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► **To cite this version:**

Guillaume Deffuant, Thibaut Roubin. Emergence of group hierarchy. Conference of Social Simulation 2022, Sep 2022, Milan, Italy. hal-03790987

HAL Id: hal-03790987

<https://hal.science/hal-03790987>

Submitted on 28 Sep 2022

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Emergence of group hierarchy

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Abstract. We consider an opinion dynamics model where, during random pair interactions, each agent modifies her opinions about both agents of the random pair and also about some other agents, chosen randomly. Moreover, each agent belongs to a single group and the opinions within the group are attracted to their average. In simulations starting from neutral opinions, we observe the emergence of a group hierarchy. We derive a moment approximation of the average opinions in groups about groups that explains this emergence.

Keywords: Opinion dynamics, Group, Hierarchy, Gossip.

1 Introduction

This paper builds on previous work [5,4,7] on an opinion dynamics model in which opinions are about the agents themselves instead of being about commercial products or political options.

Indeed in most opinion dynamics models ([10,11,6,3,12,9], for a recent review see: [8]), apart from a few exceptions [1,2], opinions about the agents themselves are not considered as deserving any specific attention. However, the opinions about agents determine the social network of positive or negative connections, hence in some respect the social structure. Moreover, it is generally recognised that this social structure has a strong influence on the agents' opinions. This suggests that opinions about agents do matter. Several opinion dynamics models include such a structure and in some cases it is evolving. This is for instance the case of some versions of the social impact model [14,13]. Moreover, other researches propose models of social structure dynamics, for instance hierarchies resulting from fights between primates [?]. In both cases, the social structure is generated by processes that are different from opinion dynamics.

In this paper, we assume that the agents of the model proposed in [4,7] belong to different groups that differentiate them. The type of groups that we have in mind would be male and female or other physical differences that can easily be observed (e.g. colour of skin). We keep all the assumption of the previous work. The agents hold an opinion (a real number between -1 and +1) about all the others and themselves. The model dynamics repeats encounters of two randomly chosen agents influencing their self-opinions and their opinions about each other. Moreover, if gossip is activated, both agents influence their opinions about some

other randomly chosen agents. The influence is attractive and the agents are more influenced by the ones that they hold in high esteem. Importantly, agents do not have a transparent access to the opinions of others; they constantly make errors of interpretation which are modelled by a random noise.

The only novelty in the dynamics is the introduction of some group conformity: the opinion of an agent of group I about an agent of group J is slightly attracted to the average opinion of agents of group I about agents of group J .

In the same line as in [7], we develop an analytical approximation of the average (first moment) evolution of the opinions in the model and the average evolution of their products (second moment), following a general approach [15]. The approximation goes one step further because it considers the average opinion of the agents of group I about the agents of group J .

The analysis of the approximate model shows the existence of biases on average group opinions that are similar to the biases on opinions about agents that are analysed in [7]. Moreover, the opinions about the groups of low status tend to decrease while the opinions about groups of high status tend to increase.

The following section describes the model and presents the patterns in more details. Section 3 is devoted to the moment approximation. Section 4 analyses the accuracy of the approximation and studies the effect of initial group hierarchies. The last section discusses the results and their limitations.

2 The model

2.1 State

The model includes n_g groups of N_g agents. Each agent $i \in \{1, \dots, n_g N_g\}$ belongs to a single group $I \in \{1, \dots, n_g\}$, has an opinion a_{ij} about each agent $j \in \{1, \dots, N_g\}$ including herself; the opinions are real values between -1, the worst opinion, and +1, the best opinion. Initially, all opinions are set to 0: agents have a neutral opinion about all the others at the beginning of the simulations.

2.2 Dynamics

At each time steps, two randomly chosen distinct agents i and j encounter and discuss their opinions about each other:

$$a_{ii}(t + 0.5) = a_{ii}(t) + h_{ij}(t)(a_{ji}(t) - a_{ii}(t) + \theta_{ii}(t)), \quad (1)$$

$$a_{ji}(t + 0.5) = a_{ji}(t) + h_{ji}(t)(a_{ii}(t) - a_{ji}(t) + \theta_{ji}(t)), \quad (2)$$

where $\theta_{ij}(t)$ is a random number uniformly drawn in $[-\delta, \delta]$ which represents errors in the evaluation of others' opinions. The function $h_{ij}(t)$ represents the influence that agent i gives agent j . This influence is high if i has a high opinion of j and low otherwise. The expression of $h_{ij}(t)$ is:

$$h_{ij}(t) = H(a_{ii}(t) - a_{ij}(t)) = \frac{1}{1 + \exp\left(\frac{a_{ii}(t) - a_{ij}(t)}{\sigma}\right)}, \quad (3)$$

where σ is a parameter of the model. The same modifications are made on the opinions a_{jj} and a_{ij} .

Then, if gossip is activated, the two agents i and j discuss their opinions about $k > 0$ randomly chosen other agents g_0, \dots, g_k :

$$a_{ig}(t + 0.5) = a_{ig}(t) + h_{ij}(t)(a_{jg}(t) - a_{ig}(t) + \theta_{ig}(t)). \quad (4)$$

After the interaction, each opinion is attracted by the average opinion of the agents of the same group:

$$a_{ii}(t + 1) = \mu \cdot a_{ii}(t + 0.5) + \frac{1 - \mu}{N_g} \sum_{p \in I} a_{pp}(t + 0.5), \quad (5)$$

$$a_{ji}(t + 1) = \mu \cdot a_{ji}(t + 0.5) + \frac{1 - \mu}{N_g(N_g - \delta_{IJ})} \sum_{p \in I} \sum_{q \in J, q \neq p} a_{qp}(t + 0.5), \quad (6)$$

where N_g the number of agents in each group.

2.3 Observed patterns

Starting from neutral opinions, when gossip is activated, we observe the emergence of a rather stable group hierarchy. Figure 1 shows a typical example of opinion matrices after a large number of interactions and of the evolution of the opinions over time. Rapidly a group hierarchy emerges and remains stable. Note that all the agents of a group have similar opinions about the agents of any group even with a value of the parameter ruling the attraction of the opinions to the group average very close to 1. As we can see this hierarchy is rather stable over time.

We propose now a mathematical analysis, based on the moment approximation technique, that explains these patterns.

3 Moment approximation

3.1 Dynamics of the average opinion in a group about agents of a group

We set x_{II} as the average self-opinion of agents of the group I and x_{JI} as the average opinion of agents of group J on agents of group I . If $I = J$, we adopt the following notation: $x_{JI} = x_{II}$. This allows us to distinguish x_{II} , the average self-opinion of the agents of group I , from x_{II} , the average opinion of agents of group I about other agents of group I .

$$x_{II}(t) = \frac{1}{N_g} \sum_{i \in I} x_{ii}(t). \quad (7)$$

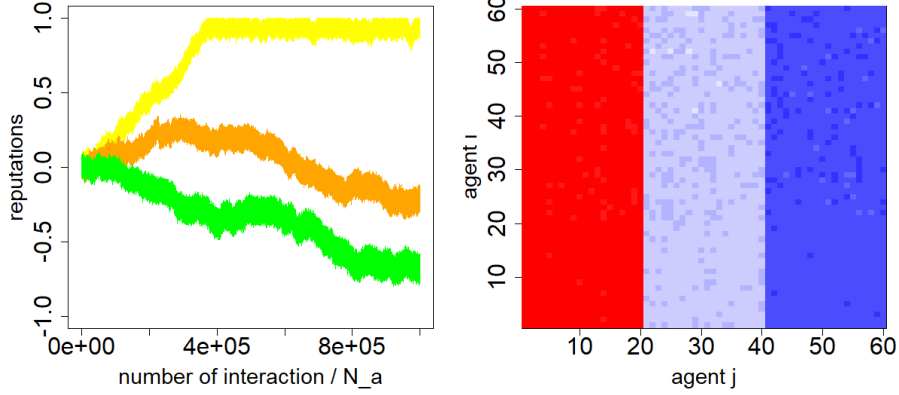


Fig. 1. Example opinion evolution (left) and opinion matrix after N_a million pair interactions (right), for 3 groups. In the matrix, blue and red squares represent respectively negative and positive opinions. The lighter the colour, the closer to 0 (neutral opinion) the represented opinion is. Agents 1 to 20 are in group 1, 21 to 40 in group 2 and 41 to 60 in group 3. Noise $\delta = 0.05$, influence parameter $\sigma = 0.3$, attraction to group average $\mu = 0.995$.

$$x_{JI}(t) = \frac{1}{N_g(N_g - \delta_{IJ})} \sum_{(i,j) \in I \times J, i \neq j} x_{ji}(t). \quad (8)$$

For a given sequence of interactions s_t , assuming that the interacting couple at t is (i, j) , let $x_{ii}(s_{t+0.5})$ be the value of x_{ii} after the interaction and before the attraction towards the average :

$$x_{ii}(s_{t+0.5}) = x_{ii}(s_t) + h_{ij}(s_t)(x_{ji}(s_t) - x_{ii}(s_t) + \theta_{ii}(s_t)). \quad (9)$$

Moreover, setting $h_{IJ}(s_t) = H(x_{II}(s_t) + a_{II}(0) - x_{IJ}(s_t) - a_{IJ}(0))$, $\overline{h_{IJ}}(t) = H(\overline{x_{II}}(t) + a_{II}(0) - \overline{x_{IJ}}(t) - a_{IJ}(0))$ and $z_{IJ}(t) = x_{II}(t) - x_{IJ}(t)$:

$$h_{ij}(s_t) = \overline{h_{IJ}}(t) + \overline{h'_{IJ}}(t)(x_{ii}(s_t) - x_{ij}(s_t) - \overline{z_{IJ}}(t)). \quad (10)$$

Therefore :

$$\begin{aligned} x_{ii}(s_{t+0.5}) &= x_{ii}(s_t) + \widehat{h_{IJ}}(t)(x_{ji}(s_t) - x_{ii}(s_t) + \theta_{ii}(s_t)) \\ &\quad + \overline{h'_{IJ}}(t)(x_{ii}(s_t) - x_{ij}(s_t))(x_{ji}(s_t) - x_{ii}(s_t) + \theta_{ii}(s_t)). \end{aligned} \quad (11)$$

Summing up over all agents in group I and dividing by N_g , the number of agents in a group, we get:

$$\begin{aligned}
x_{II}(s_{t+0.5}) &= x_{II}(s_t) + \frac{\widehat{h}_{IJ}(t)}{N_g}(x_{ji}(s_t) - x_{ii}(s_t)) \\
&+ \frac{\overline{h'_{IJ}}(t)}{N_g}(x_{ii}(s_t) - x_{ij}(s_t))(x_{ji}(s_t) - x_{ii}(s_t) + \theta_{ii}(s_t)).
\end{aligned} \tag{12}$$

Then, averaging over all sequences s_t , i.e. over all possibilities of couples (i, j) each having probability $\frac{2}{N_c}$ to be drawn with $N_c = N_a(N_a - 1)$, we get:

$$\begin{aligned}
\overline{x_{II}}(t + 0.5) &= \overline{x_{II}}(t) + \sum_J \frac{2}{N_g N_c} \sum_{(i,j) \in I \times J, i \neq j} \left(\widehat{h}_{IJ}(t)(\overline{x_{ji}}(t) - \overline{x_{ii}}(t)) \right. \\
&\left. + \overline{h'_{IJ}}(t)(\overline{x_{ji}(t) \cdot x_{ii}(t)} - \overline{x_{ii}^2(t)} + \overline{x_{ij}(t) \cdot x_{ii}(t)}) - \overline{x_{ij}(t) \cdot x_{ji}(t)} \right).
\end{aligned} \tag{13}$$

Similarly:

$$\begin{aligned}
\overline{x_{ji}}(s_{t+0.5}) &= \overline{x_{ji}}(s_t) + \widehat{h}_{JI}(t)(\overline{x_{ii}}(s_t) - \overline{x_{ji}}(s_t)) + \overline{h'_{JI}}(t)(\overline{x_{ji}^2}(s_t) \\
&- \overline{x_{ii}(s_t) \cdot x_{ji}(s_t)} + \overline{x_{ii}(s_t) \cdot x_{jj}(s_t)} - \overline{x_{jj}(s_t) \cdot x_{ji}(s_t)}).
\end{aligned} \tag{14}$$

Hence, summing up over all sequences such that $i \in I$ and $j \in J$, and on $J \in G$ the possible groups, and dividing by $N_g(N_g - \delta_{IJ})$ (with $\delta_{IJ} = 1$ iff $I = J$) we get:

$$\begin{aligned}
\overline{x_{JI}}(s_{t+0.5}) &= \overline{x_{JI}}(s_t) + \frac{1}{N_g(N_g - \delta_{IJ})} \left(\widehat{h}_{JI}(t)(\overline{x_{ii}}(s_t) - \overline{x_{ji}}(s_t)) + \overline{h'_{JI}}(t)(\overline{x_{ji}^2}(s_t) \right. \\
&\left. - \overline{x_{ii}(s_t) \cdot x_{ji}(s_t)} + \overline{x_{ii}(s_t) \cdot x_{jj}(s_t)} - \overline{x_{jj}(s_t) \cdot x_{ji}(s_t)}) \right).
\end{aligned} \tag{15}$$

Averaging over all the possible sequences s_t hence all possible choices of (i, j) :

$$\begin{aligned}
\overline{x_{JI}}(t + 0.5) &= \overline{x_{JI}}(t) + \frac{2}{N_c N_g(N_g - \delta_{IJ})} \sum_{(i,j) \in I \times J, i \neq j} \left(\widehat{h}_{JI}(t)(\overline{x_{ii}}(t) - \overline{x_{ji}}(t)) \right. \\
&\left. + \overline{h'_{JI}}(t)(\overline{x_{ji}^2}(t) - \overline{x_{ii}(t) \cdot x_{ji}(t)} + \overline{x_{ii}(t) \cdot x_{jj}(t)} - \overline{x_{jj}(t) \cdot x_{ji}(t)}) \right).
\end{aligned} \tag{16}$$

We now introduce the average products of opinions at group level (second moments):

$$x_{III}^2(t) = \frac{1}{N_g} \sum_{i \in I} x_{ii}(t)x_{ii}(t); \tag{17}$$

$$x_{JII}^2(t) = \frac{1}{N_g(N_g - \delta_{IJ})} \sum_{(i,j) \in I \times J, i \neq j} x_{ji}(t)x_{ji}(t); \tag{18}$$

The other products $x_{IIJI}^2(t)$, $x_{IIJJ}^2(t)$, x_{IIJJ}^2 , are defined similarly.

Then:

$$\begin{aligned} \overline{x_{II}}(t+0.5) &= \overline{x_{II}}(t) + \sum_{J \in G} \frac{2(N_g - \delta_{IJ})}{N_c} \left(\widehat{h_{IJ}}(t)(\overline{x_{JI}}(t) - \overline{x_{II}}(t)) \right. \\ &\quad \left. + \overline{h'_{IJ}}(t)(\overline{x_{IIJI}^2}(t) - \overline{x_{IIII}^2}(t) + \overline{x_{IIJJ}^2}(t) - \overline{x_{JJJI}^2}(t)) \right). \end{aligned} \quad (19)$$

Similarly:

$$\begin{aligned} \overline{x_{JI}}(t+0.5) &= \overline{x_{JI}}(t) + \frac{2}{N_c} \left(\widehat{h_{JI}}(t)(\overline{x_{II}}(t) - \overline{x_{JI}}(t)) \right. \\ &\quad \left. + \overline{h'_{JI}}(t)(\overline{x_{JIIJ}^2}(t) - \overline{x_{IIJI}^2}(t) + \overline{x_{IIJJ}^2}(t) - \overline{x_{JJJI}^2}(t)) \right). \end{aligned} \quad (20)$$

We can add the effect of gossip:

$$\begin{aligned} \overline{x_{JI}}(t+0.5) &= \overline{x_{JI}}(t) + \frac{2}{N_c} \left(\widehat{h_{JI}}(t)(\overline{x_{II}}(t) - \overline{x_{JI}}(t)) \right. \\ &\quad \left. + \overline{h'_{JI}}(t)(\overline{x_{JIIJ}^2}(t) - \overline{x_{IIJI}^2}(t) + \overline{x_{IIJJ}^2}(t) - \overline{x_{JJJI}^2}(t)) \right) \\ &\quad + \frac{2k}{N_t} \sum_{P \in G} (N_g - \delta_{IP} - \delta_{JP}) \widehat{h_{JP}}(t)(\overline{x_{PI}}(t) - \overline{x_{JI}}(t)), \end{aligned} \quad (21)$$

with:

$$N_t = N_a(N_a - 1)(N_a - 2) \quad (22)$$

After each interaction, we also reduce the distance to the average group opinion of each opinion:

$$\overline{x_{ii}}(t+1) = \mu \overline{x_{ii}}(t+0.5) + (1 - \mu) \overline{x_{II}}(t+0.5), \forall i \in \{1, \dots, N_a\}. \quad (23)$$

That does not change the value of x_{II} , so finally, we obtain:

$$\overline{x_{II}}(t+1) = \overline{x_{II}}(t+0.5). \quad (24)$$

Similarly:

$$\overline{x_{JI}}(t+1) = \overline{x_{JI}}(t+0.5). \quad (25)$$

We also express the time evolution of average products of group opinions, before attraction to the average, and then the effect of the attraction to the average. In general, the attraction to the average modifies the expression of the products. An example is provided in the appendix. It should be noted that more products should be considered than in the model without groups.

3.2 Evolution of the equilibrium average opinion about the agents of group

A remarkable feature of the dynamics that we observe on simulations is that the different opinions $\overline{x_{JI}}(t)$ of groups J about group I tend to be parallel after a while. Following the same idea as in [7], we now define an equilibrium opinion about group I , whose evolution represents this average evolution of the opinions of all groups about group I . The equilibrium opinion is obtained by summing up the average self-opinion in group I $\overline{x_{II}}(t)$ and, for all $J \in G$, $\overline{x_{JI}}(t)$ the average opinion of group J about group I multiplied by a coefficient determined so that the sum of the first order changes is 0. Let $e_I(t)$ be this average equilibrium opinion. We have:

$$e_I(t) = \frac{1}{1 + S_I(t)} \left(\overline{x_{II}}(t) + \sum_{J \in G} \frac{(N_g - \delta_{IJ}) \widehat{h_{IJ}}(t)}{\widehat{h_{JI}}(t)} \overline{x_{JI}}(t) \right), \quad (26)$$

where $S_I(t)$ is defined as follows:

$$S_I(t) = \sum_{J \in G} \frac{(N_g - \delta_{IJ}) \widehat{h_{IJ}}(t)}{\widehat{h_{JI}}(t)}. \quad (27)$$

Therefore the evolution of $e_I(t)$ is given by:

$$\begin{aligned} e_I(t+1) = e_I(t) + \frac{1}{1 + S_I(t)} \sum_{J \in G} \frac{2(N_g - \delta_{IJ})}{N_c} & \left(\overline{h'_{IJ}}(t) (\overline{x_{IIJI}^2}(t) - \overline{x_{IIII}^2}(t)) \right. \\ & \left. + \overline{x_{IIIJ}^2}(t) - \overline{x_{IJIJ}^2}(t) \right) \\ & + \frac{\widehat{h_{IJ}}(t)}{\widehat{h_{JI}}(t)} \overline{h'_{JI}}(t) (\overline{x_{JIIJ}^2}(t) - \overline{x_{IIJI}^2}(t) + \overline{x_{IIJJ}^2}(t) - \overline{x_{JJJI}^2}(t)), \end{aligned} \quad (28)$$

We recognise an expression which is very similar to the one found in [7] on the opinions about agents (instead of groups). Similarly, the terms $\overline{h'_{IJ}}(t) (\overline{x_{IIJI}^2}(t) - \overline{x_{IIII}^2}(t) + \overline{x_{IIIJ}^2}(t) - \overline{x_{IJIJ}^2}(t))$ are positive and can be interpreted as a "positive bias" about the self-opinions of the groups and the terms $\overline{h'_{JI}}(t) (\overline{x_{JIIJ}^2}(t) - \overline{x_{IIJI}^2}(t) + \overline{x_{IIJJ}^2}(t) - \overline{x_{JJJI}^2}(t))$ are negative and can be interpreted as a "negative bias" of the opinions of groups about other groups and others in their own group.

The tendency of the evolution of the opinion about group I is thus the result of the interplay of the positive and negative biases weighted by the coefficients $\frac{\widehat{h_{IJ}}(t)}{\widehat{h_{JI}}(t)}$. This coefficient is large when group J is lower than group I in the group hierarchy (group J has a high opinion of group I and group I a low opinion of group J). Therefore, the negative bias is stronger for the groups that are low in the hierarchy.

Moreover, when there is gossip, the term $\overline{x_{JIIJ}^2}(t)$ which is essential in the negative bias, is increased, which increases the negative bias, especially for the

groups that are low in the hierarchy. When gossip is activated, the equation of $e_i(t+1)$ remains the same except that the following first order term coming from gossip is added:

$$\frac{1}{1+S_I(t)} \sum_{J \in G} \frac{\widehat{h_{IJ}}(t)}{\widehat{h_{JI}}(t)} \sum_{P \in G} \frac{2k(N_g - \delta_{IJ})(N_g - \delta_{IP} - \delta_{JP})}{N_t} \widehat{h_{JP}}(t) (\overline{x_{PI}}(t) - \overline{x_{JI}}(t)). \quad (29)$$

In the case of a single group I and without gossip, the evolution of the equilibrium opinion about group I is:

$$e_I(t+1) = e_I(t) + \frac{\overline{h'_{II}}(t)}{N_a} \left(\overline{x^2_{II^j II^j}}(t) - \overline{x^2_{III}}(t) + \overline{x^2_{III^j I^j}}(t) - \overline{x^2_{II^j I^j I}}(t) \right). \quad (30)$$

It seems that, even with gossip, $\overline{x_{III}}(t) > \overline{x_{II^j II^j}}(t)$ and $\overline{x^2_{III^j I^j}}(t) > \overline{x^2_{II^j I^j I}}(t)$, therefore, the opinion is always increasing (remember that $h'_{II}(t) < 0$). Actually, this corresponds to the case already identified in [7] where the opinions are kept close to each other, leading to an increase of the opinions with or without gossip.

4 Results

Figure 2 shows the evolution of average self-opinions of group I agents $\overline{x_{II}}(t)$ (left panel) and average opinions of group I about group J agents $\overline{x_{JI}}(t)$ (right panel) for 3 groups of 10 agents. Initially, all the agents have the opinion -0.5 about agents of group 0, the opinion 0 about agents of group 1, opinion 0.5 about agents of group 2. The panels show the average value over 100,000 simulations (dots) and the moment approximation. We can observe that the approximation is reasonably accurate. It is also noticeable that the average opinion about some groups increases while the opinion about other groups decreases.

The left panel of Figure 3 shows the relative values of the positive and negative biases as defined formally in section 3.2 for group 0 in the case of 2 groups (0 and 1). The horizontal axis represents the initial difference between the opinions about group 1 and the opinions about group 0, assuming that initially all the agents have the same opinion about the agents of one group. Hence, when for the negative values of the horizontal axis, group 0 is dominated by group 1 (opinions about group 0 are lower than opinions about group 1) and on the contrary for positive values, group 0 dominates group 1 (opinions about group 0 are higher than opinions about group 1). The graph shows that the biases from in-group interactions are stronger than the biases from out-group interactions when group 0 dominates. The in-group and out-group biases are of similar amplitude when group 0 is dominated. Importantly, when group 0 is dominated the sum of all biases (green curve) is negative, hence the opinions about group 0 decrease, and when group 0 dominates, the sum of all biases is positive and the opinions about group 0 increase. This explains that, once there is a random difference of opinion between two groups, this difference tends to increase.

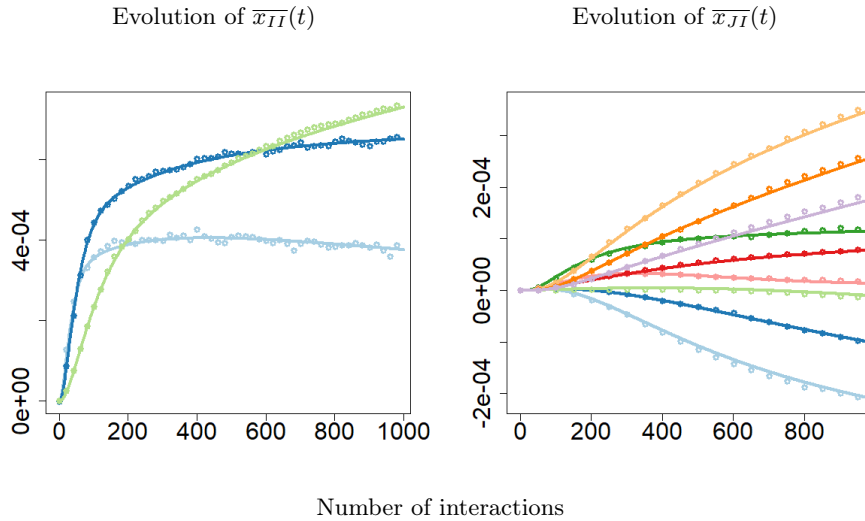


Fig. 2. Examples of evolution of average group opinions for 3 groups of 10 agents. The lines are obtained with the moment approximation and the points by averaging the results of 100 thousands simulations. Noise $\delta = 0.05$, influence parameter $\sigma = 0.3$, attraction to group average $\mu = 0.995$, gossip $k = 2$. See main text for details

The right panel of figure 3 shows the change of equilibrium opinion after 1000 random interactions, predicted by the moment approximation, in the case of 3 groups. The panel shows $e_0(1000) - e_0(999)$ and $e_1(1000) - e_1(999)$, for different initial differences $e_0(0) - e_2(0)$ and assuming that the hierarchy is initially regular: $e_1(0) = \frac{1}{2}(e_0(0) + e_2(0))$. We note that e_0 decreases when $e_0(0)$ is significantly lower than $e_2(0)$ (approximately $e_0(0) - e_2(0) < -0.3$) and e_1 decreases (slightly) only when $e_0(0)$ much lower than $e_2(0)$ (approximately $e_0(0) - e_2(0) < -1.5$). Therefore, when the difference between the two extreme groups becomes high, the opinions about the intermediate group tend to decrease, as we can see in the example show on Figure 1.

5 Discussion

In this model, the dominance of a group comes from random differences that tend to increase because the highest in the hierarchy tends to get higher while the lowest tends to get lower. The process is very similar to the emergence of a hierarchy of agents in the model without group. We should investigate the model with larger numbers of groups to confirm if groups are unstable at the top of the hierarchy as individuals are. In any case, the main conclusion of this work is that, like the agents have a positive bias on their self-opinion, the average self-opinion of the agents of a group has a positive bias and like the agents have a negative bias on their opinion about other agents, the average opinion of the agents of a group about the agents of another group has a negative bias. Similarly again to

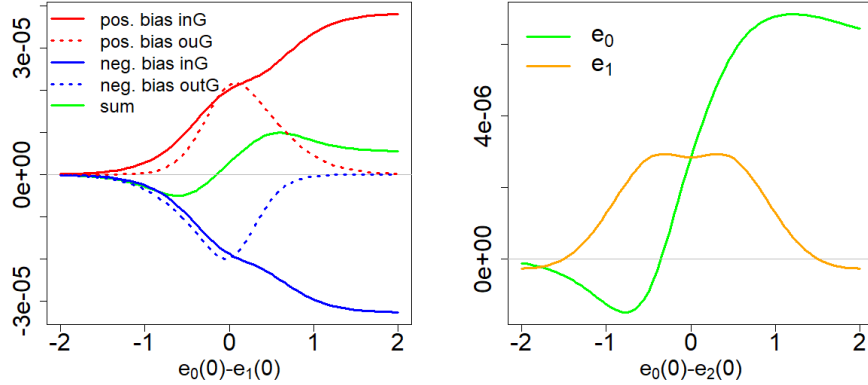


Fig. 3. Slope of different opinion components trajectory at $t = 1000$ ($100 \times e_0(1000) - e_0(999)$) computed by the moment approximation. In the left panel, the number of groups is $n_g = 2$ and the curves are the Positive bias (red) and negative bias (blue), from in-group interactions (line), out-group interactions (dotted), and the sum of this 4 components (green line). In the right panel, the number of groups is $n_g = 3$ and the curves are the equilibrium opinion of the groups 0 and 1, depending on $e_0(0) - e_2(0)$. Number of agent per group $Ng = 20$, Noise $\delta = 0.05$, influence parameter $\sigma = 0.3$, attraction to group average $\mu = 0.995$, gossip parameter $k = 5$.

the agents without groups, the relative effect of the negative bias is stronger for the groups which are low in the hierarchy. Therefore, the model suggests that, even if the differences between groups are purely anecdotal and have no impact on their intrinsic qualities, any society tends to establish a group hierarchy, purely because of increasing advantages or disadvantages that are initially randomly established.

The limitations mentioned in [7] of course remain valid for this work. Mainly, the model does not pretend to be realist. On the contrary, it is simplified on purpose, in order to make the effect of some hypotheses clearer. Moreover, in the model with groups, the process by which the opinions in a group are attracted to their average is probably more artificial than the others. It is indeed designed in order to facilitate the derivation of the moment approximation. Nevertheless, it seems possible to design dynamics which are more clearly related to observed social processes and that tend to homogenise the opinions within a group. The conclusions drawn with our simplified process are likely to remain valid when replacing this simplified process by a better grounded one.

6 Acknowledgement

This work has been partly supported by the Agence Nationale de la Recherche through the ToRealSim project (ORA call).

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7 Appendix: equations of the evolution of average products

This appendix provides an example of equations ruling the approximate average evolution of products of average group opinions. We consider the product $x_{III}^2(t)$.

$$\begin{aligned} \overline{x_{ii}^2}(t+0.5) = \overline{x_{ii}^2}(t) + \frac{2}{N_c} \sum_{J \in G} \sum_{j \in J, j \neq i} \left((\widehat{h_{jJ}^2}(t) - 2\widehat{h_{IJ}}(t)) \overline{x_{ii}^2}(t) + \widehat{h_{IJ}}(t)^2 \overline{x_{ji}^2}(t) \right. \\ \left. + 2(1 - \widehat{h_{IJ}}(t)) \widehat{h_{IJ}}(t) \overline{x_{ii}(t)x_{ji}(t)} + \widehat{h_{IJ}}(t)^2 \frac{\delta^2}{3} \right). \end{aligned} \quad (31)$$

Therefore:

$$\begin{aligned} \overline{x_{III}^2}(t+0.5) &= \overline{x_{III}^2}(t) + \sum_{J \in G} \frac{2(N_g - \delta_{IJ})}{N_c} \left((\widehat{h_{IJ}^2}(t) - 2\widehat{h_{IJ}}(t)) \overline{x_{III}^2}(t) \right. \\ &\quad \left. + \widehat{h_{IJ}}(t)^2 \overline{x_{IIJI}^2}(t) + 2(1 - \widehat{h_{IJ}}(t)) \widehat{h_{IJ}}(t) \overline{x_{IIJI}^2}(t) + \widehat{h_{IJ}}(t)^2 \frac{\delta^2}{3} \right). \end{aligned} \quad (32)$$

Note that the expression $\frac{\delta^2}{3}$ appearing in the previous equation comes from the average of the squared noise $\overline{\theta^2}(t) = \frac{\delta^2}{3}$ (see [7] for details). Now, we express how these quantities change because of the process of attraction of the average opinion.

$$\overline{x_{ii}^2}(t+1) = \left(\mu \overline{x_{ii}}(t+0.5) + \frac{1-\mu}{N_g} \sum_{j \in I} \overline{x_{jj}}(t+0.5) \right)^2 \quad (33)$$

$$\begin{aligned} \overline{x_{ii}^2}(t+1) &= \mu^2 \overline{x_{ii}^2}(t+0.5) + \frac{2\mu(1-\mu)}{N_g} \overline{x_{ii}}(t+0.5) \sum_{j \in I} \overline{x_{jj}}(t+0.5) \\ &\quad + \frac{(1-\mu)^2}{N_g^2} \sum_{j \in I} \sum_{p \in I} \overline{x_{jj}(t+0.5)x_{pp}(t+0.5)} \end{aligned} \quad (34)$$

$$\begin{aligned} \frac{1}{N_g} \sum_{i \in I} \overline{x_{ii}^2}(t+1) &= \frac{\mu^2}{N_g} \sum_{i \in I} \overline{x_{ii}^2}(t+0.5) \\ &\quad + \frac{2\mu(1-\mu)}{N_g^2} \sum_{i \in I} \sum_{j \in I} \overline{x_{ii}(t+0.5)x_{jj}(t+0.5)} \\ &\quad + \frac{(1-\mu)^2}{N_g^2} \sum_{j \in I} \sum_{p \in I} \overline{x_{jj}(t+0.5)x_{pp}(t+0.5)} \end{aligned} \quad (35)$$

Therefore:

$$\overline{x_{III}^2}(t+1) = \left(\mu^2 + \frac{1-\mu^2}{N_g} \right) \overline{x_{III}^2}(t+0.5) + \frac{(1-\mu^2)(N_g-1)}{N_g} \overline{x_{IIIjIj}^2}(t+0.5) \quad (36)$$

With:

$$\overline{x_{IIIjIj}^2}(t) = \frac{1}{N_g(N_g-1)} \sum_{(i,j) \in I^2, i \neq j} \overline{x_{ii}(t)x_{jj}(t)}; \quad (37)$$

There is a dozen of such products for which the same equations are derived to complete the approximate model.