

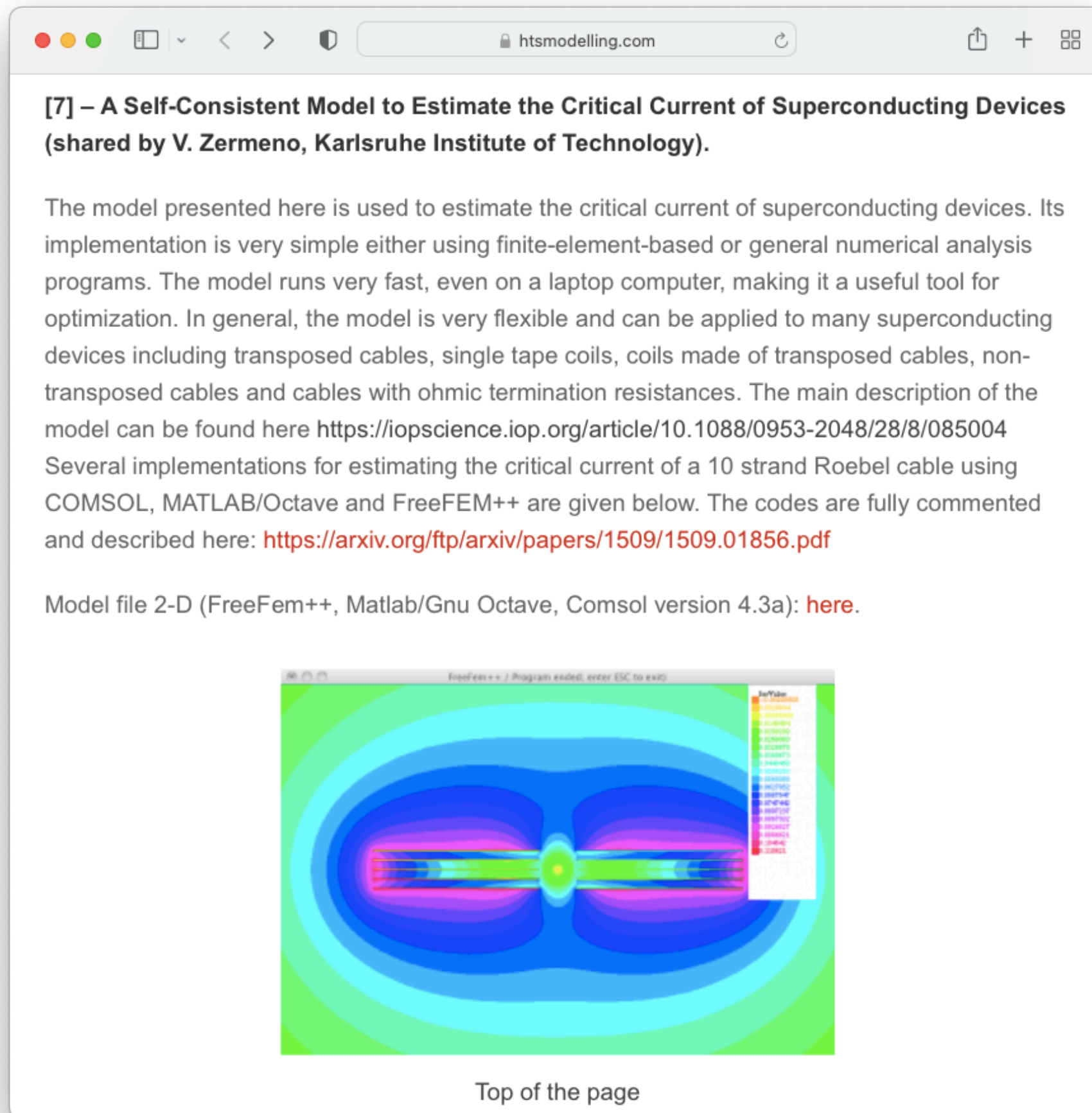
On Static H-formulation Adapted to the Critical Current Model of HTS Conductors

Dong Keun Oh

with KFE (Korea institute of Fusion Energy)

Motivation

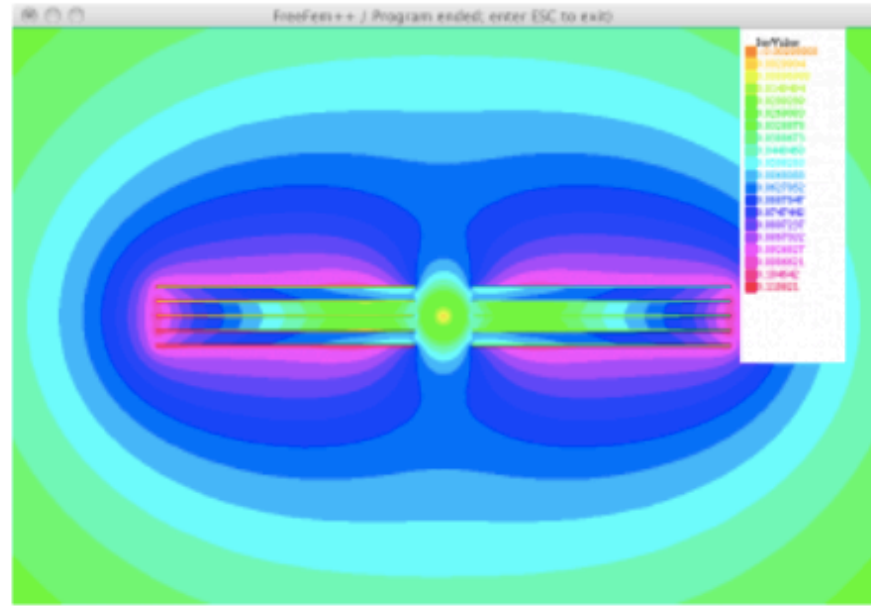
: *A self-consistent critical current model*



[7] – A Self-Consistent Model to Estimate the Critical Current of Superconducting Devices (shared by V. Zermeno, Karlsruhe Institute of Technology).

The model presented here is used to estimate the critical current of superconducting devices. Its implementation is very simple either using finite-element-based or general numerical analysis programs. The model runs very fast, even on a laptop computer, making it a useful tool for optimization. In general, the model is very flexible and can be applied to many superconducting devices including transposed cables, single tape coils, coils made of transposed cables, non-transposed cables and cables with ohmic termination resistances. The main description of the model can be found here <https://iopscience.iop.org/article/10.1088/0953-2048/28/8/085004>. Several implementations for estimating the critical current of a 10 strand Roebel cable using COMSOL, MATLAB/Octave and FreeFEM++ are given below. The codes are fully commented and described here: <https://arxiv.org/ftp/arxiv/papers/1509/1509.01856.pdf>.

Model file 2-D (FreeFem++, Matlab/Gnu Octave, Comsol version 4.3a): [here](#).



Top of the page

← Static Current Modeling

- A simplified approach : 2-d planar model with $\vec{J} // \hat{z}$

← Efficient Evaluation for the Nonlinear Solution

- Indirect method using the P indicator of E -field uniformity

$$E(J) = E_c \left| \frac{J}{J_c(\vec{B})} \right|^n \rightarrow \begin{cases} J = J_c(\vec{B}) \times P \\ P = |E/E_c|^{1/n} \end{cases}$$

✓ *It has been introduced as a magnetostatic model in A-formulation, ...*

https://www.htsmodelling.com/?page_id=748#Ps_model

✓ *It's a magnetostatic model in A-formulation, ...*

$$P^{(i)} = \frac{I^{(i)}}{\int_{\Omega^{(i)}} J_c(\vec{B}) dv}$$

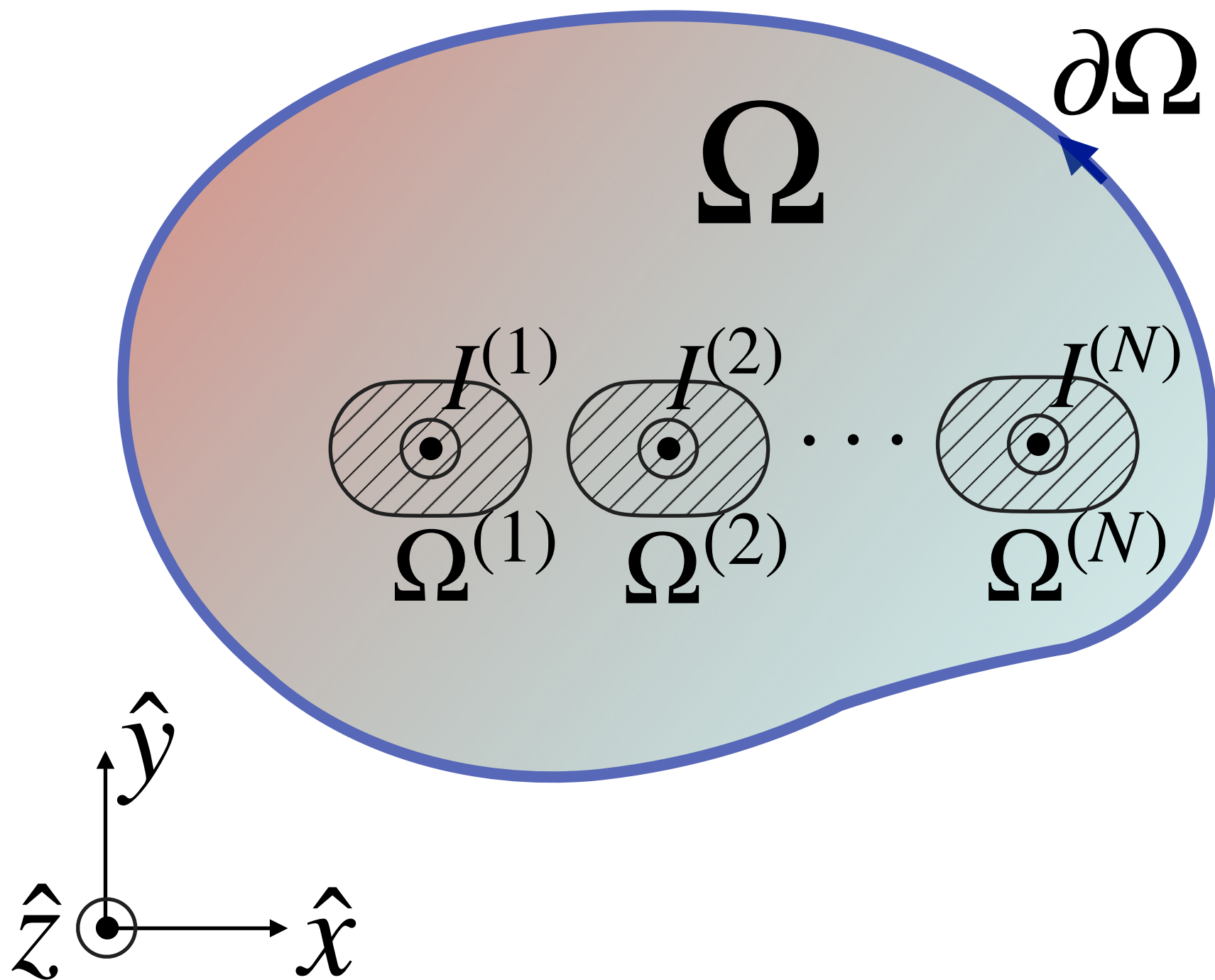
← the component-wise indicator of *E*-field uniformity

$$\nabla^2 A_z - \mu_0 J_c(B) \cdot P^{(i)} = 0$$

← **A-formulation simply leads an elliptic PDE**

✓ *Any alternative in H-formulation?!*

$$\begin{cases} \frac{\partial \mu \vec{H}}{\partial t} + \vec{\nabla} \times (\rho \vec{\nabla} \times \vec{H}) = 0 \\ \rho(J) = E_c J_c(\vec{B}) |P^{(i)}|^{n-1} \end{cases}$$



✓ Quite long time ago..

J. Phys. III France 3 (1993) 995–1004

MAY 1993, PAGE 995

Classification
 Physics Abstracts
 41.90 — 02.60

An h -formulation for the computation of magnetostatic fields. Implementation by combining a finite element method and a boundary element method.

B. Bandelier ^(1,2), C. Daveau ⁽²⁾ and F. Rioux-Damidau ⁽¹⁾

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(Received 27 November 1993, revised 1 February 1993, accepted 8 February 1993)

Abstract. — A new formulation of magnetostatics is given: it uses the magnetic field h as variable and a penalty technique. For its discretization, a finite element method inside the magnetic materials is combined with a boundary integral method which describes the exterior domain. Numerical tests are presented. The value to be chosen for the penalty parameter and a criterion of validity of the computation are given.

➔ A Weak Form on the Divergence-free Field

- The integral implies the total magnetic energy in the whole space

$$\int_{\Omega} \vec{B} \cdot \vec{H}' dx^2 - \oint_{\partial\Omega} \left(\vec{A} \times \vec{H}' \right) \cdot \hat{n} dl - \int_{\Omega} \vec{A} \cdot \left(\vec{\nabla} \times \vec{H}' \right) dx^2 = 0$$

➔ Penalization to impose the Ampère's law

- The penalized vector potential just makes the magnetic model involve the Ampère's law.

$$\vec{A} = \frac{1}{\nu} (\vec{J} - \vec{\nabla} \times \vec{H}) \text{ in } \Omega$$

➔ Free Boundary Solution in the Outside

$$\vec{A} = (a + a_0)\hat{z} \text{ at } \partial\Omega, \text{ where } \begin{cases} \vec{H} - \vec{H}_0 = \vec{\nabla} \times (a\hat{z}) \\ \vec{H}_0 = \vec{\nabla} \times (a_0\hat{z}) \end{cases}$$

✓ So..

J. Phys. III France 3 (1993) 995-1004

MAY 1993, PAGE 995

Nonlinear permeability

No source current

Classification
Physics Abstracts
41.90 — 02.60

$$\int_{\Omega} \mu_r \vec{H} \cdot \vec{H}' dx^2 + \frac{1}{\mu_0 \nu} \int_{\Omega} (\nabla \times \vec{H}) \cdot (\nabla \times \vec{H}') dx^2$$

An *h*-formulation for the computation of magnetostatic fields. Implementation by combining a finite element method and a boundary element method.

$$+ \oint_{\partial\Omega} \left(\frac{\partial\phi}{\partial n} + \vec{H}_0 \cdot \hat{n} \right) \phi' dl = 0$$

B. Bandelier ^(1,2), C. Daveau ⁽²⁾ and F. Rioux-Damidau ⁽¹⁾

⁽¹⁾ Institut d'Electronique et de Physique, Universités Paris VI et XI, URA CNRS 022, Bât 220, 91405 Orsay, France

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(Received 27 November 1993, revised 1 February 1993, accepted 8 February 1993)

Abstract. — A new formulation of magnetostatics is given: it uses the magnetic field *h* as variable and a penalty technique. For its discretization, a finite element method inside the magnetic materials is combined with a boundary integral method which describes the exterior domain. Numerical tests are presented. The value to be chosen for the penalty parameter and a criterion of validity of the computation are given.

Magnetic scalar potential

➔ **Static H-formulation for HTS modeling**

$$\int_{\Omega} \vec{H} \cdot \vec{H}' dx^2 + \frac{1}{\mu_0 \nu} \int_{\Omega} (\nabla \times \vec{H} - \vec{J}) \cdot (\nabla \times \vec{H}') dx^2$$

$$+ \oint_{\partial\Omega} (a + a_0) \vec{H}' \cdot \hat{\tau} dl = 0$$

$$\oint_{\partial\Omega} \frac{\partial a}{\partial n} a' dl + \oint_{\partial\Omega} (\vec{H} \cdot \hat{\tau} - \vec{H}_0 \cdot \hat{\tau}) a' dl = 0$$

$$\mathbf{h}' \cdot [\mathbf{F}] \cdot \mathbf{h} - \mathbf{h}' \cdot [\mathbf{V}] \cdot \mathbf{j} + \mathbf{h}' \cdot [\mathbf{C}]^T \cdot (\mathbf{a} + \mathbf{a}_0) = \mathbf{0}$$

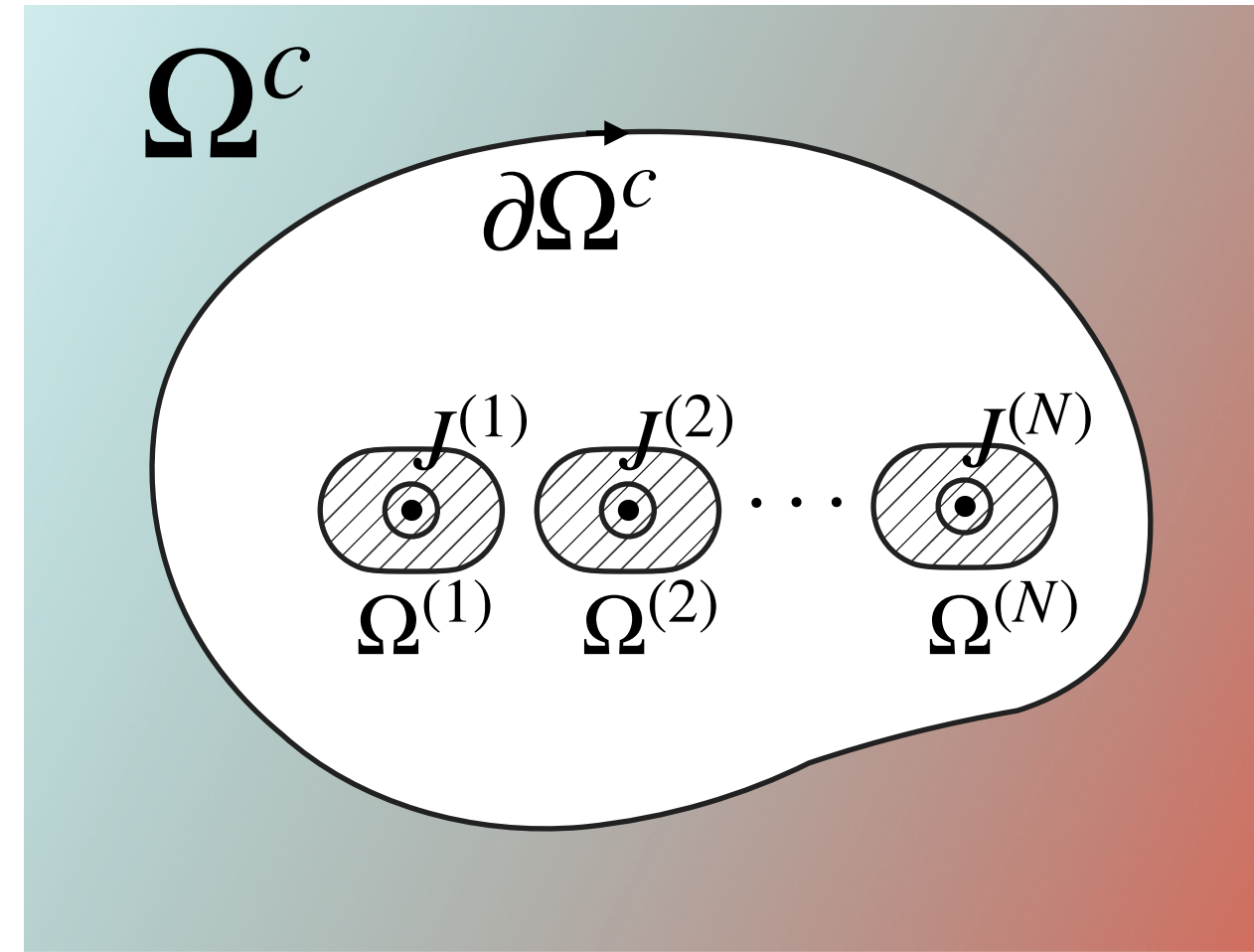
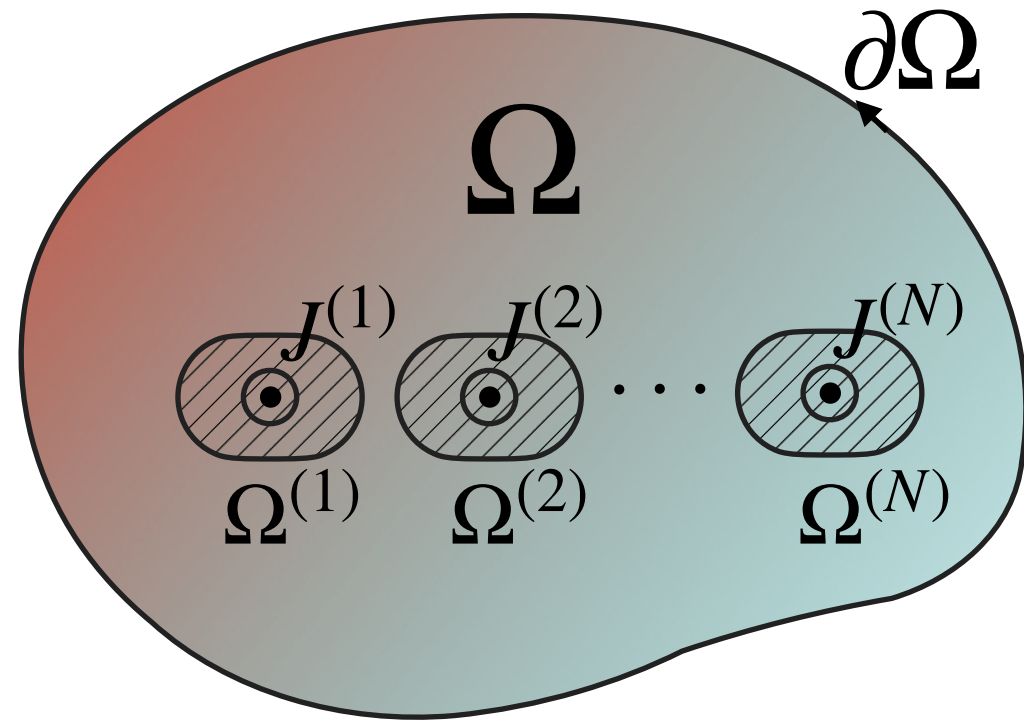
$$\mathbf{a}' \cdot [\mathbf{B}] \cdot \mathbf{a} + \mathbf{a}' \cdot [\mathbf{C}] \cdot (\mathbf{h} - \mathbf{h}_0) = \mathbf{0}$$

✓ Let's close it by setting up the boundary problem...



✓ *What about the boundary problem?*

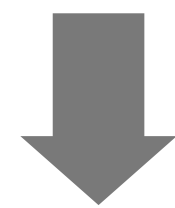
$$\oint_{\partial\Omega} \frac{\partial a}{\partial n} a' dl + \oint_{\partial\Omega} (\vec{H} \cdot \hat{\tau} - \vec{H}_0 \cdot \hat{\tau}) a' dl = 0$$



← We like to solve the Laplace's equation in the rest of the free space.

$$\mathbf{a}' \cdot [\mathbf{B}] \cdot \mathbf{a} = \mathbf{a}' \cdot [\mathbf{H}] \cdot \boldsymbol{\kappa}$$

$$\mathbf{a}' \cdot [\mathbf{Q}] \cdot \mathbf{a} = \mathbf{a}' \cdot [\mathbf{K}] \cdot \boldsymbol{\kappa}$$



$$[\mathbf{B}] = [\mathbf{H}][\mathbf{K}]^{-1}[\mathbf{Q}]$$

$$\oint_{\partial\Omega^c} \frac{\partial a(\vec{r})}{\partial n} a'(\vec{r}) dl = -\frac{1}{2} \oint_{\partial\Omega^c} \kappa(\vec{r}) a'(\vec{r}) dl + \oint_{\partial\Omega^c} dl_1 \oint_{\partial\Omega^c} dl_2 \kappa(\vec{r}_1) a'(\vec{r}_2) \frac{\partial G(\vec{r}_1, \vec{r}_2)}{\partial n_1}$$

$$\oint_{\partial\Omega^c} a'(\vec{r}) a'(\vec{r}) dl = \oint_{\partial\Omega^c} dl_1 \oint_{\partial\Omega^c} dl_2 \kappa(\vec{r}_1) a'(\vec{r}_2) G(\vec{r}_1, \vec{r}_2)$$

← the Green's function formulae

✓ Modeling in FreeFEM++ language

$$\int_{\Omega} \vec{H} \cdot \vec{H}' dx^2 + \frac{1}{\mu_0 \nu} \int_{\Omega} \left(\vec{\nabla} \times \vec{H} - \vec{J} \right) \cdot \left(\vec{\nabla} \times \vec{H}' \right) dx^2$$

$$\longrightarrow \mathbf{h}' \cdot [\mathbf{F}] \cdot \mathbf{h} - \mathbf{h}' \cdot [\mathbf{V}] \cdot \mathbf{j}$$

$$\frac{1}{\mu_0} \oint_{\partial\Omega} (\vec{A} \times \vec{H}') \cdot \hat{n} dl = \oint_{\partial\Omega} (\vec{H} \cdot \hat{\tau}) a' dl$$

$$\longrightarrow \mathbf{h}' \cdot [\mathbf{C}]^T \cdot \mathbf{a}$$

$$\oint_{\partial\Omega^c} dl_1 \oint_{\partial\Omega^c} dl_2 \kappa(\vec{r}_1) a'(\vec{r}_2) \frac{\partial G(\vec{r}_1, \vec{r}_2)}{\partial n_1} - \frac{1}{2} \oint_{\partial\Omega^c} \kappa(\vec{r}) a'(\vec{r}) dl$$

$$\longrightarrow \mathbf{a}' \cdot [\mathbf{H}] \cdot \kappa$$

$$\oint_{\partial\Omega^c} dl_1 \oint_{\partial\Omega^c} dl_2 \kappa(\vec{r}_1) a'(\vec{r}_2) G(\vec{r}_1, \vec{r}_2)$$

$$\longrightarrow \mathbf{a}' \cdot [\mathbf{K}] \cdot \kappa$$

$$\oint_{\partial\Omega^c} a'(\vec{r}) a'(\vec{r}) dl$$

$$\longrightarrow \mathbf{a}' \cdot [\mathbf{Q}] \cdot \mathbf{a}$$

```

varf F([Hx, Hy], [hx, hy])
= int2d(Th)(Hx*hx + Hy*hy)
+ int2d(Th)((dy(Hx)-dx(Hy))*(dy(hx)-dx(hy))/v)
- int2d(Th)(J(Hxp,Hyp)*(dy(hx)-dx(hy))/v);

```

```

varf C([Hx, Hy], [a])
= int1d(ThL)((Hx*Tl.x+Hy*Tl.y)*a);

```

```

varf H(a, ka) = int1dx1d(ThL)(ThL)
(BEM(BemKernel("DL", k=0i), a, ka))
-int1d(ThL)(0.5*a*ka);

```

```

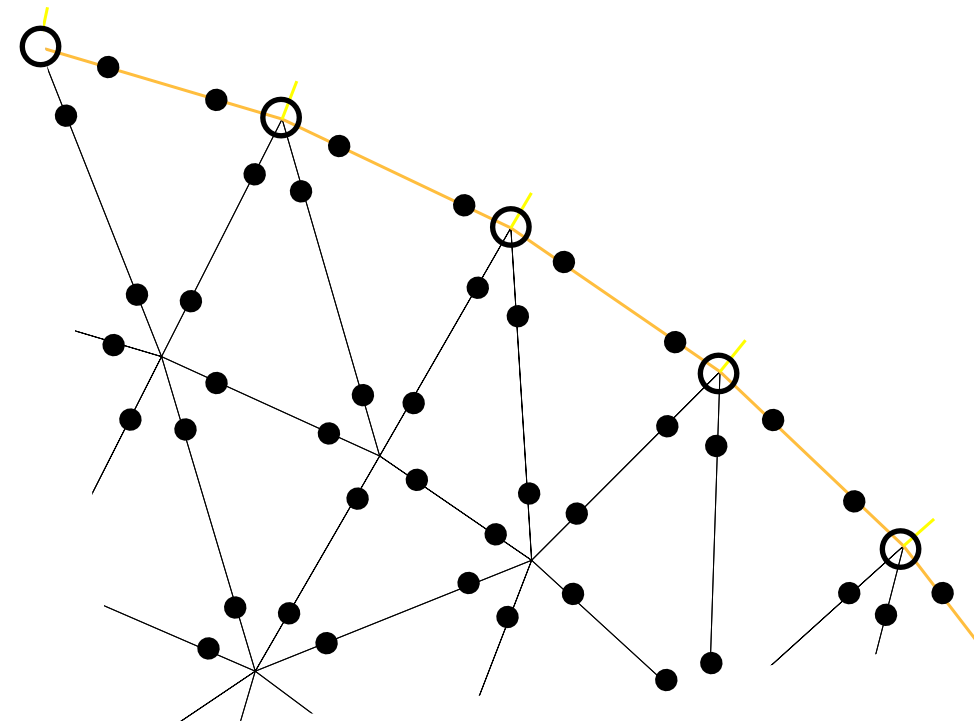
varf K(a, ka) = int1dx1d(ThL)(ThL)
(BEM(BemKernel("SL", k=0i), a, ka));

```

```

varf Q(a1, a2) = int1d(ThL)(a1*a2);

```

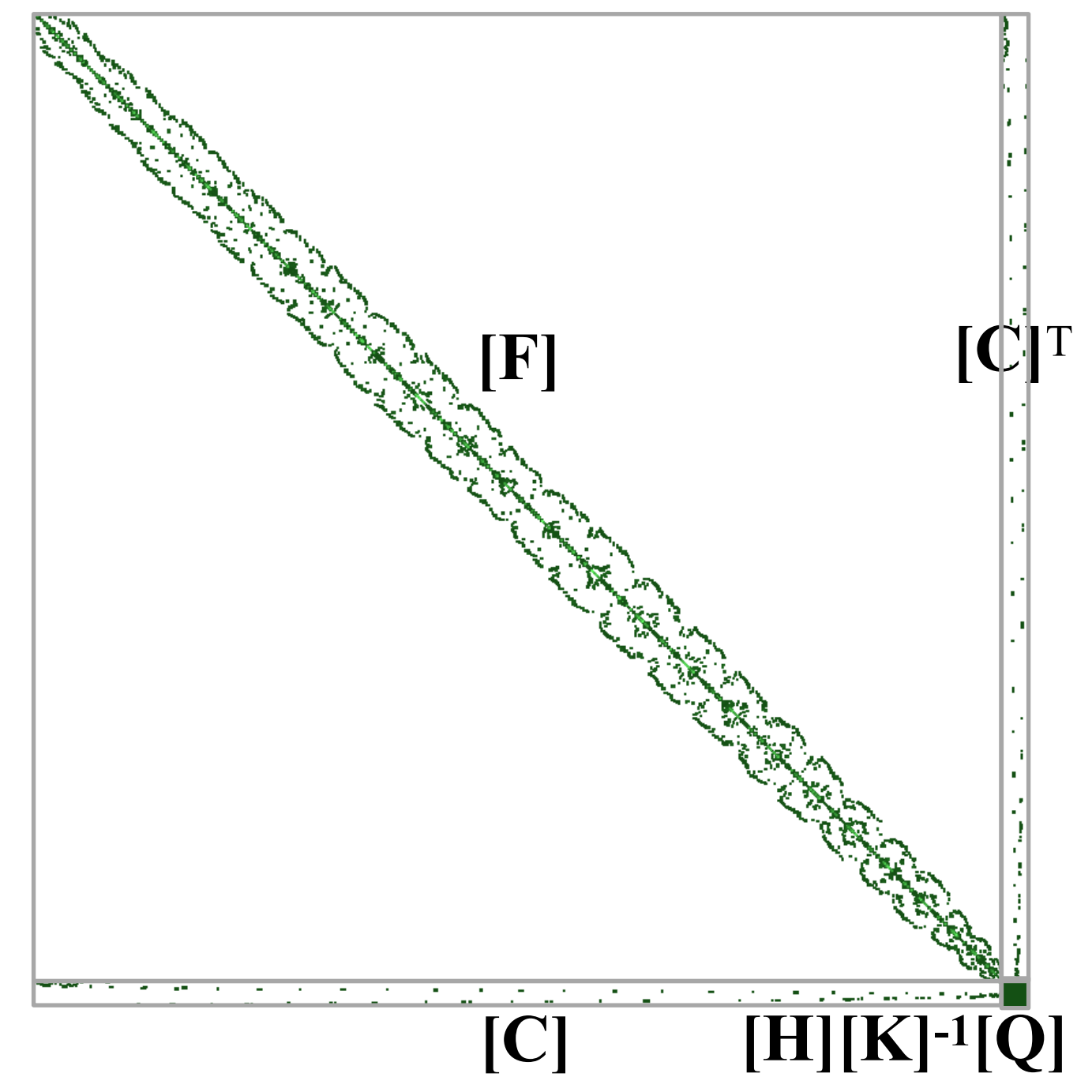
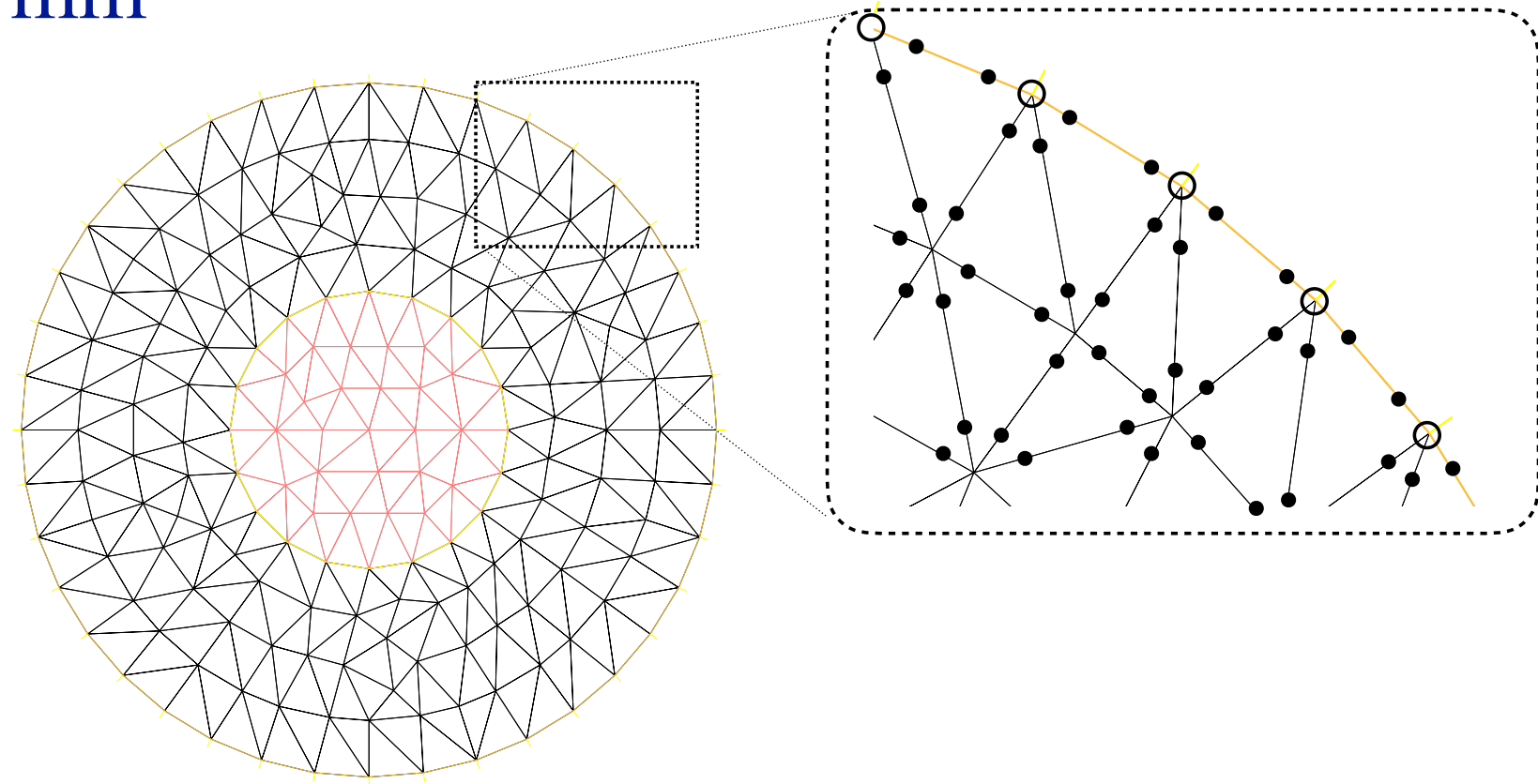


Testing the numerical scheme (1/2)

$$\begin{bmatrix} [\mathbf{F}] & [\mathbf{C}]^T \\ [\mathbf{C}] & [\mathbf{H}][\mathbf{K}]^{-1}[\mathbf{Q}] \end{bmatrix} \begin{bmatrix} \mathbf{h} \\ \mathbf{a} + \mathbf{a}_0 \end{bmatrix} = \begin{bmatrix} [\mathbf{V}] \cdot \mathbf{j} \\ 2[\mathbf{C}] \cdot \mathbf{h}_0 \end{bmatrix}$$

↑ the background field on the right side

- Dia. superconductor (red) : 0.2 mm
- Dia. of air volume (black) : 0.5 mm
- $J_{c0} = 45 \text{ kA/mm}^2$
- $B_c = 3500 \text{ Gauss}$
- $n = 21$
- $b = 0.6$

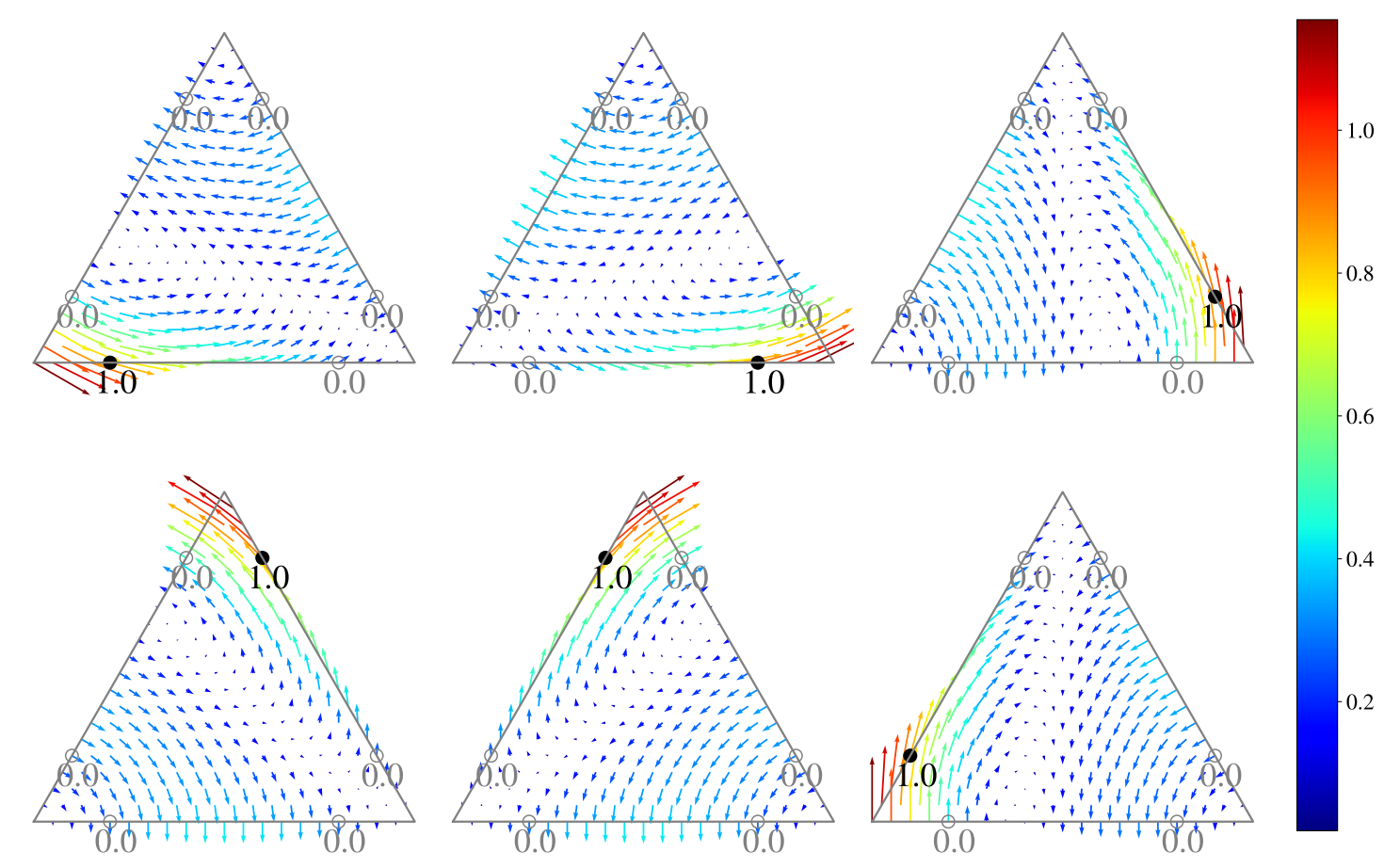


$$J_c(\vec{B}) = J_{c0} \left(1 + \frac{B_c}{|\vec{B}|} \right)^b$$

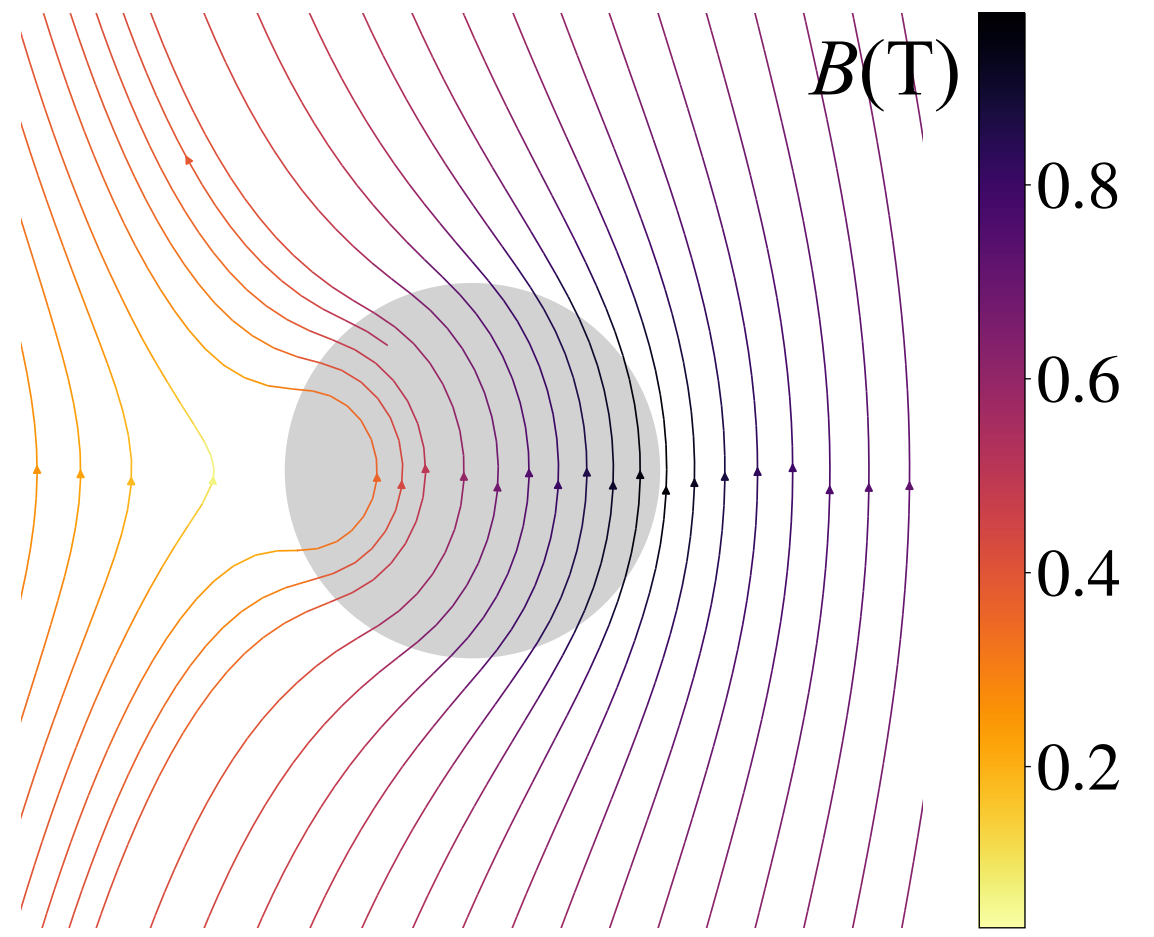
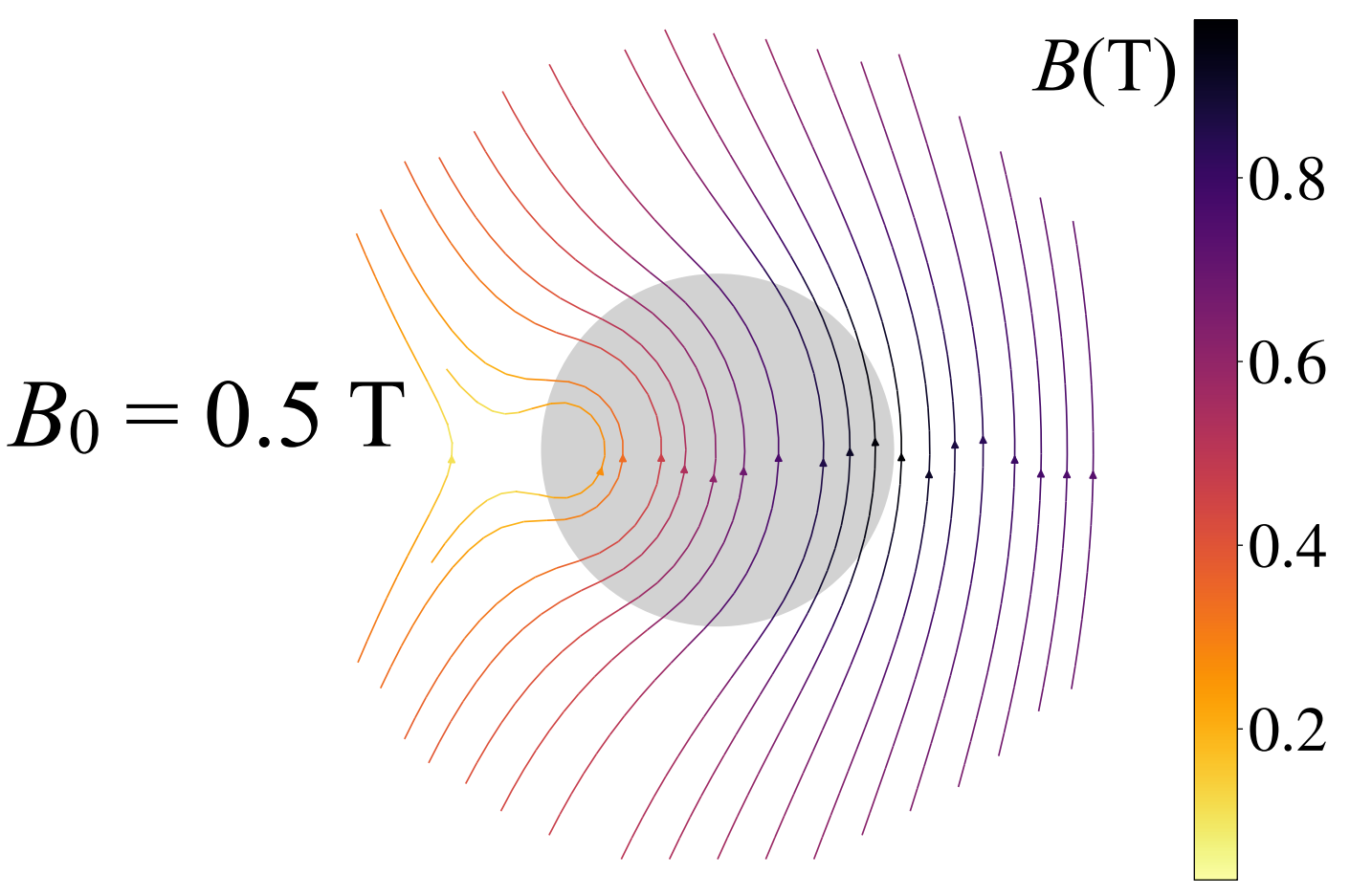
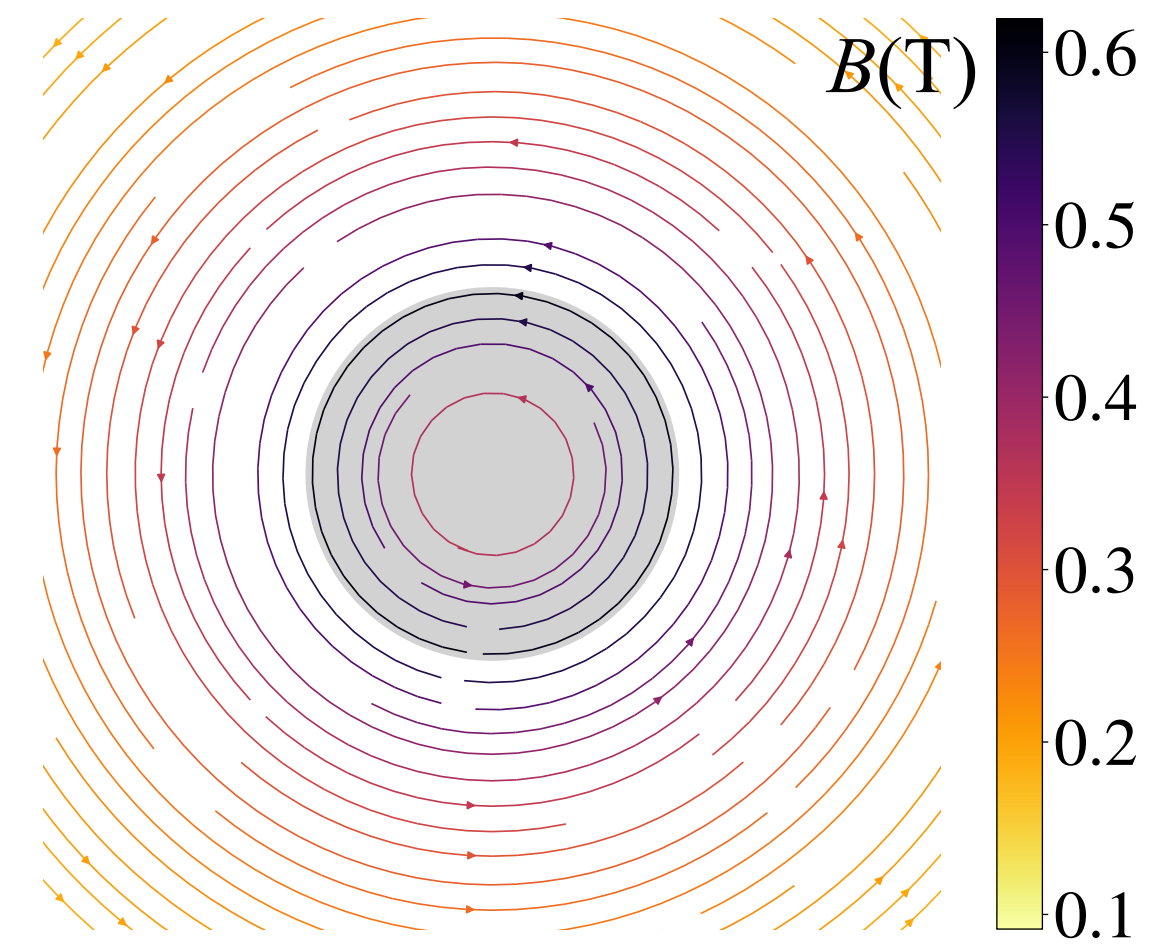
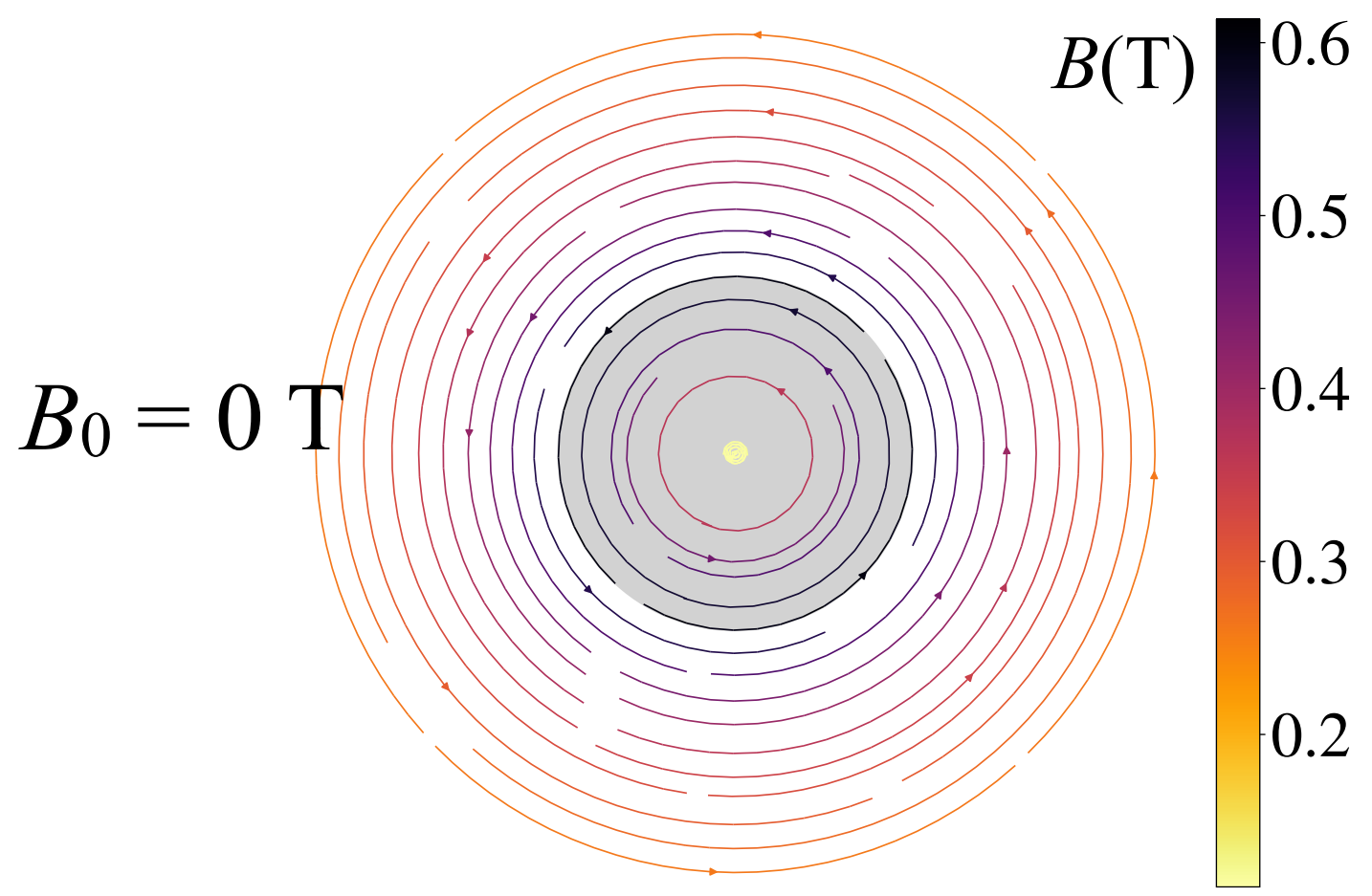
$$J = J_c(\vec{B}) P$$

$$E/E_c = \left| J/J_c \right|^n$$

188 vertices for 344 triangular elements
: "RT1Ortho" (2-d Nédélec) elements
and "P1" (piecewise linear) boundary elements

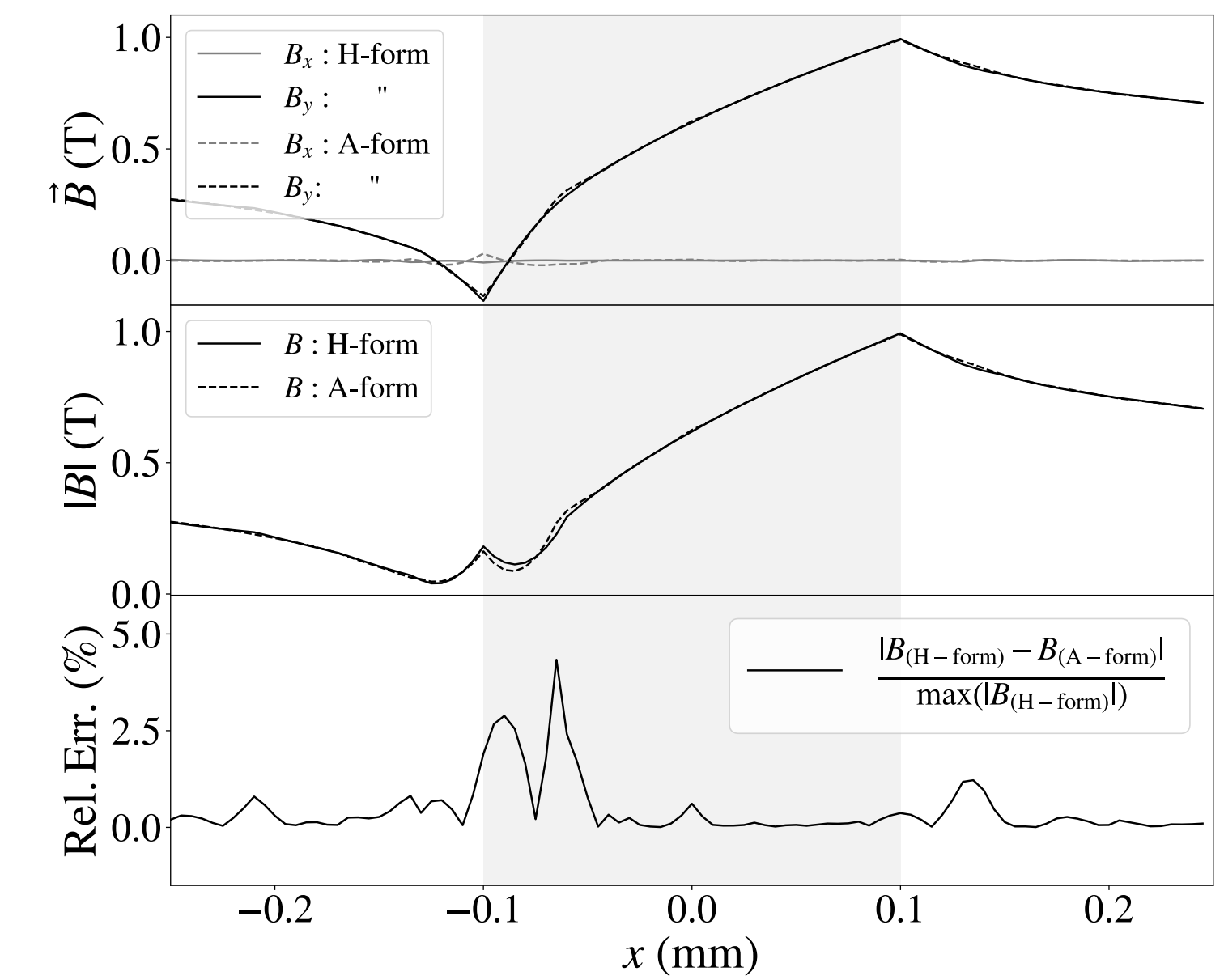
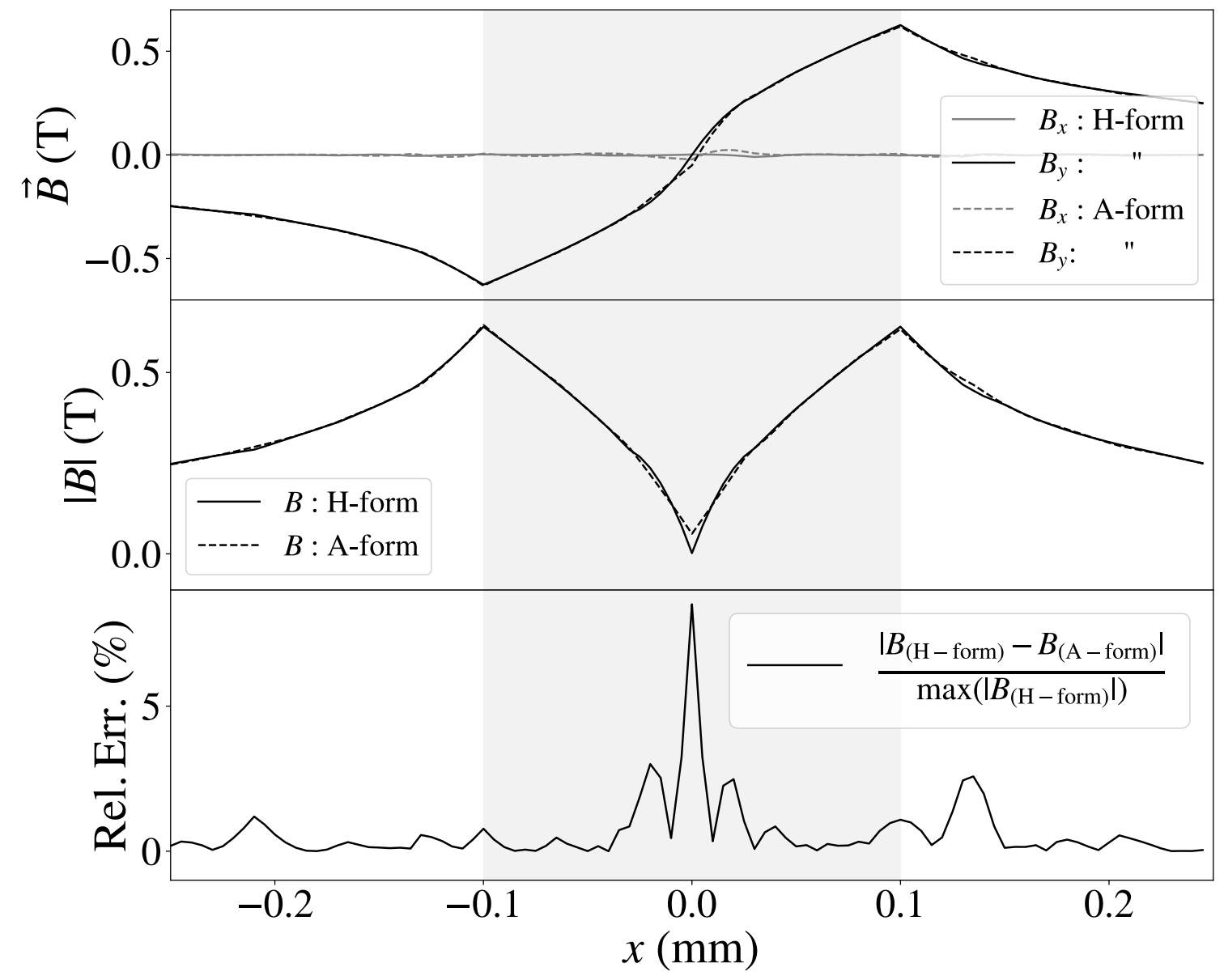


Testing the numerical scheme (2/2)



H-formulation

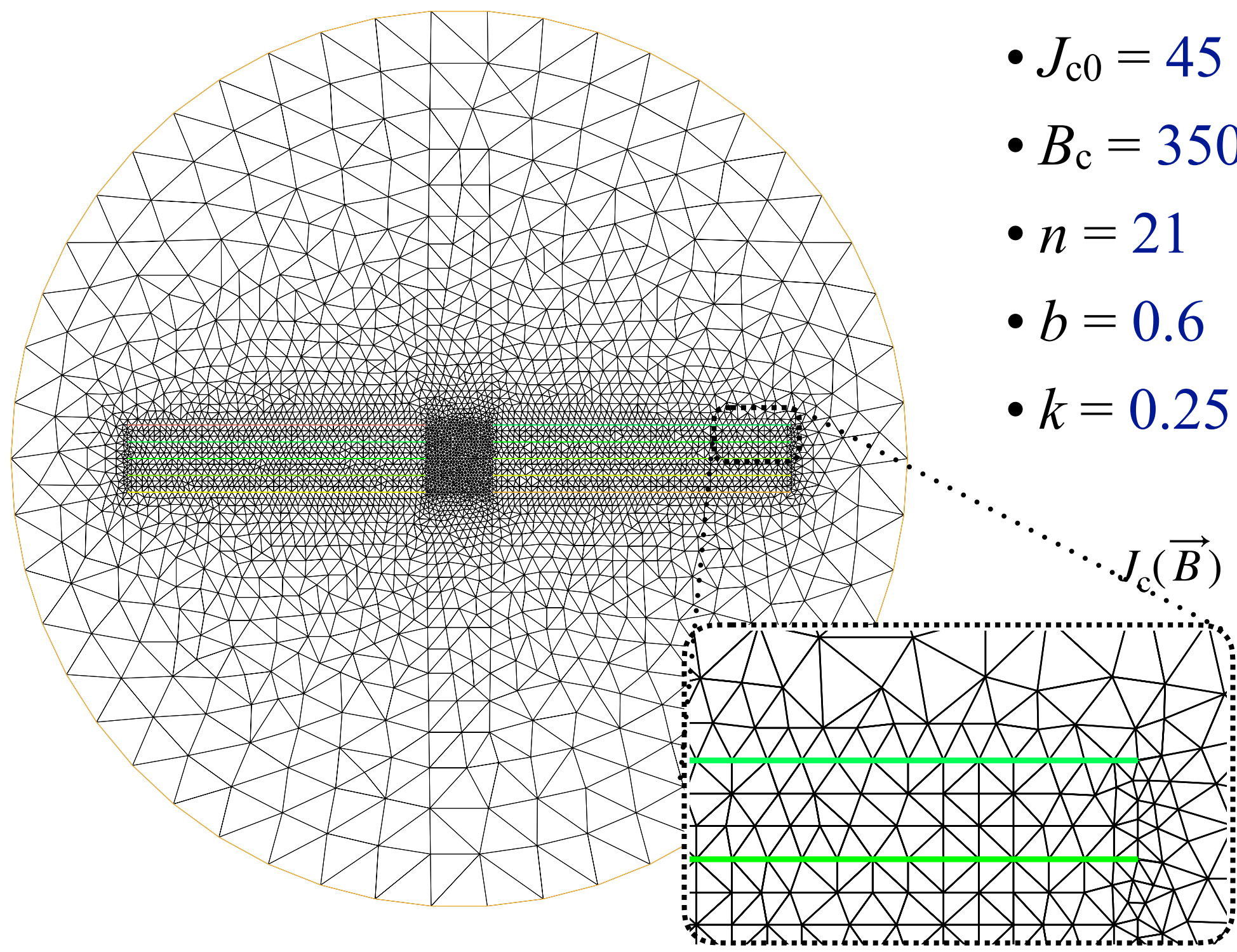
A-formulation



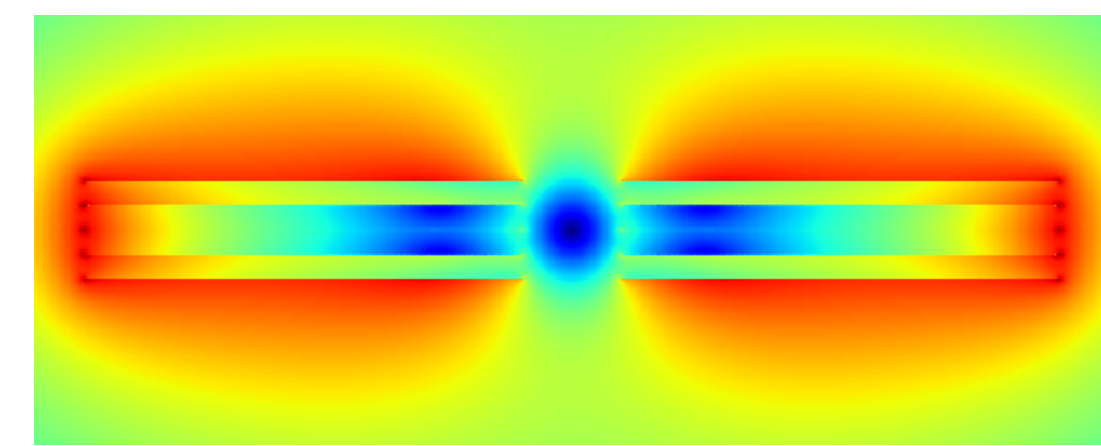
Benchmark Study (1/2)

✓ A Roebel Cable with 10 strands

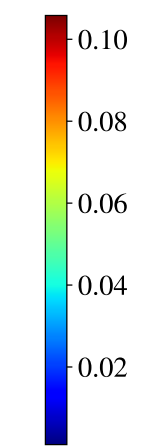
- $J_{c0} = 45 \text{ kA/mm}^2$
- $B_c = 3500 \text{ Gauss}$
- $n = 21$
- $b = 0.6$
- $k = 0.25$ (anisotropic)



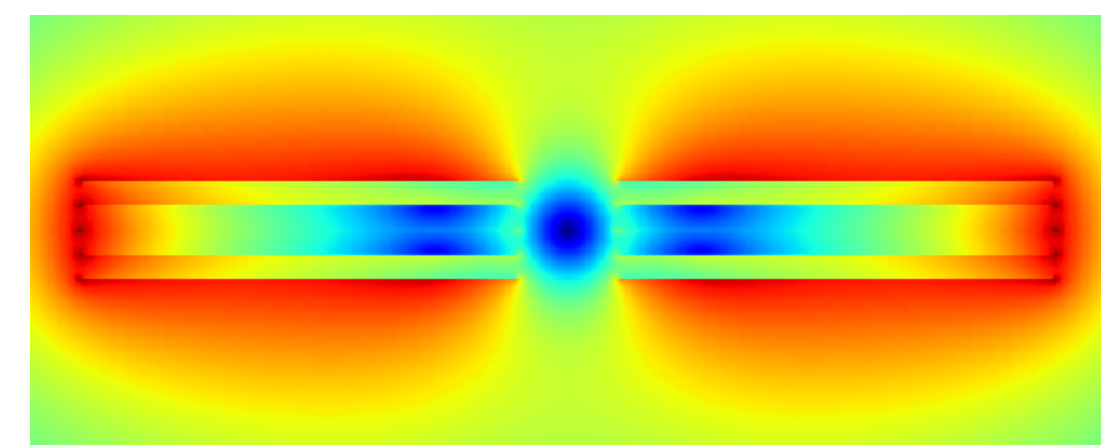
$$J_c(\vec{B}) = J_{c0} \left(1 + \frac{B_c}{\sqrt{k^2 B_x^2 + B_y^2}} \right)^b$$



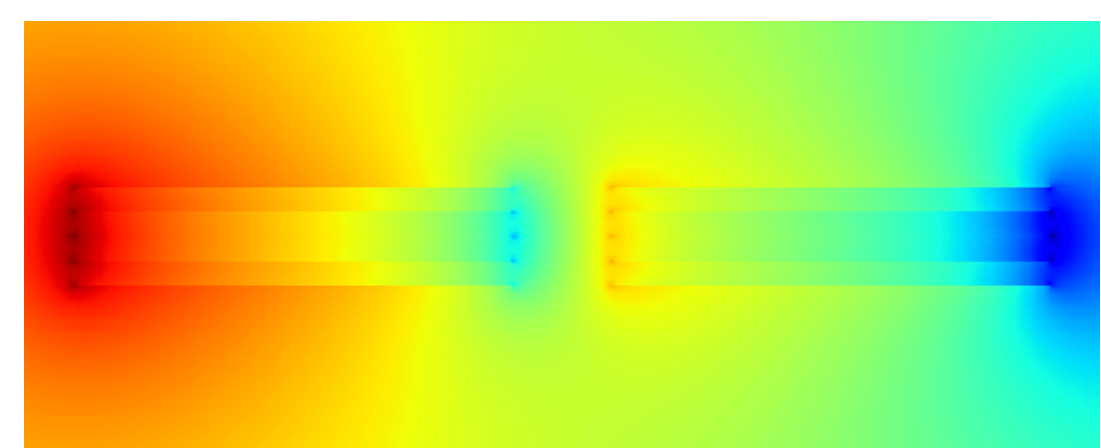
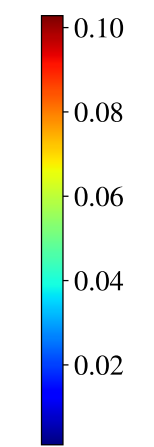
H-formulation



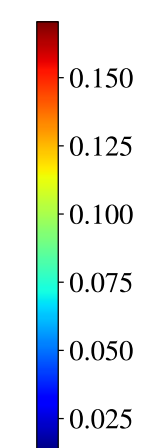
$B_0 = 0 \text{ T}$



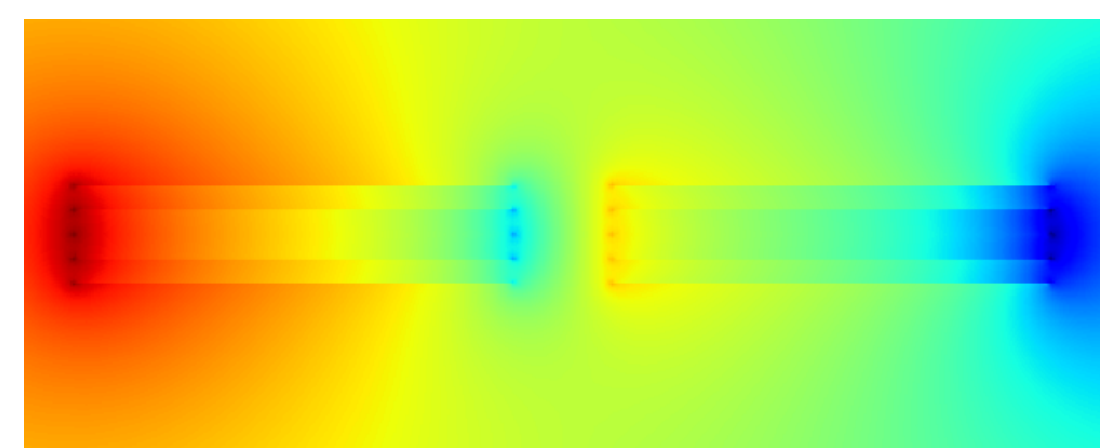
A-formulation



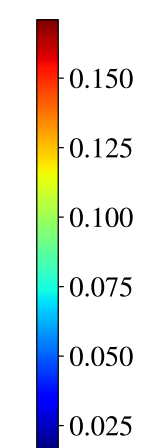
H-formulation



$B_0 = 0.1 \text{ T}$



A-formulation



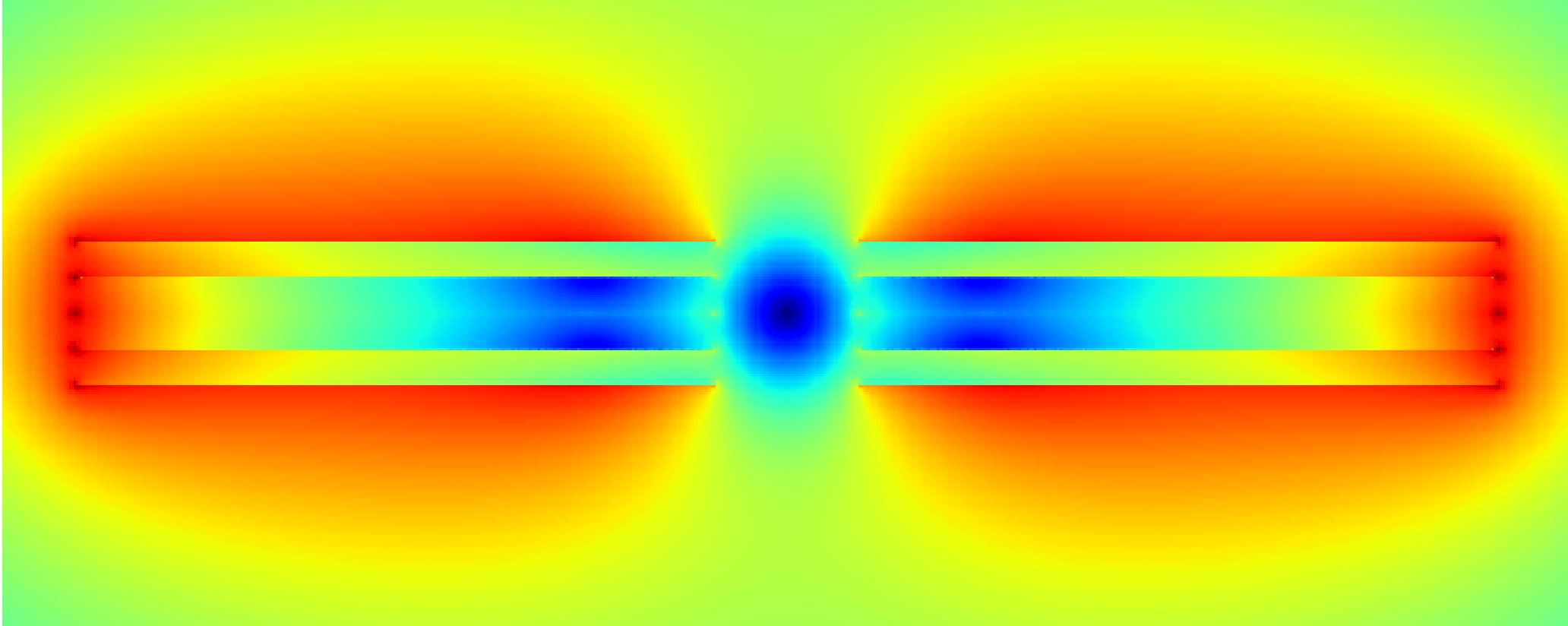
← The same mesh structure to the reference of A-formulation

https://www.htsmodelling.com/?page_id=748#Ps_model

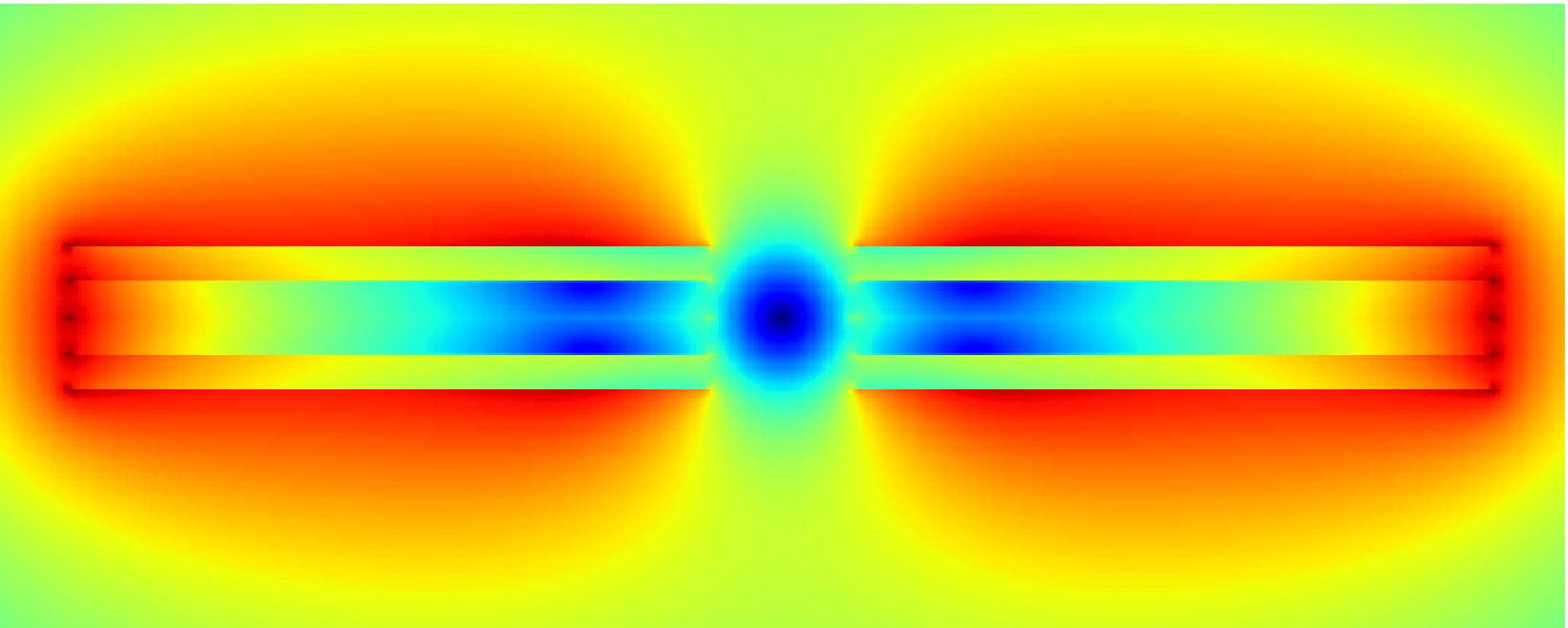


Benchmark Study (2/2)

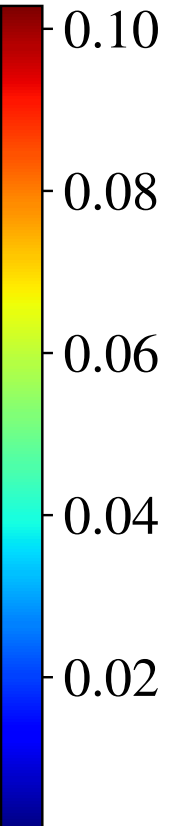
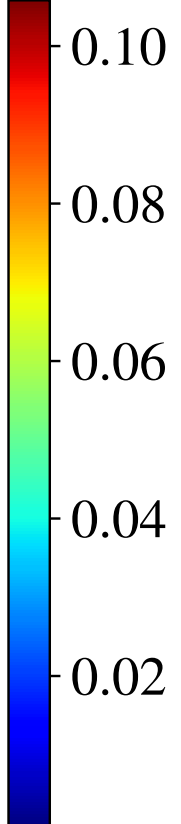
$B_0 = 0 \text{ T}$



H-formulation



A-formulation



Background Field (T)	Formulation	I_c^{MAX} (A)	CPU time of I_c^{MAX} (s)	I_c^{AVG} (A)	CPU time of I_c^{AVG} (s)
0.0	H-formulation	535.64	14.77	539.05	15.89
	A-formulation	535.76	20.91	539.14	20.01
0.1	H-formulation	353.10	12.99	382.57	7.88
	A-formulation	352.88	27.11	382.34	34.05

D. K. Oh, "AN ALTERNATIVE IN H-FORMULATION TO THE CRITICAL CURRENT MODEL OF HTS CONDUCTORS," in *IEEE Transactions on Applied Superconductivity (in press)*.

doi: 10.1109/TASC.2022.3179709.

What to the next?

○ **A COMSOL model for complement**

: Thanks to the advice of the experts (Linus Andersson and Boggavarapu Sairam of the COMSOL support team)..

✓ **Another way to assign the background field to the domain of cross section instead of the boundary**

: The idea is clear, but I'm not sure to justify the efficacy.

✗ **Application to the 3-D modeling**

: Not yet..

A COMSOL model

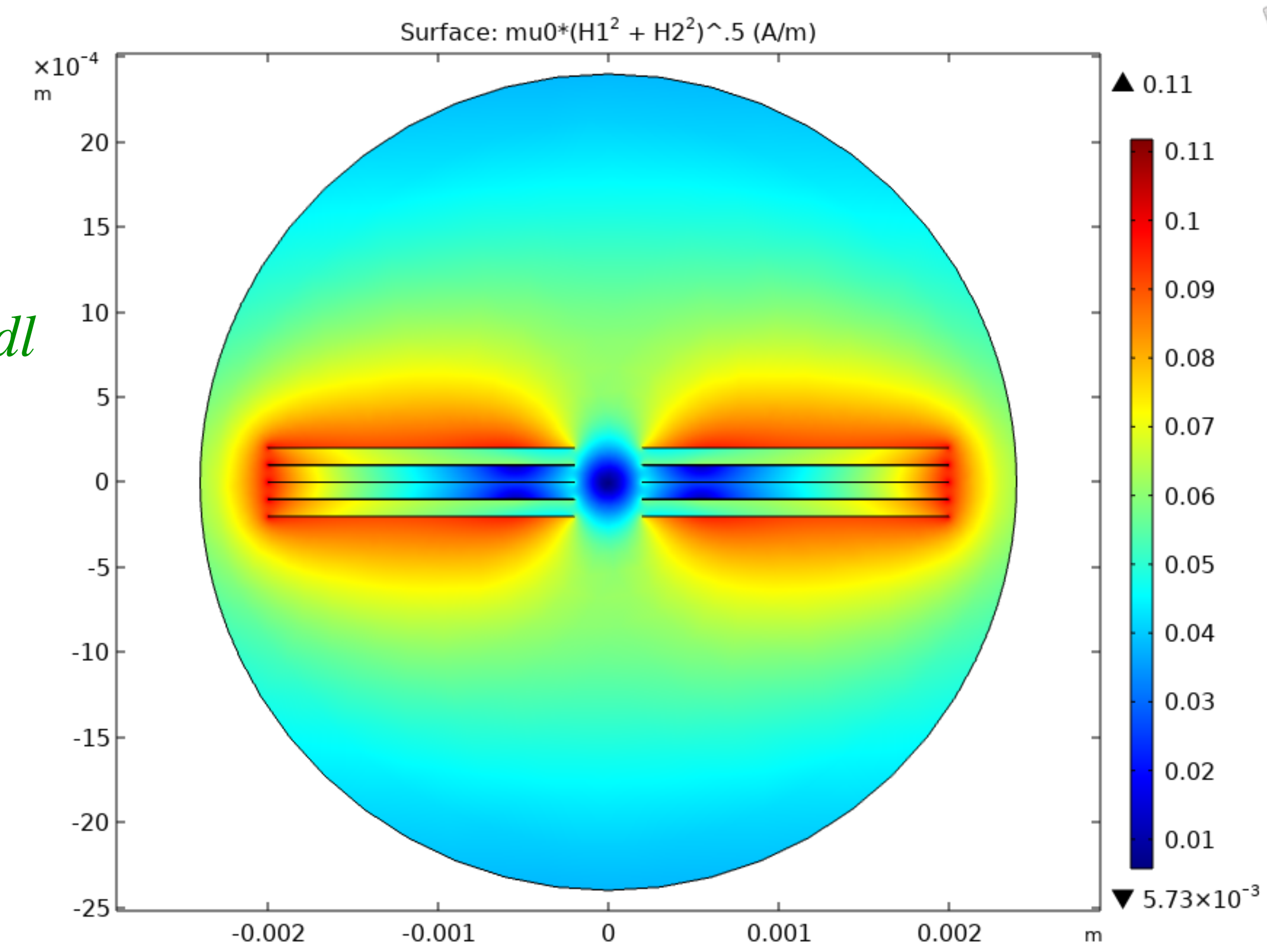
$$\int_{\Omega} \vec{H} \cdot \vec{H}' dx^2 + \frac{1}{\mu_0 \nu} \int_{\Omega} \left(\nabla \times \vec{H} - \vec{J} \right) \cdot \left(\nabla \times \vec{H}' \right) dx^2$$

$$\oint_{\partial\Omega} (a + a_0) \vec{H}' \cdot \hat{\tau} dl$$

$$\nabla^2 (a + a_0) = 0$$

$$-\frac{\partial}{\partial n} a = (\vec{H} - \vec{H}_0) \cdot \hat{\tau}$$

Weak Expressions Ω
 weak
 H1*test(H1)+ H1y*(test(H1y)-test(H2x))/mu0nu
 H2*test(H2) - H2x*(test(H1y)-test(H2x))/mu0nu
 Weak Contribution
 Weak expression:
 -JcB*P*(test(H1y)-test(H2x))/mu0nu
 Weak Contribution
 Weak expression:
 (test(H1)*tcurvx+test(H2)*tcurvy)*Az/mu0
 Equation
 -c∇·(∇Az) + aAz = 0
 Coefficients
 Diffusion coefficient:
 c 1 1
 Absorption coefficient:
 a 0 1/m²
 Boundary Flux/Source
 g mu0*(H1*tcurvx + H2*tcurvy) T



✓ Background field scheme

: The idea is quite clear, but I need to justify its use.

$$\begin{bmatrix} [\mathbf{F}] & [\mathbf{C}]^T \\ [\mathbf{C}] & [\mathbf{B}] \end{bmatrix} \begin{bmatrix} \mathbf{h} \\ \mathbf{a} + \mathbf{a}_0 \end{bmatrix} = \begin{bmatrix} [\mathbf{V}] \cdot \mathbf{j} \\ 2[\mathbf{C}] \cdot \mathbf{h}_0 \end{bmatrix}$$

← A kind of projection to the boundary

$$\begin{bmatrix} [\mathbf{F}] & [\mathbf{C}]^T \\ [\mathbf{C}] & [\mathbf{B}] \end{bmatrix} \begin{bmatrix} \mathbf{h} \\ \mathbf{a} - \mathbf{a}_0 \end{bmatrix} = \begin{bmatrix} [\mathbf{V}] \cdot \mathbf{j} - 2[\mathbf{C}]^T [\mathbf{B}]^{-1} [\mathbf{C}] \cdot \mathbf{h}_0 \\ 0 \end{bmatrix}$$

← Superposed to the domain of the cross section

✓ *In COMSOL, the former approach is preferable, as it is absolutely natural to assign the background field to the “flux/source” item of the boundary.*

✘ Application to the 3-D modeling

: Any idea of collaboration is welcome.

