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Effect of the initial conditions on the performance of a non-harmonically driven bistable Vibration Energy Harvester

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Abstract—This paper presents a study of the behavior of a bistable Vibration Energy Harvester (VEH) subjected to non-harmonic vibrations, for different initial conditions. We show that bistable VEHs are very sensitive to initial conditions even for realistic mechanical vibrations and that they can drastically impact their performance.

I. INTRODUCTION
The response adopted by the bistable energy harvester depends on its initial conditions (initial position and speed of the mass). This dependence has already been studied in literature. For example, criterions were put in place to describe the sensitivity of the snap-through response to perturbations equivalent to changes of initial conditions [1]. This dependence is also studied with attraction basins, which depict the steady-state behavior of the bistable energy harvester as a function of its initial conditions, as proposed by Udani et al. Moreover, an analytical approach was proposed to predict the transient responses for a harmonically forced Duffing oscillator [2] and characterizations of the stability of the snap-through response for a bistable structure subjected to a Gaussian white noise using an analytical criterion have been made [3].

However, for real signals, there is seldom information in the energy harvesting community about the effect of the initial conditions on the response of a bistable structure; the purpose of this paper is to determine the sensitivity of bistable energy harvesters to its initial conditions for real vibration excitation. The excitation studied in this article is railway vibrations.

II. BISTABLE VEH MODELING AND EXCITATION SIGNAL
A representation of the design of the bistable VEH as well as a picture of an existing prototype is represented in Figure 1. The mass’s movements trigger the stress on the Amplified Piezoelectric Actuator (APA), generating voltage. The behavior of the bistable VEH follows the model described by equation 1, provided $x_0 << L$.

It is, in brief, a forced and damped Duffing-type mechanical oscillator equation coupled with a piezoelectric element and a resistance as defined by (Liu et al. 2013).

\[
\begin{align*}
MY &= M\ddot{x} - Kx^3 + D\dot{x} + \frac{2a}{L}x + \frac{2a}{\gamma}x^2 + C_F\dot{x} \\
\frac{d}{dt} &= \frac{2a}{L}x + \frac{2a}{\gamma}x^2 - C_F\dot{x}
\end{align*}
\]

The structural parameters of the bistable VEH that appear in equation 1 are defined in Table 1.

Figure 1 A representation of the bistable VEH’s design (left) and a picture of a bistable VEH prototype (right).

### TABLE 1. THE STRUCTURAL PARAMETERS OF A BISTABLE AND MONOSTABLE VEH

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value for the bistable VEH</th>
<th>Value for the monostable VEH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable equilibrium positions</td>
<td>±$x_0$</td>
<td>0.26 mm</td>
<td>3 mm</td>
</tr>
<tr>
<td>Mass-frame distance</td>
<td>$L$</td>
<td>17 mm</td>
<td>17 mm</td>
</tr>
<tr>
<td>Inertial mass</td>
<td>$M$</td>
<td>29 g</td>
<td>29 g</td>
</tr>
<tr>
<td>Structure stiffness</td>
<td>$K$</td>
<td>0.342 N/µm</td>
<td>0.342 N/µm</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>$D$</td>
<td>0.026 N/m/s</td>
<td>0.026 N/m/s</td>
</tr>
<tr>
<td>Piezoelectric force factor</td>
<td>$\alpha$</td>
<td>0.096 N/V</td>
<td>0.096 N/V</td>
</tr>
<tr>
<td>Piezoelectric clamped capacitance</td>
<td>$C_F$</td>
<td>1.55 µF</td>
<td>1.55 µF</td>
</tr>
<tr>
<td>Load resistance</td>
<td>$R$</td>
<td>6.41 kΩ</td>
<td>0.54 kΩ</td>
</tr>
</tbody>
</table>

In the following study, the generators’ performances are investigated for a reference recording, representative of the real accelerations the generator can be subjected to. Figure 2 represents the Power Spectral Density (PSD) of the reference recording. The APA’s stiffness $K$, the piezoelectric clamped capacitance $C_F$ and force factor $\alpha$ are determined by the choice of Cedrat technology’s amplified piezoelectric actuator APA120S. The product was chosen because it showed satisfying performances for its size [5].

The mass-frame distance $L$ and inertial mass $M$ are fixed to 17 mm and 29 g to stay within the fixed volume requirement of 50 cm$^3$.

The value of the load resistance is set to match the modulus of the output impedance of the harvester:

\[
R = \frac{1}{C_F\omega_0^2}
\]

$\omega_0$ being the resonance frequency of the harvester. (see equation 3)
\[ \omega_0 = \frac{\omega}{L} \sqrt{\frac{4+K}{M}} \]  

As for the damping coefficient \( D \), its value is difficult to predict analytically. An empirical value of 0.026 is used for this performance evaluation, based on [5].

**Figure 2. The PSD of the reference recordings**

**Figure 3. Variation of the energy generated and jump number for different value of the stable equilibrium position**

*Figure 3* represents the energy variation for the fixed parameters previously mentioned, as well as the jump number, which is the number of snap-through movements that occur. The optimal bistable VEH is associated with the optimum of energy for values of \( x_0 \ll L \). We can observe a local extremum of energy in figure 3 at around 0.2 mm. The value of the stable equilibrium position is then fixed to that value. For comparison purposes, the monostable generator for the same fixed inertial mass \( M \), the same APA and the same Mass frame distance \( L \) is considered. The optimum on the right around 3mm in *figure 3* is the optimal monostable generator chosen. In this case, the architecture is still the one of a bistable generator, but for this value of \( x_0 \) the ambient acceleration is not strong enough to trigger the snap-through response and the generator behaves like a linear one oscillating around \( x_0 \). The values of the structural parameters for both generators are cited in Table 1.

**III. THE INITIAL CONDITIONS’ INFLUENCE ON A GENERATOR’S PERFORMANCE FOR A RAILWAYS EXCITATION SIGNAL**

In order to assess the performance of the bistable generator, the normalized average power on \( T = 4 \) seconds (see equation 4) is observed for different initial conditions. The results are obtained through numerical simulations of the previous model and are represented in *figure 4*.

\[ P_{\text{averaged}}(t) = \frac{1}{T} \int_{t}^{t+T} \frac{v(t)^2}{2R} \left( \frac{A_{\text{excitation}}(t)}{A_{\text{excitation}}(t)} \right)^2 dt \]  

**Figure 4. Normalized averaged power for the optimal linear VEH (black) and the optimal bistable VEH for different initial conditions (blue, violet, yellow, green, orange)**

Even if the performances obtained with the monostable generator are superior to those obtained with the bistable generator, it should be remembered that the behavior of the bistable generator being wider band, it is foreseeable that it adapts better to the variations of the ambient vibrations between different recordings. The performance of the bistable and monostable VEH for numerous recordings is to be made in order to determine that.

**IV. CONCLUSIONS AND PERSPECTIVES**

An optimal bistable VEH was determined for a real mechanical vibration corresponding to railway environment. Its performance was then compared to that of a monostable optimal generator with the same dynamic mass and the effect of different initial conditions was observed on the behavior of the bistable generator.

The results show significant change in power levels for different initial conditions for a real mechanical excitation, in the case of the bistable VEH. Although the monostable generator shows a better performance on this particular recording, the effect of the initial conditions in the case of the bistable VEH suggest that the use of techniques such as orbit jumps strategy could improve performances in the case of real mechanical vibrations.

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**REFERENCES**


