Structural monitoring of an aerial tramway system during operation: modeling and simulation strategy with experimental data validation

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ABSTRACT

The present study aims at building and calculating a global static model of a long-distance single span aerial tramway with non-symmetric tracks and anchored cables at extremities. From a design engineering point of view, aerial cable transport system is a complex structure involving several theoretical and practical issues due to diversity of phenomena at stake. Hence, appropriate framework is required to have relevant results according to the aim of the study. Influence of structural parameters modification occurring during system operation is analyzed by a static and quasistatic calculation. It provides information on parametric evolution through cable length unwound at drive bull-wheel as a previous step of dynamics study with a cable velocity. Hypotheses are discussed and modelling choices are confirmed by a comparison with experimental data measured on a real system.

Keywords: Aerial cable transport systems, Flexible structures, Nonlinear statics, Catenary curve, Test/Model.

1. INTRODUCTION

Aerial tramway is a well-established means of aerial cable transport system in mountainous regions where high vertical elevations must be ranged while having a growth potential of use in urban areas. Design studies dedicated to standard cable transport system engineering applications are based on statics assumptions with the cable at rest \cite{1,2,3}. On the other hand, their dynamics is widely studied in scientific literature in a simplified way without considering the entire system but only one cable portion, excluding cable translation movement \cite{4}, or several spans coupled by supports \cite{5}, with a cable translation velocity \cite{6}.

In this work, the statics cable model is based on catenary equations with elasticity correction considering cable elongation due to loading. The system under consideration is illustrated in Fig. 1. Vehicles are represented by punctual masses carried by track ropes and pulled by a haul rope. At the bottom station, haul rope loop is coupled between the two tracks by a drive bull-wheel connected to a driving system which controls cable position. A return bull-wheel closes the loop at the top station. Due to asymmetry of cable characteristics along the haul rope loop and carrier loading, some practical consequences appear as a shift of vehicles docking at stations.
An engineering challenge is the monitoring of the haul rope loop length during aerial tramway operation. The haul cable length changes due to permanent strain mainly during the first years after installation. Also, temperature and loading variations are the cause of cable length and sag modifications. To prevent cable damage or its contact with obstacles along the track, monitoring of cable length evolution provides important information for the operator to plan maintenance operation like cable shortening or replacement.

In this study, vehicles are clamped on haul rope loop with grips by a friction mechanism. Vehicles are often uncoupled from the cable to prevent any slip along the haul rope due to clamping force decrease or cable damage at the contact surface. Vehicles are moved to a new position along the haul loop. So upper and lower haul rope length separating the two carriers can change during these maintenance tasks.

In absence of dedicated sensors to measure haul cable length evolution with enough precision, a common practice of skilled person is to go through intermediate quantity measures. Thus, cable angles at stations without vehicles along the cable loop or shift length of vehicles docking at stations are directly measured on the system for specific known configuration. Then, these experimental data are put in a suitable model of the system which allows to access haul cable length value.

2. MODELING

2.1. Cable

A cable element model is chosen to represent each steel cable length located between two singular points of the system as station or vehicle. Cable model hypotheses are:

- force per unit length is supposed to be a uniform and planar distribution in \((O, e_x, e_y)\) basis: the cable is hanging under gravity action as \(-\mu g e_y\);
- the cable mechanical and geometric properties are uniform along the cable represented by a line;
- the cable is inextensible;
- the cable flexural rigidity is negligible: the contact force in the cable is only tension \(T > 0\).

\[
(x_A, y_A) \quad \rightarrow \quad (x_B, y_B)
\]

\[
\mathbf{F}_x, e_x \quad \mathbf{F}_y, e_y
\]

\[
-\mu g e_y
\]

Catenary element is an algebraic equation describing cable statics equilibrium position \(y(x), x \in [x_A, x_B]\) under previous assumptions [3] given in the form:
\[ y(x) = \tau \cosh \left( \frac{x}{\tau} + K_1 \right) + K_2, \]  

with catenary parameters \( \tau = \frac{x_A}{\mu g} K_1 = \text{argsinh} \left( \frac{y_A}{\tau} \right) - \frac{\mu g x_A}{\tau}, \) \( K_2 = -\frac{x_A}{\mu g} \sqrt{1 + \left( \frac{y_A}{\tau} \right)^2} + y_A \) depending on boundary conditions such as \((x_A, y_A)\) position of left catenary extremity, \(x_B\) abscissa of right catenary extremity and \(F = F_x e_x + F_y e_y\) the load at the left extremity (see Fig. 2).

An elastic correction of catenary Eq. (1) allows to consider cable stretching according to an elastic constitutive law

\[ T = E A \varepsilon, \]  

with \(E\) the Young’s modulus (Pa), \(A\) the cable section area (m\(^2\)) and \(\varepsilon\) the linear strain measure such as current cable length

\[ l = \int_{x_0}^{L_R} (1 + \varepsilon) \, dS, \]  

with \(L_R\) the unstretched reference cable length.

It’s usual to install a heavier cable on the upper part of the haul rope loop acting as counterweight on the system. So, the haul rope characteristics (weight per unit length, section, length) are not uniform along the loop. A modeling strategy is to considerer two different cable elements respectively of \(L_{c1}\) and \(L_{c2}\) unstretched length, coupled with tension continuity equations at each element extremities (see Fig. 3).

![Figure 3. Haul rope loop diagram.](image)

2.2. Vehicles

Vehicles are represented by point masses \((M_{v1}, M_{v2})\) (see Fig. 4) carried by one track rope for each side and pulled by a haul rope loop. Vehicles are coupled with cables by a cart rolling without friction on the track rope and clamped on the haul loop. Suspended vehicle equilibrium equation is provided in [4] and is coupled with cables by the catenary tension given in the following form, for \(x \in [x_A, x_B]\)

\[ T(x) = \mu g \tau \cosh \left( \frac{x}{\tau} + K_1 \right). \]
2.3. Tracks coupling

In the aerial tramway model, tracks 1 and 2 are coupled by the haul rope loop by intermediary of anchored pulleys located at bottom and top stations. A drive bull-wheel connected to a driving system controls the cable length $\Delta l_c$ unwound at the bottom station. The quantity $\Delta l_c$ is a model input given by the designer. At the top station, the return bull-wheel is free to rotate according to a friction contact law involving cable tension values in entrance $T^+$ and exit $T^-$ such as

$$T^+ = T^- \exp(\pm c_f \phi),$$  \hspace{1cm} (5)

with $c_f$ rolling resistance coefficient at return bull-wheel and $\phi$ cable winding angle around the pulley. The two track ropes on each side of the system are independent, embedded at extremities and respectively of $L_{p1}$ and $L_{p2}$ unstretched lengths (see Fig. 4).

![Figure 4. Haul and track ropes with vehicles diagram.](image)

3. STATIC CALCULATION

The problem of full system equilibrium is written applying statics equations to vehicles and boundary conditions at each catenary extremity. It yields to a set of nonlinear equations with catenary parameters $(\tau_0, K_{1k}, K_{2k})$ of each cable portion and vehicle positions as unknown. Model inputs are system geometry (top and bottom stations positions), cable properties (length, Young’s modulus, section area), vehicle masses $(M_{v1}, M_{v2})$ and cable unwinding control value $\Delta l_c$ at driving station. In the next subsections, the system behavior without and with inclusions of vehicles will be discussed.

3.1. Without vehicles

The first model is investigated without vehicles along cables. This model is equivalent to the practical situation where vehicles are removed from haul rope for cable angles measure on each track at top and bottom stations. Thus, tracks are only coupled by the haul rope loop and track ropes are not involved in the haul rope loop statics equilibrium.

$\beta$ parameter is introduced as a correction parameter of haul rope unstretched length input considering for uncertainty on the current cable length of the aerial tramway system. The total unstretched length of haul rope loop model input $L_t$ becomes

$$L_t = \beta(L_{t1} + L_{t2})$$  \hspace{1cm} (6)
According to Fig. 5, angle sensitivity is higher at the bottom station: for $\beta \in [0.983,1.06]$, angle $\alpha_b \in [-1.3,1]$ rad at the bottom station and $\alpha_t \in [0.64,1]$ rad at the top station on track 1.

3.2. With vehicles

The full model of the system includes vehicles and track ropes. It is supposed that the heaviest haul rope cable is located between vehicles on the upper part of the loop with a $L_{tt}$ length and the lightest one on the lower part with a $L_{tb}$ length.

$\gamma$ parameter is introduced as a correction considering for uncertainty on vehicles position along haul rope loop due to maintenance vehicle removing. The lengths of lower part $L_{tb}$ and upper part $L_{tt}$ of haul rope loop separating the two vehicles become

$$L_{tb}^{\text{cor}} = \beta \times L_{tb} + \gamma$$

$$L_{tt}^{\text{cor}} = \beta \times L_{tt} - \gamma$$

(7)

(8)

Shift docking length $\Delta \text{dock}$ characterizes the non-simultaneity of vehicles arrivals at top and bottom station due to track asymmetry (see Fig. 1). Indeed, cable geometry and mechanical characteristics are not the same on the top part and on the bottom part of the haul rope loop: $L_{tb}^{\text{cor}} \neq L_{tt}^{\text{cor}}$, $\mu_1 \neq \mu_2$, $A_1 \neq A_2$. For practical reasons, the vehicle located at the driving bull-wheel (bottom station) enters the last one while the top vehicle is already at the top station in mechanical stop: shift docking length is the remaining cable length to unwind by driving pulley while the top vehicle is already stopped. This quantity $\Delta \text{dock}$ can be easily measured on the real system. In Fig. 6, shift docking length is calculated for different values of $\beta$ parameter while in Fig. 7 shift docking length is calculated for different $\gamma$ parameter values with $\beta$ fixed. A linear variation of $\Delta \text{dock}$ can be observed with respect to $\beta$. 

Figure 5. Haul rope angles at stations without vehicles on the cable loop, as a function of $\beta$ parameter.
Figure 6. Shift docking length at bottom station as a function of $\beta$ parameter ($\gamma = 0$).

Figure 7. Shift docking length at bottom station as a function of $\gamma$ parameter ($\beta = 0.99$ [red], $\beta = 1.03$ [blue], $\beta = 1.06$ [yellow]).

4. EXPERIMENTAL VALIDATION

A set of experimental data collected on a real aerial tramway allows to address the inverse problem of identification. It consists in estimating model parameters from experimental data. The schematize of a direct and inverse process is depicted in Fig. 8.

First, haul rope angles were measured at rest on each track in stations without vehicles on the line and for a given control length $\Delta l_c$ at the driving station (see Table 1). Then, once empty vehicles were clamped on the haul loop, a shift docking length $\Delta dock = 25.2\ m$ was observed during an operating cycle.

Table 1. Cable angle measurement at stations.
<table>
<thead>
<tr>
<th>Measurement position</th>
<th>Cable slope [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom station - track 1</td>
<td>0.9</td>
</tr>
<tr>
<td>Bottom station - track 2</td>
<td>6.25</td>
</tr>
<tr>
<td>Top station – track 1</td>
<td>52.0</td>
</tr>
<tr>
<td>Top station – track 2</td>
<td>48.9</td>
</tr>
</tbody>
</table>

In the direct problem, unknown $\mathbf{Y}$ is catenary parameter vector for each $n_{\text{portion}}$ cable portion and vehicle positions depending on model input $\mathbf{X}$ and model internal parameter $\mathbf{p}$. Unknown vector is calculated solving a set of nonlinear equations of the model, such as $\mathbf{H}(\mathbf{X}, \mathbf{Y}, \mathbf{p}) = \mathbf{0}$.

The inverse problem comes from a partial lack of knowledge about the model internal parameters $\mathbf{p}$. Adding information on the model exit $\mathbf{Y}_{\text{obs}}$ (measures) allows to rebuild missing information. Inverse problem consists in finding $\mathbf{p}$ such as $\mathbf{H}(\mathbf{X}, \mathbf{Y}_{\text{obs}}, \mathbf{p}) = \mathbf{0}$. For the system application, $\mathbf{p} = (\beta, \gamma)^T$, $\mathbf{Y}_{\text{obs}}$ is the set of 4 cable slope measures $\alpha_i = g_i(t_k, K_{1k}, K_{2k}), i \in [1,4], k \in [1,n_{\text{portion}}]$ and $\Delta_{\text{dock}}$ value. Inverse problem can be reformulated as a minimization problem such as

$$\mathbf{p}^* = \arg\min_{\mathbf{p}} J(\mathbf{p}) = \arg\min_{\mathbf{p}} \| \mathbf{g}(\mathbf{Y}(\mathbf{X}, \mathbf{p})) - \mathbf{Y}_{\text{obs}} \|$$

$$\mathbf{H}(\mathbf{X}, \mathbf{Y}, \mathbf{p}^*) = \mathbf{0}$$

4.1. Identification of $\beta$

The inverse problem defined by Eq. (9) and Eq. (10) is applied for identification of $\beta$ parameter using the model without vehicles and the cable slope experimental measures collected on the system. It provides $\beta = 0.9972$.

4.2. Identification of $\gamma$

Identification of $\gamma$ parameter consists in using the full model of the system with vehicles and plotting shift docking length as a function of $\gamma$ parameter for $\beta = 0.9972$ (result of previous identification). Then, the $\gamma$ value associated with the experimental $\Delta_{\text{dock}} = 25.2$ m is read by interpolation. $\gamma = 4.34$ m is obtained.

5. CONCLUSIONS

A statics model of an aerial tramway system based on analytical catenary equation is proposed to follow cable length modification from experimental values collected on a real system. The experimental validation of the numerical statics model opens perspectives to use this tool in optimal research, for instance to optimize cycle duration anticipating shift docking length and by adjusting cable control law according to vehicle loading (see Fig. 9). Quasistatic calculation using control length $\Delta l_c$ at driving pulley as evolution parameter gives information about the system when haul loop cable is moving. As perspectives of this work, a modal analysis around statics equilibrium will give information about mode shapes and frequencies evolution with a parametrization by $\Delta l_c$ to account for time varying system modification when vehicles are moving. From an applied perspective, modal information is useful to predict dynamic response content during transient dynamics, to design acceleration and braking control curves of the cable reducing cable oscillation amplitudes, to detect eventual instabilities of the system by a stability analysis, etc.

![Figure 9. Shift docking length at bottom station as a function of vehicle load for $\beta = 0.9972$ and $\gamma = 4.34$ m.](image_url)
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REFERENCES


