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# Strategic Communication with Cost-Dependent Decoders via the Gray-Wyner Network

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**Abstract**—In decentralized decision-making problems, communicating agents choose their actions based on locally available information and knowledge about decision rules or strategies of other agents. In this work, we consider a strategic communication game between an informed encoder and two decoders communicating via a Gray-Wyner network. All three agents are assumed to be rational and endowed with distinct objectives captured by non-aligned cost functions. The encoder selects and announces beforehand the compression scheme to be implemented. Then, it transmits three signals: a public signal, and a private signal to each decoder inducing a Bayesian game among the decoders. We are interested in designing an achievable coding scheme that minimizes the encoder’s long run cost function subject to the challenges imposed by the Gray-Wyner network.

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## I. INTRODUCTION

We study the two-receiver strategic communication with mismatched objectives via a Gray-Wyner network, as in Fig. 1. This paper extends our work in [1] to the case where decoders are mutually cost-dependent, i.e. the cost function of each decoder depends on the action of the other decoder, and each decoder observes one private signal in addition to the public signal. Knowing the cost functions of all agents, the goal of each player is to minimize its respective cost.

Originally referred to as the sender-receiver game, the problem was formulated in the game theory literature with no restrictions on the amount of information transmitted. The Nash equilibrium solution of the cheap talk game was investigated by Crawford and Sobel in [2]. In [3], Kamenica and Gentzkow formulate the Stackelberg version of the strategic communication game. This setting, referred to as the Bayesian persuasion game, is the one under study in this paper by considering a Gray-Wyner network with two decoders. The Gray-Wyner network was formulated in the seminal work [4] with a characterization of the region of attainable rates. The optimal region of second-order coding for the lossy Gray-Wyner network was derived in [5].

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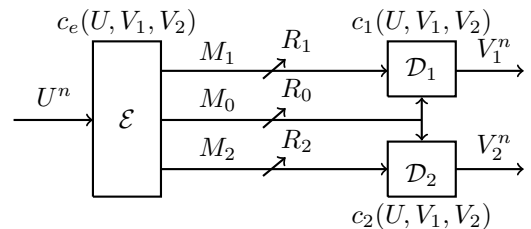


Fig. 1: Gray-Wyner network with cost-dependent decoders.

Information design with multiple designers interacting with a set of agents is studied in [6]. In [7], [8], the Nash equilibrium solution is investigated for multi-dimensional sources and quadratic cost functions, whereas the Stackelberg solution is studied in [9]. The strategic communication problem with a noisy channel is investigated in [10], [11], [12], [13], and [14]. The case where the decoder privately observes a signal correlated to the state, also referred to as the Wyner-Ziv setting [15], is studied in [16], [17] and [18]. Vora and Kulkarni investigate the achievable rates for the strategic communication problem in [19], [20] where the decoder is the Stackelberg leader.

In this paper, we assume that the encoder  $\mathcal{E}$  commits to and reveals an encoding strategy before observing the source symbols. Each commitment of the encoder induces a Bayesian game among the decoders  $\mathcal{D}_1$  and  $\mathcal{D}_2$ . This Bayesian game admits Bayes-Nash equilibria [21]. We assume that decoders will select the pair of output sequences that minimizes their respective costs and maximizes the encoder’s cost. Our goal is to characterize the encoder’s minimal long-run cost function subject to the optimal compression scheme that satisfies both decoders incentives constraints.

### A. Notations

Notations  $U^n$  and  $V_i^n$  denote the sequences of random variables of source information  $u^n = (u_1, \dots, u_n) \in \mathcal{U}^n$ , and decoder  $\mathcal{D}_i$ ’s actions  $v_i^n \in \mathcal{V}_i^n$  respectively for  $i \in \{1, 2\}$ . Calligraphic fonts  $\mathcal{U}$  and  $\mathcal{V}_i$  denote the alphabets and lowercase letters  $u$  and  $v_i$  denote the realizations. Notation  $X \oplus Y \oplus Z$  stands for the Markov chain  $\mathcal{P}_{Z|XY} = \mathcal{P}_{Z|Y}$ . The information source  $U$  follows the independent and identically

distributed (i.i.d) probability distribution  $\mathcal{P}_U \in \Delta(\mathcal{U})$ , where  $\Delta(\mathcal{U})$  denotes the probability simplex over  $\mathcal{U}$ .

## II. SYSTEM MODEL

In this section, we introduce the coding problem and formulate the Bayesian game induced by each encoding function.

**Definition 1.** Let  $R_0, R_1, R_2 \in \mathbb{R}_+^3 = [0, +\infty[^3$ , and  $n \in \mathbb{N}^* = \mathbb{N} \setminus \{0\}$ . The encoding  $\sigma$  and the decoding  $\tau_i$  of the encoder  $\mathcal{E}$  and decoders  $\mathcal{D}_i$  for  $i \in \{1, 2\}$  respectively, are random functions given by

$$\begin{aligned} \sigma : \mathcal{U}^n &\mapsto \Delta(\{1, \dots, 2^{\lfloor nR_0 \rfloor}\} \times \{1, \dots, 2^{\lfloor nR_1 \rfloor}\} \times \{1, \dots, 2^{\lfloor nR_2 \rfloor}\}), \\ \tau_i : (\{1, 2, \dots, 2^{\lfloor nR_0 \rfloor}\} \times \{1, 2, \dots, 2^{\lfloor nR_i \rfloor}\}) &\mapsto \Delta(\mathcal{V}_i^n). \end{aligned}$$

The coding triplets  $(\sigma, \tau_1, \tau_2)$  are stochastic and induce a joint probability distribution

$$\begin{aligned} \mathcal{P}_{U^n M_0 M_1 M_2 V_1^n V_2^n}^{\sigma, \tau_1, \tau_2} = \\ \left( \prod_{t=1}^n \mathcal{P}_{U_t} \right) \mathcal{P}_{M_0 M_1 M_2 | U^n}^{\sigma} \mathcal{P}_{V_1^n | M_0 M_1}^{\tau_1} \mathcal{P}_{V_2^n | M_0 M_2}^{\tau_2}. \quad (1) \end{aligned}$$

**Definition 2.** Single-letter cost functions  $c_e : \mathcal{U} \times \mathcal{V}_1 \times \mathcal{V}_2 \mapsto \mathbb{R}$  of the encoder and  $c_i : \mathcal{U} \times \mathcal{V}_1 \times \mathcal{V}_2 \mapsto \mathbb{R}$  of decoder  $\mathcal{D}_i$  for  $i \in \{1, 2\}$  induce long-run cost functions

$$\begin{aligned} c_e^n(\sigma, \tau_1, \tau_2) &= \mathbb{E}_{\sigma, \tau_1, \tau_2} \left[ \frac{1}{n} \sum_{t=1}^n c_e(U_t, V_{1,t}, V_{2,t}) \right] \\ &= \sum_{u^n, v_1^n, v_2^n} \mathcal{P}_{U^n V_1^n V_2^n}^{\sigma, \tau_1, \tau_2}(u^n, v_1^n, v_2^n) \cdot \left[ \frac{1}{n} \sum_{t=1}^n c_e(u_t, v_{1,t}, v_{2,t}) \right], \\ c_i^n(\sigma, \tau_1, \tau_2) &= \mathbb{E}_{\sigma, \tau_1, \tau_2} \left[ \frac{1}{n} \sum_{t=1}^n c_i(U_t, V_{1,t}, V_{2,t}) \right]. \end{aligned}$$

where  $\mathcal{P}_{U^n V_1^n V_2^n}^{\sigma, \tau_1, \tau_2}$  denotes the marginal distributions of  $\mathcal{P}^{\sigma, \tau_1, \tau_2}$  over the  $n$ -sequences  $(U^n, V_1^n, V_2^n)$ .

Each encoding function  $\sigma$  induces a Bayesian game  $G^\sigma(M_0, M_1, M_2, V_1^n, V_2^n)$  among the decoders which is defined below. We introduce the concept of decoders types in order to avoid hierarchy of Bayesian beliefs.

**Definition 3.** For each encoding  $\sigma$ , the finite Bayesian game  $G^\sigma(M_0, M_1, M_2, V_1^n, V_2^n)$  consists of:

- the decoders  $\mathcal{D}_i, i \in \{1, 2\}$  as the players of the game,
- $\mathcal{V}_i^n$  is the set of action sequences of  $\mathcal{D}_i$ ,
- $(M_0, M_i)$  is the type of decoder  $\mathcal{D}_i$ ,
- $\tau_i$  is a behavior strategy of decoder  $\mathcal{D}_i$ ,
- the belief of decoder  $\mathcal{D}_1$  (resp.  $\mathcal{D}_2$ ) over the type of decoder  $\mathcal{D}_2$  (resp.  $\mathcal{D}_1$ ) is given by  $\mathcal{P}_{M_2 | M_0 M_1}^{\sigma}$  (resp.  $\mathcal{P}_{M_1 | M_0 M_2}^{\sigma}$ ).
- $C_i^\sigma : \{1, 2, \dots, 2^{\lfloor nR_0 \rfloor}\} \times \{1, 2, \dots, 2^{\lfloor nR_1 \rfloor}\} \times \{1, 2, \dots, 2^{\lfloor nR_2 \rfloor}\} \times \mathcal{V}_1^n \times \mathcal{V}_2^n \mapsto \mathbb{R}$  is the  $\sigma$ -cost function of  $\mathcal{D}_i$  such that

$$\begin{aligned} C_i^\sigma(m_0, m_1, m_2, v_1^n, v_2^n) &= \sum_{u^n} \mathcal{P}^\sigma(u^n | m_0, m_1, m_2) \times \\ &\left[ \frac{1}{n} \sum_{t=1}^n c_i(u_t, v_{1,t}, v_{2,t}) \right], \quad \forall v_1^n, v_2^n, m_0, m_1, m_2. \end{aligned}$$

- For a fixed strategy profile  $(\tau_1, \tau_2)$ , the expected  $\sigma$ -costs  $\Psi_1^\sigma(\tau_1, \tau_2, m_0, m_1)$  of  $\mathcal{D}_1$  with type  $(m_0, m_1)$  is given by

$$\begin{aligned} \Psi_1^\sigma(\tau_1, \tau_2, m_0, m_1) &= \sum_{m_2} \mathcal{P}^\sigma(m_2 | m_0, m_1) \times \\ &\sum_{v_1^n, v_2^n} \mathcal{P}^{\tau_1}(v_1^n | m_0, m_1) \mathcal{P}^{\tau_2}(v_2^n | m_0, m_2) \times \\ &C_1^\sigma(v_1^n, v_2^n, m_0, m_1, m_2). \end{aligned}$$

Similarly,  $\Psi_2^\sigma(\tau_1, \tau_2, m_0, m_2)$  can be defined.

**Definition 4.** Given  $\sigma$ , for each behavior strategy  $\tau_2$ , decoder  $\mathcal{D}_1$ , computes the sets  $BR_1^\sigma(\tau_2)$  of best-response strategies

$$BR_1^\sigma(\tau_2) = \left\{ \tau_1, \Psi_1^\sigma(\tau_1, \tau_2, m_0, m_1) \leq \Psi_1^\sigma(\tilde{\tau}_1, \tau_2, m_0, m_1), \right. \\ \left. \forall \tilde{\tau}_1, m_0, m_1 \right\}.$$

Similarly,  $\mathcal{D}_2$  computes  $BR_2^\sigma(\tau_1)$ .

The Bayesian game  $G^\sigma(M_0, M_1, M_2, V_1^n, V_2^n)$  is finite, the players use behavioral strategies and Nash Theorem [22] ensures the existence of at least one Bayes-Nash equilibrium. In the following, we define the set of such equilibria.

**Definition 5.** For each encoding strategy  $\sigma$ , we define the set  $BNE(\sigma)$  of Bayes-Nash equilibria  $(\tau_1, \tau_2)$  of  $G^\sigma(M_0, M_1, M_2, V_1^n, V_2^n)$  as follows

$$BNE(\sigma) = \{(\tau_1, \tau_2), \tau_1 \in BR_1^\sigma(\tau_2) \text{ and } \tau_2 \in BR_2^\sigma(\tau_1)\}.$$

The communication game goes in the following order:

- The encoder  $\mathcal{E}$  chooses, announces the encoding  $\sigma$ .
- The sequence  $U^n$  is drawn i.i.d with distribution  $\mathcal{P}_U$ , and the game  $G^\sigma(M_0, M_1, M_2, V_1^n, V_2^n)$  begins.
- The messages  $(M_0, M_1, M_2)$  are encoded according to  $\mathcal{P}_{M_0 M_1 M_2 | U^n}^{\sigma}$ .
- Knowing  $\sigma$ , the decoders select the worst  $BNE(\sigma)$  for the encoder's cost.
- The cost values are given by  $c_e^n(\sigma, \tau_1, \tau_2)$ ,  $\Psi_1^\sigma(\tau_1, \tau_2, m_0, m_1)$ ,  $\Psi_2^\sigma(\tau_1, \tau_2, m_0, m_2)$ .

For  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$  and  $n \in \mathbb{N}^*$ , the encoder has to solve the following coding problem.

$$\Gamma_e^n(R_0, R_1, R_2) = \inf_{\sigma} \max_{(\tau_1, \tau_2) \in BNE(\sigma)} c_e^n(\sigma, \tau_1, \tau_2). \quad (2)$$

## III. MAIN RESULT

We consider three auxiliary random variables  $W_0 \in \mathcal{W}_0$ ,  $W_1 \in \mathcal{W}_1$  and  $W_2 \in \mathcal{W}_2$  with  $|\mathcal{W}_0| = |\mathcal{V}_1| \times |\mathcal{V}_2| + 1$ , and  $|\mathcal{W}_i| = |\mathcal{V}_i|$ , for  $i \in \{1, 2\}$ .

**Definition 6.** For  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , we define

$$\mathbb{Q}_0(R_0, R_1, R_2) = \left\{ \begin{aligned} &\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|UW_0} \mathcal{Q}_{W_2|UW_0}, \\ &R_0 \geq I(U; W_0), \quad R_1 \geq I(U; W_1|W_0), \\ &R_2 \geq I(U; W_2|W_0) \end{aligned} \right\}, \quad (3)$$

$$\hat{\mathbb{Q}}_0(R_0, R_1, R_2) = \left\{ \begin{aligned} &\mathcal{Q}_{W_0W_1W_2|U}, \\ &R_0 \geq I(U; W_0), \quad R_0 + R_1 \geq I(U; W_1, W_0), \\ &R_0 + R_2 \geq I(U; W_2, W_0) \end{aligned} \right\}. \quad (4)$$

**Definition 7.** For each distribution  $\mathcal{Q}_{W_0W_1W_2|U} \in \Delta(\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2)^{|\mathcal{U}|}$ , the auxiliary single-letter Bayesian game is given as follows:

- $(w_0, w_i)$  is the type of decoder  $\mathcal{D}_i$ ,  $i \in \{1, 2\}$ ,
- the belief of decoder  $\mathcal{D}_1$  (resp.  $\mathcal{D}_2$ ) over the type of decoder  $\mathcal{D}_2$  (resp.  $\mathcal{D}_1$ ) is given by  $\mathcal{Q}_{W_2|W_0W_1}$  (resp.  $\mathcal{Q}_{W_1|W_0W_2}$ ).
- $C_i^* : \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2 \times \mathcal{V}_1 \times \mathcal{V}_2 \mapsto \mathbb{R}$  is the single-letter cost of  $\mathcal{D}_i$  such that  $\forall w_0, w_1, w_2, v_1, v_2$

$$C_i^*(w_0, w_1, w_2, v_1, v_2) = \sum_u \mathcal{Q}(u|w_0, w_1, w_2) c_i(u, v_1, v_2).$$

- for each pair  $(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2})$  and profile  $(w_0, w_1)$ , the single-letter expected costs  $\Psi_1^*(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_1)$  of  $\mathcal{D}_1$  is given by

$$\begin{aligned} \Psi_1^*(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_1) = \\ \sum_{w_2} \mathcal{Q}(w_2|w_0, w_1) \sum_{v_1, v_2} \mathcal{Q}(v_1|w_0, w_1) \times \\ \mathcal{Q}(v_2|w_0, w_2) C_1^*(w_0, w_1, w_2, v_1, v_2). \end{aligned}$$

Similarly, we get  $\Psi_2^*(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_2)$ .

For each distribution  $\mathcal{Q}_{W_0W_1W_2|U} \in \Delta(\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2)^{|\mathcal{U}|}$ , the auxiliary set of Bayes-Nash equilibria is given by

$$\begin{aligned} \mathbb{BNE}(\mathcal{Q}_{W_0W_1W_2|U}) = \left\{ (\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}), \right. \\ \forall w_0, w_1, w_2, \Psi_1^*(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_1) \leq \\ \Psi_1^*(\tilde{\mathcal{Q}}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_1) \quad \forall \tilde{\mathcal{Q}}_{V_1|W_0W_1}, \\ \text{and } \Psi_2^*(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}, w_0, w_2) \leq \\ \left. \Psi_2^*(\mathcal{Q}_{V_1|W_0W_1}, \tilde{\mathcal{Q}}_{V_2|W_0W_2}, w_0, w_2) \quad \forall \tilde{\mathcal{Q}}_{V_2|W_0W_2} \right\}. \end{aligned}$$

The encoder's optimal cost is defined w.r.t.  $\mathbb{Q}_0(R_0, R_1, R_2)$  and  $\hat{\mathbb{Q}}_0(R_0, R_1, R_2)$  respectively as follows

$$\Gamma_e^*(R_0, R_1, R_2) = \inf_{\substack{\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \\ \mathcal{Q}_{W_2|W_0U} \in \mathbb{Q}_0(R_0, R_1, R_2)}} \max_{(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}) \in \mathbb{BNE}(\mathcal{Q}_{W_0W_1W_2|U})} \mathbb{E} \left[ c_e(U, V_1, V_2) \right], \quad (5)$$

$$\hat{\Gamma}_e(R_0, R_1, R_2) = \inf_{\substack{\mathcal{Q}_{W_0W_1W_2|U} \\ \in \hat{\mathbb{Q}}_0(R_0, R_1, R_2)}} \max_{\mathbb{BNE}(\mathcal{Q}_{W_0W_1W_2|U})} \mathbb{E} \left[ c_e(U, V_1, V_2) \right], \quad (6)$$

where the distribution in (5) satisfies the following Markov chain

$$W_1 \text{---} (U, W_0) \text{---} W_2.$$

The expectation in (6) is evaluated with respect to  $\mathcal{P}_U \mathcal{Q}_{W_0W_1W_2|U} \mathcal{Q}_{V_1|W_0W_1} \mathcal{Q}_{V_2|W_0W_2}$ .

**Remark 1.** The random variables  $U, W_0, W_1, W_2, V_1$  and  $V_2$  satisfy the following Markov chains

$$\begin{aligned} (U, W_2, V_2) \text{---} (W_0, W_1) \text{---} V_1, \\ (U, W_1, V_1) \text{---} (W_0, W_2) \text{---} V_2. \end{aligned}$$

**Theorem 1.** Let  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , we have

$$\begin{aligned} \forall \varepsilon > 0, \exists \hat{n} \in \mathbb{N}, \forall n \geq \hat{n}, \\ \Gamma_e^n(R_0, R_1, R_2) \leq \Gamma_e^*(R_0, R_1, R_2) + \varepsilon, \\ \forall n \in \mathbb{N}, \Gamma_e^n(R_0, R_1, R_2) \geq \hat{\Gamma}_e(R_0, R_1, R_2). \end{aligned}$$

**Lemma 1.** Let  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , and consider  $c_{e1} : \mathcal{U} \times \mathcal{V}_1 \mapsto \mathbb{R}$ , and  $c_{e2} : \mathcal{U} \times \mathcal{V}_2 \mapsto \mathbb{R}$ . If for all  $(u, v_1, v_2)$ ,  $c_e(u, v_1, v_2) = c_{e1}(u, v_1) + c_{e2}(u, v_2)$ , then

$$\Gamma_e^*(R_0, R_1, R_2) = \hat{\Gamma}_e(R_0, R_1, R_2) \quad (7)$$

Using Fekete's Lemma for the sub-additive sequence  $(n\Gamma_e^n(R_0, R_1, R_2))_{n \in \mathbb{N}^*}$  [1, Lemma 1] we get

$$\lim_{n \rightarrow \infty} \Gamma_e^n(R_0, R_1, R_2) = \inf_n \Gamma_e^n(R_0, R_1, R_2) = \Gamma_e^*(R_0, R_1, R_2).$$

#### IV. SKETCH OF ACHIEVABILITY PROOF OF THEOREM 1

The full proof of achievability can be found in [23, Section IV]. We restrict the optimization to a dense subset of target distributions inducing essential equilibria in order to ensure convergence. Then, for a given essential target distribution, we generate the codebook and show that the probability of error over the codebook is small. Finally, we outline the passage from the block game to the single-letter game by analyzing Bayes-Nash equilibria of all intermediate Bayesian games.

##### A. Essential Equilibria

**Definition 8.** Given  $\mathcal{Q}_{W_0W_1W_2|U} \in \Delta(\mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2)^{|\mathcal{U}|}$ , an equilibrium  $(\mathcal{Q}_{V_1|W_1, W_0}, \mathcal{Q}_{V_2|W_2, W_0}) \in \mathbb{BNE}(\mathcal{Q}_{W_0W_1W_2|U})$  is said to be essential if for all  $\varepsilon > 0$ , there exists an open neighborhood  $\Omega$  of  $\mathcal{Q}_{W_0W_1W_2|U}$  such that for all  $\hat{\mathcal{Q}}_{W_0W_1W_2|U} \in \Omega$ ,

$$\begin{aligned} (\hat{\mathcal{Q}}_{V_1|W_1, W_0}, \hat{\mathcal{Q}}_{V_2|W_2, W_0}) \in \mathbb{BNE}(\hat{\mathcal{Q}}_{W_0W_1W_2|U}) \implies \\ \|\mathcal{Q}_{V_1|W_0, W_1} - \hat{\mathcal{Q}}_{V_1|W_0, W_1}\| + \|\mathcal{Q}_{V_2|W_0, W_2} - \hat{\mathcal{Q}}_{V_2|W_0, W_2}\| \leq \varepsilon. \end{aligned}$$

We denote by  $\mathbb{EBNE}(\mathcal{Q}_{W_0W_1W_2|U})$  the set of essential Bayes-Nash equilibria.

**Definition 9.** For  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , we define the set

$$\begin{aligned} \tilde{\mathcal{Q}}_0(R_0, R_1, R_2) = & \left\{ \mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U}, \right. \\ & \min_{u, w_0, w_1, w_2} \mathcal{Q}(w_0|u) \mathcal{Q}(w_1|w_0, u) \mathcal{Q}(w_2|w_0, u) > 0, \\ & R_0 > I(U; W_0), R_1 > I(U; W_1|W_0), R_2 > I(U; W_2|W_0), \\ & \text{BNE}(\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U}) = \\ & \left. \text{EBNE}(\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U}) \right\}. \end{aligned}$$

**Lemma 2.** For  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , we have

$$\Gamma_e^*(R_0, R_1, R_2) = \inf_{\substack{\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U} \\ \in \tilde{\mathcal{Q}}_0(R_0, R_1, R_2)}} \max_{\substack{(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}) \in \mathcal{L} \\ \text{EBNE}(\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U})}} \mathbb{E} \left[ c_e(U, V_1, V_2) \right].$$

The proof of Lemma 2 follows from [21, Theorem 4.2], [23, Lemma 3], and since  $\tilde{\mathcal{Q}}_0(R_0, R_1, R_2)$  is dense in  $\mathcal{Q}_0(R_0, R_1, R_2)$  [23, Lemma 4].

For simplicity, we denote by  $\mathcal{T}_\delta^n$  the sets of typical sequences evaluated with respect to  $\mathcal{P}_U \mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U}$ .

### B. Codebook Generation

Fix a conditional probability distribution  $\mathcal{Q}_{W_0|U} \mathcal{Q}_{W_1|W_0U} \mathcal{Q}_{W_2|W_0U} \in \tilde{\mathcal{Q}}_0(R_0, R_1, R_2)$ . There exists  $\eta > 0$  such that

$$R_0 = I(U; W_0) + \eta, \quad (8)$$

$$R_1 = I(U; W_1|W_0) + \eta, \quad (9)$$

$$R_2 = I(U; W_2|W_0) + \eta. \quad (10)$$

Randomly and independently generate  $2^{\lfloor nR_0 \rfloor}$  sequences  $w_0^n(m_0)$  for  $m_0 \in [1 : 2^{\lfloor nR_0 \rfloor}]$ , according to the i.i.d distribution  $\prod_{t=1}^n \tilde{\mathcal{Q}}_{W_0}(w_{0t})$ . For each  $(m_1, m_0) \in [1 : 2^{\lfloor nR_1 \rfloor}] \times [1 : 2^{\lfloor nR_0 \rfloor}]$  generate a sequence  $w_1^n(m_1, m_0)$  randomly and conditionally independently according to the i.i.d conditional distribution  $\prod_{t=1}^n \mathcal{Q}_{W_1|W_0}(w_{1t}|w_{0t}(m_0))$ . For each  $(m_2, m_0) \in [1 : 2^{\lfloor nR_2 \rfloor}] \times [1 : 2^{\lfloor nR_0 \rfloor}]$  generate a sequence  $w_2^n(m_2, m_0)$  randomly and conditionally independently according to the i.i.d conditional distribution  $\prod_{t=1}^n \mathcal{Q}_{W_2|W_0}(w_{2t}|w_{0t}(m_0))$ .

Coding algorithm: Encoder  $\mathcal{E}$  observes  $u^n$  and looks in the codebook for a triplet  $(m_0, m_1, m_2)$  such that  $(u^n, w_1^n(m_1, m_0), w_2^n(m_2, m_0), w_0^n(m_0)) \in \mathcal{T}_\delta^n$ . If such a jointly typical tuple doesn't exist, the source encoder sets  $(m_0, m_1, m_2)$  to  $(1, 1, 1)$ . Then, it sends  $(m_0, m_1)$  to decoder  $\mathcal{D}_1$ , and  $(m_0, m_2)$  to decoder  $\mathcal{D}_2$ . Decoder  $\mathcal{D}_1$  declares  $v_1^n$  and decoder  $\mathcal{D}_2$  declares  $v_2^n$  according to  $\tau_1$  and  $\tau_2$ .

### C. Analysis of error probability

We define the following error events

$$\mathcal{F}_0 = \{(U^n, W_0^n(m_0)) \notin \mathcal{T}_\delta^n, \forall m_0\},$$

$$\forall m_0, \mathcal{F}_1(m_0) = \{(U^n, W_0^n(m_0), W_1^n(m_1, m_0)) \notin \mathcal{T}_\delta^n, \forall m_1\},$$

$$\forall m_0, \mathcal{F}_2(m_0) = \{(U^n, W_0^n(m_0), W_2^n(m_2, m_0)) \notin \mathcal{T}_\delta^n, \forall m_2\}.$$

By the covering lemma [24, Lemma 3.3],  $\mathcal{P}(\mathcal{F}_0)$  tends to zero as  $n \rightarrow \infty$  if  $R_0 \geq I(U; W_0) + \eta$ ,  $\mathcal{P}(\mathcal{F}_1(M_0)|\mathcal{F}_0^c)$  goes

to zero by the covering lemma if  $R_1 \geq I(U; W_1|W_0) + \eta$ , and  $\mathcal{P}(\mathcal{F}_2(M_0)|\mathcal{F}_0^c)$  goes to zero by the covering lemma if  $R_2 \geq I(U; W_2|W_0) + \eta$ .

The expected probability of error over the codebook being small means that for all  $\varepsilon_2 > 0$ , for all  $\eta > 0$ , there exist  $\bar{\delta} > 0$  and  $\bar{n} \in \mathbb{N}$ , such that for all  $\delta \leq \bar{\delta}$ , and for all  $n \geq \bar{n}$ , we have:

$$\mathbb{E}[\mathcal{P}(\mathcal{F}_0)] \leq \varepsilon_2, \quad \mathbb{E}[\mathcal{P}(\mathcal{F}_1(m_0)|\mathcal{F}_0^c)] \leq \varepsilon_2, \quad (11)$$

$$\mathbb{E}[\mathcal{P}(\mathcal{F}_2(m_0)|\mathcal{F}_0^c)] \leq \varepsilon_2, \quad (12)$$

### D. Analysis of Bayes-Nash Equilibria

For each encoding  $\sigma$ , we denote by  $\mathcal{G}^{\sigma, t}(M_0, M_1, M_2, V_{1,t}, V_{2,t})$  the Bayesian game of stage  $t \in \{1, 2, \dots, n\}$ , where  $(M_0, M_i)$  is the type of decoder  $\mathcal{D}_i$ , the respective beliefs of the decoders are given by  $\mathcal{P}_{M_2|M_0M_1}^\sigma$  and  $\mathcal{P}_{M_1|M_0M_2}^\sigma$ , their actions by  $V_{1,t}, V_{2,t}$ , and the  $\sigma$ -cost functions are given for all  $(m_0, m_1, m_2, v_{1,t}, v_{2,t})$  by  $\sum_{u_t} \mathcal{P}^\sigma(u_t|m_0, m_1, m_2) \times c_i(u_t, v_{1,t}, v_{2,t})$ ,  $i \in \{1, 2\}$ .

We denote by  $\text{BNE}(\sigma, t)$  its set of Bayes-Nash equilibria  $(\tau_{1,t}, \tau_{2,t})$  at stage  $t$  [23, Def. 11].

**Lemma 3.** 1. If  $(\tau_1, \tau_2) \in \text{BNE}(\sigma)$ , then  $(\tau_{1,t}, \tau_{2,t}) \in \text{BNE}(\sigma, t)$  for all  $t \in \{1, 2, \dots, n\}$ .

2. If  $(\tau_{1,t}, \tau_{2,t}) \in \text{BNE}(\sigma, t)$  for all  $t \in \{1, 2, \dots, n\}$ , then  $(\prod_{t=1}^n \tau_{1,t}, \prod_{t=1}^n \tau_{2,t}) \in \text{BNE}(\sigma)$ .

The proof of Lemma 3 is stated in [23, Lemma 5].

We introduce the indicator of error events  $E_\delta \in \{0, 1\}$  defined as follows

$$E_\delta = \begin{cases} 1, & \text{if } (u^n, w_1^n, w_2^n, w_0^n) \notin \mathcal{T}_\delta^n. \\ 0, & \text{otherwise.} \end{cases}$$

We control the Bayesian belief of decoder  $\mathcal{D}_1$  (resp.  $\mathcal{D}_2$ ) about the type of  $\mathcal{D}_2$  (resp.  $\mathcal{D}_1$ ). Let  $\mathcal{P}_{W_2, t}^{w_0^n, w_1^n} = \mathcal{P}_{W_2, t|W_0^n, W_1^n}(\cdot | w_0^n, w_1^n) \in \Delta(\mathcal{W}_2)$  and  $\mathcal{P}_{W_1, t}^{w_0^n, w_2^n} = \mathcal{P}_{W_1, t|W_0^n, W_2^n}(\cdot | w_0^n, w_2^n) \in \Delta(\mathcal{W}_1)$ . In a similar fashion, we denote by  $\mathcal{Q}_{W_2}^{w_0, w_1}$  and  $\mathcal{Q}_{W_1}^{w_0, w_2}$  the distributions  $\mathcal{Q}_{W_2|W_0, W_1}(\cdot | w_0, w_1)$  and  $\mathcal{Q}_{W_1|W_0, W_2}(\cdot | w_0, w_2)$  respectively.

**Lemma 4.** For all  $w_0^n, w_1^n, w_2^n, w_0, w_1, w_2$ , we have

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{t=1}^n D(\mathcal{P}_{W_2, t}^{w_0^n, w_1^n} \| \mathcal{Q}_{W_2}^{w_0, w_1}) \Big| E_\delta = 0 \right] = 0, \quad (13)$$

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{t=1}^n D(\mathcal{P}_{W_1, t}^{w_0^n, w_2^n} \| \mathcal{Q}_{W_1}^{w_0, w_2}) \Big| E_\delta = 0 \right] = 0. \quad (14)$$

*Proof.* The proof of Lemma 4 is stated in [23, App. A].  $\square$

We denote the Bayesian posterior beliefs  $\mathcal{P}_{U_t}^{\sigma, M_1 M_2 M_0}(\cdot | m_1, m_2, m_0) \in \Delta(\mathcal{U})$  by  $\mathcal{P}_{U_t}^{m_1 m_2 m_0}$ , and by  $\mathcal{Q}_{U}^{w_1 w_2 w_0}$  the single-letter belief  $\mathcal{Q}_{U|W_1 W_2 W_0}(\cdot | w_1, w_2, w_0)$ .

**Lemma 5.** For all  $\varepsilon > 0$ , for all  $m_0, m_1, m_2$ , we have

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{1}{n} \sum_{t=1}^n D(\mathcal{P}_{U_t}^{m_0 m_1 m_2} \| \mathcal{Q}_{U}^{w_1 w_2 w_0}) \Big| E_\delta = 0 \right] \leq \varepsilon. \quad (15)$$

*Proof.* The proof of Lemma 5 is stated in [23, App. B].  $\square$

Denote by  $\tilde{G}^{\sigma,t}(M_0, M_1, M_2, V_{1,t}, V_{2,t})$  the essential Bayesian game of stage  $t \in \{1, 2, \dots, n\}$ , where  $(M_0, M_i)$  is the type of decoder  $\mathcal{D}_i$ , the respective beliefs of the decoders are given by  $\mathcal{P}_{M_2|M_0M_1}^\sigma$  and  $\mathcal{P}_{M_1|M_0M_2}^\sigma$ , their actions by  $V_{1,t}, V_{2,t}$ , and the  $\sigma$ -cost functions are given for all  $(m_0, m_1, m_2, v_{1,t}, v_{2,t})$  by  $\sum_{u_t} \mathcal{Q}^\sigma(u_t|w_{0,t}(m_0), w_{1,t}(m_0, m_1), w_{2,t}(m_0, m_2)) \times c_i(u_t, v_{1,t}, v_{2,t})$ . We denote by  $EBNE(\sigma, t)$  its set of essential Bayes-Nash equilibria  $(\tau_{1,t}, \tau_{2,t})$ . It follows from Lemma 5 that for all  $\sigma, t$ , every equilibrium in  $BNE(\sigma, t)$  induces an equilibrium in  $EBNE(\sigma, t)$ . Similarly, denote by  $G_w^{\sigma,t}(W_{0,t}, W_{1,t}, W_{2,t}, V_{1,t}, V_{2,t})$  the game of stage  $t$  where the type of  $\mathcal{D}_i$  is  $W_{0,t}, W_{i,t}$ , and respective beliefs are the marginals  $\mathcal{P}_{W_{2,t}|W_{0,t}W_{1,t}}^\sigma$  and  $\mathcal{P}_{W_{1,t}|W_{0,t}W_{2,t}}^\sigma$ . This game directly derives from  $\tilde{G}^{\sigma,t}(M_0, M_1, M_2, V_{1,t}, V_{2,t})$  by considering components  $W_{0,t}, W_{1,t}$  and  $W_{2,t}$  of  $W_0^n(M_0), W_1^n(M_0, M_1)$  and  $W_2^n(M_0, M_2)$  respectively. Finally, consider the game  $\tilde{G}_w^{\sigma,t}(W_{0,t}, W_{1,t}, W_{2,t}, V_{1,t}, V_{2,t})$  of stage  $t$  where the type of  $\mathcal{D}_i$  is  $W_{0,t}, W_{i,t}$ , and respective beliefs are  $\mathcal{Q}_{W_{2,t}|W_{0,t}W_{1,t}}$  and  $\mathcal{Q}_{W_{1,t}|W_{0,t}W_{2,t}}$ . It follows from Lemma 4 that equilibria of both games  $G_w^{\sigma,t}(W_{0,t}, W_{1,t}, W_{2,t}, V_{1,t}, V_{2,t})$  and  $\tilde{G}_w^{\sigma,t}(W_{0,t}, W_{1,t}, W_{2,t}, V_{1,t}, V_{2,t})$  are arbitrarily close.

This concludes the achievability proof of Theorem 1. More details are provided in [23, Section IV].  $\square$

## V. CONVERSE PROOF OF THEOREM 1

Let  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$  and  $n \in \mathbb{N}^*$ . Fix  $(\sigma, \tau_1, \tau_2)$ , and consider a random variable  $T$  uniformly distributed over  $\{1, 2, \dots, n\}$  and independent of  $(U^n, M_0, M_1, M_2, V_1^n, V_2^n)$ . We introduce the auxiliary random variables  $W_0 = (M_0, T)$ ,  $W_1 = M_1$ ,  $W_2 = M_2$ ,  $(U, V_1, V_2) = (U_T, V_{1,T}, V_{2,T})$ , distributed according to  $\mathcal{P}_{UW_0W_1W_2V_1V_2}^{\sigma\tau_1\tau_2}$  defined for all  $(u, w_0, w_1, w_2, v_1, v_2) = (u_t, m_0, m_1, m_2, t, v_{1,t}, v_{2,t})$  by

$$\begin{aligned} & \mathcal{P}_{UW_0W_1W_2V_1V_2}^{\sigma\tau_1\tau_2}(u, w_0, w_1, w_2, v_1, v_2) = \\ & \mathcal{P}_{UTM_0M_1M_2TV_{1T}V_{2T}}^{\sigma\tau_1\tau_2}(u_t, m_0, m_1, m_2, t, v_{1,t}, v_{2,t}) \\ & = \frac{1}{n} \sum_{\substack{u^{t-1} \\ u^{t+1}}} \sum_{\substack{v_1^{t-1}, v_{1,t+1} \\ v_2^{t-1}, v_{2,t+1}}} \left( \prod_{t=1}^n \mathcal{P}_U(u_t) \right) \mathcal{P}_{M_0M_1M_2|U^n}^\sigma(m_0, m_1, m_2|u^n) \\ & \times \mathcal{P}_{V_1^n|M_0M_1}^{\tau_1}(v_1^n|m_0, m_1) \mathcal{P}_{V_2^n|M_0M_2}^{\tau_2}(v_2^n|m_0, m_2). \end{aligned}$$

**Lemma 6.** *The distribution  $\mathcal{P}_{UW_0W_1W_2V_1V_2}^{\sigma\tau_1\tau_2}$  has marginal on  $\Delta(U)$  given by  $\mathcal{P}_U$  and satisfies the Markov chain properties*

$$\begin{aligned} (U, V_2) & \text{---} (W_0, W_1) \text{---} V_1; \\ (U, W_1, V_1) & \text{---} (W_0, W_2) \text{---} V_2. \end{aligned}$$

*Proof.* [Lemma 6] The i.i.d. property of the source ensures that the marginal distribution is  $\mathcal{P}_U$ . By the definition of the decoding functions  $\tau_1$  and  $\tau_2$  we have

$$\begin{aligned} (U_T, V_{2,T}) & \text{---} (M_1, M_0, T) \text{---} V_{1,T}, \\ (U_T, M_1, V_{1,T}) & \text{---} (M_2, M_0, T) \text{---} V_{2,T}. \end{aligned}$$

$\square$  Therefore  $\mathcal{P}_U \mathcal{P}_{W_0|U}^\sigma \mathcal{P}_{W_1|W_0U}^\sigma \mathcal{P}_{W_2|W_0U}^\sigma \mathcal{P}_{V_1|W_0W_1}^{\tau_1} \mathcal{P}_{V_2|W_0W_2}^{\tau_2} =$

**Lemma 7.** *Given  $(R_0, R_1, R_2) \in \mathbb{R}_+^3$ , for all  $\sigma$ , the distribution  $\mathcal{P}_{W_0W_1W_2|U}^\sigma \in \mathbb{Q}_0(R_0, R_1, R_2)$ .*

*Proof.* [Lemma 7] We consider an encoding strategy  $\sigma$ , then

$$nR_0 \geq H(M_0) \geq I(M_0; U^n) \quad (16)$$

$$= \sum_{t=1}^n I(U_t; M_0|U^{t-1}) = nI(U_T; M_0|U^{T-1}, T) \quad (17)$$

$$= nI(U_T; M_0, U^{T-1}, T) \geq nI(U_T; M_0, T) \quad (18)$$

$$= nI(U; W_0). \quad (19)$$

Similarly,

$$nR_1 \geq H(M_1) \geq I(U^n; M_1|M_0) \quad (20)$$

$$\geq nI(U_T; M_1|M_0, T) = nI(U; W_1|W_0). \quad (21)$$

Similarly,  $nR_2 \geq nI(U; W_2|W_0)$ .  $\square$

**Lemma 8.** *For all  $(\sigma, \tau_1, \tau_2)$  and  $i \in \{1, 2\}$ , we have*

$$c_e^n(\sigma, \tau_1, \tau_2) = \mathbb{E}[c_e(U, V_1, V_2)], \quad (22)$$

$$c_i^n(\sigma, \tau_1, \tau_2) = \mathbb{E}[c_i(U, V_1, V_2)]. \quad (23)$$

*evaluated with respect to  $\mathcal{P}_U \mathcal{P}_{W_0W_1W_2|U}^\sigma \mathcal{P}_{V_1|W_0W_1}^{\tau_1} \mathcal{P}_{V_2|W_0W_2}^{\tau_2}$ . Moreover, for each  $(m_0, m_1, m_2, v_1^n, v_2^n)$ , we have*

$$C_i^\sigma(m_0, m_1, m_2, v_1^n, v_2^n) = C_i^*(w_0, w_1, w_2, v_1, v_2), \quad (24)$$

$$\Psi_i^\sigma(\tau_1, \tau_2, m_0, m_i) = \mathbb{E}_{\mathcal{P}_U}[\Psi_i^*(\mathcal{P}_{V_1|W_0W_1}^{\tau_1}, \mathcal{P}_{V_2|W_0W_2}^{\tau_2}, w_0, w_i)].$$

The proof of Lemma 8 is provided in [23, Lemma 15].

**Lemma 9.** *For all  $\sigma$ , we have*

$$\text{BNE}(\mathcal{P}_{W_0W_1W_2|U}^\sigma) = \left\{ (\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}), \right.$$

$$\left. \exists(\tau_1, \tau_2), \tau_1 \in BR_1^\sigma(\tau_2), \tau_2 \in BR_2^\sigma(\tau_1), \right.$$

$$\left. \mathcal{Q}_{V_1|W_0W_1} = \mathcal{P}_{V_1|W_0W_1}^{\tau_1}, \mathcal{Q}_{V_2|W_0W_2} = \mathcal{P}_{V_2|W_0W_2}^{\tau_2} \right\}. \quad (25)$$

The proof of Lemma 9 is provided in [23, Lemma 16]. For any strategy  $\sigma$ , we have

$$\max_{\tau_1, \tau_2} c_e^n(\sigma, \tau_1, \tau_2) = \max_{\tau_1, \tau_2} \mathbb{E}_{\mathcal{P}_{V_1|W_0W_1}^{\tau_1} \mathcal{P}_{V_2|W_0W_2}^{\tau_2}} \mathcal{P}_{W_0W_1W_2|U}^\sigma [c_e(U, V_1, V_2)] \quad (26)$$

$$= \max_{(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}) \in \text{BNE}(\mathcal{P}_{W_0W_1W_2|U}^\sigma)} \mathbb{E}_{\mathcal{Q}_{V_1|W_0W_1} \mathcal{Q}_{V_2|W_0W_2}} \mathcal{P}_{W_0W_1W_2|U}^\sigma [c_e(U, V_1, V_2)] \quad (27)$$

$$\geq \inf_{\mathcal{Q}_{W_0W_1W_2|U} \in \mathbb{Q}_0(R_0, R_1, R_2)} \max_{(\mathcal{Q}_{V_1|W_0W_1}, \mathcal{Q}_{V_2|W_0W_2}) \in \text{BNE}(\mathcal{P}_{W_0W_1W_2|U}^\sigma)} \mathbb{E} [c_e(U, V_1, V_2)] \quad (28)$$

$$= \hat{\Gamma}_e(R_0, R_1, R_2). \quad (29)$$

Equations (26) and (27) follow from Lemma 8, whereas (28) comes from Lemma 7. This concludes the converse proof of Theorem 1.

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