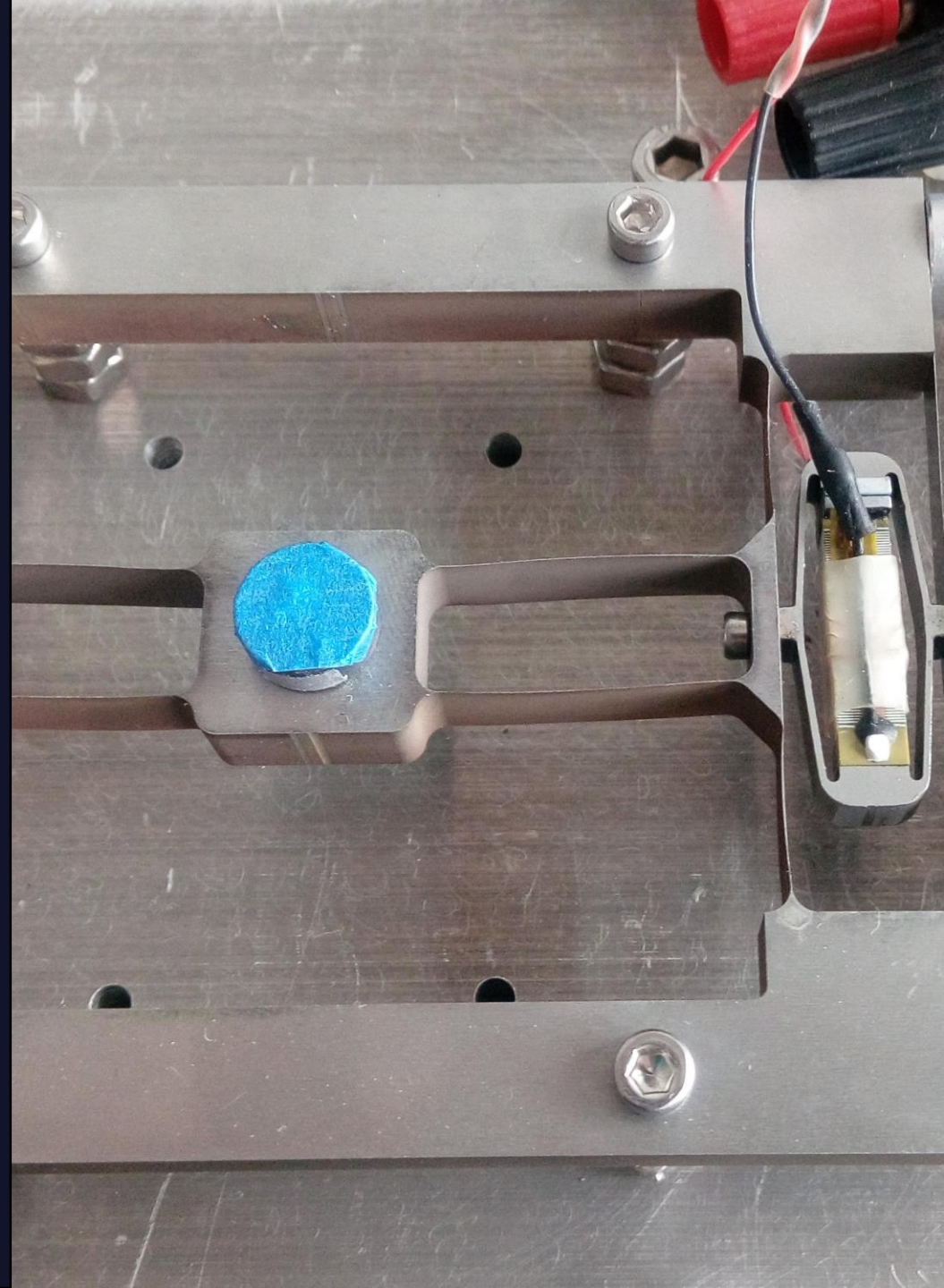


Aya Benhemou, Thomas Huguet , David Gibus, Aurélien Carré,
Camille Saint-Martin, Emile Roux, Ludovic Charleux, Adrien Badel
SYMME, Université Savoie Mont Blanc (France)

“ Combination of a static finite elements approach and dynamic lumped model for bistable vibration energy harvester modelling ”



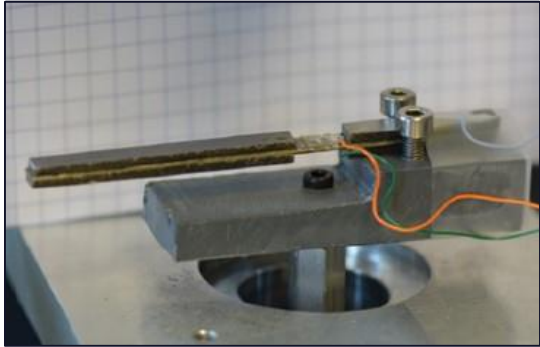
This project has received funding from the European Union’s Horizon
2020 research and innovation program under grant agreement No
862289



Technologies used for vibration energy harvesting

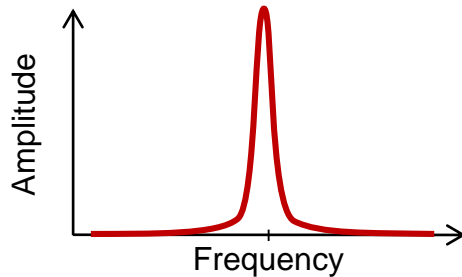
Linear structures

Cantilever beam
(Badel et al., 2014)



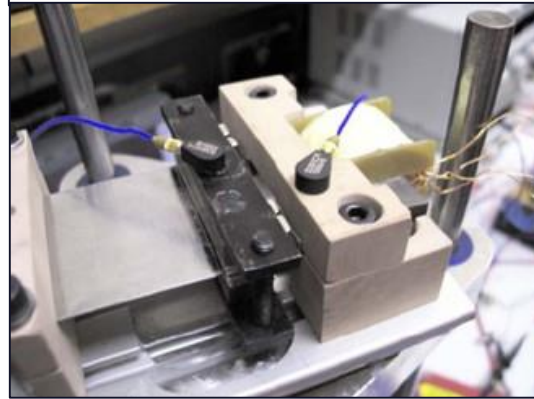
Simple architectures

Low frequency band



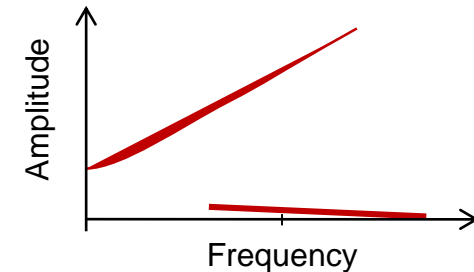
Bistable structures

Repulsive magnetic mechanism
(Barton et al. 2010)

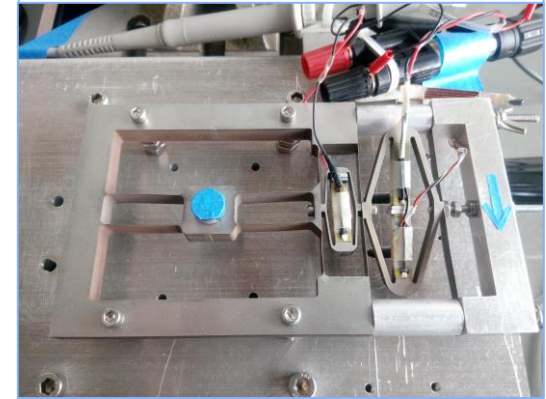


Complex architectures

Wide frequency band

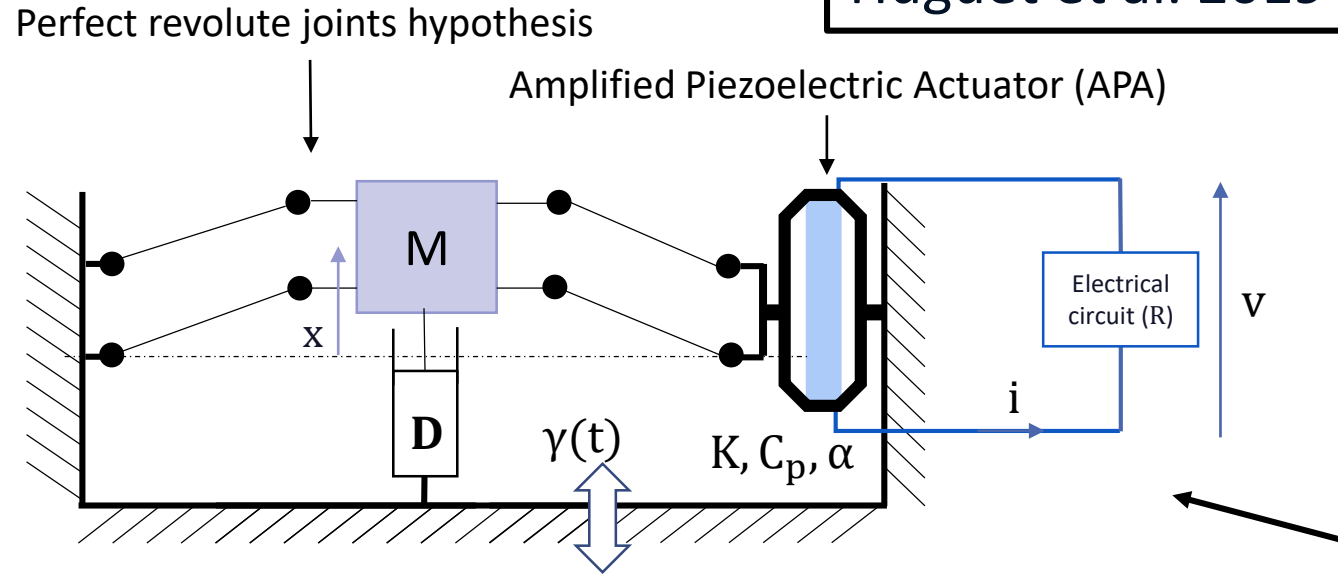


One-side clamped buckled beam
(Huguet et al. 2019)

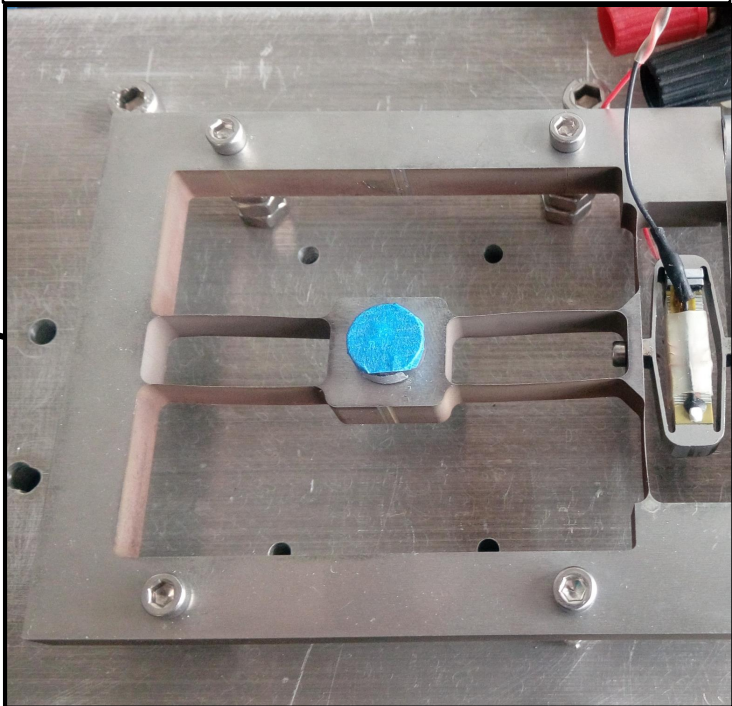


Bistable vibration energy harvester modelling

Huguet et al. 2019



One-side clamped buckled beam (Huguet et al. 2019)



$$M\dot{\gamma} = M\ddot{x} - 2K\frac{x_0^2}{L^2}x + 2\frac{K}{L^2}x^3 + D\dot{x} + \frac{2\alpha}{L}xv$$

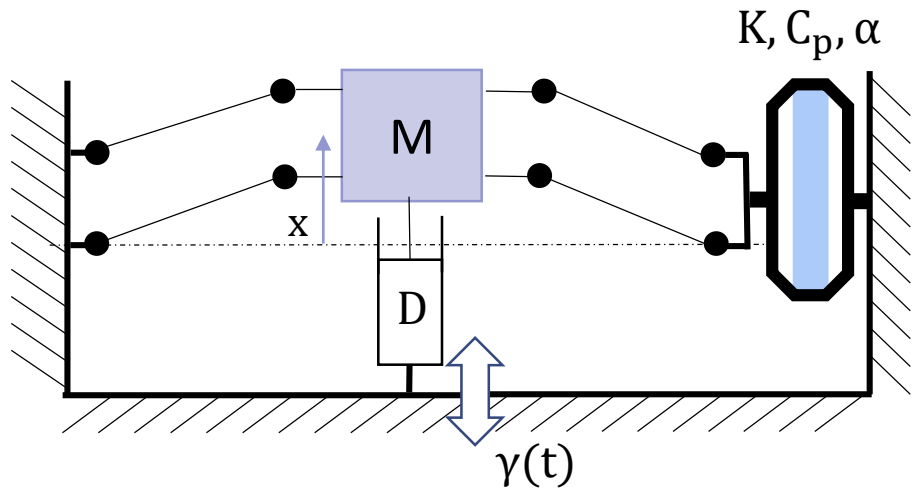
Mechanical equation

$$\dot{i} = \frac{2\alpha}{L}x\dot{x} - C_p\dot{v}$$

Electrical equation

➡ What is the effect of the buckled beams ?

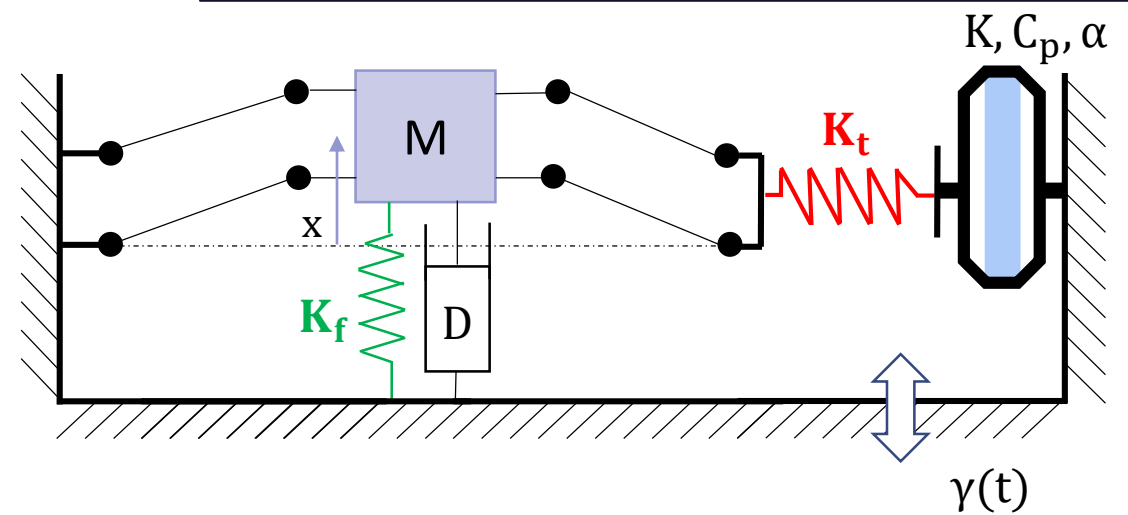
Perfect revolute joints hypothesis



Modelling beam elasticity

K_f = Beam bending stiffness

K_t = Beam extension/compression stiffness



$$M\gamma = M\ddot{x} - 2K \frac{x_0^2}{L^2} x + 2 \frac{K}{L^2} x^3 + D\dot{x} + \frac{2\alpha}{L} xv$$

$$M\gamma = M\ddot{x} - \left(2K\beta_{K_t} \frac{x_0^2}{L^2} - K_f \right) x + \frac{2K\beta_{K_t} x^3}{L^2} + D\dot{x} + \frac{2\alpha\beta_{K_t} xv}{L}$$

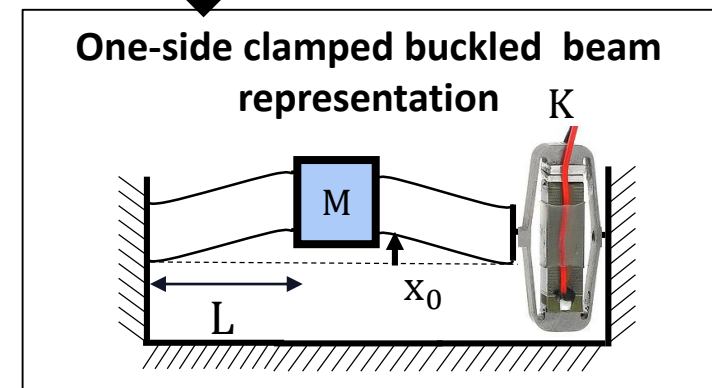
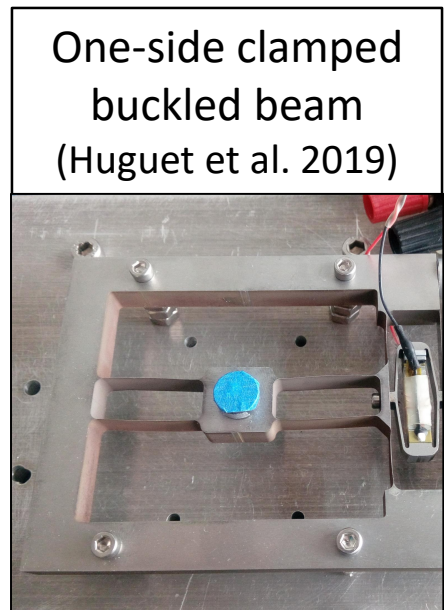
$$i = \frac{2\alpha}{L} x\dot{x} - C_p \dot{v}$$

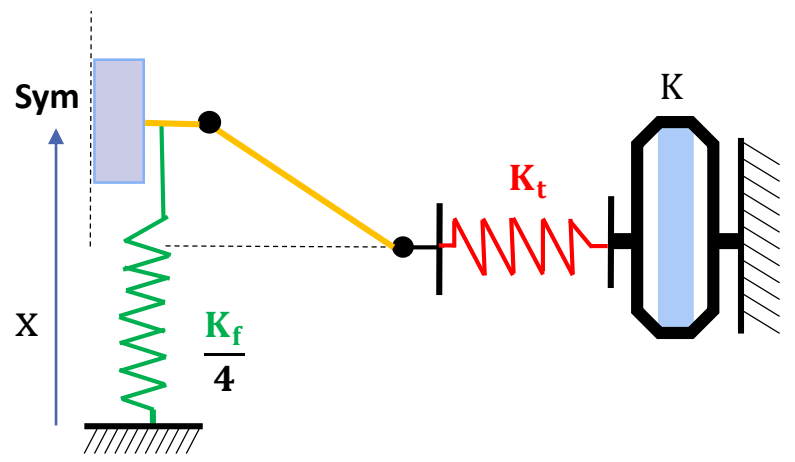
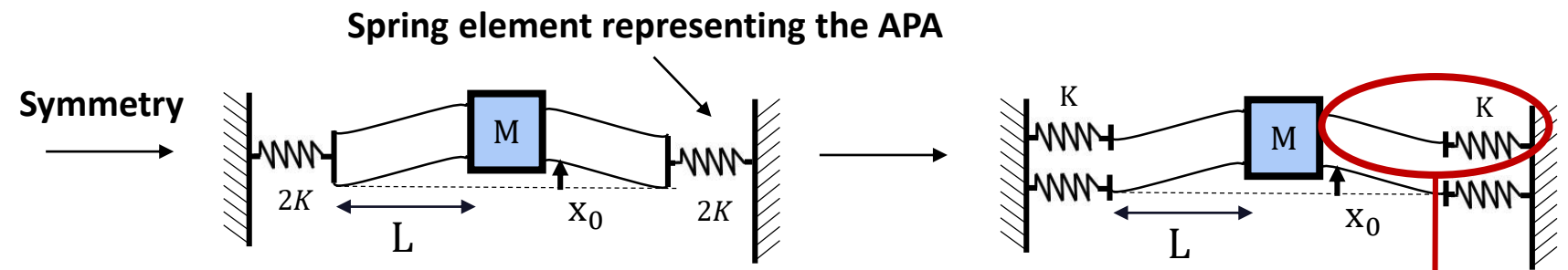
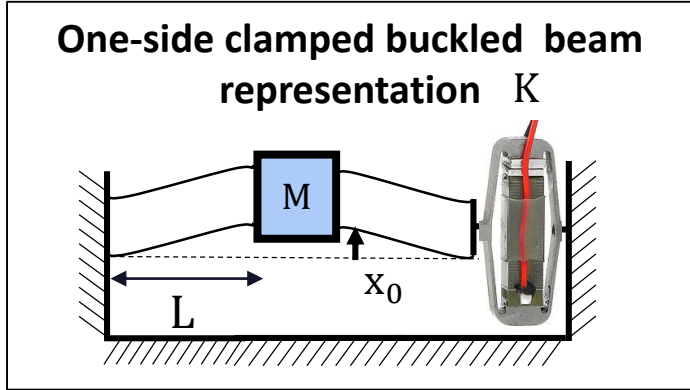
$$i = \frac{2\alpha\beta_{K_t} x\dot{x}}{L} - C_p \dot{v}$$

with: $\beta_{K_t} = \left(\frac{1}{1 + \frac{K}{K_t}} \right)$

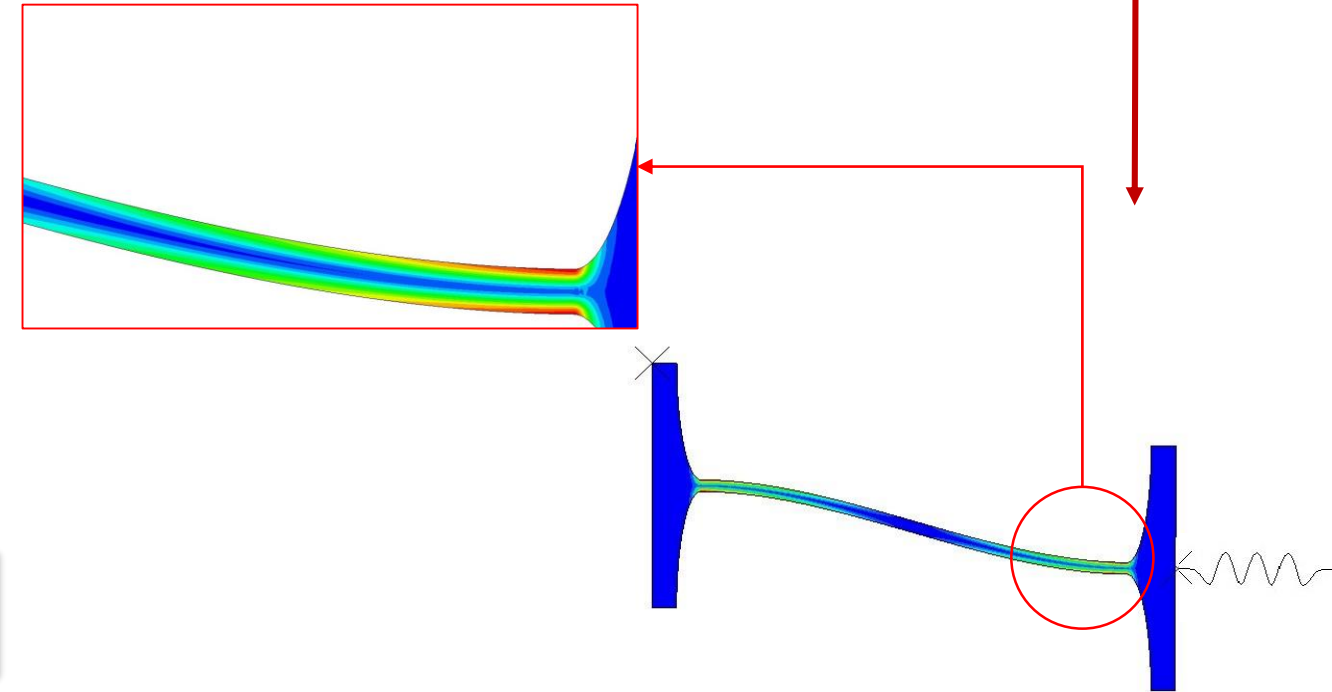
Case study characteristics

	Parameters	Value
APA parameters (CEDRAT technology's APA120S)	APA stiffness K	0.342 N/ μ m
Buckled beam parameters	Beam depth	5 mm
	Beam thickness	0.2 mm
	Beam Length L	35 mm
	Beam buckling level x_0	1 mm

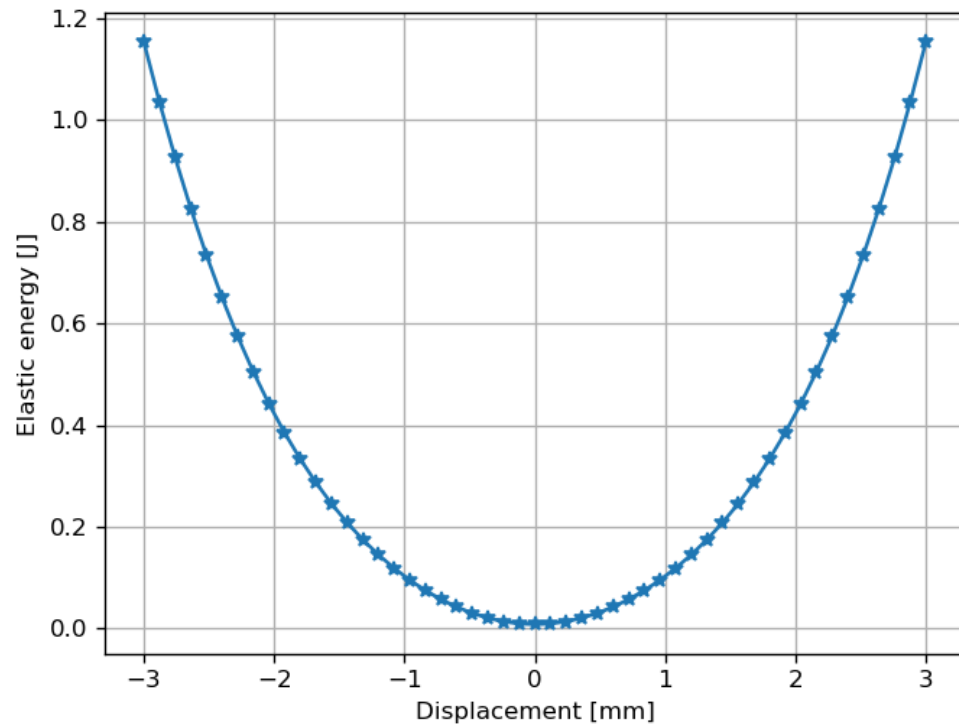




$$E_{\text{beam}} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{\text{APA}}}{K_t} E_{\text{APA}}$$

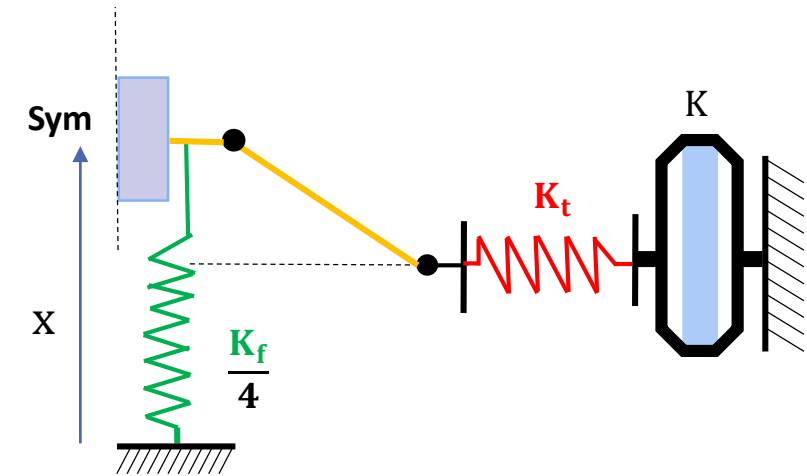


➔ The FEA results show that the elastic energy stored in the beams can reach 60% of the total energy

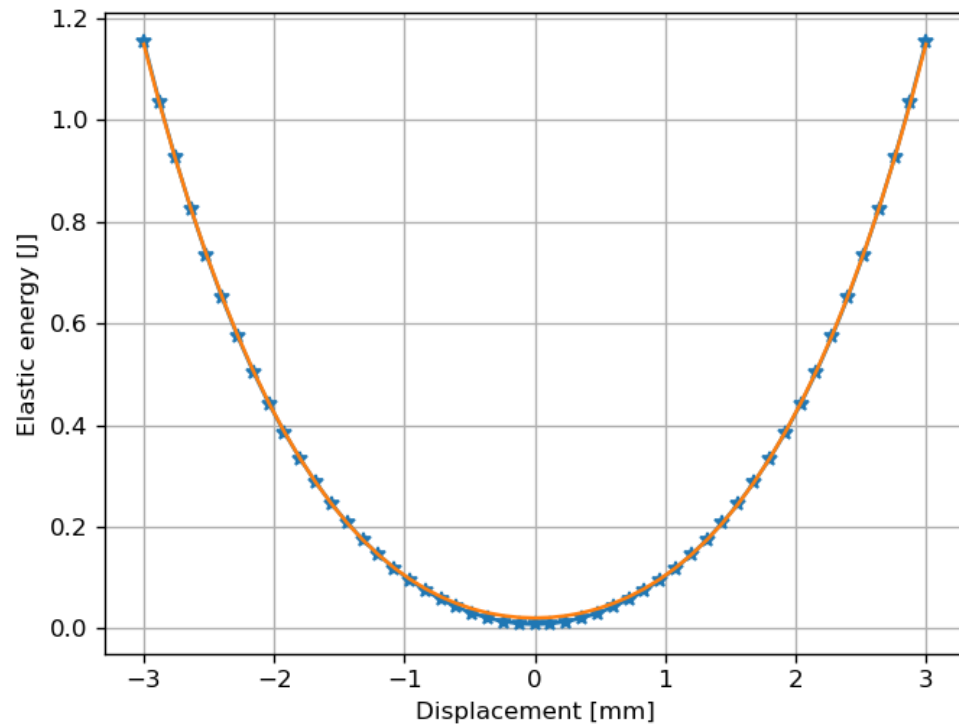


Energy stored in the beam predicted by :

- the [FEA](#)

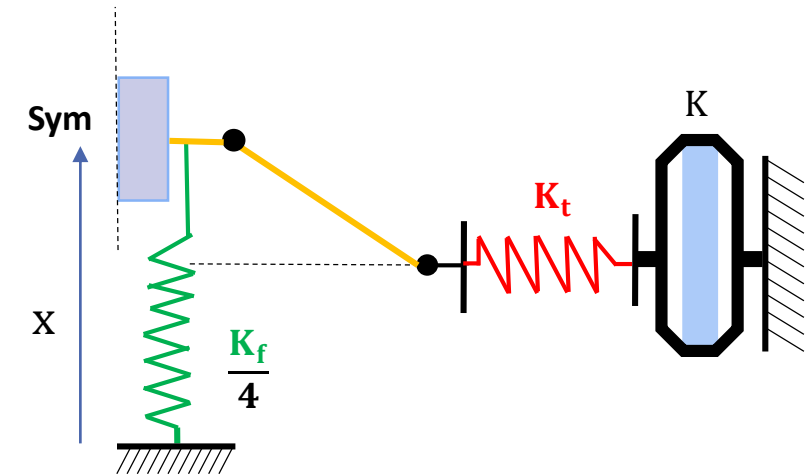


$$E_{\text{beam}} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{\text{APA}}}{K_t} E_{\text{APA}}$$

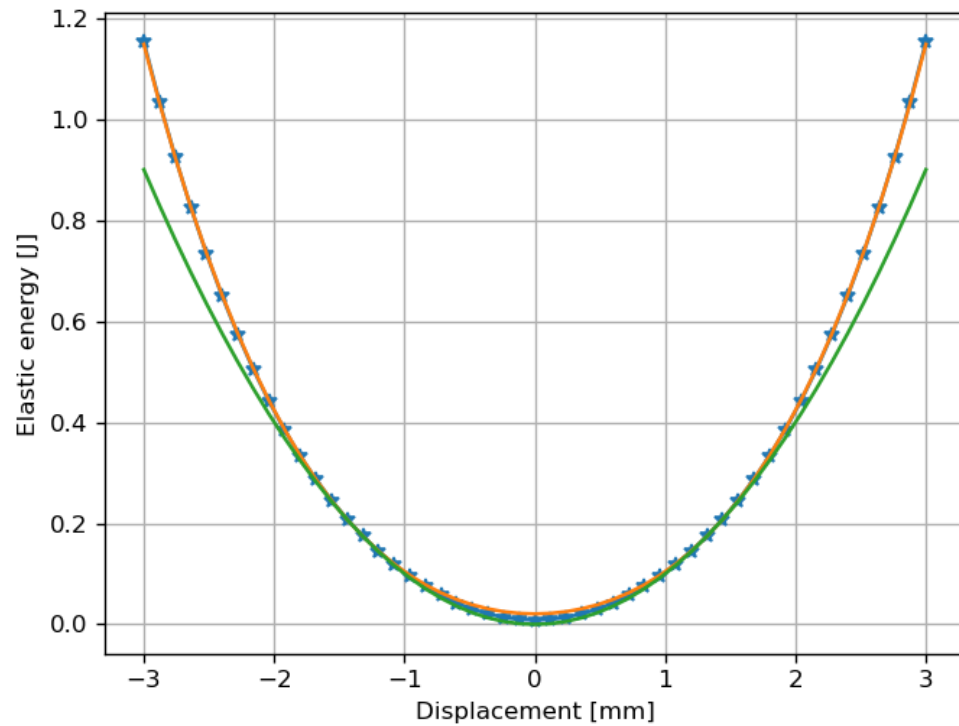


Energy stored in the beam predicted by :

- the FEA
- The lumped model

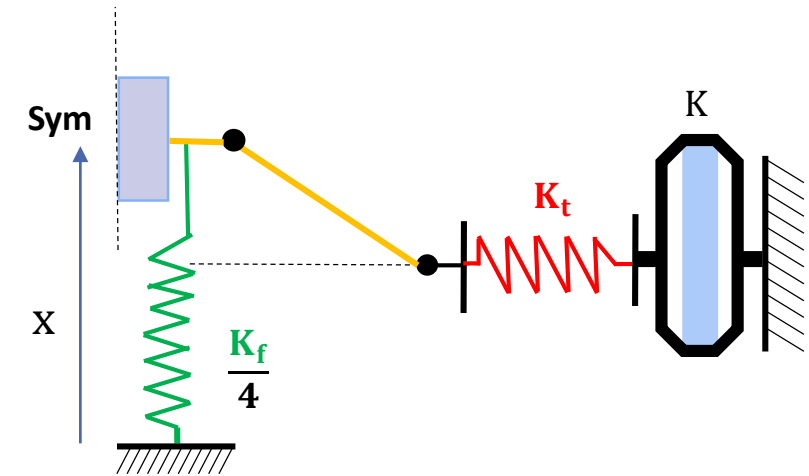


$$E_{beam} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{APA}}{K_t} E_{APA}$$

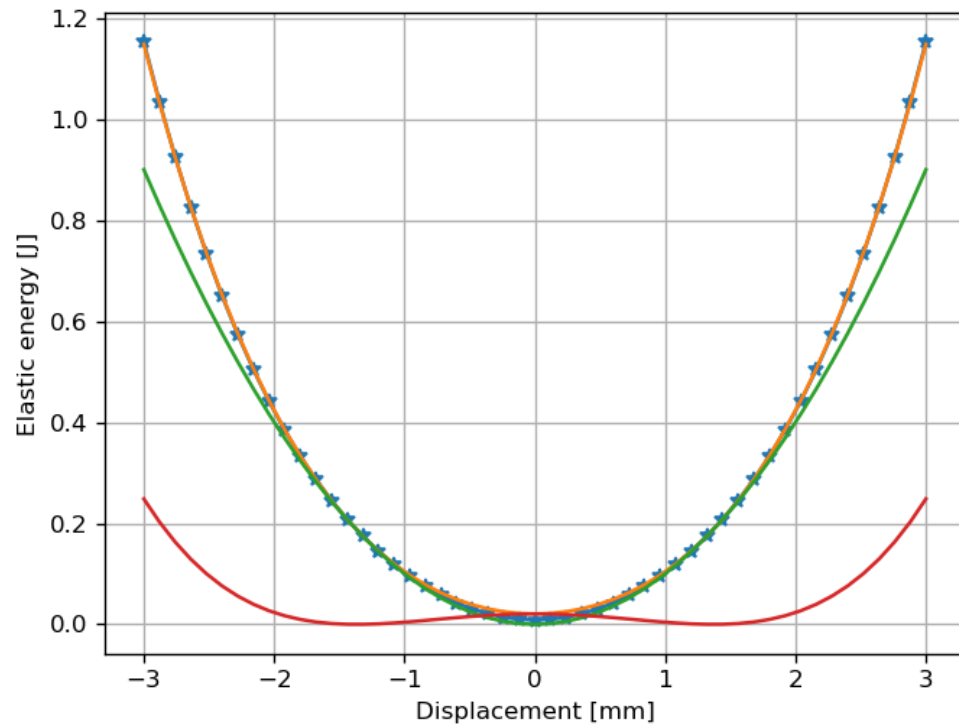


Energy stored in the beam predicted by :

- the FEA
- The lumped model
 - By bending

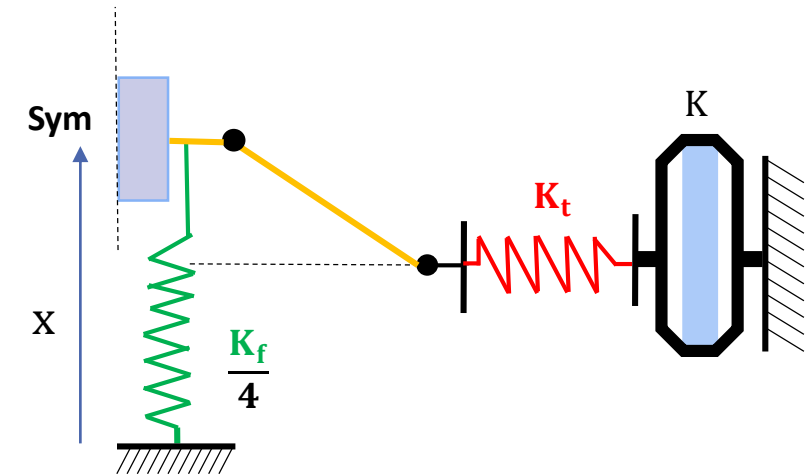


$$E_{beam} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{APA}}{K_t} E_{APA}$$

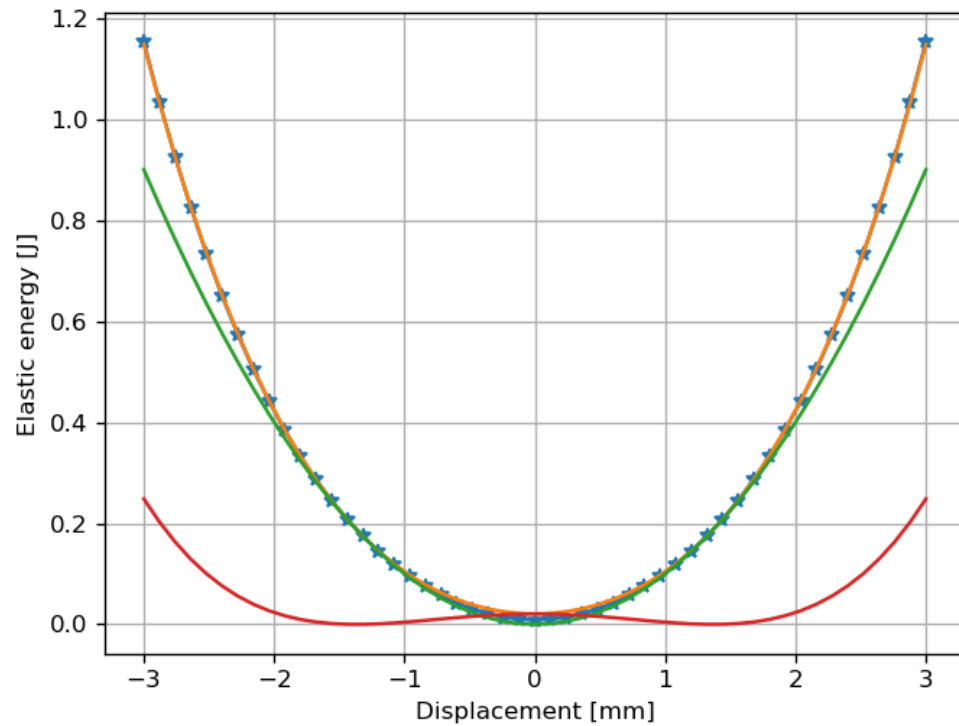


Energy stored in the beam predicted by :

- the FEA
- The lumped model
 - By bending
 - By compression

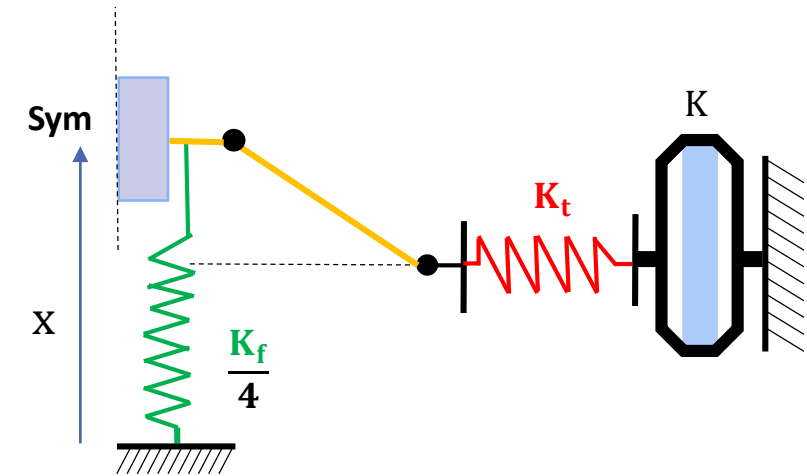


$$E_{\text{beam}} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{\text{APA}}}{K_t} E_{\text{APA}}$$



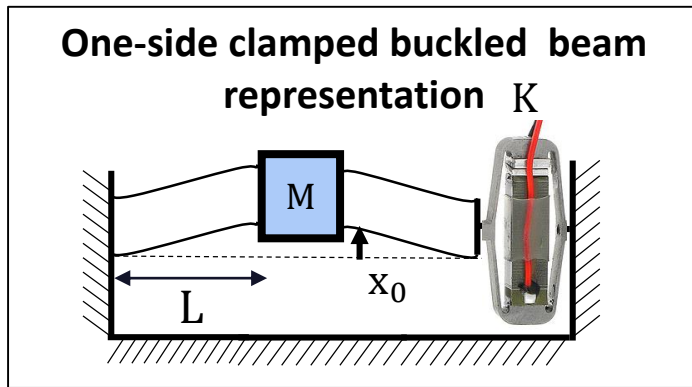
Energy stored in the beam predicted by :

- the FEA
- The lumped model
 - By bending
 - By compression



$$E_{\text{beam}} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{\text{APA}}}{K_t} E_{\text{APA}}$$

➔ The error between the FEA model and the lumped model is less than 10%

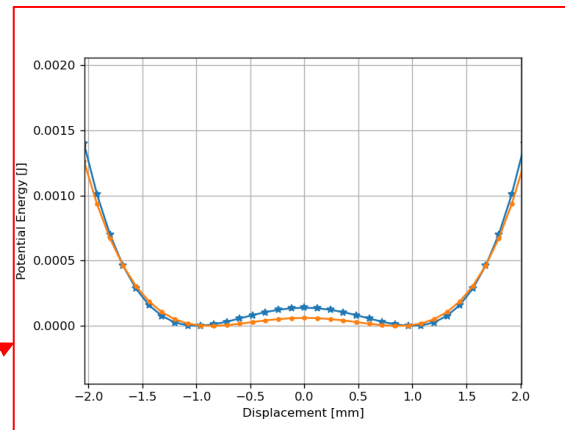
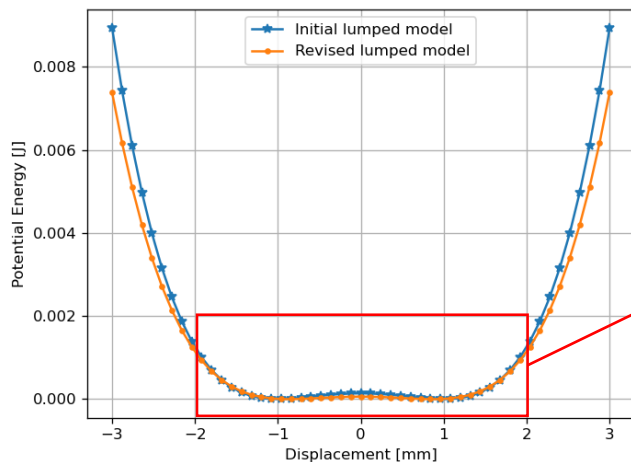


$$M\ddot{y} = M\ddot{x} - \left(2K\beta_{K_t} \frac{x_0^2}{L^2} - K_f \right) x + \frac{2K\beta_{K_t} x^3}{L^2} + D\dot{x} + \frac{2\alpha\beta_{K_t} xv}{L}$$

$$\dot{i} = \frac{2\alpha\beta_{K_t} x \dot{x}}{L} - C_p \dot{v}$$

with : $\beta_{K_t} = \left(\frac{1}{1 + \frac{K}{K_t}} \right)$

Potential energy predicted by the initial lumped model and the revised lumped model for the same buckling level*



Characteristic parameters

- Buckling level : $x_{01} = x_0 \left(\sqrt{1 - \frac{K_f L^2}{K\beta_{K_t} x_0^2}} \right)$
- Angular resonant frequency : $\omega_{01} = \omega_0 \left(\sqrt{1 - \frac{K_f L^2}{K\beta_{K_t} x_0^2}} \right) \sqrt{\beta_{K_t}}$
- Electromechanical coupling: $k_{e1} = k_e \beta_{K_t}$

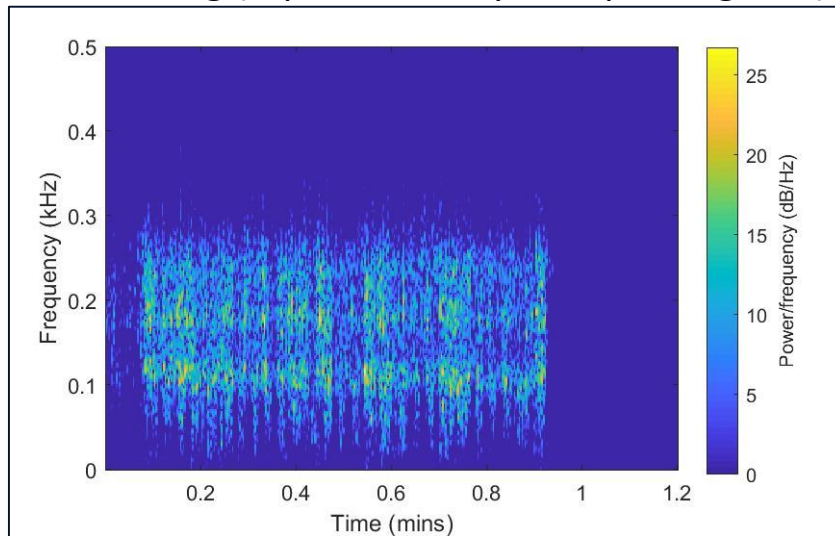
*: in reality, the revised buckling level x_{01} is re-adjusted to the original x_0 by decreasing the L.

➔ Experimental validations of the lumped model will be realized next

This model allows us to:

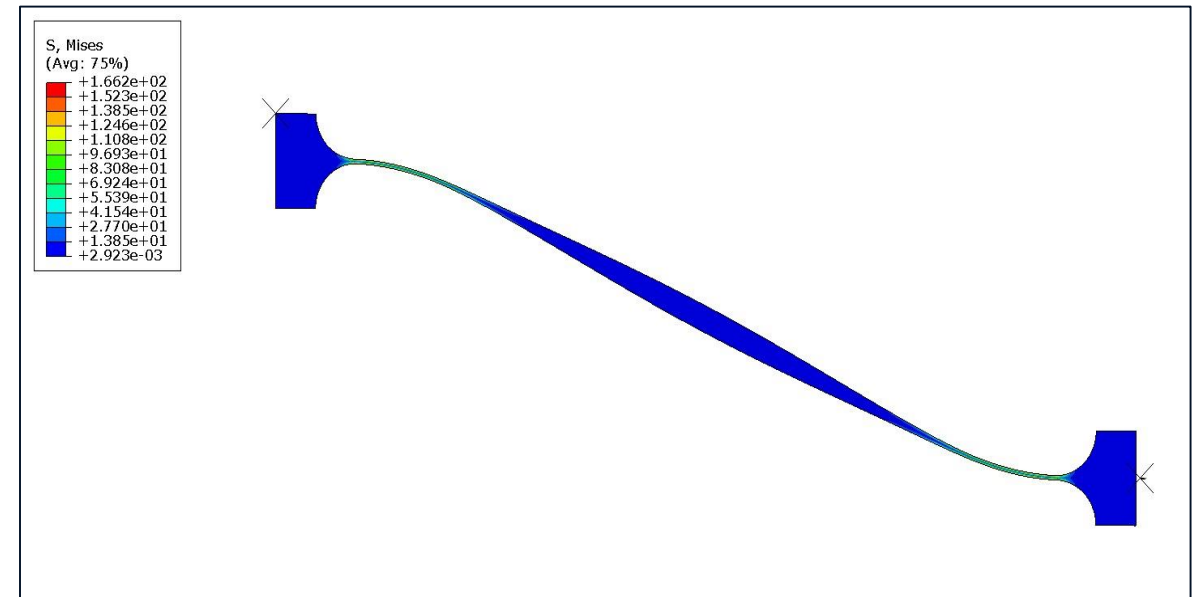
Study the effect of the beams on performance

Example : harvested power for a railway's vibration recording (represented by the spectrogram)



Initial lumped model power output prediction = **1.7 mW**
Revised lumped model power output prediction = **1.6 mW**

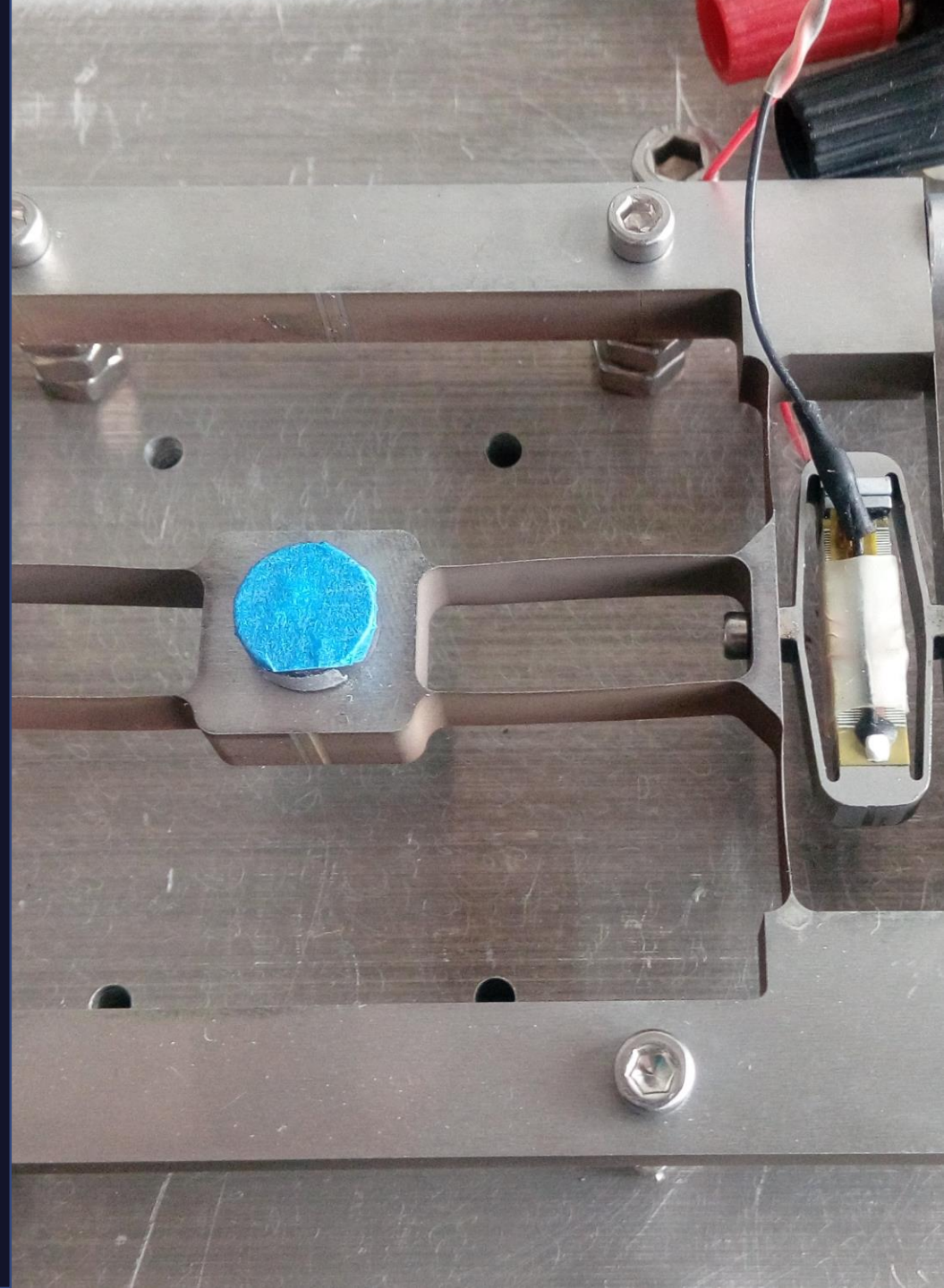
Study complex geometries



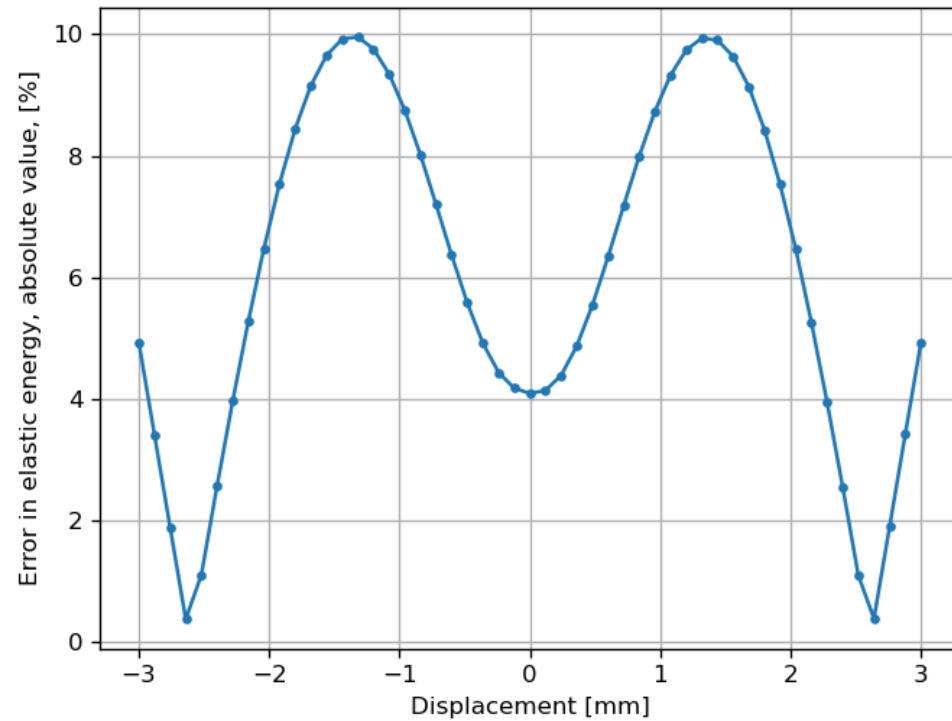
Thank you for your attention



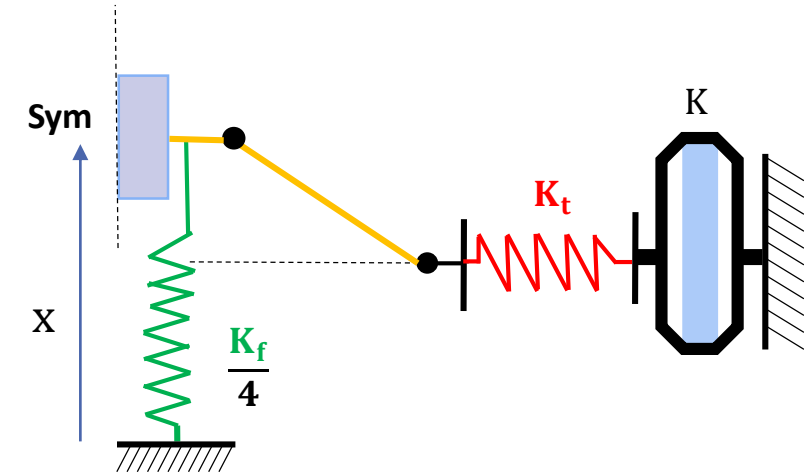
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Appendix 1 : Error between the lumped model and FEA predicted energies



Error between the FEA model and the Lumped model formula for E_{beam}

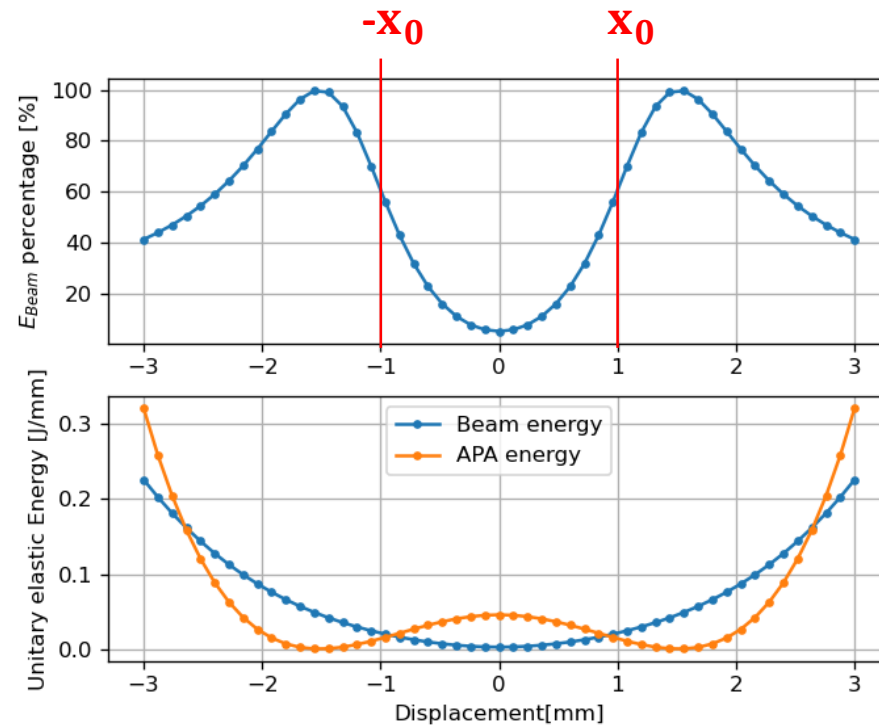


$$E_{\text{beam}} = \frac{1}{2} \left(\frac{K_f}{4} \right) x^2 + \frac{K_{\text{APA}}}{K_t} E_{\text{APA}}$$

➔ The error between the FEA model and the lumped model is less than 10%

Appendix 2 : Beam energy pourcentage on the total elastic energy

Percentage of energy stored in the beam on the total elastic energy of the system



The maximum value reached by the energy percentage for displacements between x_0 and $-x_0$ is 60% !