) with gains given by (12). Furthermore, the obtained observer is exponential with α as convergence rate and overshoot $\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$.

Claim 2: Theorem 3 can be restated as a convex optimization problem targeting to minimize the overshoot. The optimisation variables would be the P positive definite matrix and the matrices S, W_j, V_{ij}, O_j. Therefore, by solving the optimization problem

$$\min_{(15),(9),(10)(11)} trace(P) \tag{16}$$

it is possible to reduce the value of $\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$, which is an upper bound of the overshoot of the exponential observer in (3).

Cosndiering the following set of LMIs

$$\begin{pmatrix} \alpha^2 P & PA_j + SCA_j - W_j C \\ \star & P \end{pmatrix} \succ 0, \tag{17}$$

Theorem 4: Given a positive scalar $\alpha \in [0, 1]$, if there exist a P positive definite matrix and matrices of appropriate dimension S, W_j, O_j, V_{ij} such that $\forall\, j \in \mathbb{I}_f; i = 1...m$ the following relations are verified : (17), (9), (10), (11), then, it is possible to synthesize an observer of the form (4) with gains given by (12). Furthermore, the obtained observer is exponential with α as convergence rate and overshoot $\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$.

Note that Claim 2 also applies to the discrete time case.

IV. Simulations

This section provides two numerical examples implemented in Matlab, one in CT and one in DT, to show the effectiveness of the proposed observers. Without loss of generality, for simplicity of exposition we will consider the case where nominal parameters are used, thus dealing with bilinear systems. Note that for both examples, a more precise modeling should include parameter variation.

A. Continuous time: DC-DC converter

Due to the increase in the utilisation of renewable energy, current research literature is recently focusing on the modeling and control of power converters, which models are bilinear and parameter varying (see for instance [20]). Indeed, converters can be described as BLPV systems, as shown in [21]. To test the proposed robust UIOs, we consider a bilinear DC–DC converter as in [21], which system is in the form of

$$\begin{aligned} \dot{x} &= Ax + Bu(t) + uGx + Dv + uFv + \Omega\omega(t), \\ y &= Cx + \Delta\delta. \end{aligned} \tag{18}$$

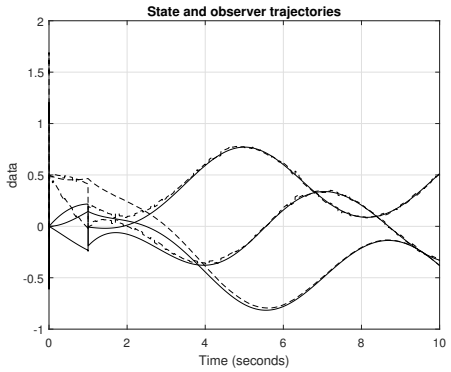

Fig. 1: system states (plain), estimated states (dashed)

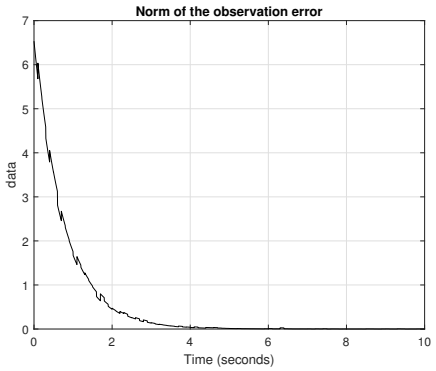

Fig. 2: Norm of the estimation error $||x(t) - \hat{x}(t)||$

The numerical values of the matrices are taken from equation (4) in [21] (using the nominal values of the parameters). In addition to them, and differently from [21] where a noiseless model with full state measurement is considered, here we choose:

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \Delta = \begin{pmatrix} 0.01 \\ 0.01 \end{pmatrix}, \Omega = 10^{-2}\,(1, 1, 1)'\,.$$

In this example, we consider the reference voltage to be the unknown input. To obtain Figure 1 and Figure 2, the input is a stabilizing controller while the unknown input is a step function starting at time $1s$. Figure 1 shows that the UIO converges to the real system. Furthermore, Figure 2 highlights that the observation error is not influenced by the unknown input while the noise has a limited impact.

B. Discrete time: PhotoVoltaic/Thermal (PVT) system

Nowadays, renewable energy for electricity production is rapidly developing, especially solar and wind energies. Solar energy can be used for electricity production through photovoltaic (PV) cell, or for thermal purposes through solar panels (SP).

We consider a complex continuous time nonlinear model using the Dymola software, then PVT model in Dymola is exported to MATLAB/Simulink as an S-function hardware in the loop platforms after being translated as a C code. The UIO system is connected to the Dymola block via MATLAB/Simulink.

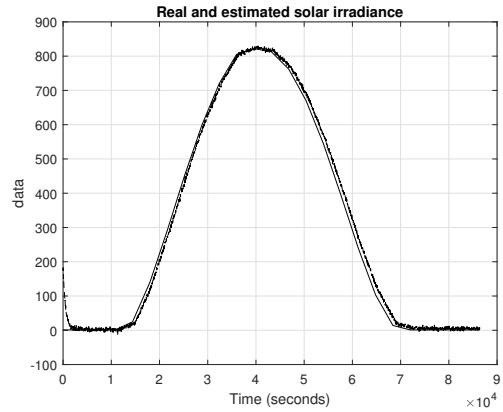

Fig. 3: solar irradiance (plain), estimated solar irradiance (dashed)

A simplified bilinear model is identified from the data generated by Dymola, in the following form:

$$\begin{cases} x^1_{k+1} = (1-a)x^1_k + cx^2_k - bu_k x^1_k + cv_k + 0.02\omega_k \\ x^2_{k+1} = x^2_k + v_k y_k = x^1_k + 0.1\delta_k \end{cases}$$

The state x is composed by the temperature difference x^1_k (in K°) between the ambient temperature and the temperature of the solar panel, and by the solar irradiance x^2_k (in W/m^2). The considered sampling time is 32 seconds, a is the thermal loss, b is the cooling induced by forced convection of air trough the panel and c capture the heating induced by solar irradiance on the panel. The input u_k is the controlled mass flow rate of air due to forced convection. The unknown input v_k is the change in solar irradiance while ω_k account for model uncertainty and δ_k is a bounded noise between -0.5 and 0.5 (K°) on the temperature sensors. The mass flow rate u belongs the the interval $[0, 0.1]kg/s$.

By solving the conditions of Theorem 2, the unknown input observer exhibits good properties in estimating the unmeasured *solar irradiance* when considering a noisy (in the temperature sensor) and uncertain model. Note that the bilinear model used for observation is a discretized and simplified model of the real system, which is a more complex nonlinear model with respect to the one used for simulations.

V. Conclusion

In this work, we have considered the problem of synthesizing UIOs for BLPV systems with modeling uncertainties and measurement noise, in both continuous and discrete time. The proposed methodology relies on convex optimization, and allows for finding UIO gains that minimize the effect of the noise while ensuring that the unknown input has no impact on the observation error dynamics. In absence of noise, the proposed results can be simplified to ensure exponential convergence of the UIOs. The effectiveness of the proposed methods has been illustrated on both continuous and discrete time energy systems. Since the proposed methodology relies on the existence of a common Lyapunov function, it is not straightforward to extend it to cases where the output is parameter dependent. Future research will focus on alleviating such restrictions.

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