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Virtual fields based-method for reconstructing the elastic modulus in quasi-static ultrasound elastography

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Abstract—Reconstructing tissue elastic properties from displacements measured in quasi-static ultrasound elastography is a challenging task. Indeed, it requires to solve an ill-posed inverse problem, with generally no available boundary information and solely 2D estimated displacements, whereas the problem is inherently three-dimensional. In this paper, a method based on the virtual work principle is investigated to reconstruct Young’s modulus maps from the knowledge of internal displacements and the force applied. The media examined are assumed to be linear elastic and isotropic. Moreover, for these first developments, the plane stress problem is investigated to overcome the lack of 3D data. The developed method is assessed with plane-stress and 3D simulations, as well as with experimental data. For all the media examined, regions of different stiffnesses are clearly revealed in the reconstructed Young’s modulus maps. The stiffness contrast between regions is accurately estimated for the plane stress simulations but underestimated for the 3D simulations, which could be expected as plane stress conditions are no longer satisfied in this last case. Finally, similar comments can be made for the phantom results, with an inclusion-to-background Young’s modulus ratio of 2.4 lower than the reference ratio of around 3, provided by the manufacturer.

Index Terms—Quasi-static ultrasound elastography, Young’s modulus map reconstruction, Inverse problem, Virtual fields method

I. INTRODUCTION

Elastography techniques have been developed in recent decades with the aim of estimating the mechanical properties of biological tissues. Indeed, local modifications in these properties can be observed in a large number of pathologies, which can be used for diagnostic purposes. This study focuses on quasi-static ultrasound elastography, that commonly produces strain images of biological media under compression. Strain imaging has proved to be a valuable tool for distinguishing regions with different stiffnesses within a medium [2], [6], [19]. Nevertheless, it only partially reflects the medium elastic properties, because of the heterogeneous stress field. Reconstructing the tissue mechanical properties in quasi-static ultrasound elastography is a challenging task, first because of their high complexity. This makes necessary to introduce some simplifying assumptions regarding tissues, and in this work, linear elastic and isotropic media will be

considered. Moreover, difficulties to solve the inverse problem result generally from the fact that only 2D (and not 3D) displacements can be estimated, and from a lack of boundary information, which explains for instance that methods for stiffness contrast mapping have been developed.

Various approaches are therefore available in the literature to reconstruct the (relative) elastic properties of a medium. A first approach we can mention is direct inversion [5], [21], [24]. These methods are based on directly solving the equilibrium equations, obtained when replacing the unknown stress terms σ_{ij} with their expression given by the material constitutive law. However, additional information is needed to obtain quantitative results (in kPa), e.g. a surface Young’s modulus [21]. Moreover, determining the solution requires to compute first and second order derivatives of the displacement, known to raise the noise present within data, and which may highly degrade the resulting reconstructed maps [1]. To overcome this problem, other studies use the weak formulation of the equilibrium equations, which allows to get rid of second order derivatives [8], [17].

Another approach consists in reformulating the inverse problem as an optimization problem, in which the parameters of interest are iteratively varied until minimizing, for example, an error between the displacements or strains determined by solving the forward problem and those measured experimentally [4], [11], [18]. Generally speaking, a theoretical representation of the problem is made with a FEM (Finite Element Method) model, using certain assumptions, notably on the tissue properties with the choice of the constitutive law. At the end of the iterative process, the mechanical parameters of the model are expected to be close to the ones of the concerned tissue. With this method, boundary conditions need to be introduced, whereas these ones are generally unknown, especially for clinical applications. Moreover, a forward problem has to be solved at each iteration, which can be time-consuming.

Recently, artificial intelligence was used to reconstruct the Young’s modulus distribution. The main advantage of this approach, as described in [15], [16], is that no prior

assumption about the material constitutive law is required because the neural network takes the place of the constitutive matrix. Nonetheless, FEM modeling and a series of force-displacement measurements are required.

A last approach to reconstruct the mechanical properties within media from the knowledge of the internal displacements and the force applied is the virtual fields method. This approach has been originally developed for the solid mechanics field, but a few articles can also be cited in MRE [10], [20], [22]. Its interest lies in its flexibility, in particular in the virtual fields used, which are determined to meet certain chosen conditions, for instance, to minimize the effect of noise [3].

In this paper, a reconstruction method based on the virtual work principle for Young's modulus mapping in quasi-static ultrasound elastography is introduced. A description of the developed method is given in section II, and the simulations and experimental data used for its assessment in section III. Results are detailed and discussed in section IV. The last section presents the conclusions of the study.

II. METHOD

The virtual work principle relates the internal, external and acceleration virtual work, the latter being null in the static case. For these first developments, and to overcome the lack of 3D data in quasi-static ultrasound elastography, the plane stress conditions are considered. Under these assumptions, the principle of the virtual work leads to:

$$\int_S \boldsymbol{\sigma} : \boldsymbol{\varepsilon}^* ds = \int_L \boldsymbol{T} \cdot \boldsymbol{u}^* dl \quad (1)$$

with $\boldsymbol{\sigma}$ the stress tensor, \boldsymbol{T} the force per unit area, and \boldsymbol{u}^* and $\boldsymbol{\varepsilon}^*$ the virtual displacement and strain tensor, respectively. The virtual displacement \boldsymbol{u}^* , that we can also call "test function", is designed to compute the desired parameters under chosen conditions. One constraint on this field is that it has to satisfy the kinematic admissibility condition.

To introduce the mechanical parameters of interest in (1), the stress tensor $\boldsymbol{\sigma}$ is rewritten using a chosen material constitutive law. Here, linear, elastic and isotropic materials are considered (2).

$$\boldsymbol{\sigma} = \frac{E}{1+\nu} \left(\boldsymbol{\varepsilon} + \frac{\nu}{1-2\nu} \text{Tr}(\boldsymbol{\varepsilon}) \boldsymbol{I} \right) \quad (2)$$

Moreover, in this study, the Poisson's ratio ν is assumed to be known and equal to 0.49 over the whole medium. Consequently, only the spatial distribution of the Young's modulus E has to be reconstructed. For this, the medium is meshed and a value of E is estimated for each element of the mesh.

Let us consider a given element k . To estimate the Young's modulus E_k , specific virtual fields are, in this work, defined piecewisely over the whole medium [23], [25]. The virtual fields over an element, \boldsymbol{u}_e^* and $\boldsymbol{\varepsilon}_e^*$, are written using the nodal

virtual displacements of this element $\tilde{\boldsymbol{u}}_e^*$ and the matrix \boldsymbol{N} containing the shape functions N_i , such as:

$$\boldsymbol{u}_e^* = \boldsymbol{N} \tilde{\boldsymbol{u}}_e^* \quad (3)$$

Here, rectangular elements with 4 nodes are used, and \boldsymbol{N} is written as follows :

$$\boldsymbol{N} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \quad (4)$$

Considering that the four nodes are locally denoted n_1 , n_2 , n_3 and n_4 , each shape function N_i is determined such that it is equal to 1 at n_i and cancels at the three other nodes n_j with $j \neq i$. The virtual strain $\boldsymbol{\varepsilon}_e^*$ is obtained by computing the derivatives of \boldsymbol{u}_e^* :

$$\boldsymbol{\varepsilon}_e^* = \boldsymbol{L} \tilde{\boldsymbol{u}}_e^* \quad (5)$$

where \boldsymbol{L} is a matrix containing the derivatives of the shape functions.

The basic idea is to determine the virtual displacement such that the left-hand side of (1) is equal to the Young's modulus E_k to be determined. And the value of E_k is then directly equal to the external virtual work (right-hand side of (1)), computed with this specific virtual displacement. In other words, estimating E_k amounts to finding a virtual displacement, which selects the contribution of the element k and cancels the contributions from all the other elements. More complex choices can of course be made, for instance by taking into account a contribution of the neighboring elements. To obtain such a field, the problem is rewritten as a linear system, where the virtual nodal displacements are this time the unknowns to be determined. It must be pointed out that the nodal virtual displacements over the whole medium are taking into account for the estimation of E_k .

More specifically, the virtual displacement is computed as the solution of an optimization problem, which allows to determine the Young's modulus value E_k for a specific element k with a weighted contribution of the neighborhood. Two constraints are introduced: 1) the virtual displacement vanishes at the domain boundary except where the force is applied, and 2) the lateral component of the virtual displacement is equal to 0 as the force is mainly applied vertically. The optimization problem is solved using CVX [13], [14]. Finally, to reconstruct a Young's modulus map, the method is repeated for each element of the medium.

III. MATERIALS AND DATA ACQUISITION

In this study, a first assessment of the method was carried out using numerical simulations. Two kinds of simulations were performed, considering media under plane stress conditions and 3D simulations (size : $60 \times 40 \times 40 - mm^3$). All media consist of two regions, a $12 - mm$ in diameter inclusion embedded in a background, the different regions being made of a linear, elastic and isotropic material. In all cases, the Young's modulus of the background is 10

kPa, whereas the one of the inclusion is 5, 20 or 30 kPa, depending on the case. The Poisson's ratio is $\nu = 0.49$. Simulation of medium compression was performed using *Comsol Multiphysics*[®] [9]. A force was applied on the top surface, vertically and downwards, while at the bottom surface vertical displacements were forbidden. Vertical surfaces were let free to move. For 3D simulations, only the fields of the median plane were retained, since the developed method is 2D.

The method was also assessed with experimental data, acquired on a *CIRS* (Computerized Imaging Reference Systems, Norfolk, VA, USA) phantom model 059, which mimics a woman breast in supine position. This phantom contains several spherical inclusions with an inclusion-to-background Young's modulus ratio R close to 3 ($R = \frac{E_i}{E_b}$, manufacturer information). The transducer was positioned so as to scan an area with one inclusion, and a typical quasi-static elastography experiment was performed, i.e. the medium was cautiously and continuously compressed and decompressed by the operator using the ultrasound probe while the RF images were acquired. Data were collected with an *Ultrasonix* ultrasound scanner equipped with an L14-5W/60 transducer. The sampling frequency was 40 MHz. The axial and lateral displacement fields were then estimated using the motion-tracking technique and the regularization method we previously developed for 2D strain tensor imaging purposes [7], [12]. In this experiment, the applied force was not available. Thus, it was set to an arbitrarily value, and only a relative Young's modulus map can therefore be estimated.

For all cases examined, the mean Young's modulus within the inclusion (E_i) and the background (E_b) were computed from two circular ROIs of identical size and positioned at the same depth. For the phantom experiment, the inclusion-to-background modulus ratio R was also determined.

IV. RESULTS AND DISCUSSION

Fig.1 displays the reconstructed Young's modulus maps for the numerical simulations. In all cases, the inclusion is clearly revealed, and both the inclusion and the background appear relatively homogeneous. The results are very accurate for the plane stress simulations (Tab.I). However, a decrease in the stiffness contrast can be observed for the 3D simulations, with for instance, a mean Young's modulus of the inclusion (22 kPa) lower than the actual value (30 kPa), while the background modulus (10 kPa) is well-preserved (Tab.I). This deviation from the theoretical Young's modulus map can be expected, as plane stress conditions are no more satisfied in the 3D simulated cases.

For the phantom case, the reconstructed relative Young's modulus map is displayed in Fig. 2d. The components of the 2D strain tensor are also shown in Fig. 2a-c, and one can note that, like the axial strain, the lateral and shear strains provide mechanical information about the region scanned and appear therefore usable for the reconstruction method. Concerning the reconstructed map, the spherical inclusion is clearly revealed

TABLE I
SIMULATION RESULTS - RECONSTRUCTED YOUNG'S MODULUS IN *kPa*.

	Actual values	Reconstruction from simulations	
		plane stress	3D
Background	10	10	10
Inclusion	5	5	7
Background	10	10	10
Inclusion	20	20	16
Background	10	10	10
Inclusion	30	29	22

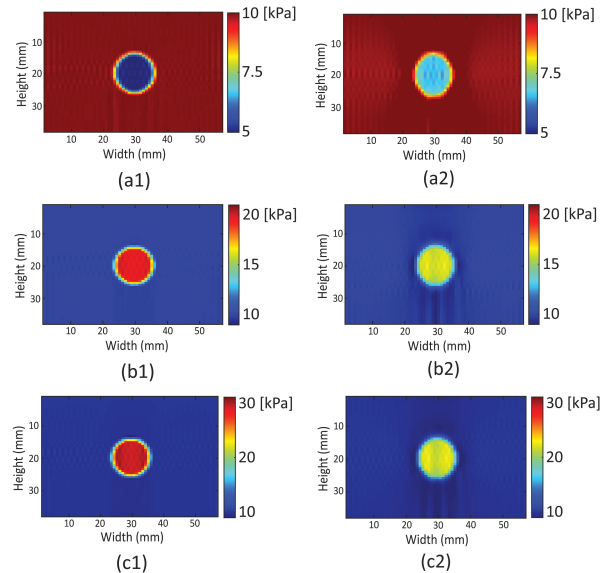


Fig. 1. Reconstructed Young's modulus maps from plane stress simulations (first column) and 3D simulations (second column). Simulated media consist of a homogeneous background of 10 kPa containing an inclusion of 5 kPa (a), 20 kPa (b) or 30 kPa (c).

and appears stiffer than the background. Both the inclusion and the background are quite homogeneous even though a "red area" can be observed at the top of the image. Nonetheless, this area is of small size and does not affect the interpretation of the result. Finally, the ratio is estimated at $R = 2.4$, which is a bit lower than the value provided by the manufacturer (around 3). Once again, this result was expected since these experimental data cannot perfectly satisfy the plane stress conditions.

For these first developments, a plane stress approach was developed to overcome the lack of 3D data, but a plane strain approach could also be investigated. However, it has to be kept in mind that neither plane stress nor plane strain enable to perfectly model the medium/tissue behavior, and will inevitably lead to errors in the reconstructed maps. One way to overcome this limit is to extend the proposed method to the third dimension, making necessary to acquire ultrasound data not restricted to images. This will be the subject of future research work.

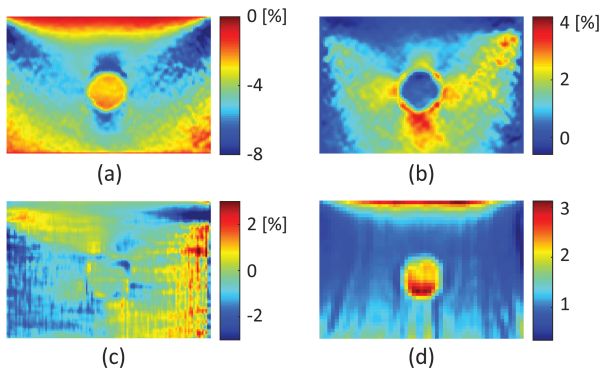


Fig. 2. Experimental results obtained with the CIRIS phantom, model 059 : axial strain (a), lateral strain (b), shear strain (c) and reconstructed relative Young's modulus map (d). The 2D strain tensor is obtained after applying a previously developed regularization method to the estimated displacements.

V. CONCLUSIONS

In this paper, reconstruction of the elastic modulus in quasi-static ultrasound elastography was investigated using a virtual fields based-method. Results show reconstructed maps where the inclusions of various stiffnesses are clearly visible. Future work will focus on going on developing this method, with in particular a deep analysis of the 2D vs 3D problem. Moreover, a system to capture the force applied in parallel to the ultrasound data, will also be designed to make possible quantitative results with experimental data.

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