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POLYTECHNIQUE MONTRÉAL
TECHNOLOGICAL UNIVERSITY

A physics-guided recurrent data-driven model for long-time prediction of nonlinear partial differential equations

J.-H. Fournier-Lupien, **F. Sirois**, C. Lacroix
(Collaborators from NRC: J. Valdes and A. Chagang)

Polytechnique Montréal, Montréal, Canada
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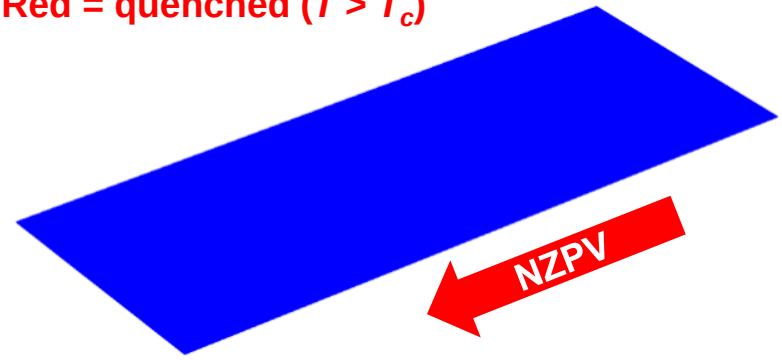
National Research Council Canada
Conseil national de recherches Canada

Numerical simulation of quench in HTS tapes

Blue = superconducting
Red = quenched ($T > T_c$)

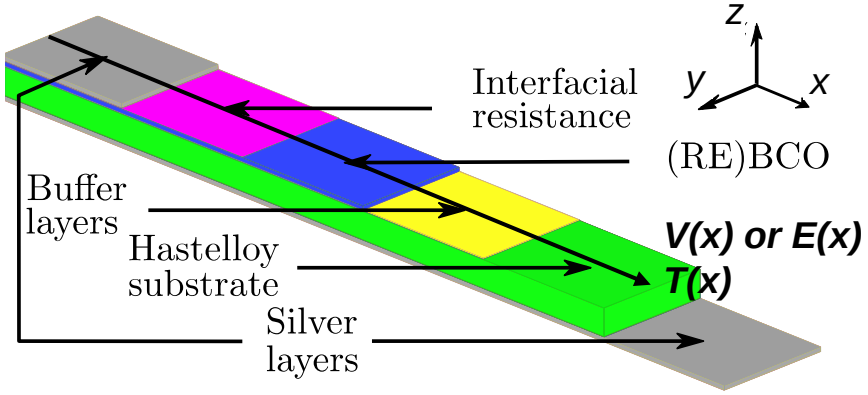


NZPV = Normal zone propagation velocity

- **Electro-thermal simulation**
- **Nonlinear E-J characteristic**
- **Full 3D mesh**

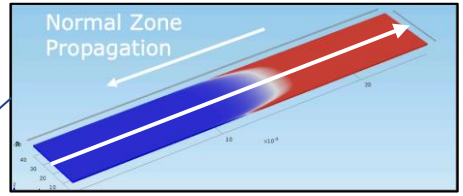
Problem with finite element simulations:

- **Computationally expensive**
- **Impossible to simulate tape > few cm**
- **Need to variations of I_c along tape length, i.e. $I_c = f(x)$**



Physics is 3-D, but output is typically 1-D

Surrogate model to substitute an expensive FEM solver



Hot spot propagation simulations (FEM solver)

$$y = f(x)$$

Inputs:

x_1
x_2
x_3
$\vdots$
x_n

Expensive

Training

Cheap

Surrogate model: Data-driven model

$$y \approx \hat{f}(x)$$

Expensive

Cheap

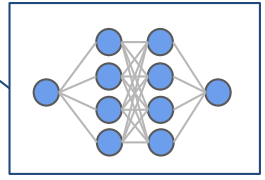
Outputs:

y_1
y_2
y_3
$\vdots$
y_n

EMTP-RV
The reference for power systems transients

Power system transient simulator

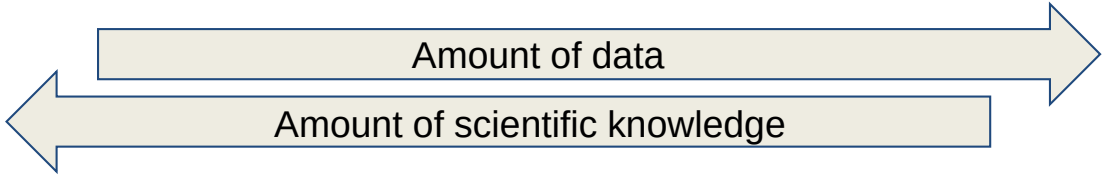
Example of use of surrogate model



Machine learning (ML) model

E(x), T(x)

Physics modeling versus Data-Driven modeling



No data regime

Numerical techniques such as Finite Element Method

Small data regime

Physics + data

Regime for Physics-Guided data-driven models

Big data regime

Machine learning techniques Deep learning

Physics and things to take into account with ML model

Problem to solve:

Very nonlinear

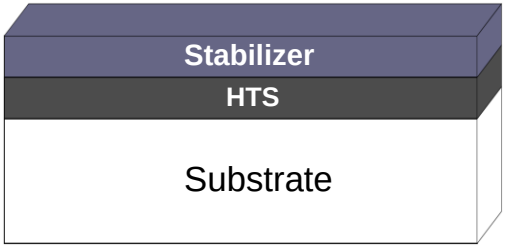
Current continuity equation: $\nabla \cdot (-\sigma(T)\nabla V) = 0$ ⇒ Solved for **electric field *E* = grad(*V*)**

Heat equation: $\rho_m C_p(T)\frac{\partial T}{\partial t} + \nabla \cdot (-k(T)\nabla T) = Q$ ⇒ Solved for **temperature *T***

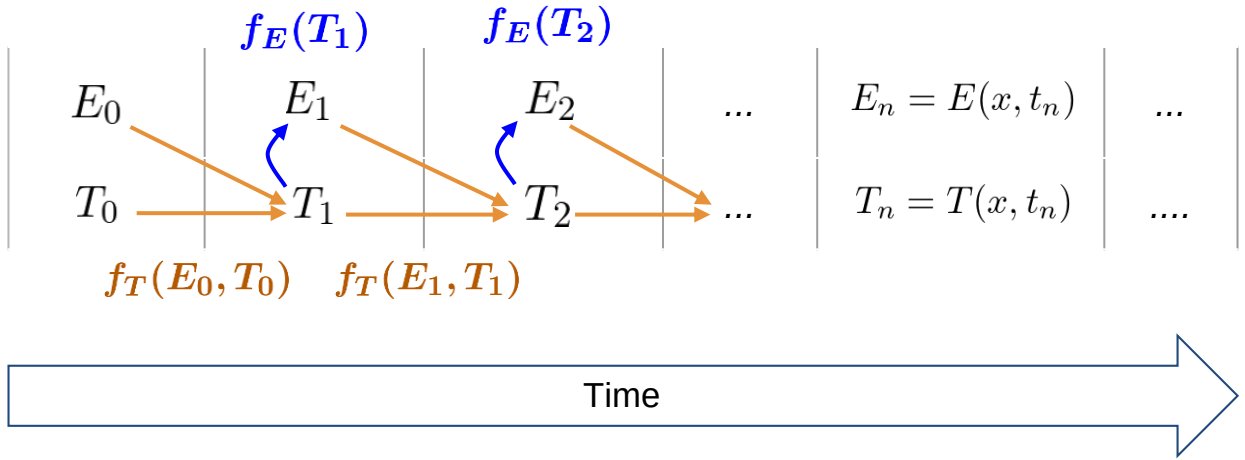
Multi-material | Time-dependent | Multi-material | External time-dependent variable

Challenges:

1) Nonlinear electric and heat equations
2) Multi-materials geometry
3) Time dependence must be solved one step at a time **in a recursive manner**



Decoupling of the electric and the thermal problems

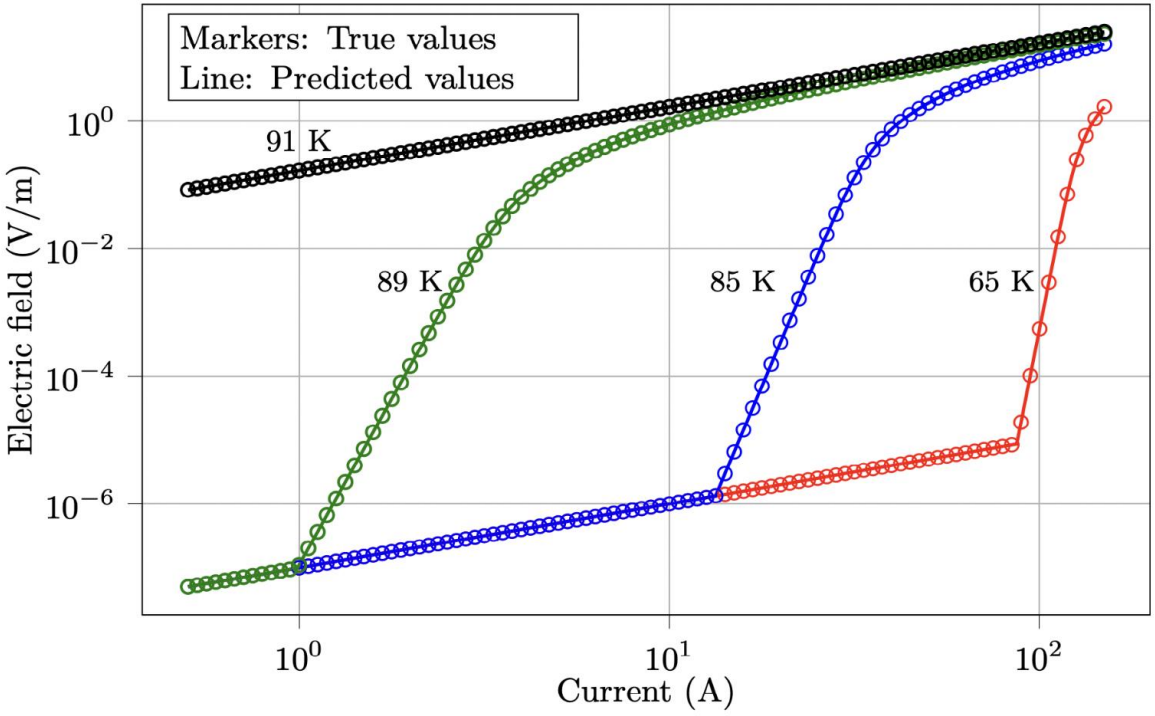


f_E : **Electric problem**

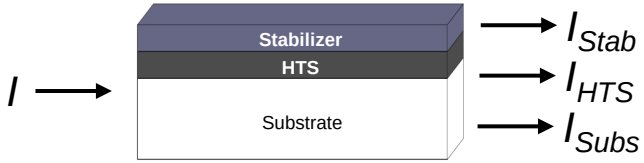
f_T : **Thermal problem**

The problem can be reduced to finding a good model for f_T

Training of the electric problem



- Fitted on 3D simulations of quench propagation
- NN architecture: [4,10,1]
- 40 simulations
- 360,000 datapoints



$\Delta x \rightarrow E = \Delta V / \Delta x$

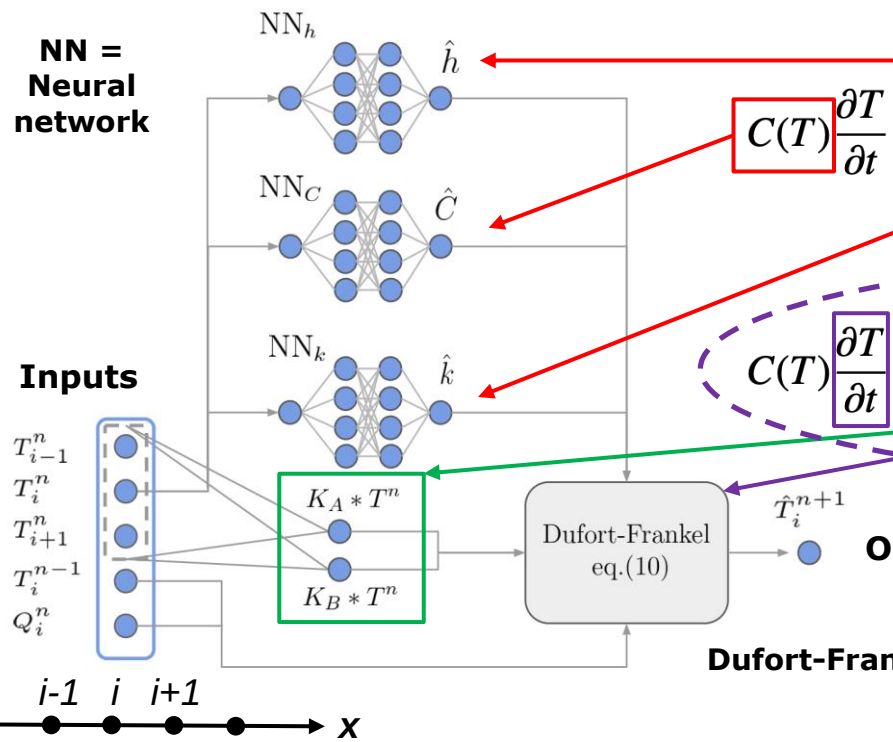
$$\hat{E} = f_E(I, I_c, T)$$

Neural network

Training of the thermal problem

Physics-Guided Recurrent Data-Driven model (PGRDD)

NN = Neural network



Nonlinear 1-D heat equation

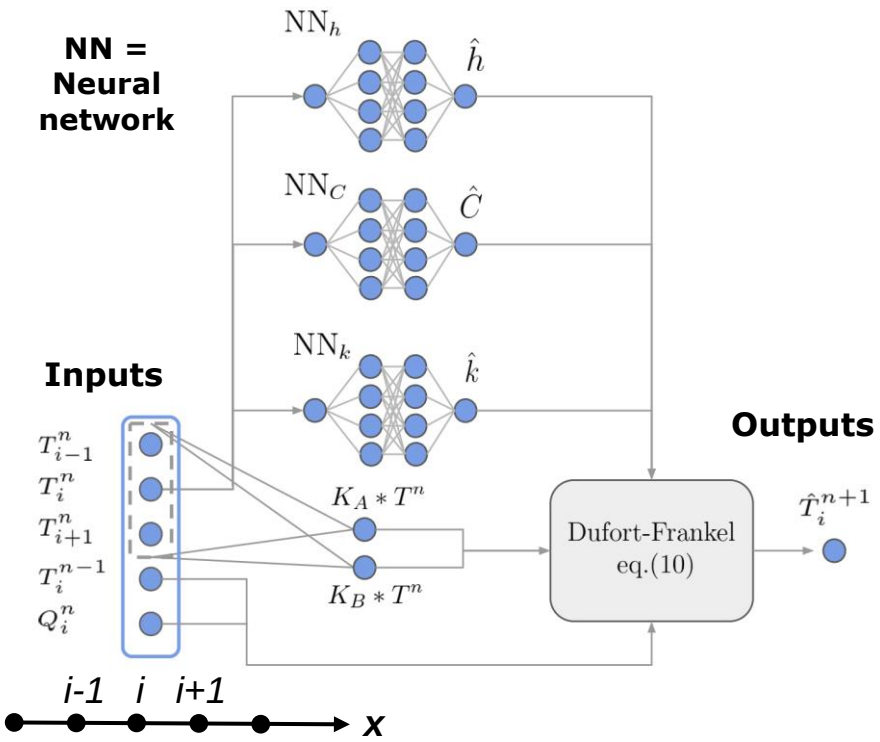
$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(x,t) - S/V \cdot h(T - T_0) \cdot (T - T_0)$$

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(x,t) - S/V \cdot h(T - T_0) \cdot (T - T_0)$$

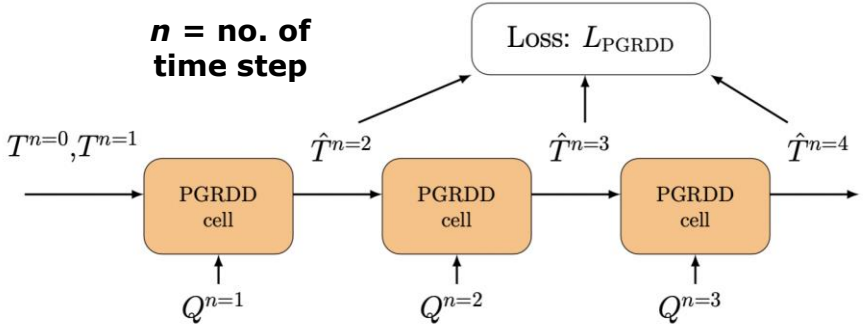
Dufort-Frankel = Two-step finite difference discretization scheme

Training for the thermal problem

Physics-Guided Recurrent Data-Driven model (PGRDD)



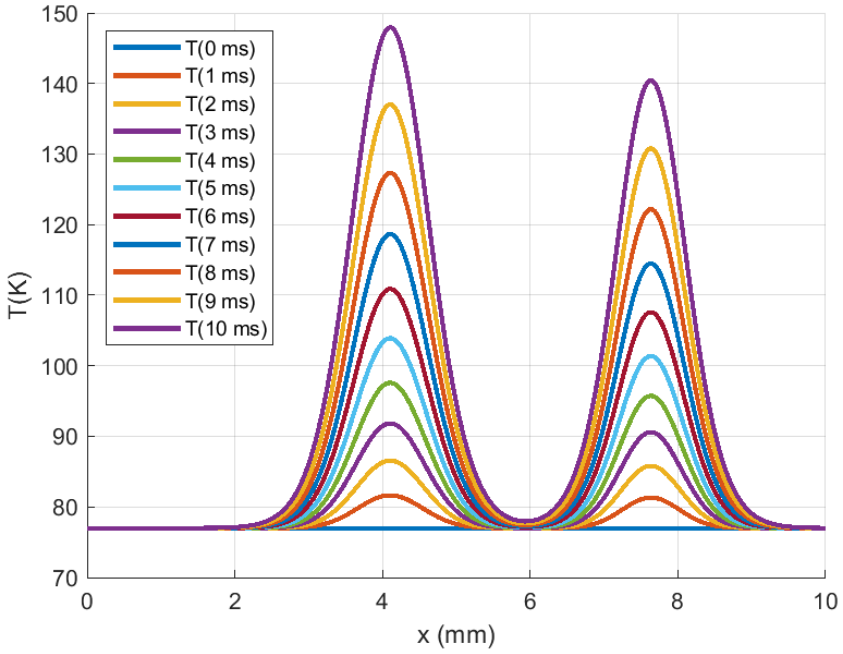
Training on recurrent predictions



- Material properties = Neural Networks (NN)
- Data-driven model with physical constraints (finite difference scheme built-in) trained on recursive predictions

→ **Physics guided recurrent data-driven model**

Data set used to train the thermal problem



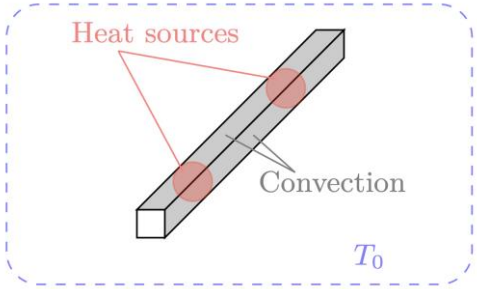
Example of a 1-D FEM solution

Nonlinear 1-D heat equation solved by FEM:

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(x,t) - S/V \cdot h(T - T_0) \cdot (T - T_0)$$

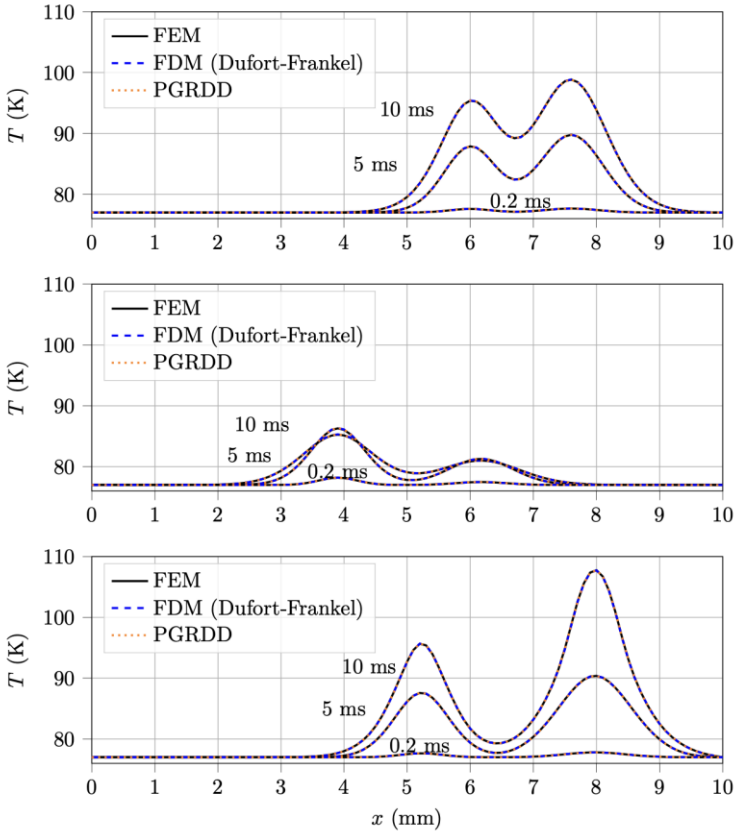
$$Q(x,t) = e^{t/\tau} \cdot \left[Q_1 e^{-\left(\frac{x - x_1}{2\sigma_1}\right)^2} + Q_2 e^{-\left(\frac{x - x_2}{2\sigma_2}\right)^2} \right]$$

*Randomly generated parameters



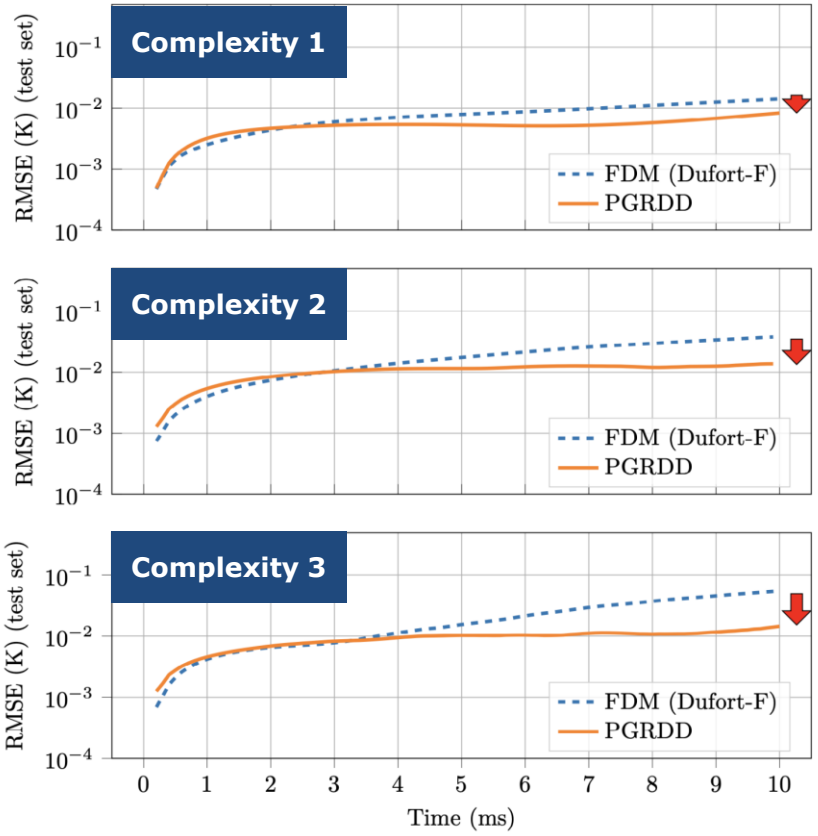
Data set:
- **100 Simulations**
- **1,000,000 datapoints**
- **5 predictors, 1 target**

Accurate predictions for different level of complexity



Equation	Heat source	
$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\mathrm{k(T)}\frac{\partial T}{\partial x}\right] + Q(x)$	$Q_1\, e^{-\left(\frac{x-x_1}{2\sigma_1}\right)^2} + Q_2\, e^{-\left(\frac{x-x_2}{2\sigma_2}\right)^2}$	Complexity 1
$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(x,t)$	$e^{t/\tau}\cdot\left(Q_1\, e^{-\left(\frac{x-x_1}{2\sigma_1}\right)^2} + Q_2\, e^{-\left(\frac{x-x_2}{2\sigma_2}\right)^2}\right)$	Complexity 2
$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + Q(x,t)$ $-S/V\cdot h(T-T_0)\cdot(T-T_0)$	$e^{t/\tau}\cdot\left(Q_1\, e^{-\left(\frac{x-x_1}{2\sigma_1}\right)^2} + Q_2\, e^{-\left(\frac{x-x_2}{2\sigma_2}\right)^2}\right)$	Complexity 3

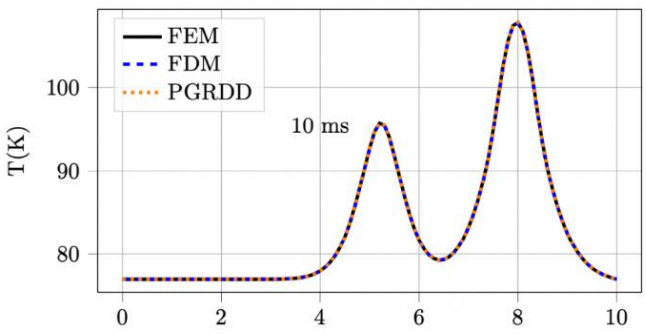
Minimization of the error accumulation

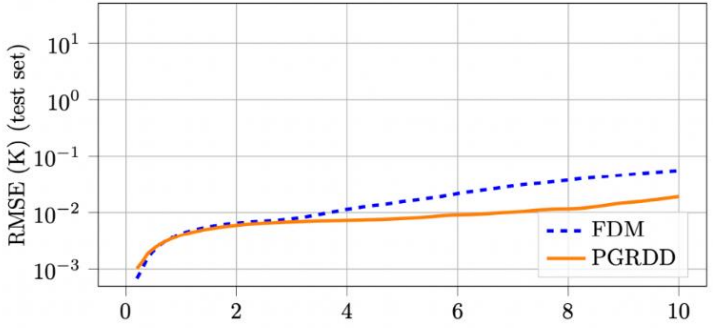


- Minimization of error accumulation
- Accurate long-time predictions
- General for any initial states and heat sources
- Prediction takes ~1.6 seconds (whereas a single 3-D simulations takes 25 minutes!)

***RMSE: error compared to FEM solution**

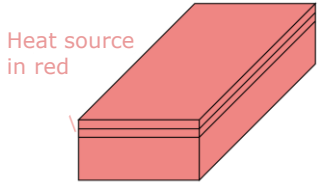
Generalization of the model to 3D effects

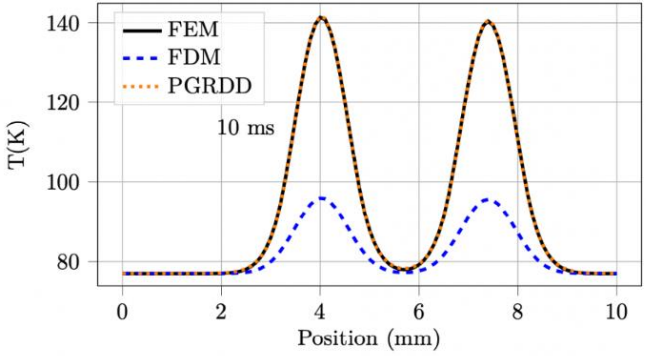


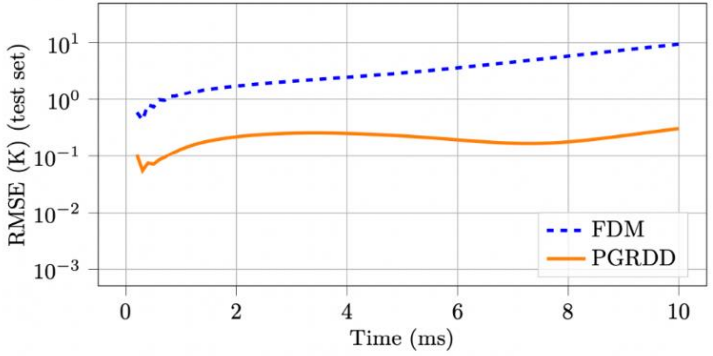


1-D simulations

(Complexity 3)

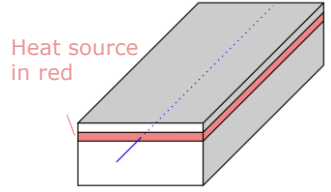






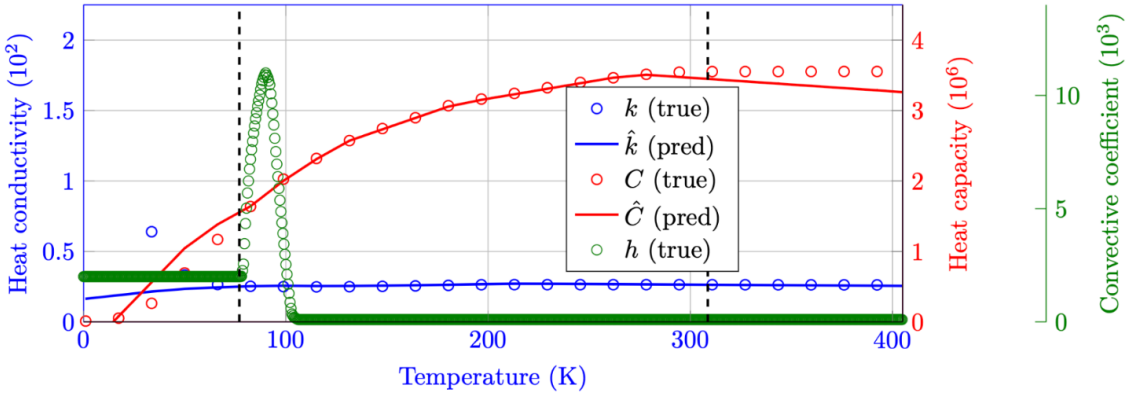
3-D simulations

(Complexity 3)

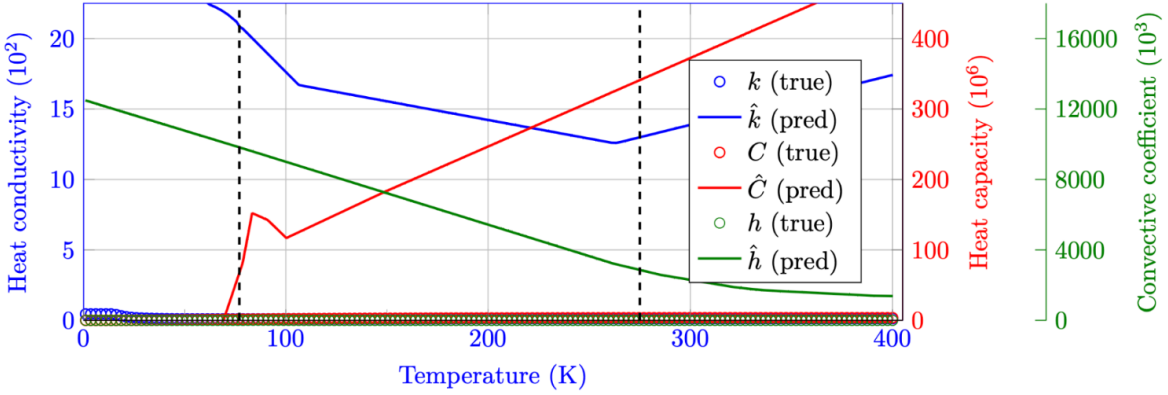


ML model is general enough to include 3D effects in 1-D model!

Material properties can be extracted from data

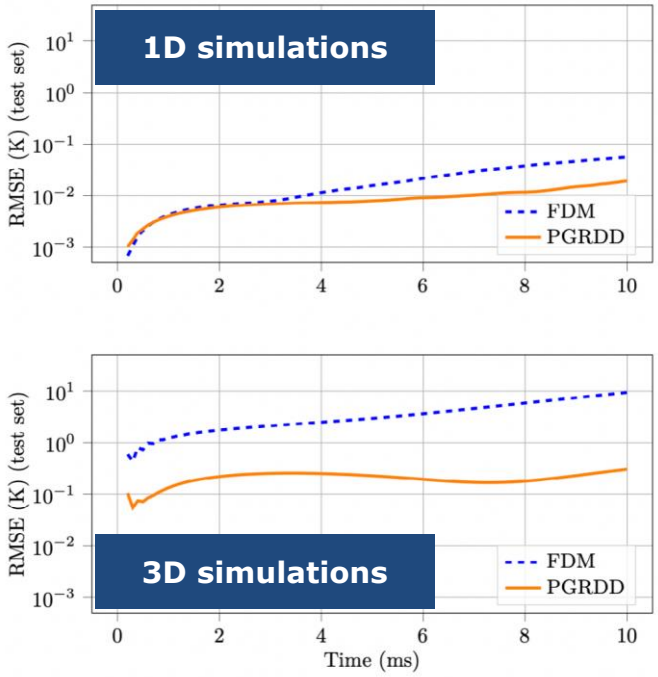


1-D simulations
(Complexity 3)



3-D simulations
(Complexity 3)

Conclusion



- Data-driven modeling is promising to develop surrogate models for 3-D quench propagation simulations in HTS tapes
- It minimizes error accumulation caused by the recurrence of the scheme
- Surrogate model: general for any initial state and heat source (no need for re-training)
- 3D simulations takes ~25 min
  Surrogate model takes ~**1.6 second**
  (~75 min training)