

# Modelling high- $T_c$ superconducting dynamos

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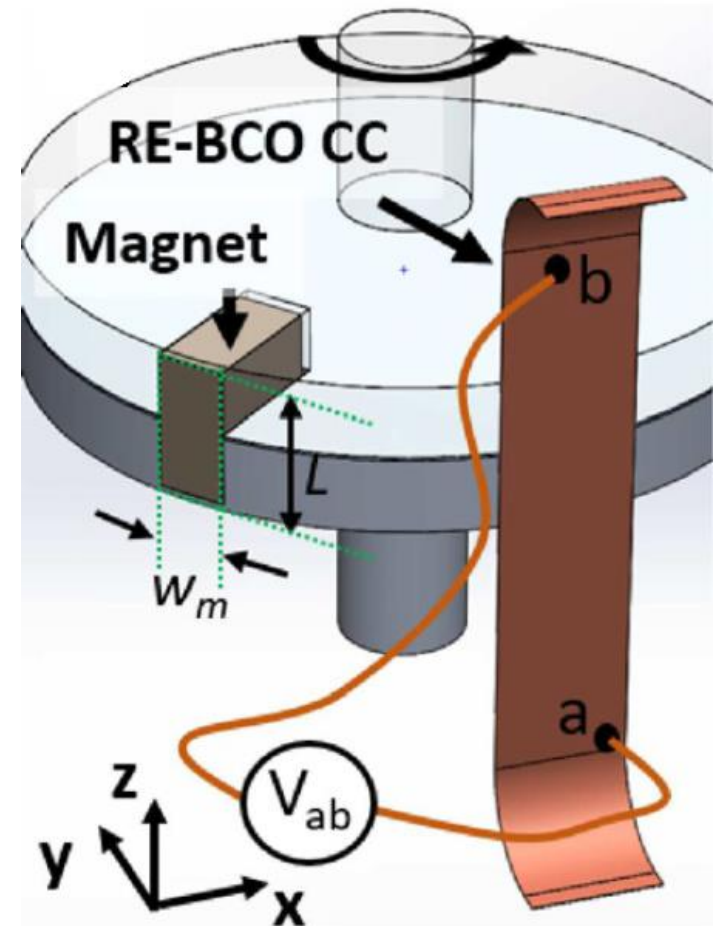
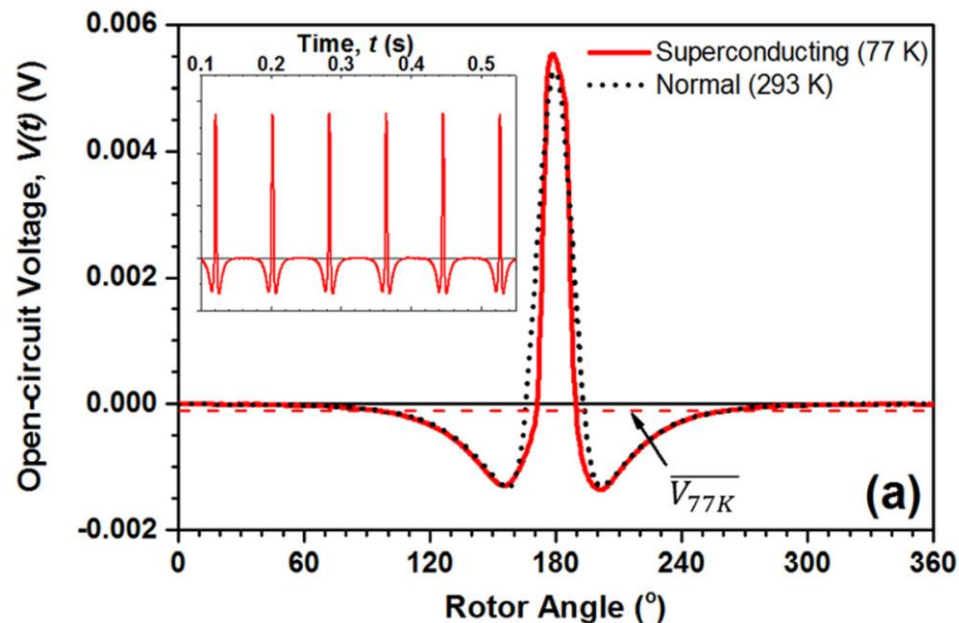
# Presentation Outline

- **The high- $T_c$  superconducting (HTS) dynamo**
- **Numerical modelling of the HTS dynamo**
  - Benchmark problem
  - Efficient 3D models
  - Modelling dynamic coil charging behaviour
  - Future view

# High- $T_c$ Superconducting (HTS) Dynamo

- Inject large DC supercurrents into a closed superconducting circuit
- Energise HTS coils in NMR/MRI magnets, superconducting rotating machines *without need for current leads*

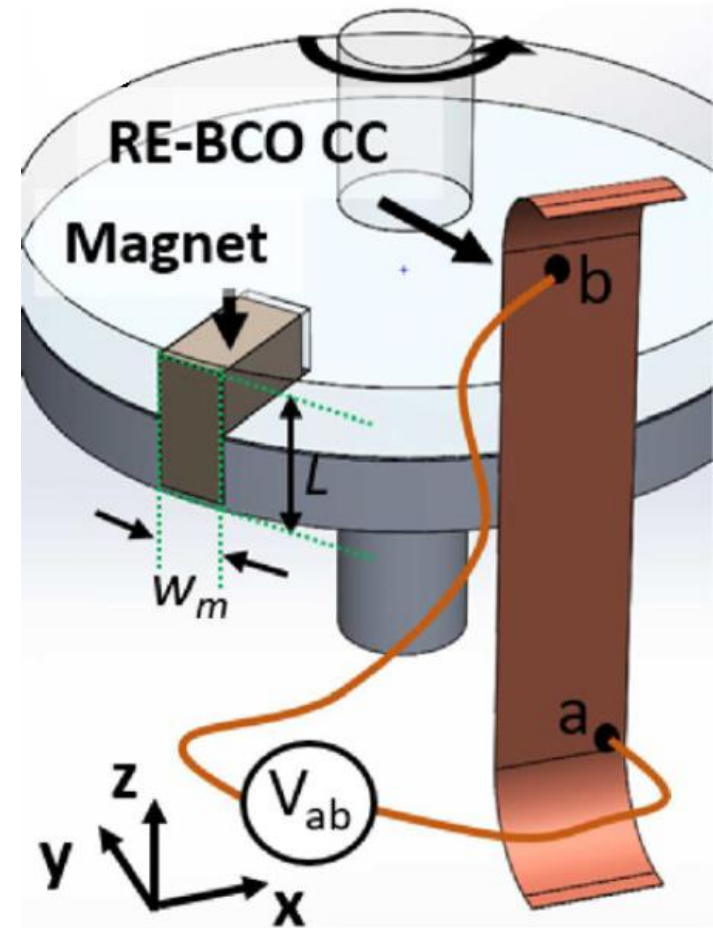
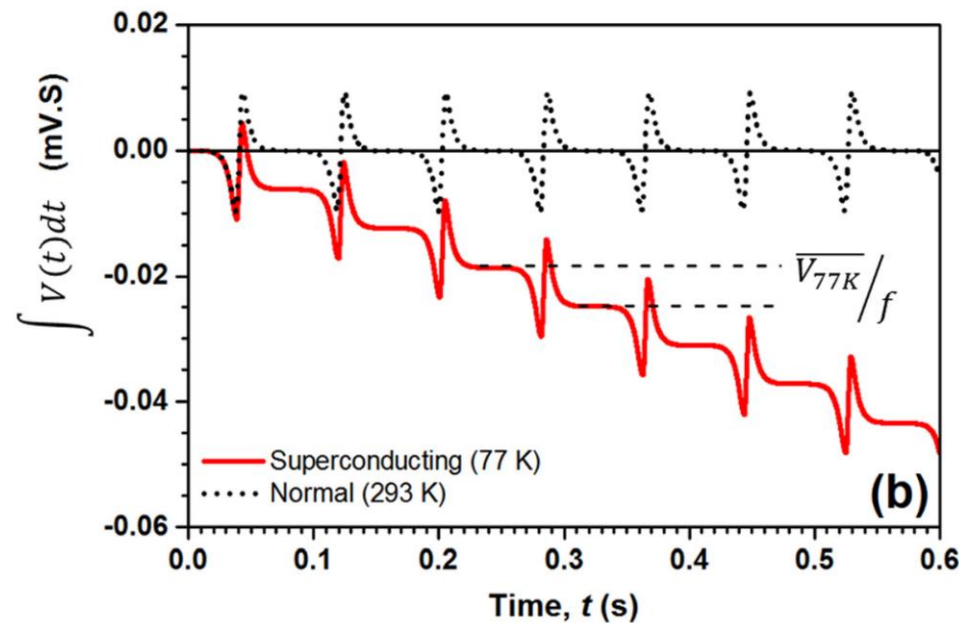
Mataira et al. *APL* 114 (2019) 112601



# HTS Dynamo

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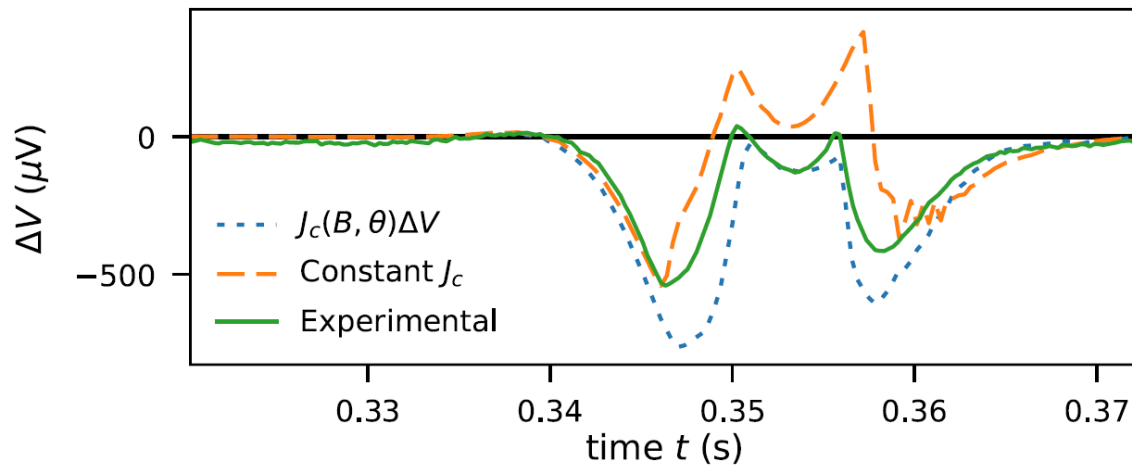
Mataira et al. *APL* 114 (2019) 112601



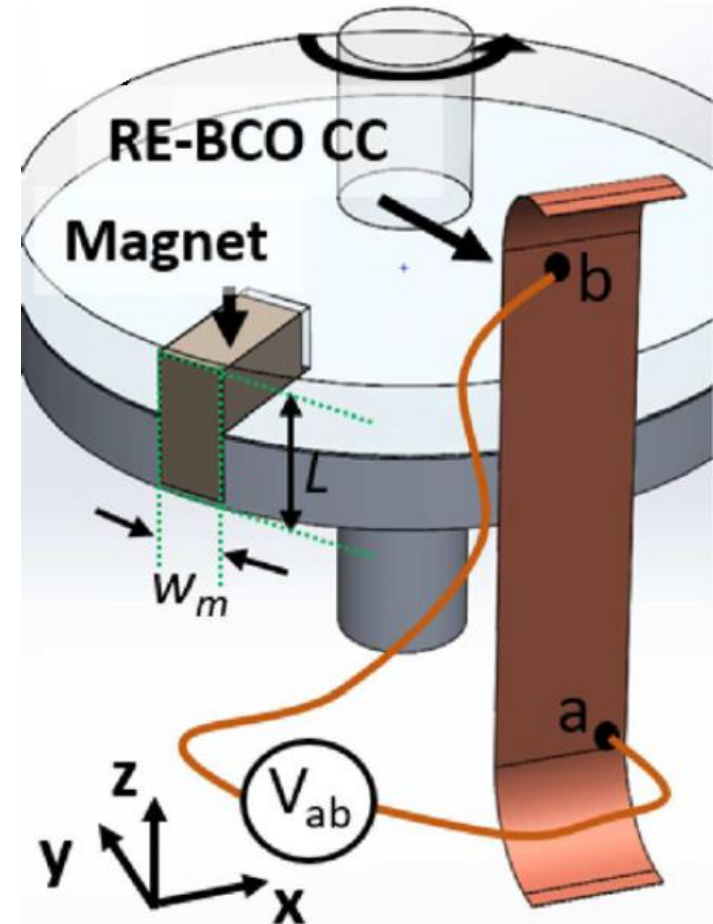
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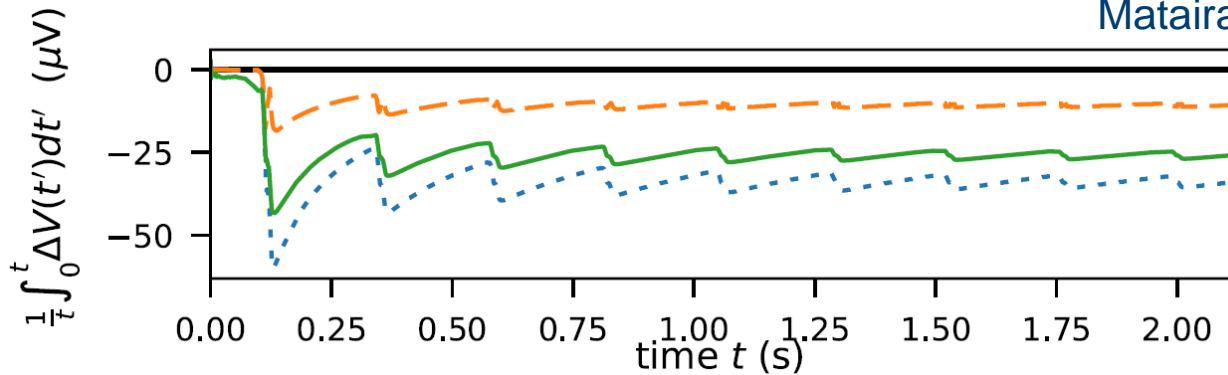
Mataira et al. *APL* 114 (2019) 112601



**Open-circuit voltage:  
models and experiments**

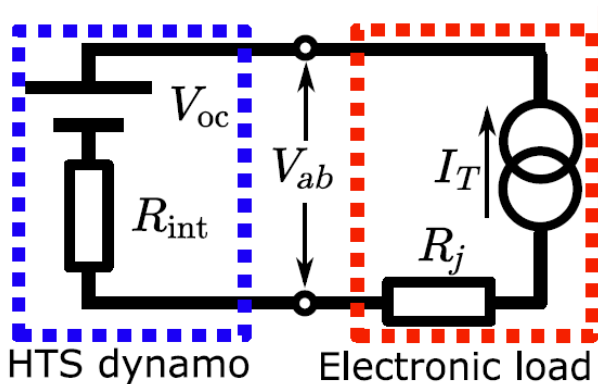


# HTS Dynamo – Device Characterisation

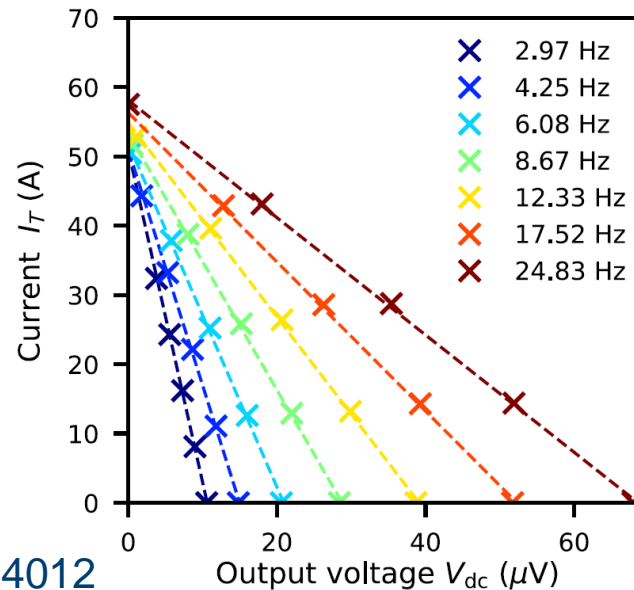


Mataira et al. *APL* 114 (2019) 112601

**Cumulative time-averaged voltage**



**Circuit diagram incl. current supply for  $I$ - $V$  characterisation**

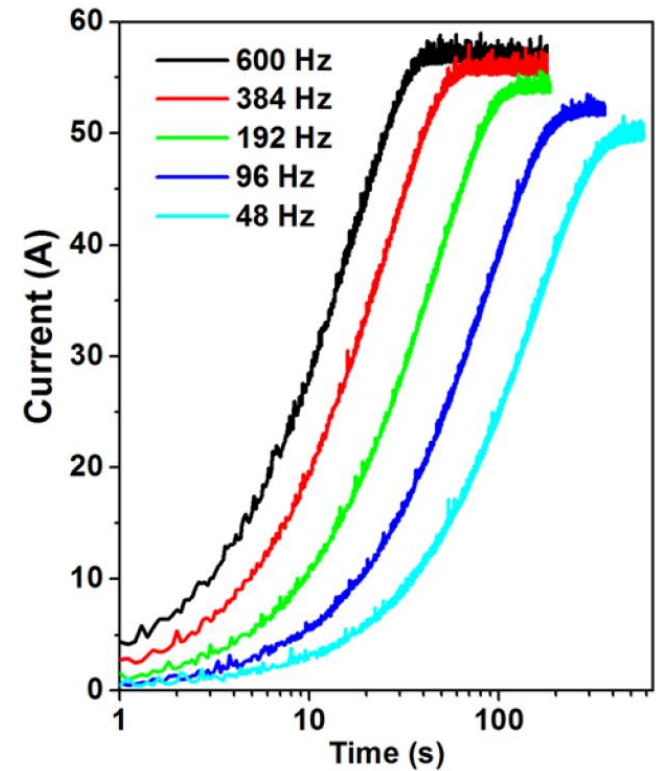
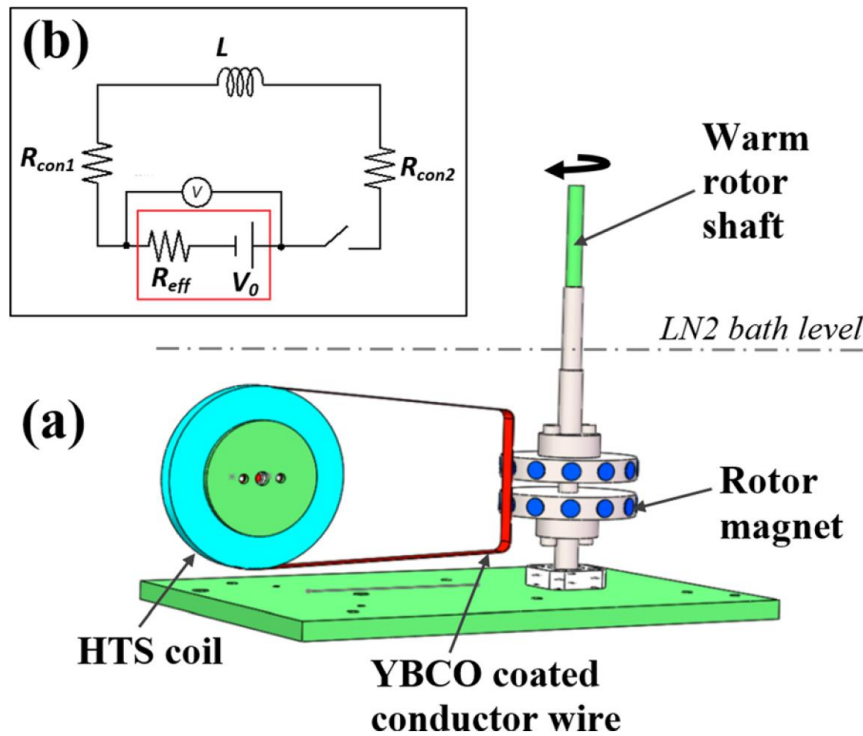


**Typical  $I$ - $V$  characterisation of HTS dynamo**

Mataira et al. *Phys Rev Appl* 14 (2020) 024012

# HTS Dynamo – Current Injection Without Leads

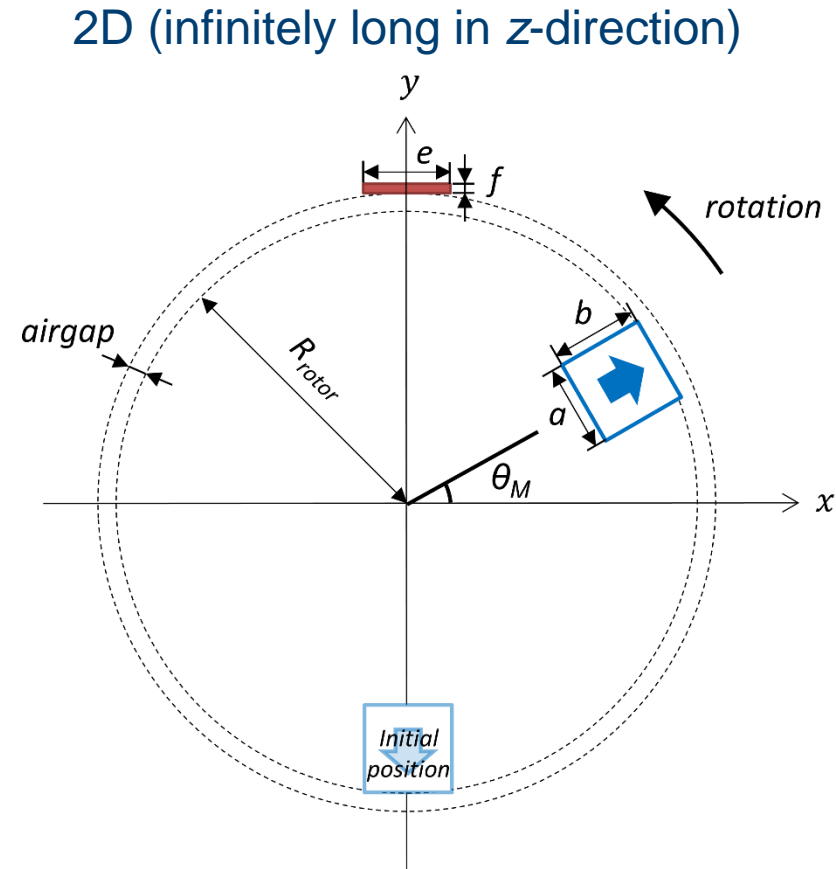
Can be used to drive large DC current into a superconducting coil, without current leads



Jiang et al. *APL* 105 (2014) 112601

# Benchmark Problem Definition

- Several numerical models have been developed recently to model the HTS dynamo
- Benchmark problem:
  - A specific simplified geometry with well-defined inputs (i.e. assumptions)
  - An expected set of outputs (i.e. the solution)
  - Allows any modelling technique to be validated & its performance critically compared



**Geometry of the HTS dynamo  
2D benchmark problem**



# Benchmark Problem Definition

Based on experimental setup in  
Badcock et al. *IEEE TAS 27* (2017) 5200905

HTS dynamo benchmark parameters

Permanent magnet (PM)	Width, $a$	6 mm
	Height, $b$	12 mm
	Active length (depth), $L$	12.7 mm
	Remanent flux density, $B_r$	1.25 T
	Width, $e$	12 mm
HTS stator wire	Thickness, $f$	1 $\mu\text{m}$
	Critical current, $I_c$ [self-field, 77 K]	283 A
	$n$ value	20
	Rotor external radius, $R_{rotor}$	35 mm
Distance between PM face and HTS surface, <i>airgap</i>	3.7 mm	
Frequency of rotation	4.25 Hz	
Number of cycles	10	

## General definitions

$E$ - $J$  power law:  $\mathbf{E} = \frac{E_0}{J_c} \left| \frac{\mathbf{J}}{J_c} \right|^{n-1} \mathbf{J}$

DC component:

$$V_{DC} = -\frac{L}{T} \int_t^{t+T} E_{ave}(t') dt'$$

Equivalent instantaneous voltage:

$$V_{eq}(t) = -L E_{ave}(t)$$

Cumulative time-averaged voltage:

$$V_{cumul}(t) = \frac{1}{t} \int_0^t V_{eq}(t) dt$$

$$I(t) = \iint_S J_z(x, y, t) dS = 0$$

# Numerical Modelling Frameworks

- **H-formulation methods**

- Coupled H-A
- H-formulation + shell current
- Segregated H-formulation

- **Coupled T-A formulation**

- 1D (line) or 2D (finite thickness) object
- ‘Pure’ T-A (vector potential only)
- Mixed scalar-vector potential

- **MEMEP**

- Variational method
- Solves  $\mathbf{J}$  by minimising a functional containing all variables of problem:  $\mathbf{A}$ ,  $\mathbf{J}$ ,  $\varphi$

- **Integral equation (IE)**

- Current distribution along 1D superconducting layer obtained from IE

- **Volume integral equation (VIE)**

- Solves  $\mathbf{E} = -\frac{\partial \mathbf{A}^{int}}{\partial t} - \mathbf{v} \times \mathbf{B}^{PM} - \nabla \varphi$

# Comparison of Key Results – $V_{eq}(t)$

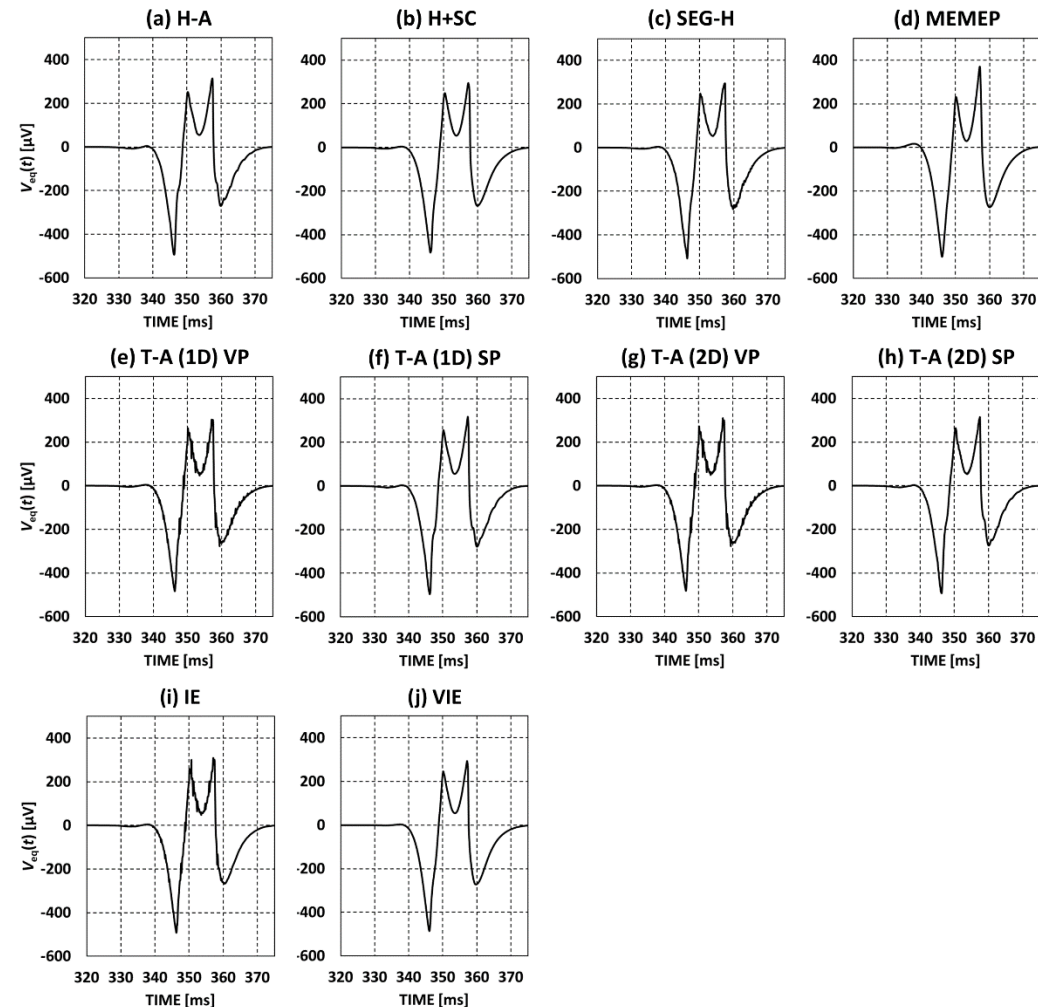
**Open-circuit  
equivalent instantaneous  
voltage**

$$V_{eq}(t) = -LE_{ave}(t)$$

2<sup>nd</sup> transit of PM past HTS wire,  
ignoring any initial transient effects  
in 1<sup>st</sup> cycle

Qualitatively, four distinct peaks  
with left-to-right asymmetry,  
as observed in experiments

Excellent quantitative agreement  
(see cumulative voltage next)



# Comparison of Key Results – $V_{cumul}(t)$

**Cumulative  
time-averaged  
equivalent voltage**

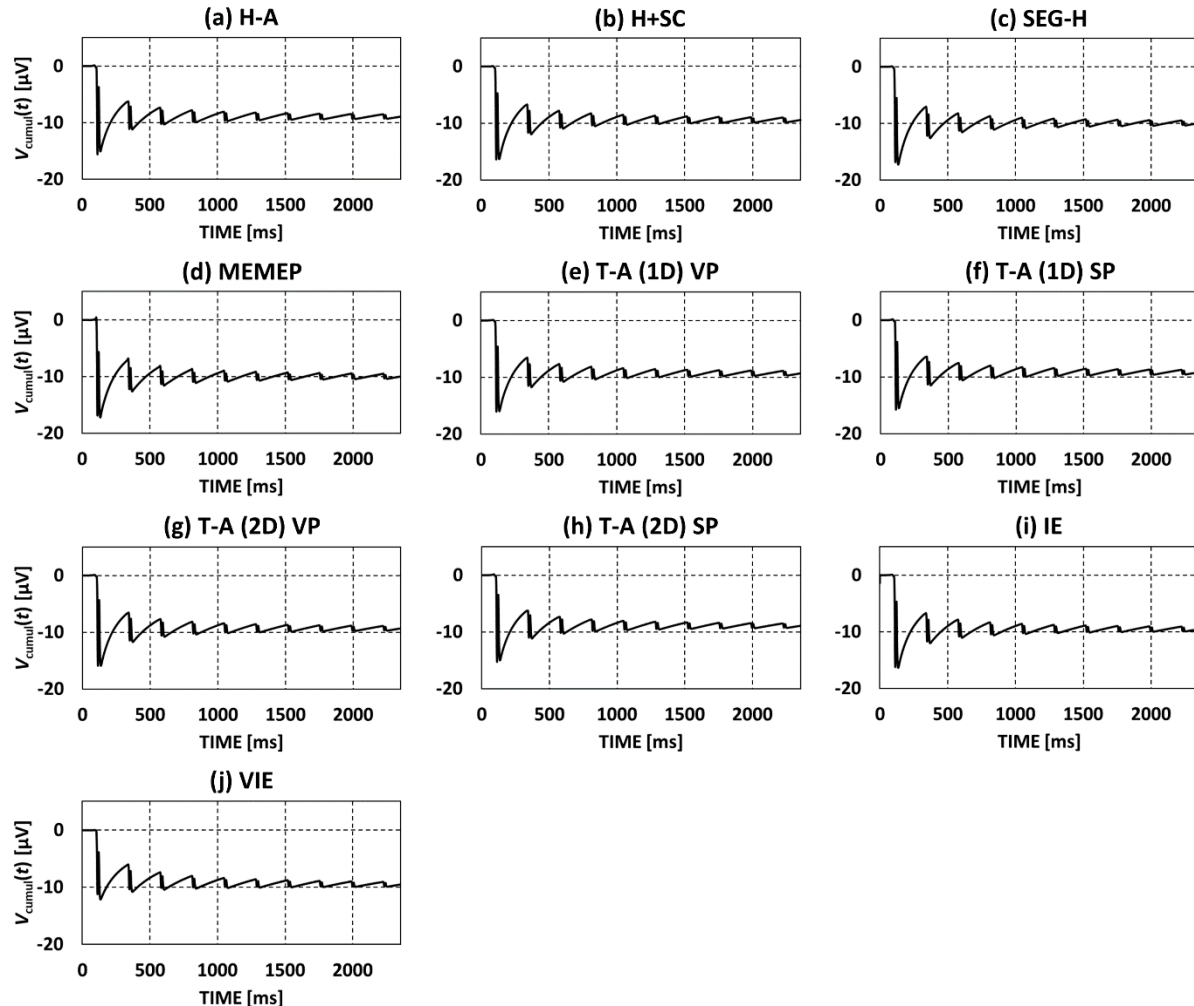
$$V_{cumul}(t) = \frac{1}{t} \int_0^t V_{eq}(t) dt$$

$V_{cumul}$  over 10 cycles

Converges to non-zero asymptotic value  $\rightarrow V_{DC}$

Excellent quantitative agreement:

–9.41  $\mu\text{V}$  average with 0.34  $\mu\text{V}$  standard deviation



# Comparison of Key Results – $J/J_{c0}$ , $E$

Normalised critical current density\*

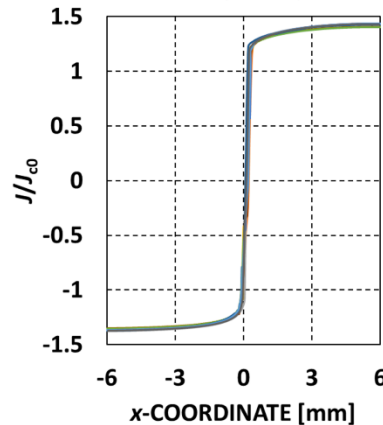
Electric field\*

\*averaged along thickness

Magnet approaching

$$\theta_M \approx 81^\circ$$

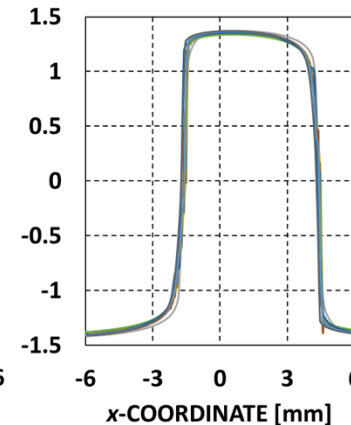
$t = 347$  ms



Magnet directly below

$$\theta_M = 90^\circ$$

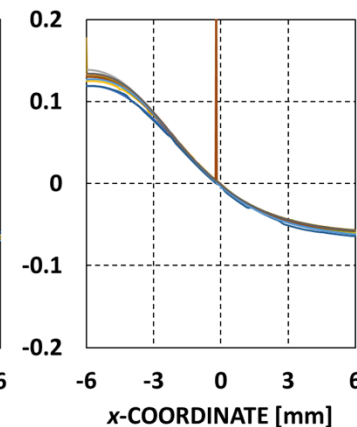
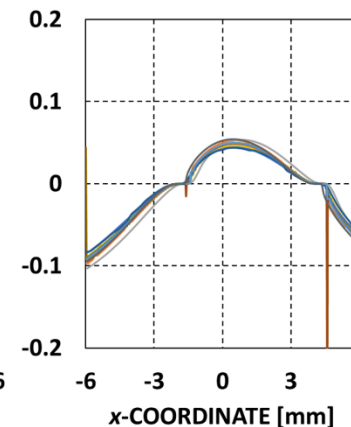
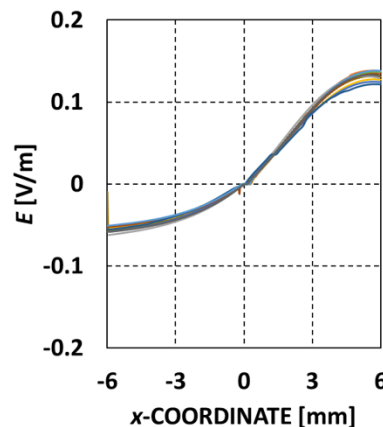
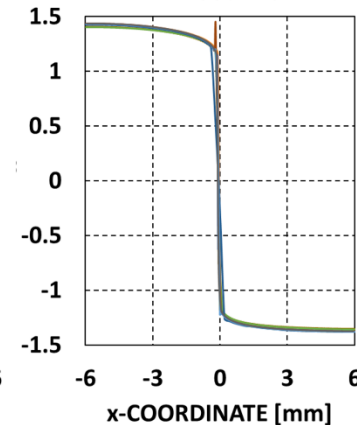
$t = 353$  ms



Magnet leaving

$$\theta_M \approx 99^\circ$$

$t = 359$  ms



- H-A
- H+SC
- SEG-H
- MEMEP
- T-A (1D) VP
- T-A (1D) SP
- T-A (2D) VP
- T-A (2D) SP
- IE
- VIE



# Comparison of Modelling Frameworks

## Methods emphasising reduction of mesh elements / DOFs performed best

Key metrics assessed for each benchmark model

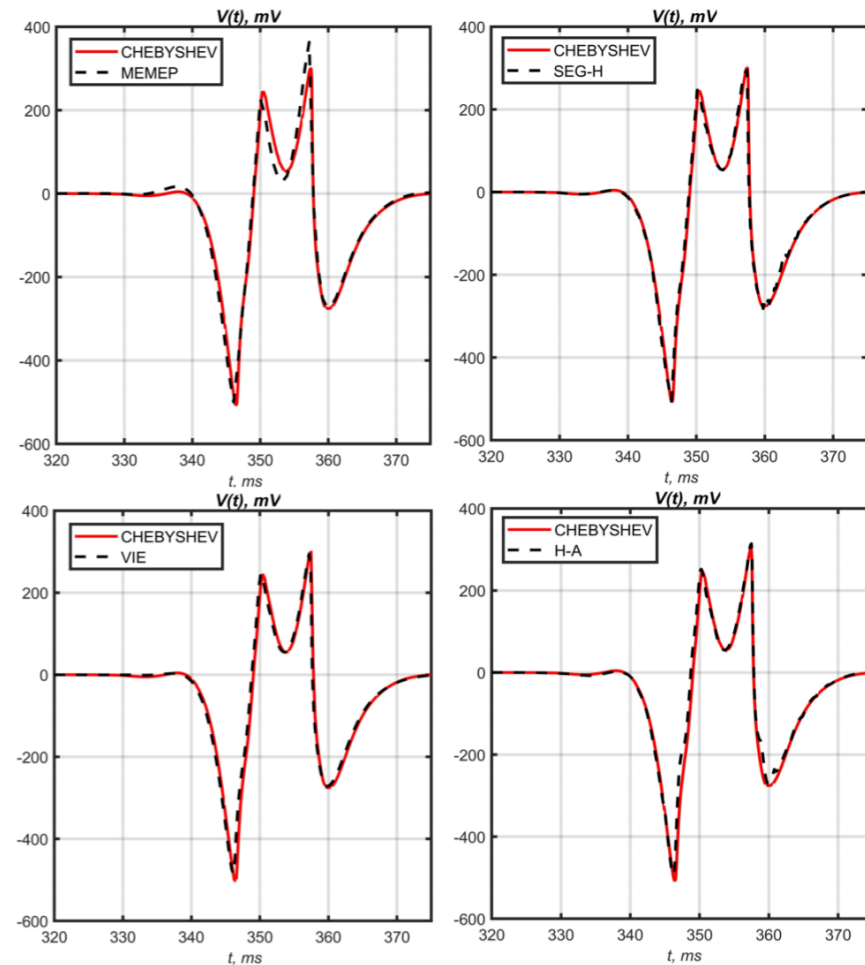
Model	Mesh (SC)	Mesh (total)	DOFs	Rel./abs. tolerance	Approx. time/cycle (min/cycle)	Software implementation
MEMEP	120 (120 × 1)	120	120	1e-4 <sup>1</sup>	<0.25 <sup>a</sup>	C++
SEG-H	120 (120 × 1)	2653	4071	1e-4/0.1	1.1 <sup>b</sup> 2.6 <sup>b</sup>	COMSOL 5.4 COMSOL 5.5
VIE	120 (120 × 1)	120	120	1e-3 <sup>2</sup> /1e-6 <sup>2</sup>	1.6 <sup>b</sup>	MATLAB
H-A	120 (120 × 1)	4176	3661	1e-4/0.1	2.1 <sup>b</sup>	COMSOL 5.5
T-A (2D) SP	240 (60 × 4)	3800	2863	1e-5/1e-4	3.9 <sup>b</sup>	COMSOL 5.5
IE	120 (120 × 1)	5932	12 451	5e-3/0.1	5.1 <sup>b</sup>	COMSOL 5.5
T-A (1D) SP	120 (120 × 1)	4876	2779	1e-5/1e-4	6.5 <sup>b</sup> 7.9 <sup>b</sup>	COMSOL 5.5 COMSOL 5.4
H+SC	120 (120 × 1)	11 272	16 988	1e-5/1e-3	> 120	COMSOL 5.5
T-A (1D) VP	120 (120 × 1)	6064	12 715	1e-4/0.1	21.6 <sup>b</sup>	COMSOL 5.5
T-A (2D) VP	240 (60 × 4)	5286	13 696	1e-4/0.1	64.6 <sup>b</sup>	COMSOL 5.5

PC specifications:<sup>a</sup>Intel® Core™ i7-8700 CPU @ 3.20 GHz, 31.1 GB RAM (10% memory used for MEMEP model), Ubuntu 16.04 LTS, 64-bit<sup>b</sup>Intel® Core™ i9-7900X CPU @ 3.30 GHz, 63.7 GB RAM, Microsoft Windows 10 Pro, 64-bit  
 Other notes:<sup>1</sup>Tolerance for the mutual interaction matrix<sup>2</sup>Default settings for MATLAB/ode23b solver

**H-A, T-A = rotating machine-like frameworks; stability issues with 'pure' T-A → use of mixed scalar-vector potential performed significantly better**

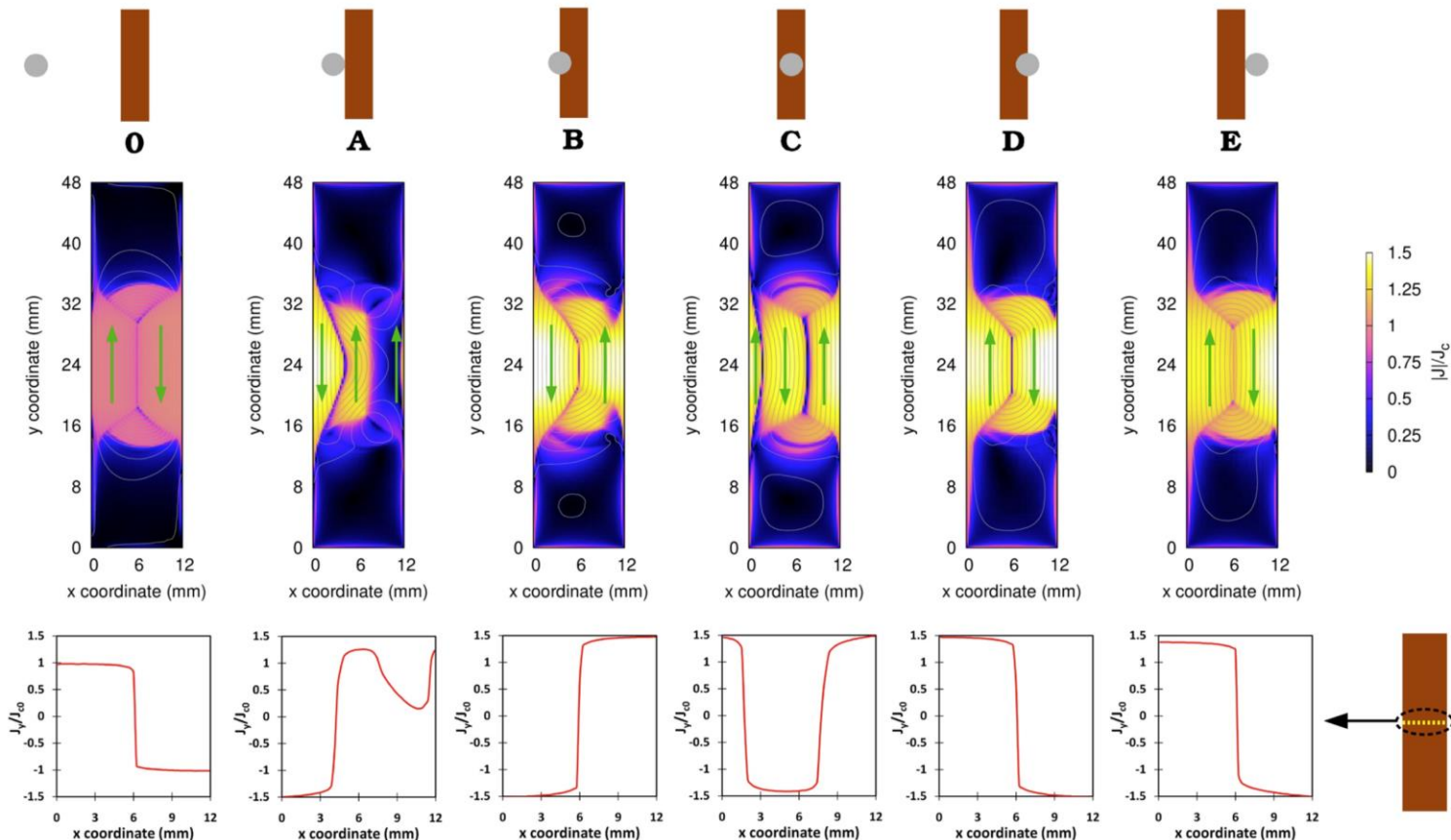
# Prigozhin Method

- **Benchmark problem also implemented by Prigozhin & Sokolovsky in 2021**
  - Uses expansions in Chebyshev polynomials for approximation in space, method of lines for integration in time
  - Offers a potentially even faster method to model the HTS dynamo
  - See *Supercond. Sci. Technol.* 34 (2021) 065006 for more details
  - **Also talk today PM (OS-M1-PM)**



# Efficient 3D Models

3D models can consider the return of the current within the wire & more complex geometries, e.g., magnet shape



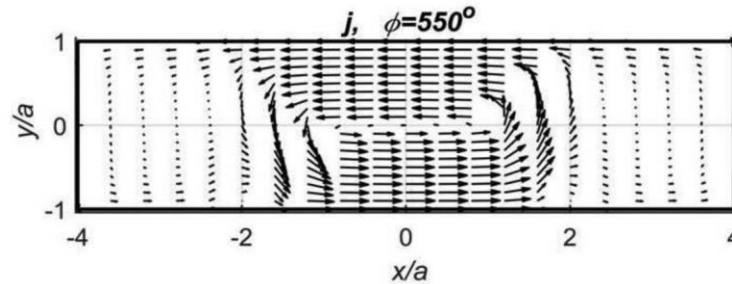
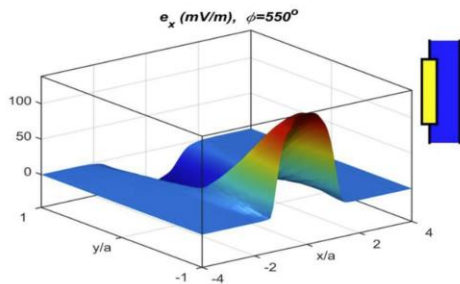
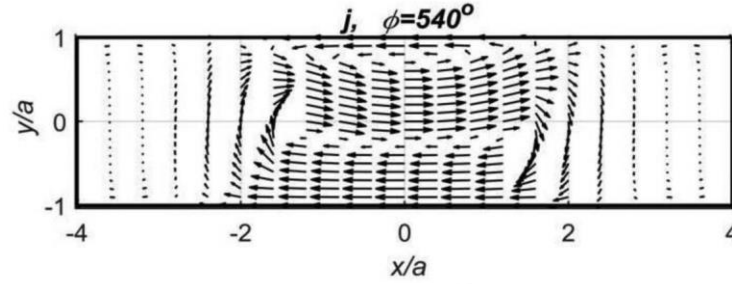
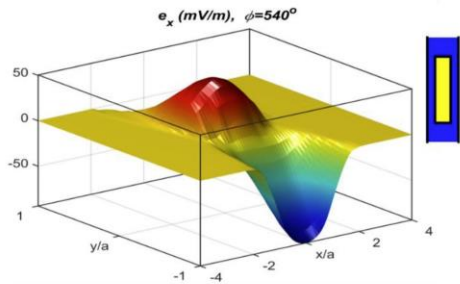
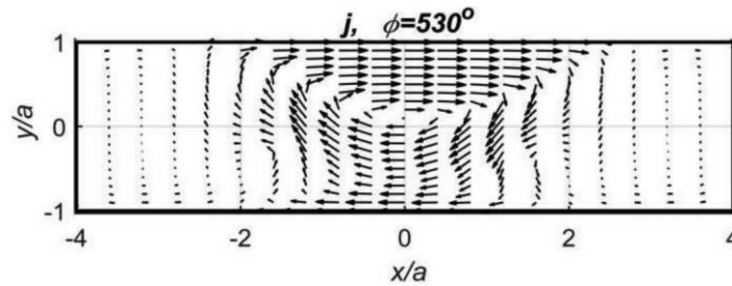
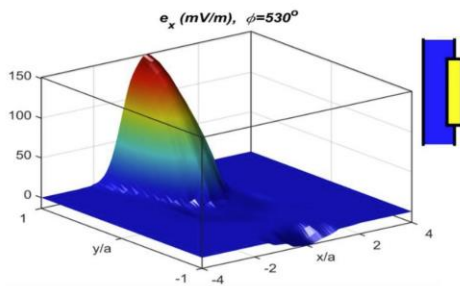
**MEMEP 3D**

Ghabeli et al.  
*Sci. Rep.* 11  
(2021) 10296



# Efficient 3D Models

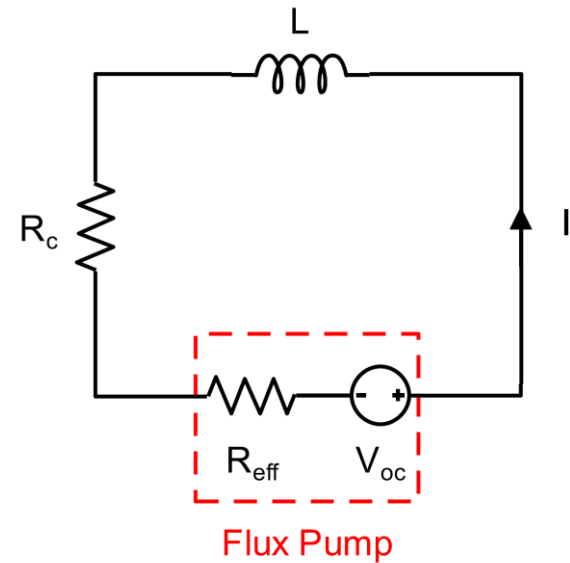
3D models can consider the return of the current within the wire & more complex geometries, e.g., magnet shape



Mixed FEM, fast  
Fourier transform  
methods

Prigozhin & Sokolovsky  
*IEEE TAS* 31 (2021)  
5201107

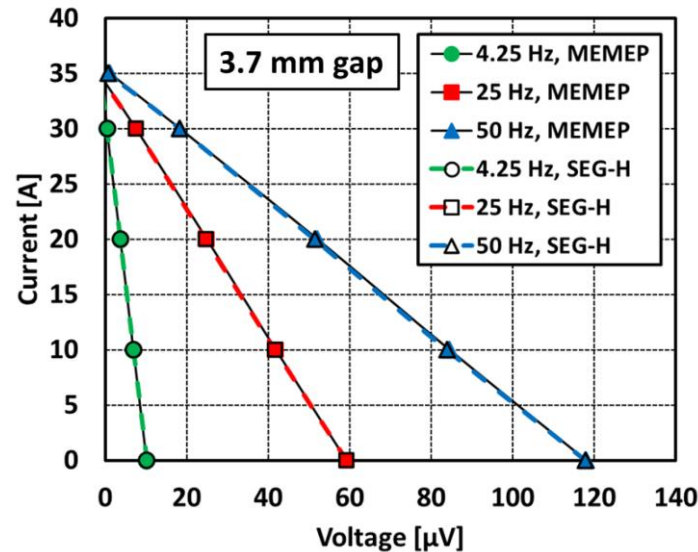
# Modelling Dynamic Coil Charging Behaviour



$$i(t) = I_{\text{sat}} (1 - e^{-t/\tau})$$

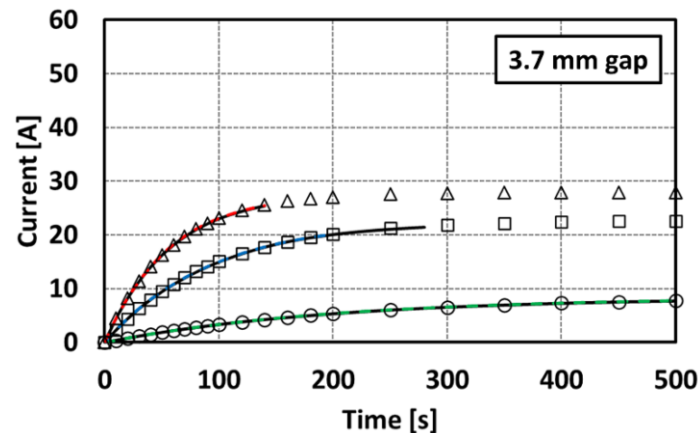
$$I_{\text{sat}} = V_{\text{oc}} / (R_c + R_{\text{eff}})$$

$$\tau = L / (R_c + R_{\text{eff}})$$



## I-V characterisation of flux pump

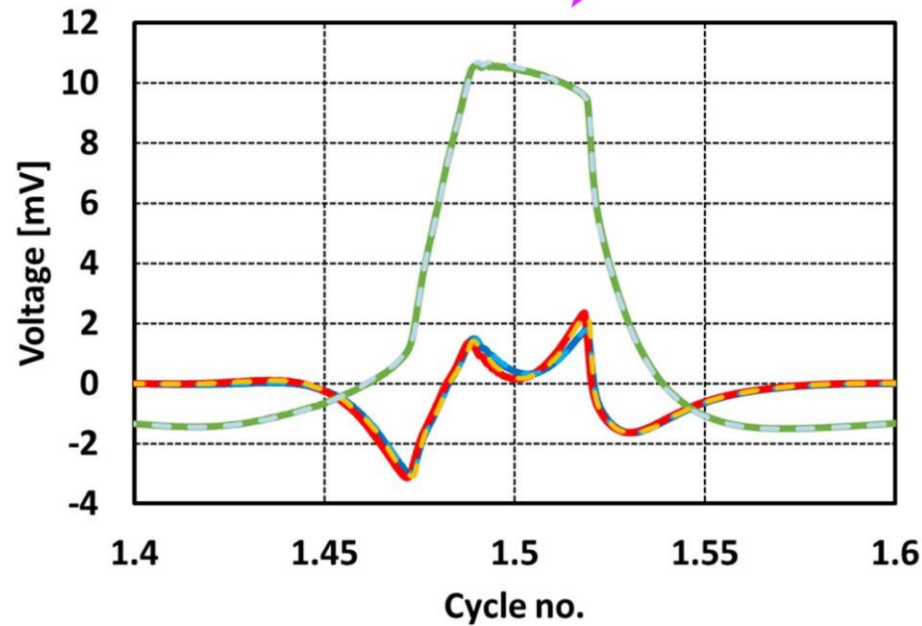
Benchmark problem  
MEMEP, SEG-H  
Airgap = 3.7 mm  
Freq. = 4.25, 25, 50 Hz



## Calculated charging curves

Analytical equation vs  
MEMEP, SEG-H

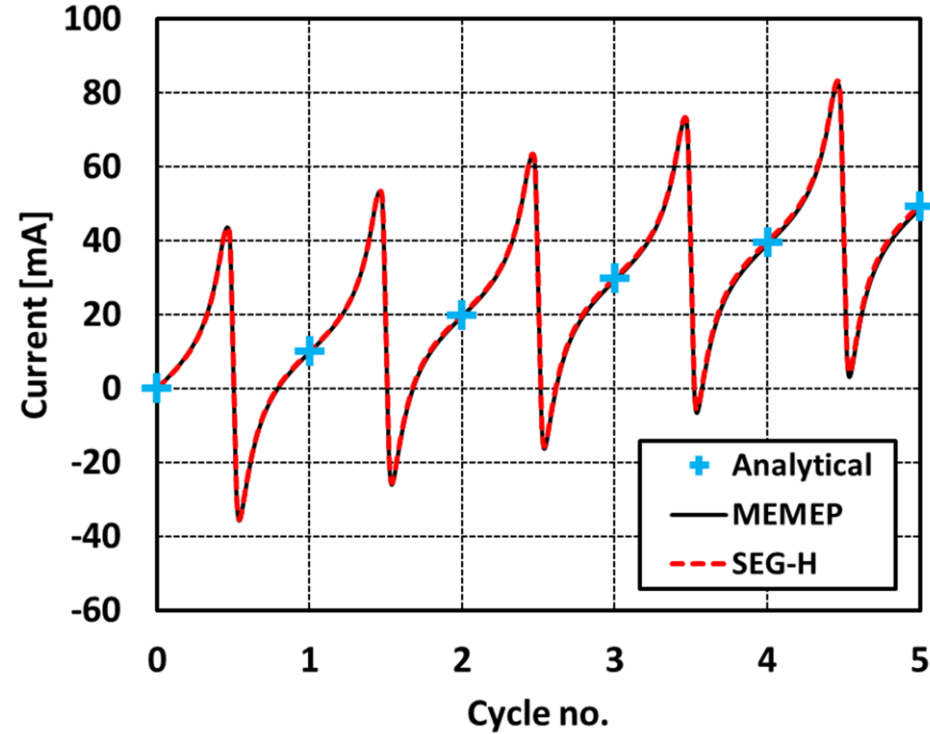
# Modelling Dynamic Coil Charging Behaviour



—  $E_{av}$  MEMEP      —  $E_{av}+A_J$  MEMEP      —  $E_{av}+A_J+A_M$  MEMEP  
- - -  $E_{av}$  SEG-H      - - -  $E_{av}+A_J$  SEG-H      - - -  $E_{av}+A_J+A_M$  SEG-H

**Total flux pump output voltage  $V(t)$   
& its constituent components:**

$$V_{eq}(t) \text{ from } E_{av} + A_{J,av} + A_{M,av}$$

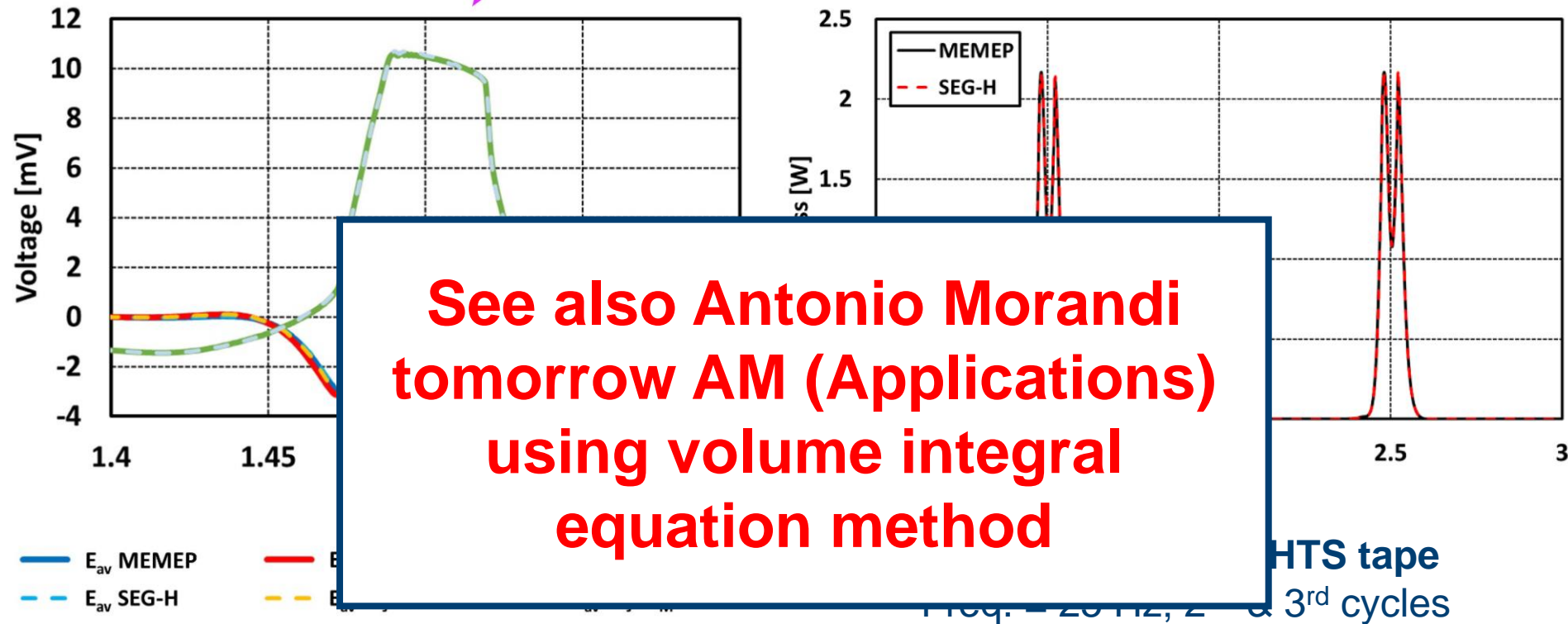


**Dynamic charging current curve**

Freq. = 25 Hz

Analytical equation vs MEMEP, SEG-H

# Modelling Dynamic Coil Charging Behaviour

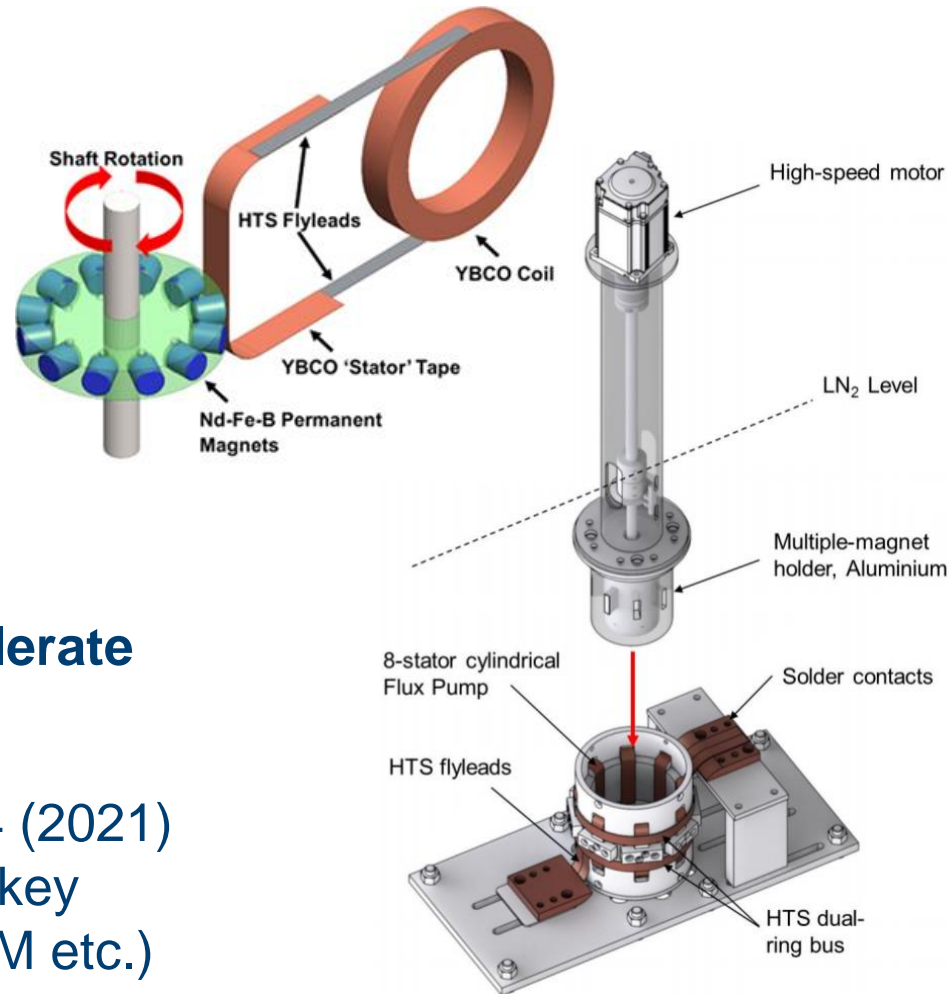


**Total flux pump output voltage  $V(t)$  & its constituent components:**  
 $V_{eq}(t)$  from  $E_{av} + A_{J,av} + A_{M,av}$

**1<sup>st</sup> five cycles: 135.4 mW, 135.7 mW**  
**5001<sup>st</sup> cycle: 135.7 mW, 135.9 mW**

# Future View

- More complicated models  
→ kA-class dynamo design
- Coupling of FEM models
  - e.g. dynamo  $\leftrightarrow$  coil
  - See Pengbo Zhou talk – tomorrow AM (Applications)
- Use of machine learning to accelerate dynamo design optimisation
  - Proposed in Wen et al. *SUST* 34 (2021) 125019 to obtain  $V_{oc}$  for various key parameters (freq, airgap,  $B_r$  of PM etc.)



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