

# Added mass in the presence of density stratification

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Added mass represents the additional inertia experienced by a body as it moves through a fluid, owing to the flow that this motion generates within the fluid. As such, added mass determines the impulse and energy of the fluid, the dipole strength of the body and the hydrodynamic forces exerted on it. When, in addition, the fluid supports the propagation of waves, added mass gets an imaginary part associated with wave drag. When viscosity is present, depending on the Stokes number, added mass combines with Stokes drag and the Basset memory integral.

We investigate the way buoyancy affects the added mass of a body oscillating in a uniformly stratified fluid. Two definitions of added mass are possible, one involved in the energy of the fluid and the forces on the body, and the other in the impulse of the fluid and the dipole strength of the body. For bodies with horizontal and vertical symmetries, the relation between the two is straightforward. Based on an earlier study of the oscillations using boundary integrals (Voisin 2021), an elliptic horizontal cylinder and a spheroid of vertical axis are considered explicitly.

Two applications of added mass are important in practice: the wave energy, called conversion rate in the context of internal tides (Garrett & Kunze 2007); and the dynamics of buoyant bodies (Cairns, Munk & Winant 1979; Goodman & Levine 1990; D'Asaro 2018). Particular attention is paid to this latter aspect. Added mass is decomposed into its value in a non-stratified fluid, plus a memory integral with kernel shown in figure 1 for vertical motion. The results are compared with the experiments of Larsen (1969) and Levitskii & Chashechkin (1999) for a buoyant sphere, and Le Gal *et al.* (2022) for a Cartesian diver.

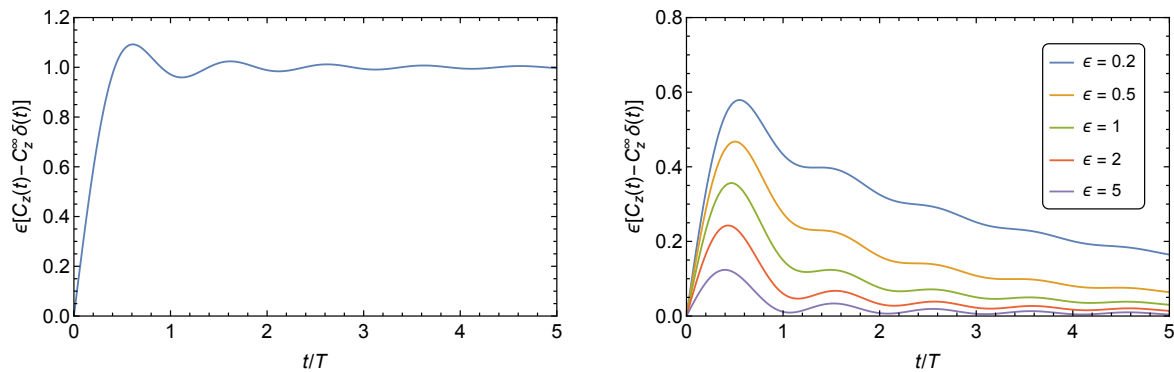


Figure 1: Memory kernels for the vertical motion of a horizontal elliptic cylinder (left) and a spheroid of vertical axis (right). Both bodies have horizontal semi-axis  $a$ , vertical semi-axis  $b$  and aspect ratio  $\epsilon = b/a$ . Time  $t$  is expressed in terms of the buoyancy period  $T$ , and the added mass coefficient is  $C_z^\infty$  without stratification and  $C_z$  with it.

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