

3D MODELLING OF THE CURRENTS INDUCED IN A SUPERCONDUCTING LINEAR HALBACH ARRAY

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Devendra Kumar Namburi³, John Durrell², Christophe Geuzaine¹,
Benoît Vanderheyden¹ and Philippe Vanderbemden¹

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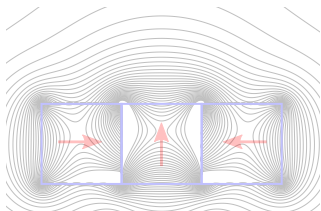
² Bulk Superconductivity Group, University of Cambridge, United Kingdom

³ Quantum Sensors Group, University of Glasgow, United Kingdom



Motivation

- Gradient of the magnetic flux density
→ Magnetic force.
- Typical method achieving significant gradient: Halbach array of permanent magnets.
- The gradient using permanent magnets is intrinsically limited by the saturation magnetization of ferromagnetic materials.

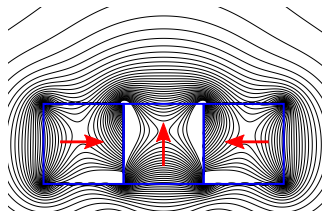


Aim of this work

Investigate the behaviour of several magnetized superconductors instead of permanent magnets in a linear Halbach array.

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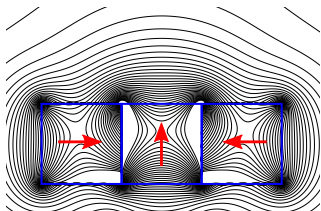
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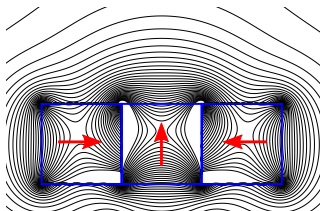


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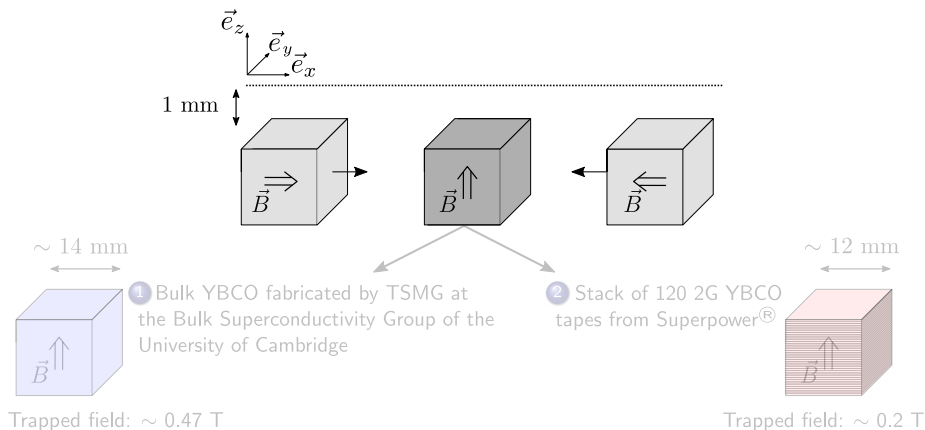


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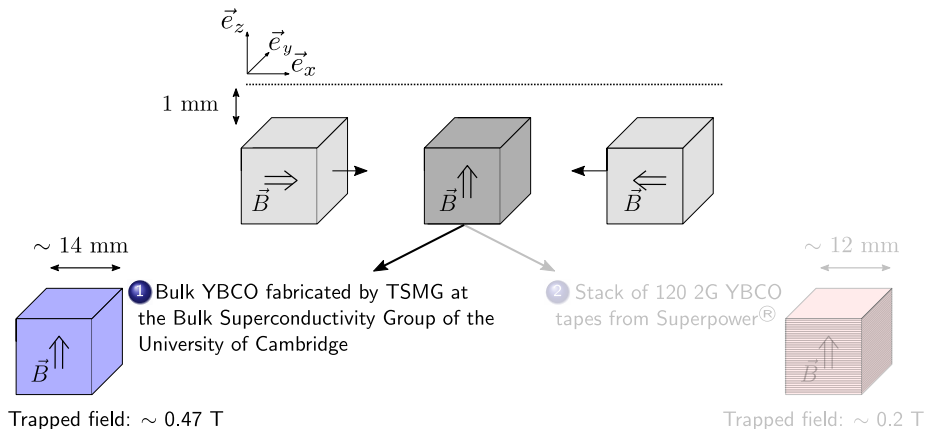
Experimental conditions

- Assembly of 3 magnetized superconductors in a Halbach configuration.
- Approach speed: ~ 1 mm/s; Temperature: 77 K.
- Measurement of the trapped field ~ 1 mm from the samples surfaces.



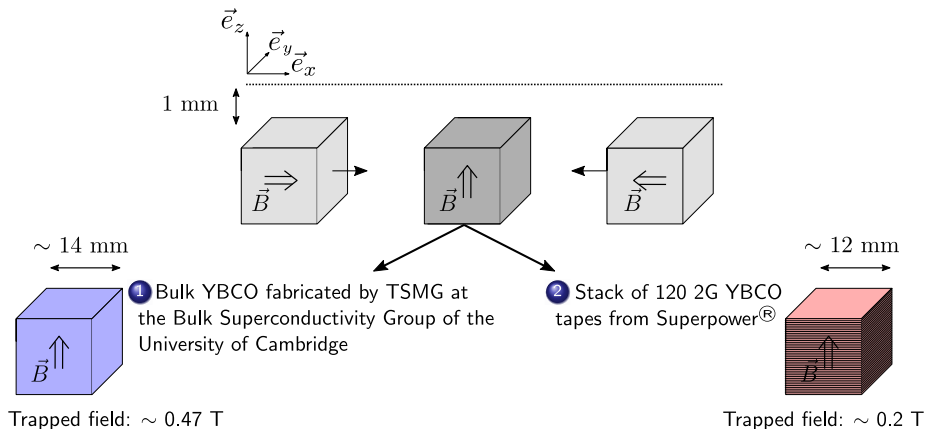
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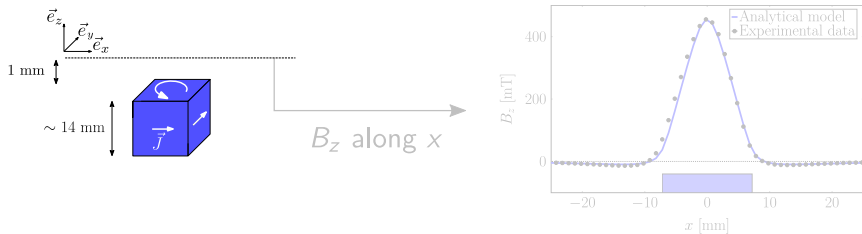
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Analytical model calculating $\vec{B}(x, y, z)$ of a single bulk superconductor

Assumptions

- The critical current density is field-independent and isotropic.
- The sample is fully penetrated $\Rightarrow ||\vec{J}|| = J_c$ everywhere.
- The current flows in square loops strictly parallel to the surface of the superconductor and perpendicular to the magnetization direction.

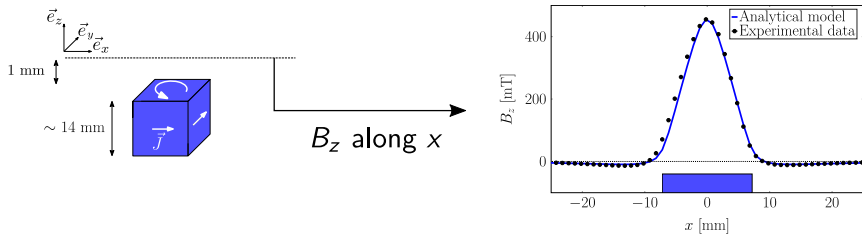


After adjusting the model: $J_c \sim 2 \cdot 10^8 \text{ Am}^{-2}$

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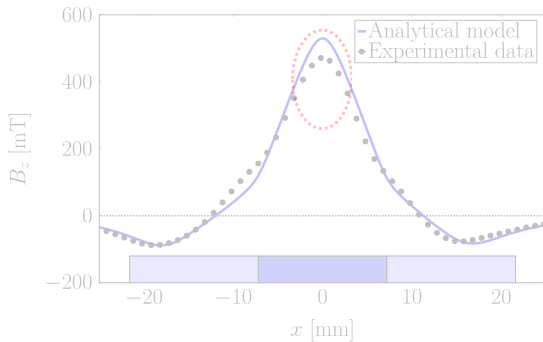
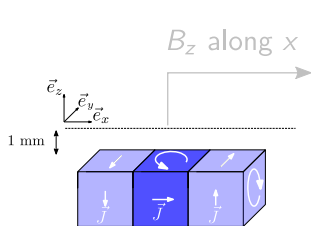


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Analytical model calculating $\vec{B}(x, y, z)$ of a superconducting Halbach array made of bulk superconductors

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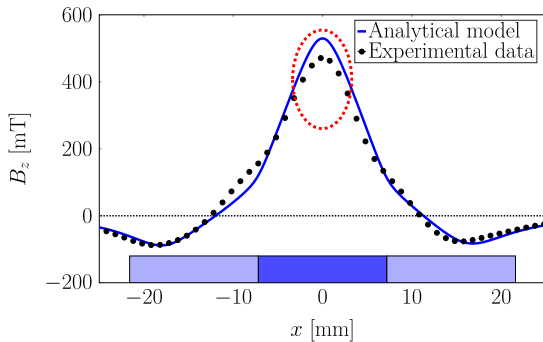
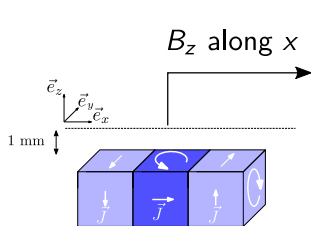


Overestimation of the central trapped field

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Overestimation of the central trapped field

Finite element model for bulk superconductors

- Magnetodynamic formulation:

$$\left\{ \begin{array}{l} \vec{H}\text{-formulation for the superconducting zones,} \\ \vec{A}\text{-formulation for the air,} \\ \text{Surface coupling terms at the interface.} \end{array} \right.$$

- Power law:

$$\vec{E} = \frac{E_c}{J_c} \left(\frac{\|\vec{J}\|}{J_c} \right)^{n-1} \vec{J}$$

$$\left\{ \begin{array}{l} E_c = 10^{-4} \text{ Vm}^{-1} \\ J_c = 2.6 \cdot 10^8 \text{ Am}^{-2} \\ n = 20 \end{array} \right.$$

- Simulations:

- a) Individual zero-field-cooled magnetization of each superconductor,
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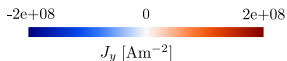
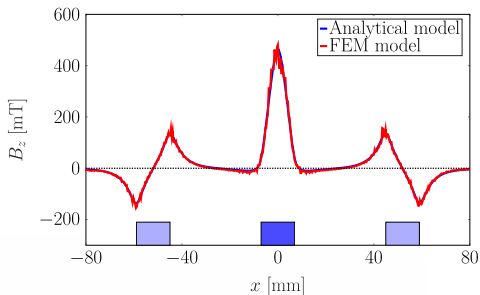
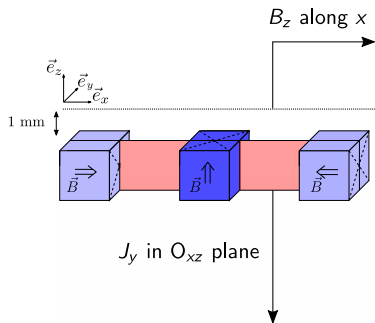
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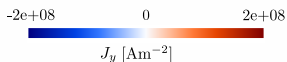
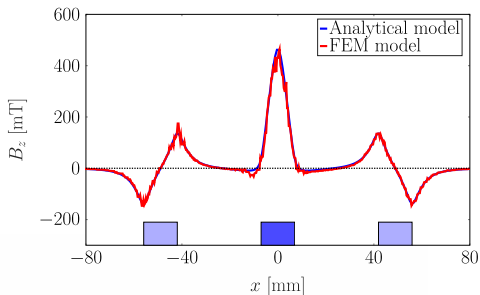
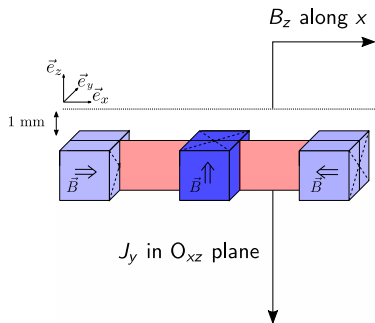
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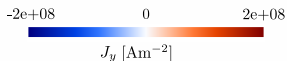
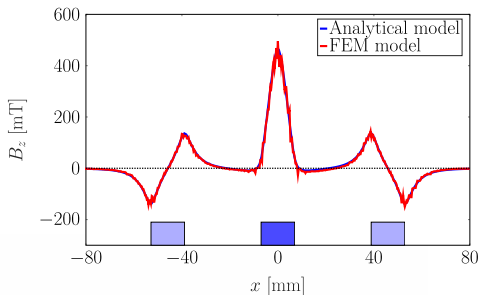
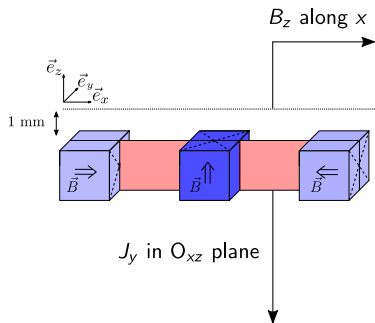
Combination of 3 bulk superconductors



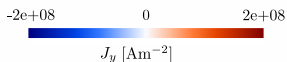
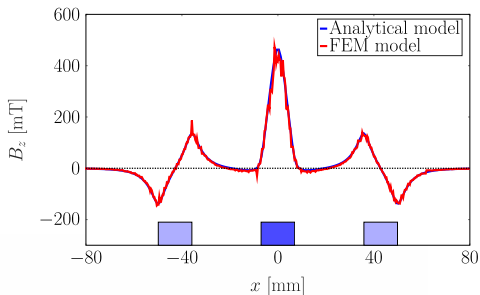
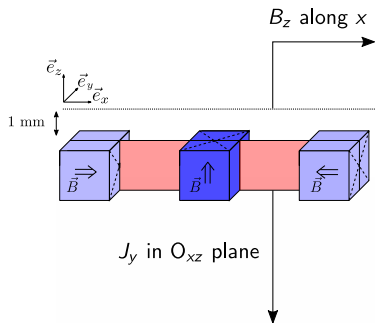
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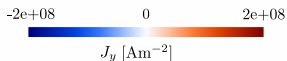
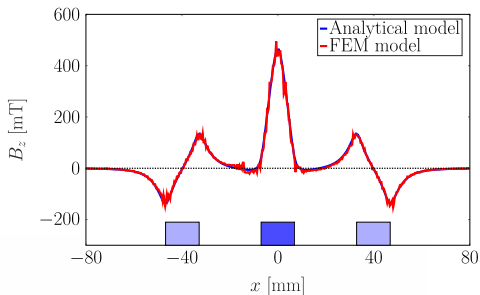
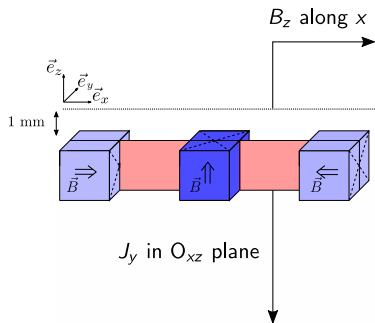
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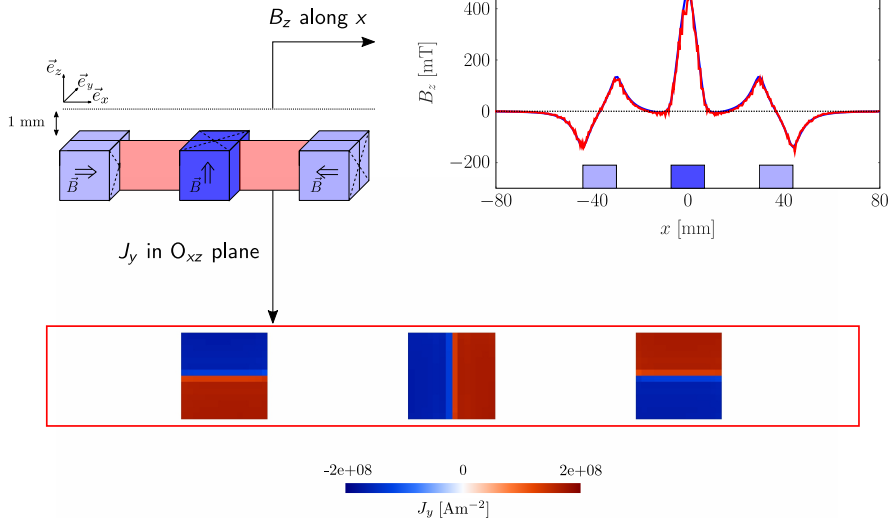
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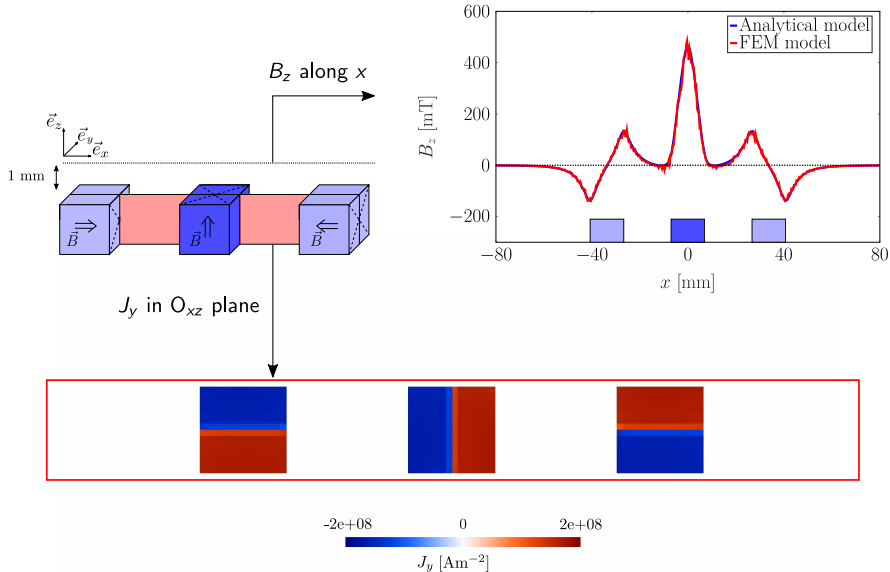
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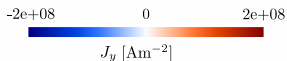
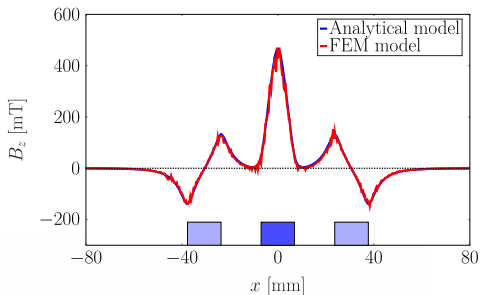
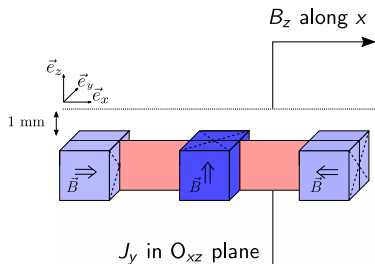
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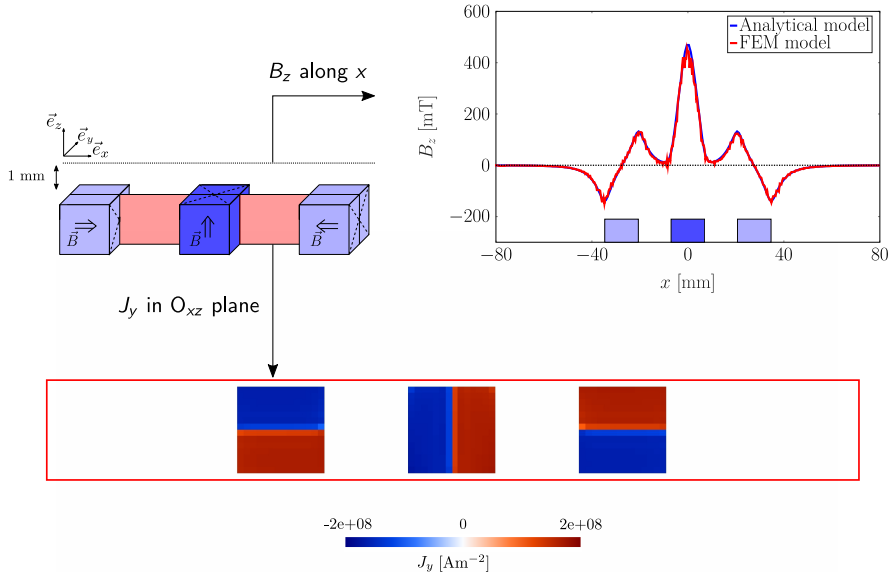
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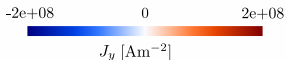
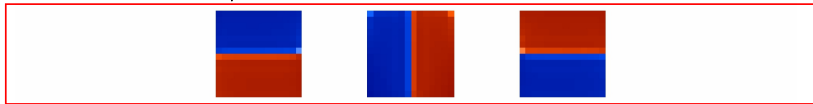
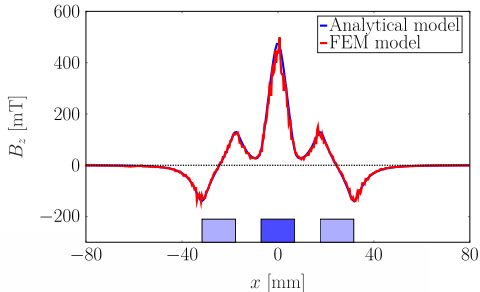
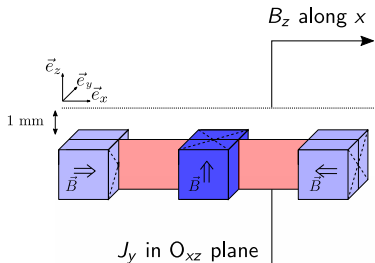
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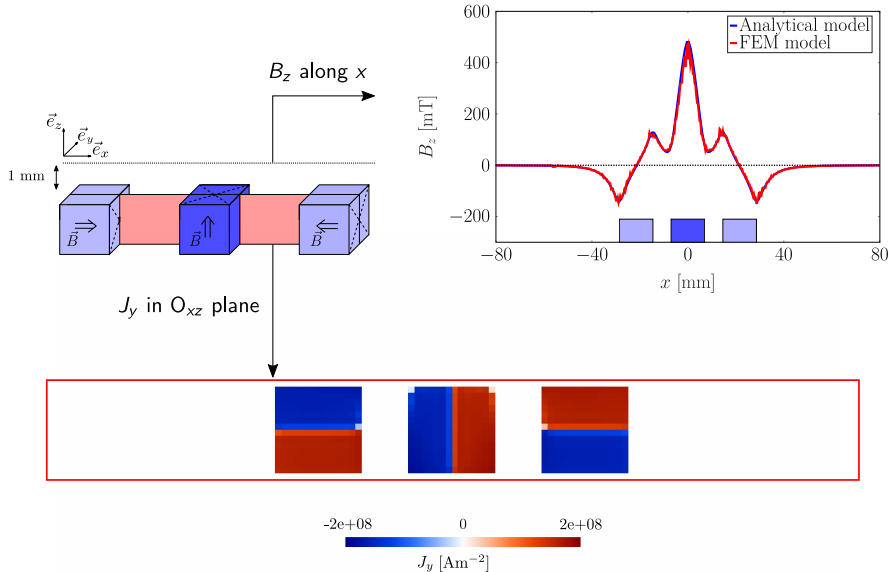
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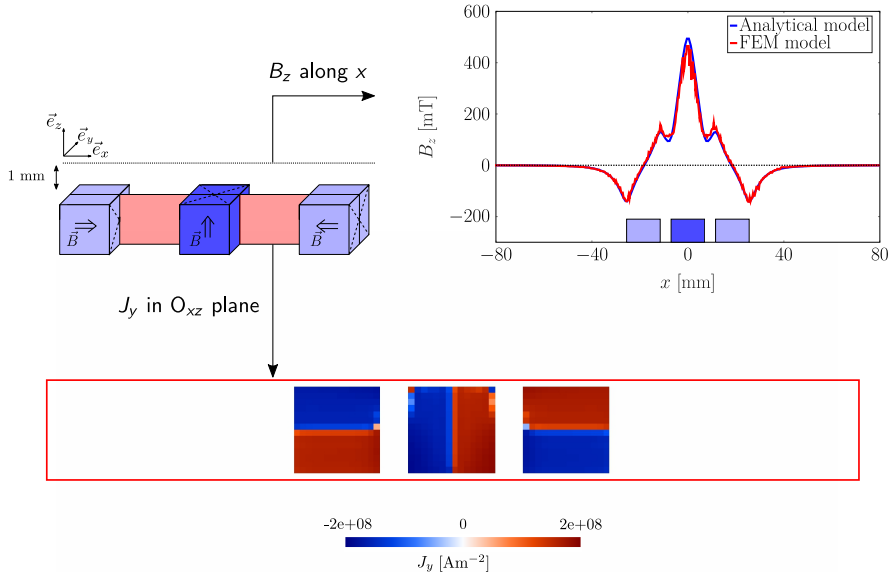
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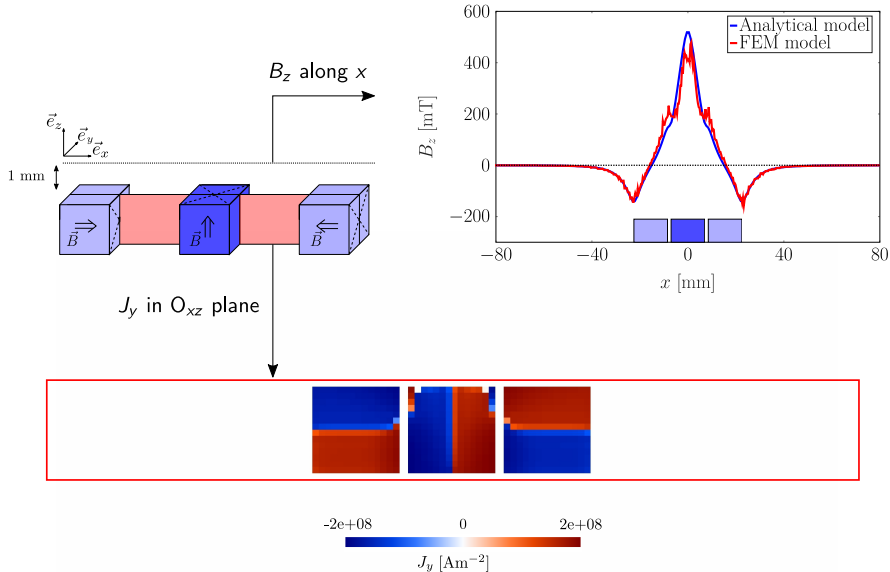
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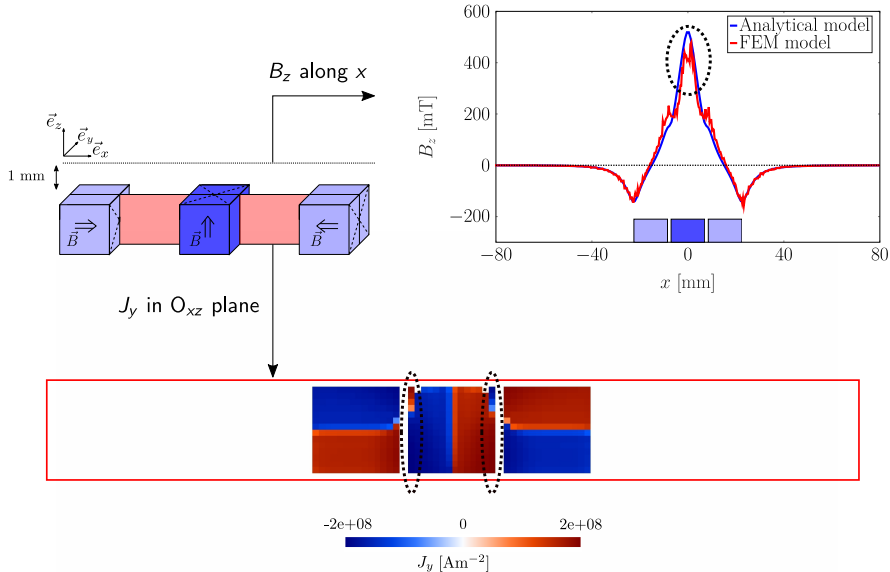
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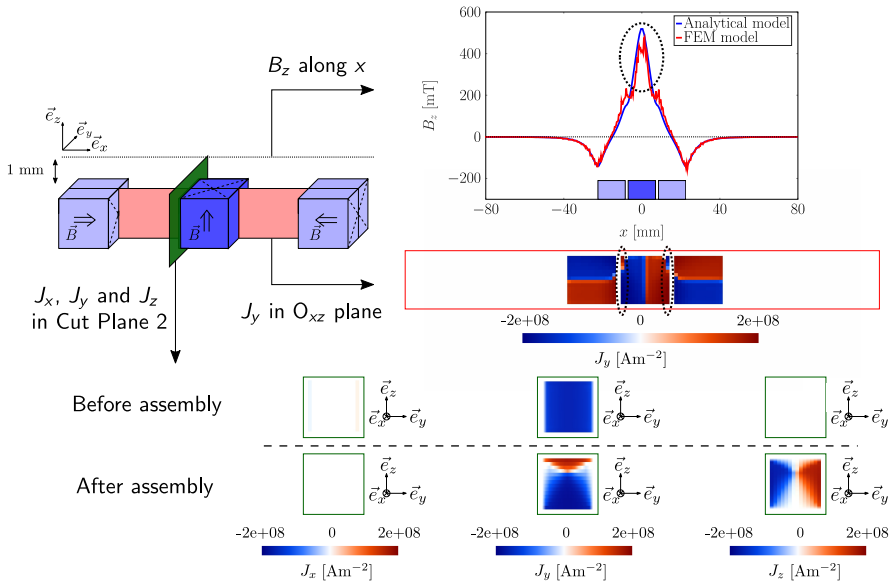
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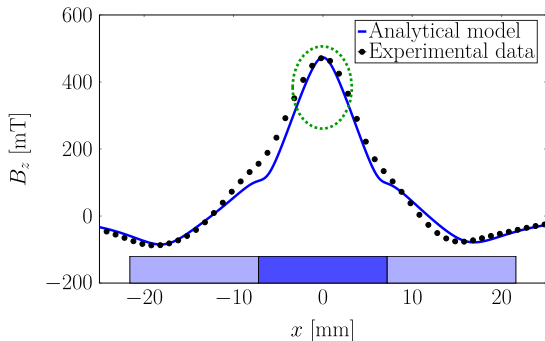
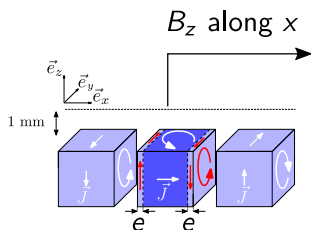
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Updated analytical model of a Halbach array made of Bulk superconductors

Assumption

- The current density distribution in the central superconductor is altered as described by the finite element simulation.



After adjusting the model: $e = 0.8$ mm

Modification of the current density distribution

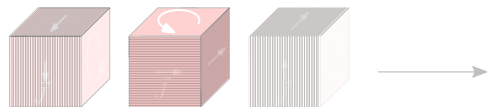
1 Halbach array of Bulk superconductors



Before Assembly

After Assembly

2 Halbach array of stacked tapes



Before Assembly

After Assembly

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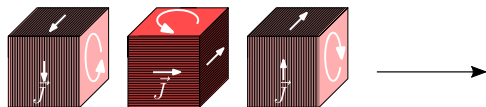
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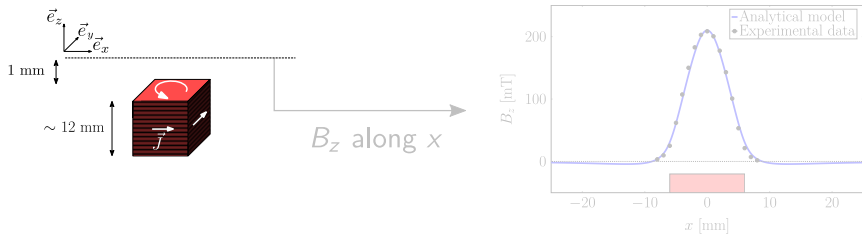
Before Assembly

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Analytical model calculating $\vec{B}(x, y, z)$ of a single stack of superconducting tapes

Assumptions

- The critical current density is field-independent and isotropic.
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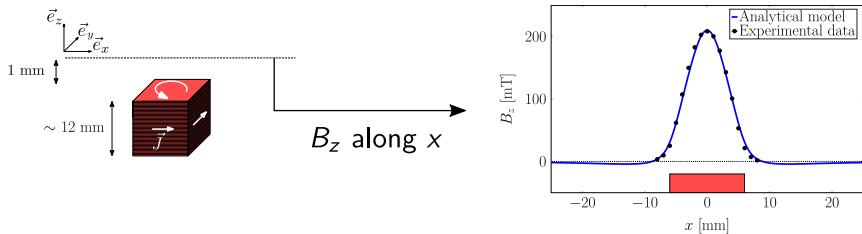


After adjusting the model: $J_c \sim 1.3 \cdot 10^8 \text{ Am}^{-2}$

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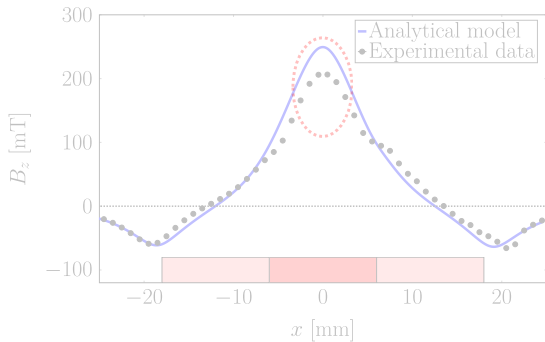
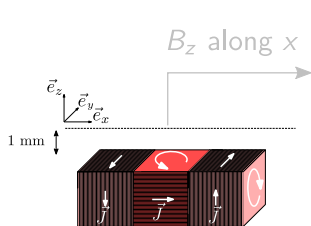


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Analytical model calculating $\vec{B}(x, y, z)$ of a superconducting Halbach array made of stack tapes

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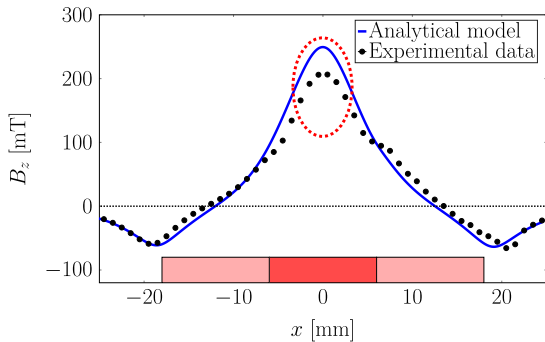
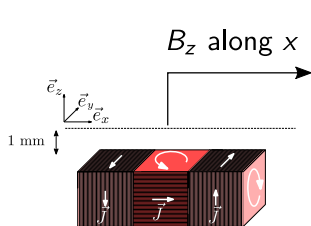


Overestimation of the central trapped field

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Finite element model for stack of superconducting tapes

- Magnetodynamic formulation:

$$\begin{cases} \vec{H}\text{-formulation for the superconducting zones,} \\ \vec{A}\text{-formulation for the air,} \\ \text{Surface coupling terms at the interface.} \end{cases}$$

- Modified power law:

$$\vec{E} = \left[\frac{E_c}{J_c} \left(\frac{\|\vec{J}\|}{J_c} \right)^{n-1} + \text{diag}(\rho_1, \rho_2, \rho_3) \right] \vec{J} \quad \begin{cases} E_c = 10^{-4} \text{ Vm}^{-1} \\ J_c = 1.7 \cdot 10^8 \text{ Am}^{-2} \\ n = 20 \end{cases}$$

$$\rho_i = \begin{cases} 10^{-8} & \Omega\text{m in the direction perpendicular to the tapes,} \\ 0 & \text{otherwise.} \end{cases}$$

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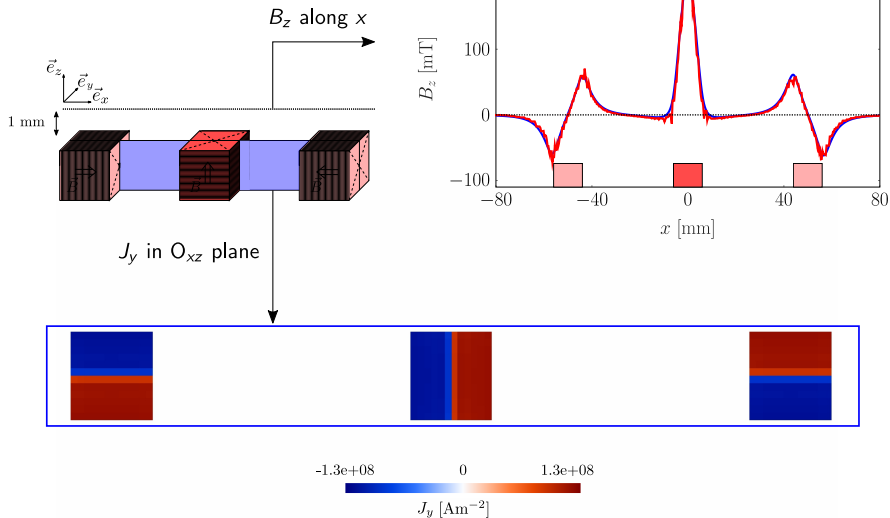
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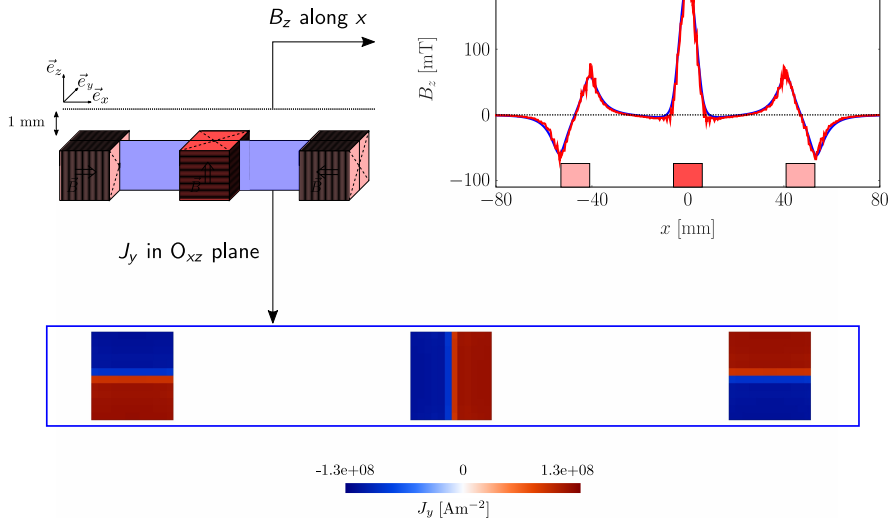
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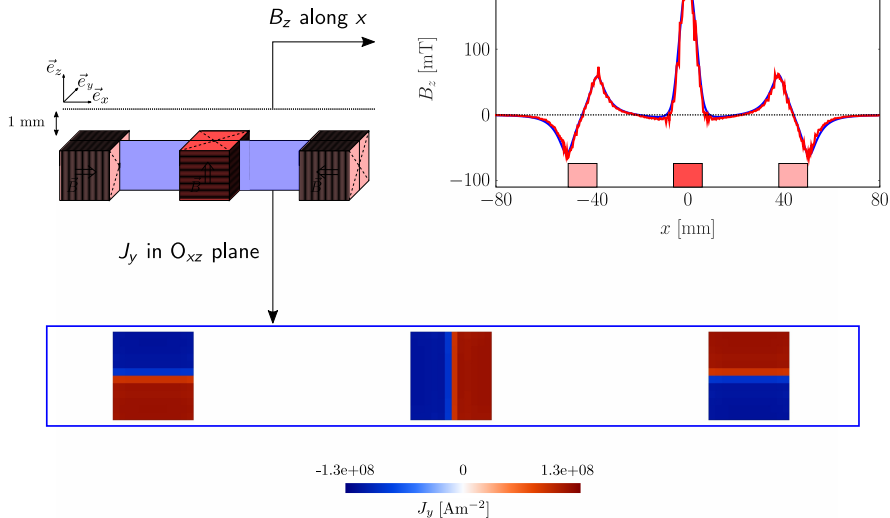
Combination of 3 stack tapes



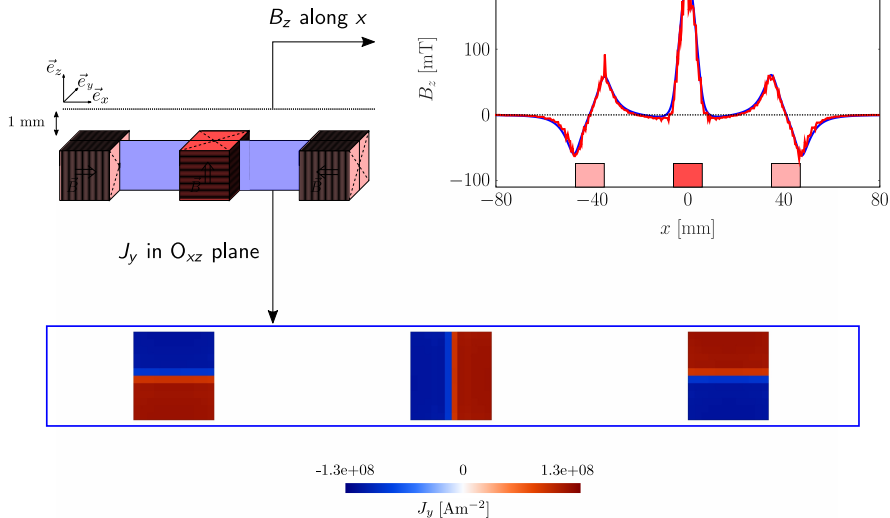
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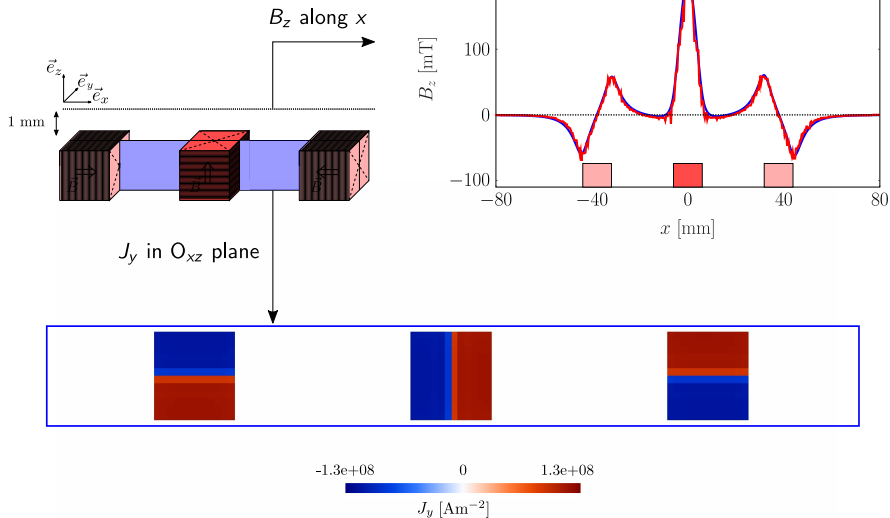
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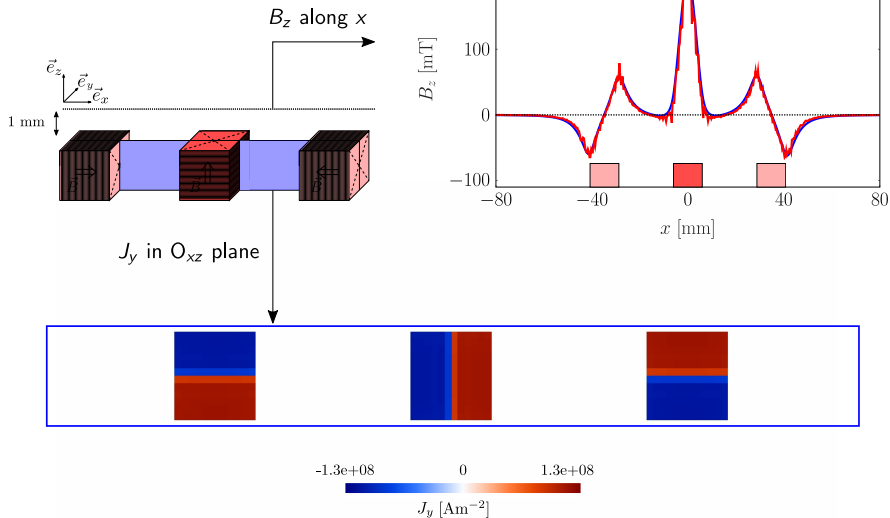
Combination of 3 stack tapes



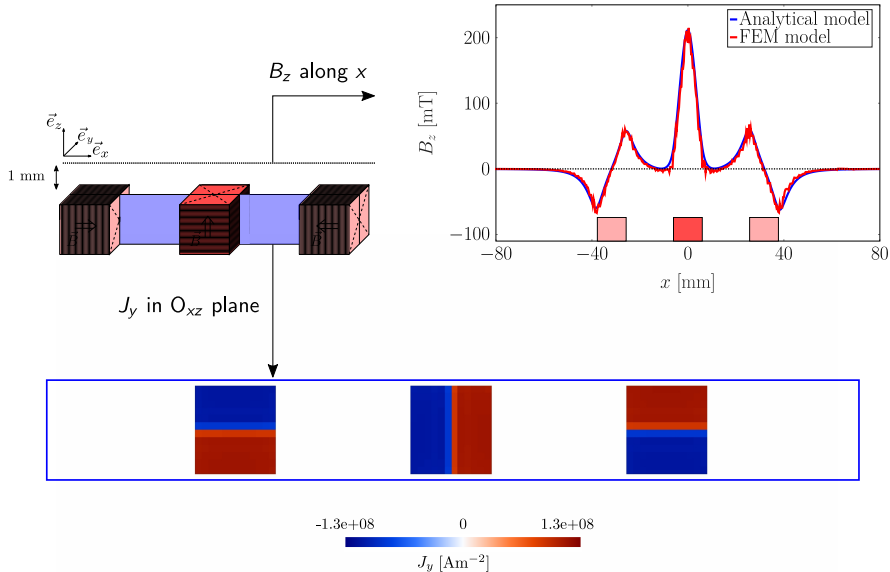
Combination of 3 stack tapes



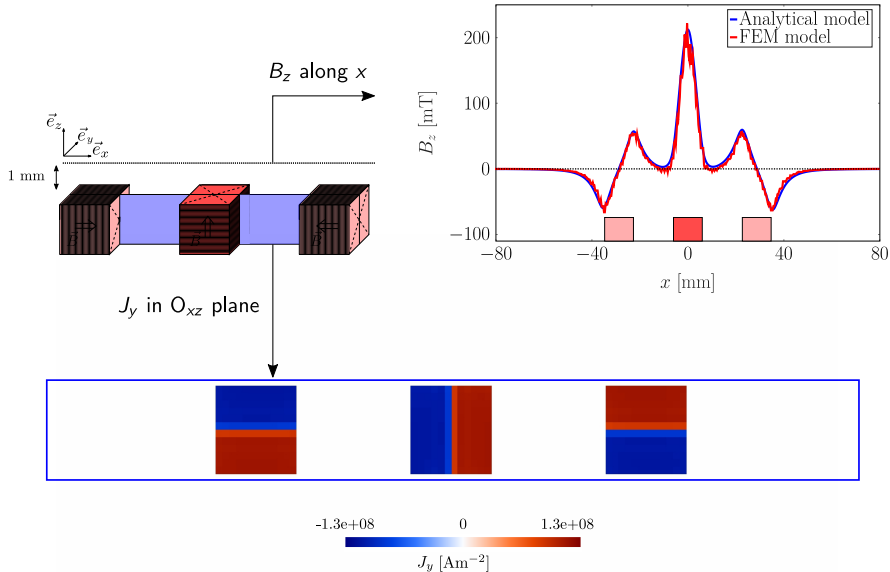
Combination of 3 stack tapes



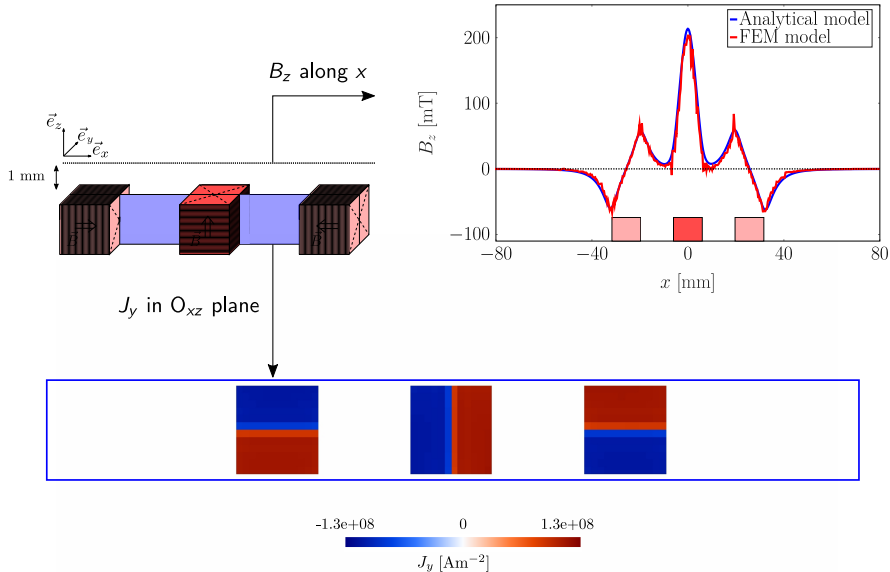
Combination of 3 stack tapes



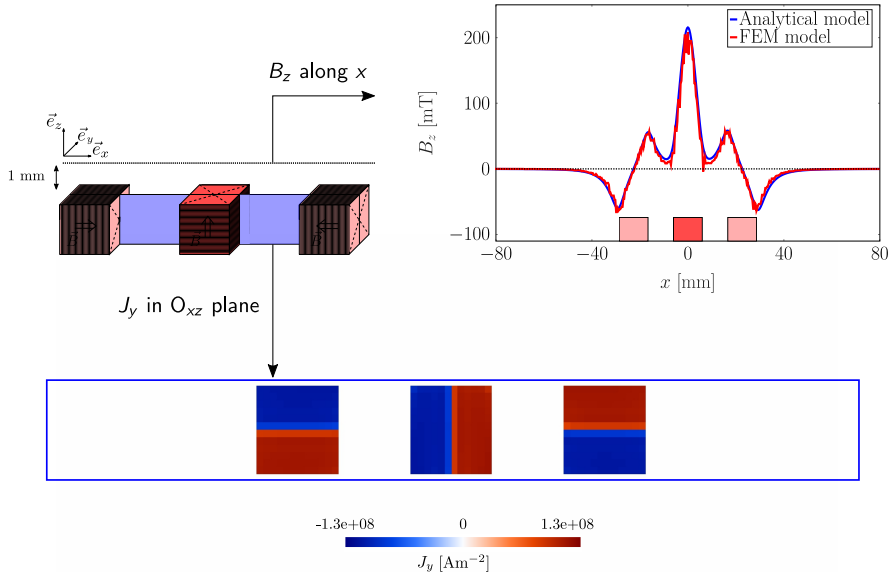
Combination of 3 stack tapes



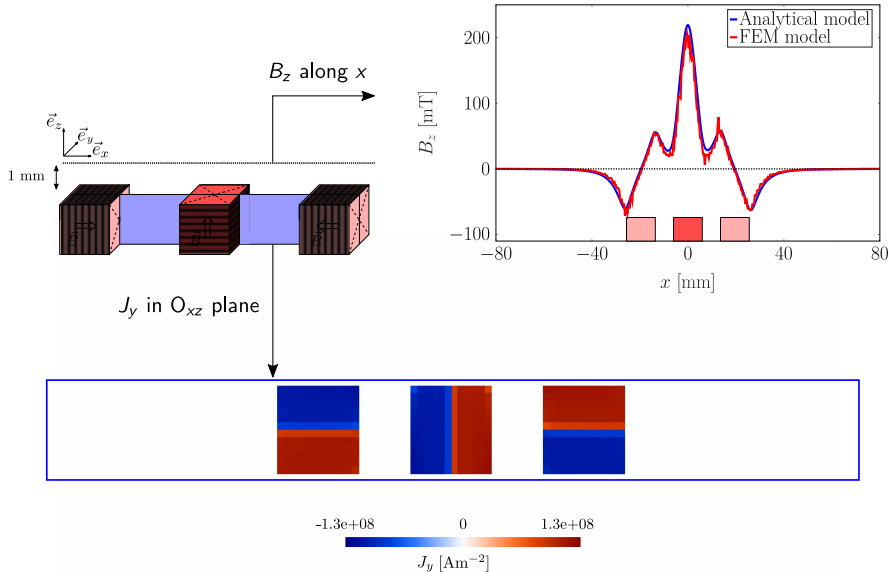
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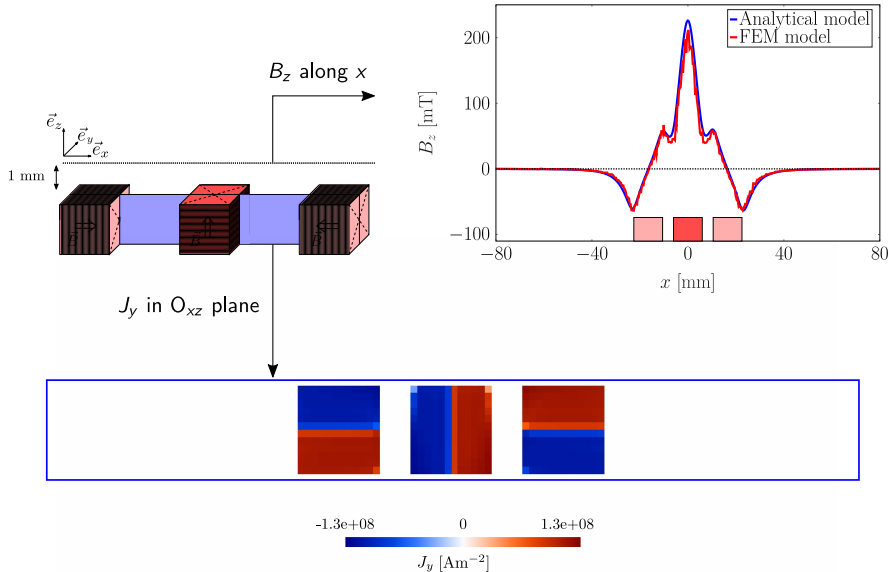
Combination of 3 stack tapes



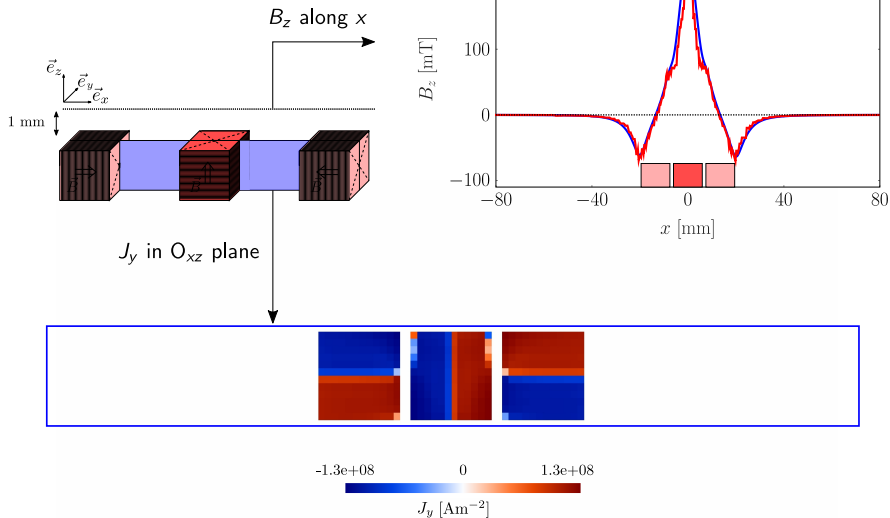
Combination of 3 stack tapes



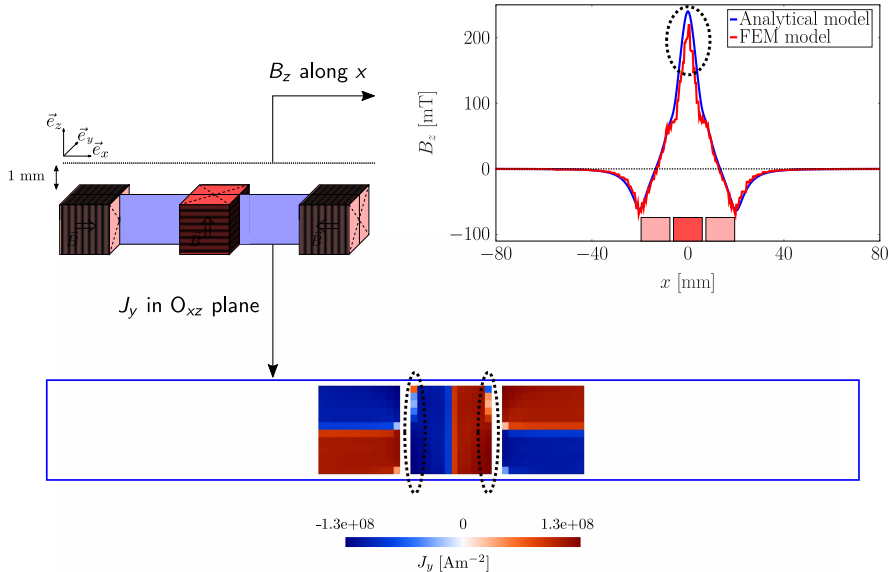
Combination of 3 stack tapes



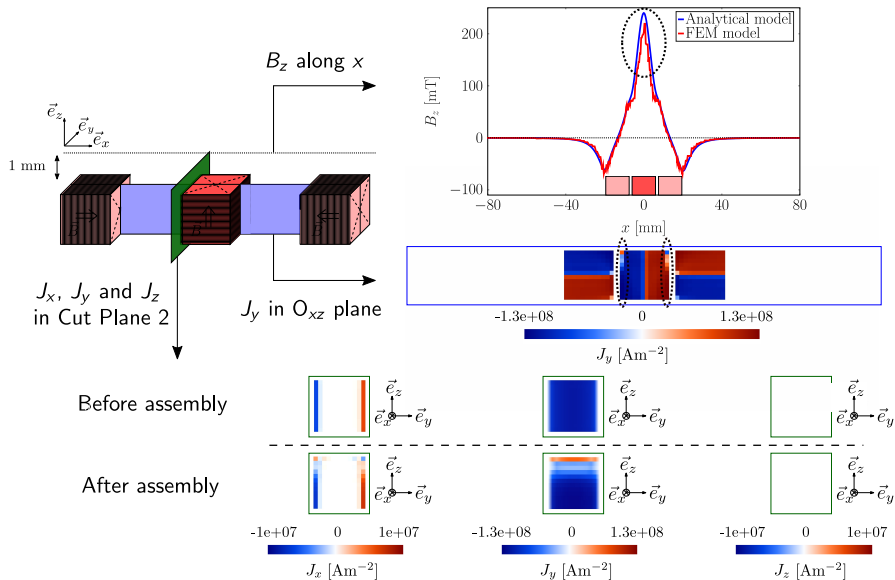
Combination of 3 stack tapes



Combination of 3 stack tapes



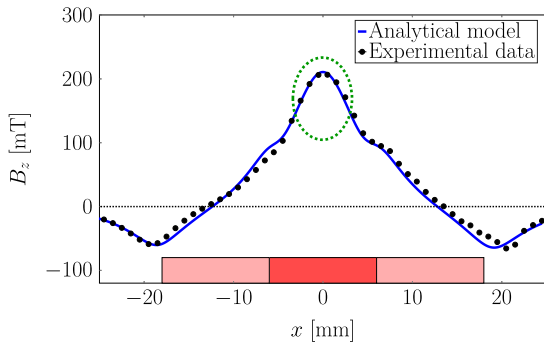
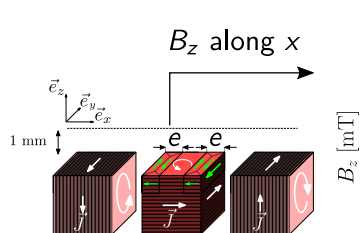
Combination of 3 stack tapes



Updated analytical model of a Halbach array made of stack tapes

Assumption

- The current density distribution in the central superconductor is altered as described by the finite element simulation.



After adjusting the model: $e = 1.1$ mm

Summary and conclusions

- A Halbach array made of 3 superconductors was successfully assembled.
- Both bulk YBCO superconductors and stacked 2G YBCO tapes were investigated.
- A finite-element model based on a coupled formulation and an analytical model based on the Biot-Savart law were developed and successfully captured the main features observed experimentally.
- The performances of the assembly are currently limited by a partial demagnetization of the central sample.
- Preventing the current redistribution would lead to a central field 10% higher.

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