

Thermal Thin Shell Approximation for 3D Finite Element Quench Simulations of Insulated HTS Coils



TECHNISCHE
UNIVERSITÄT
DARMSTADT

8th International Workshop on Numerical Modelling of High-Temperature Superconductors

Erik Schnaubelt^{1,2}, Mariusz Wozniak¹ and Sebastian Schöps²

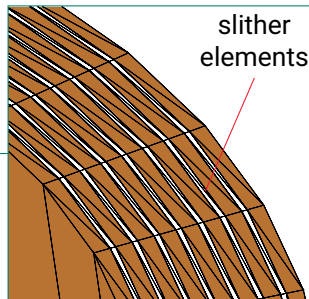
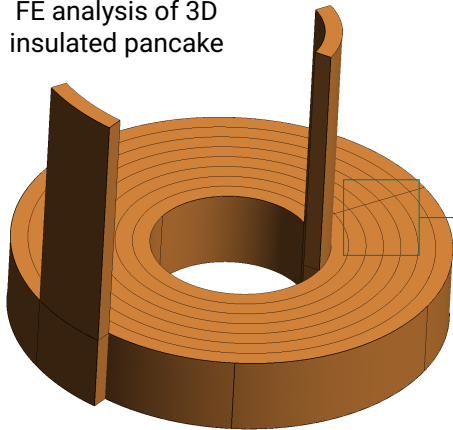
¹CERN ²TU Darmstadt



Motivation: Thin Layers in HTS Applications

Example: HTS Pancake

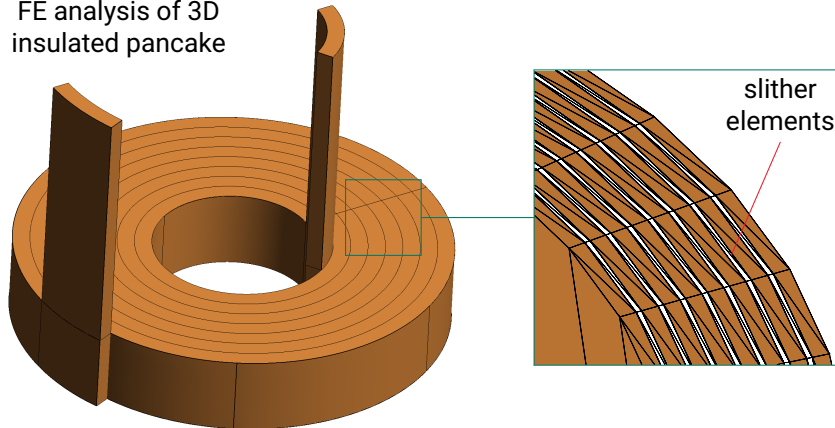
FE analysis of 3D
insulated pancake



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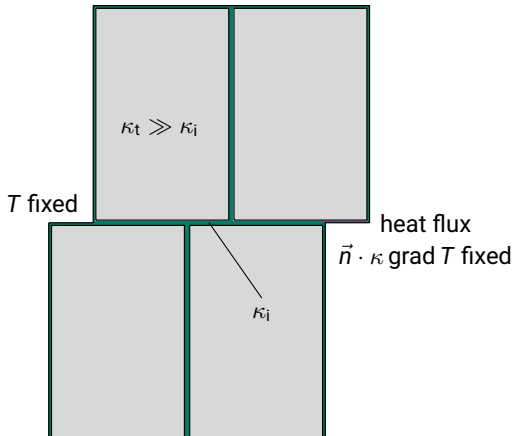
FE analysis of 3D insulated pancake



- Thin insulation, contact res., supercond. layer of coated conductors...

Simple Example: Brick Wall Geometry

Influence of Slither Elements on Accuracy

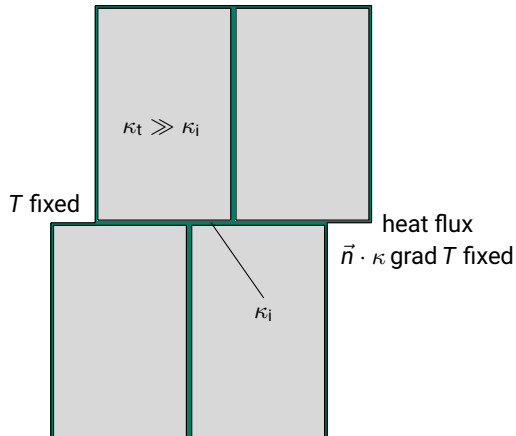


- Stationary heat equation

$$\text{div} (\kappa \text{ grad } T) = 0$$

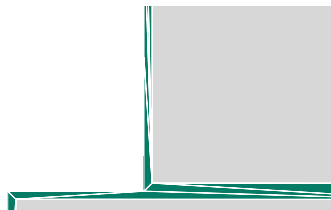
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Simple Example: Brick Wall Geometry

Influence on Numerical Solution



Structured mesh (1400 DoF)

Simple Example: Brick Wall Geometry

Influence on Numerical Solution



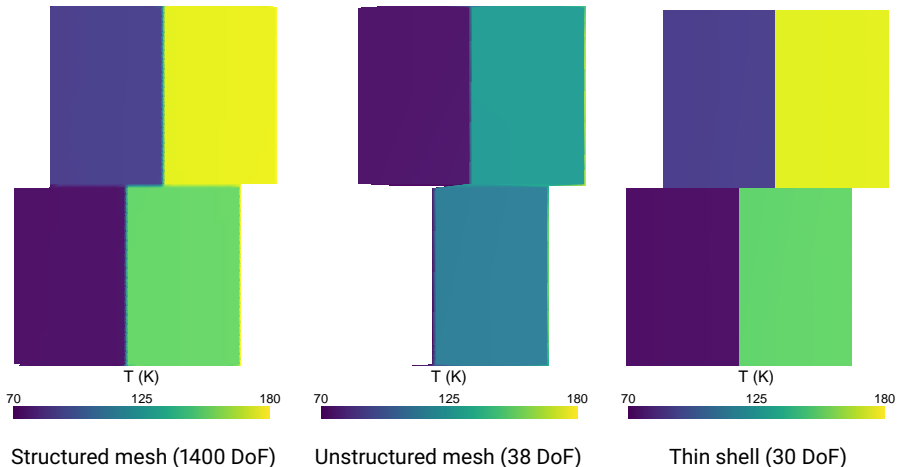
Structured mesh (1400 DoF)



Unstructured mesh (38 DoF)

Simple Example: Brick Wall Geometry

Influence on Numerical Solution



Benefits of Thin Shell Approximations

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- **Thin shell approximation** does not require volumetric mesh!
- Better condition number of finite element matrices
- Potentially fewer degrees of freedom
- Simplification of geometry
 - Less effort for user and/or mesh software

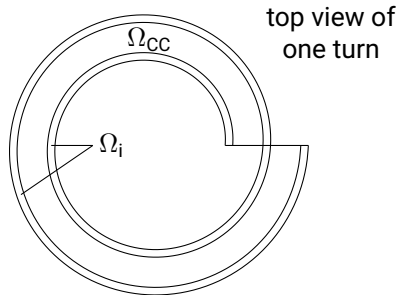
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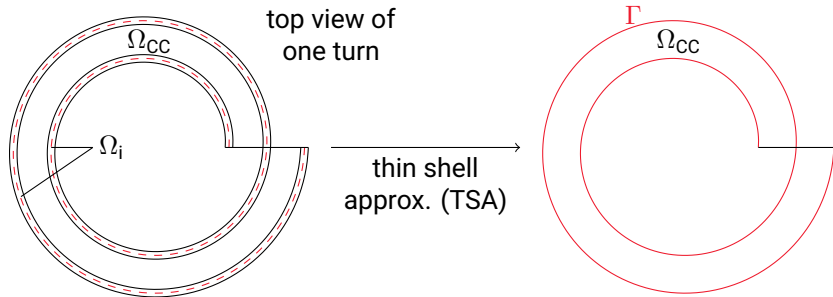
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Thin Shell Approximation: Derivation I

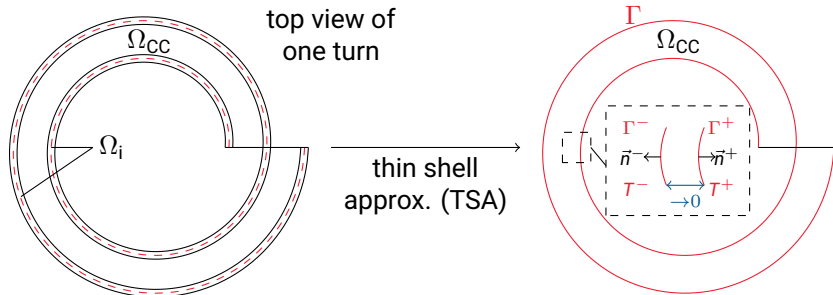


Thin Shell Approximation: Derivation I



1. Replace volumetric layer Ω_i by surface Γ

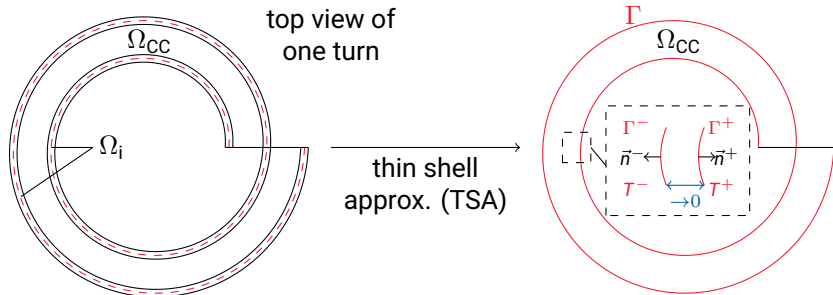
Thin Shell Approximation: Derivation I



2. Need to represent discontinuous temperatures across Γ

- ▣ Duplicate $\Gamma \rightarrow \Gamma^+ \neq \Gamma^- \rightarrow T^+ \neq T^-$

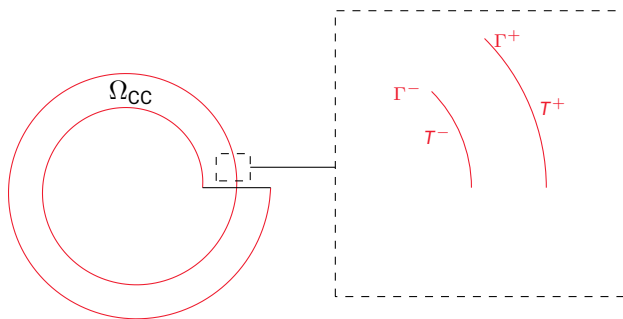
Thin Shell Approximation: Derivation I



3. Additional integral with jump operator $[b] := b|_{\Gamma^+} - b|_{\Gamma^-}$

$$\begin{aligned}
 & (\kappa \operatorname{grad} T, \operatorname{grad} T')_{\Omega_{CC}} + (\rho C_p \partial_t T, T')_{\Omega_{CC}} \\
 & + \langle [\vec{n} \cdot \kappa \operatorname{grad} T], T' \rangle_{\Gamma} = (Q, T')_{\Omega_{CC}} \quad \forall T' \in H_{\text{dir}}^1(\Omega_{CC})
 \end{aligned}$$

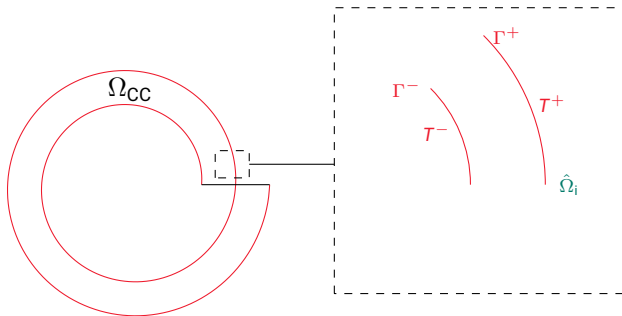
Thin Shell Approximation: Derivation II



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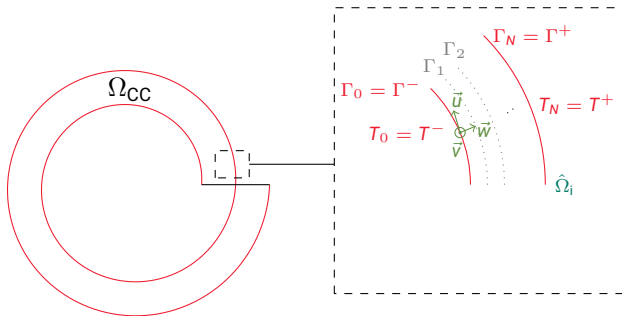
Thin Shell Approximation: Derivation II



4. Virtual discretization of Ω_i : $\hat{\Omega}_i$

$$\langle [\vec{n} \cdot \kappa \text{grad} T], T' \rangle_{\Gamma} = (\kappa \text{grad} T, \text{grad} T')_{\hat{\Omega}_i} + (\rho C_p \partial_t T, T')_{\hat{\Omega}_i}$$

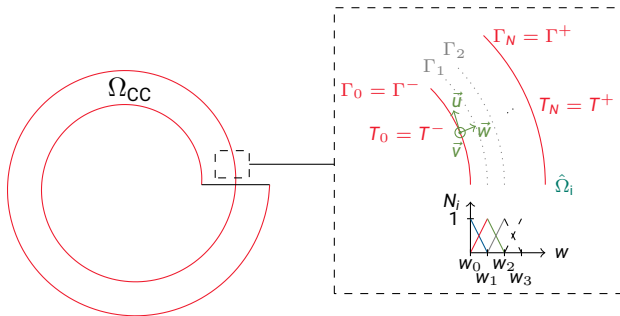
Thin Shell Approximation: Derivation II



5. $\hat{\Omega}_i$ split into N slabs $\hat{\Omega}_i^{(k)} := \Gamma \times [w_{k-1}, w_k]$

$$\langle [\vec{n} \cdot \kappa \text{grad} T], T' \rangle_{\Gamma} = \sum_{k=1}^N (\kappa \text{grad} T, \text{grad} T')_{\hat{\Omega}_i^{(k)}} + \sum_{k=1}^N (\rho C_p \partial_t T, T')_{\hat{\Omega}_i^{(k)}}$$

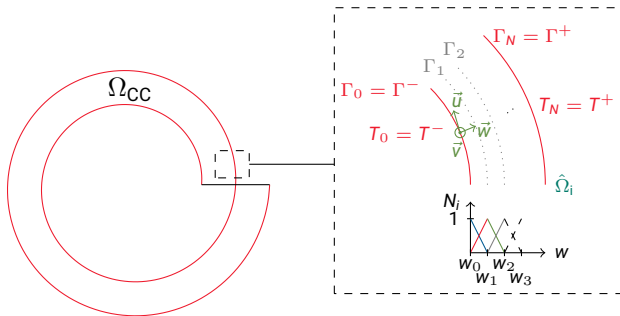
Thin Shell Approximation: Derivation II



6. 1D discretization of T along normal direction \vec{w}

$$T|_{\hat{\Omega}_i^{(k)}}(u, v, w, t) = \sum_{j=k-1}^k T|_{\Gamma_j}(u, v, t) N_j(w) \quad \text{with, e.g., } N_j \text{ first-order Lagrange basis}$$

Thin Shell Approximation: Derivation II



7. Decomposition into surface integrals and 1D matrices

$$(\rho C_p \partial_t T, T')_{\hat{\Omega}_i} = \sum_{k=1}^N \sum_{j=k-1}^k \langle \mathbf{M}_{lj, C_V}^{(k)} \partial_t T|_{\Gamma_j}, T'|_{\Gamma_j} \rangle_{\Gamma} \text{ with } \mathbf{M}_{lj, C_V}^{(k)} := \int_{w_{k-1}}^{w_k} \rho C_p N_l N_j dw$$

Thin Shell Approximation: Summary and Remarks

- No volumetric mesh representation needed!

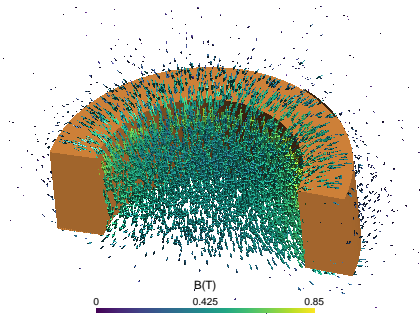
Thin Shell Approximation: Summary and Remarks

- No volumetric mesh representation needed!
- Generality due to FE and choice of N
 - ▣ Multi-layered domains (e.g., HTS coated conductors)
 - ▣ Discretization of eddy current¹ (and coupled!) problems

¹B. de Sousa Alves et al. "Thin-shell approach for modeling superconducting tapes in the H- ϕ finite-element formulation". In: *Supercond Sci Technol* (2021).

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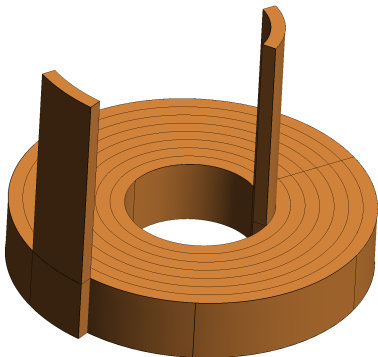
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3 Numerical Experiments

4 Conclusion and Outlook

Insulated HTS Pancake

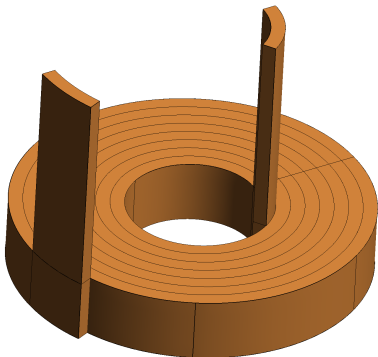
Simple Quench Model



- Insulated, adiabatic HTS pancake
 - ▣ Homogenized Shanghai Creative Superconductor Technologies 2G HTS
 - ▣ Bare CC thickness $t_{CC} = 121 \mu\text{m}$
 - ▣ Ins. thickness betw. turns, $t_{ins} = 40 \mu\text{m}$ or $80 \mu\text{m}$ or $120 \mu\text{m}$

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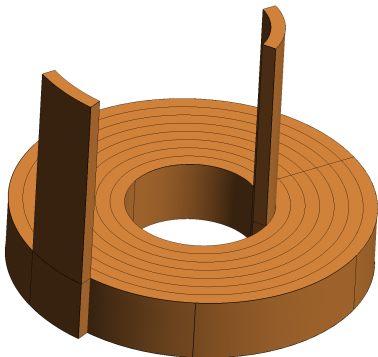
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 - ▣ Constant transport current, $I_t = 210 \text{ A}$ ($\approx 60\%$ of short sample)

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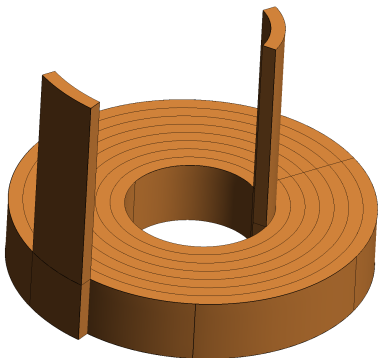
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 - External field: $B_{ext} = 5 \text{ T} \parallel$ to coil axis
 - At B_{ext} : $T_{cs} = 25 \text{ K}$ and $T_c = 85 \text{ K}$
- Transient thermal simulation considered
 - Local defect with $I_{c, defect} = 0 \text{ A}$
 - Linear scaling of I_c between T_c and T_{cs}

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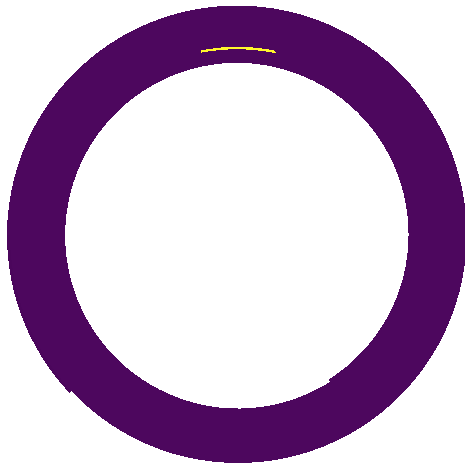
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- NB: scaling of κ and ρC_p of insulation due to increased volume of Ω_{CC}

Insulated HTS Pancake

Local Defect

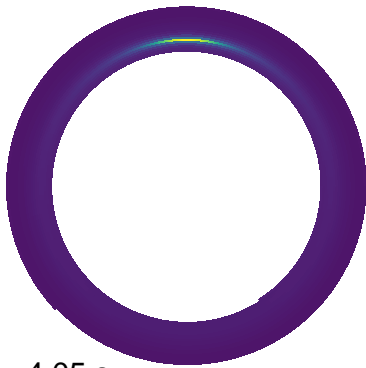
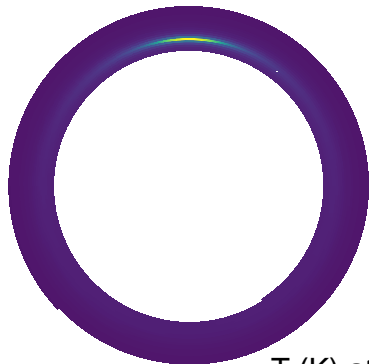


Insulated HTS Pancake

Resulting Hot Spot for $t_{\text{ins}} = 40 \mu\text{m}$

Ref. (volumetric mesh)

TSA



T (K) at time, $t = 4.05 \text{ s}$

15

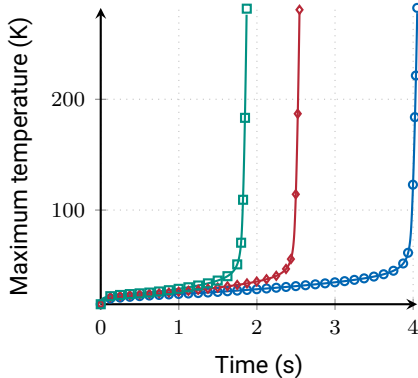
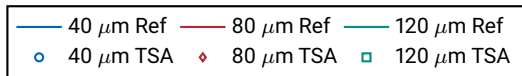
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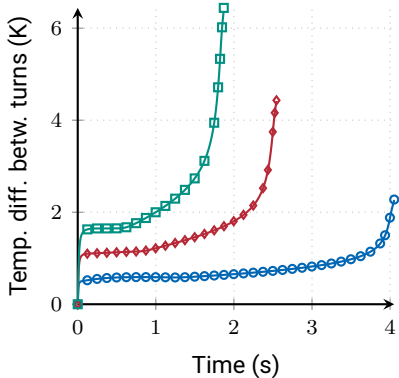
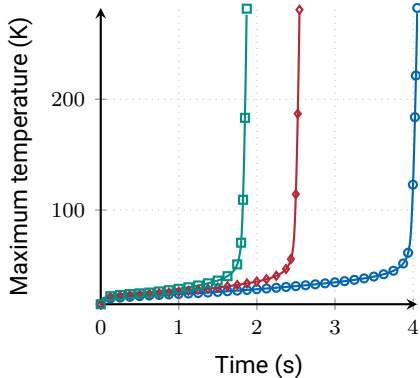
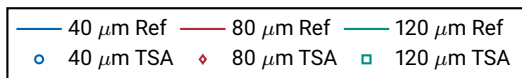
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Influence of the Insulation Thickness



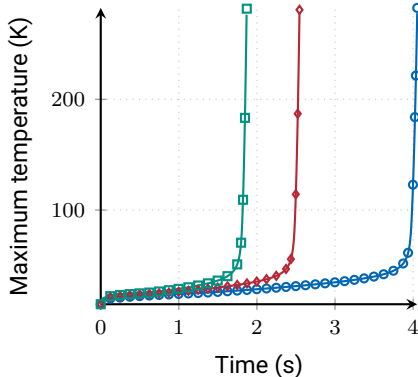
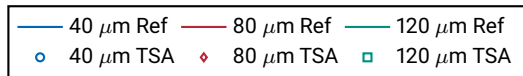
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Influence of the Insulation Thickness



□ $t_{\text{ins}} = 40 \mu\text{m}$: $\text{NZPV}_{\text{longitudinal}} \approx 2.4 \text{ cm/s}$

□ $t_{\text{ins}} = 80 \mu\text{m}$: $\text{NZPV}_{\text{longitudinal}} \approx 2.8 \text{ cm/s}$

□ $t_{\text{ins}} = 120 \mu\text{m}$: $\text{NZPV}_{\text{longitudinal}} \approx 3.6 \text{ cm/s}$

□ TSA and reference same results

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³cern.ch/steam

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Thank you for your attention!

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