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Adaptive Control/Identification for Hybrid Systems, Part II: with Linear-growth-order Discrete Regressor

Mohamed Maghenem Adnane Saoud Antonio Loría

Abstract—In this and the companion paper [1] we propose a direct-adaptive-control framework for hybrid dynamical systems with unknown parameters. The approach addresses both the tracking-control and the parameter-estimation problems and relies on Lyapunov theory for hybrid systems. In this paper, we extend the main results of [1] to deal with hybrid systems that contain a regressor that is of linear order of growth, thereby relaxing the boundedness restriction imposed in [1]. As in the latter reference, the statements rely on Lyapunov theory for hybrid systems and we establish uniform global asymptotic stability in closed loop. In particular, parameter-estimation convergence is guaranteed when a generic hybrid persistence of excitation condition on the pair of discrete and continuous regressor functions holds. On the other hand, the relaxation of the boundedness assumption relies on a higher-order adaptation law.

I. INTRODUCTION

Adaptive control algorithms have been developed in the literature for continuous, discrete, and hybrid systems. In most existing works, the focus is on either state convergence only or parameter estimation only; see for example [2], [3], [4], [5], [6]. In few other works, the tracking-plus-estimation problem is considered [7], [8], [1]. For example, for continuous-time systems, uniform global asymptotic stability of the subset corresponding to a null value of the tracking and estimation errors is established in [7]. Moreover, stability plus convergence to the latter subset, for discrete-time case, is guaranteed in [8], where boundedness of the regressor is required. This requirement is replaced in [4] by a linear growth condition on the regressor function. It is important to note that relaxing the regularities of the regressor function, in the discrete-time case, usually requires a high-dimension adaptation law [3].

Motivated by the aforementioned discussion, in this paper, we revisit the hybrid adaptive-control framework proposed in Part I [1] while relaxing the boundedness of the discrete regressor. That is, we allow the discrete regressor to have a linear growth with respect to the tracking error. We solve the tracking-plus-estimation problem in two steps. First, we design the discrete-time adaptation law based on existing adaptive-control literature for discrete-time systems [3], [4],

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[9]. Furthermore, we propose a new continuous-time adaptation law so that we can guarantee global stability of the closed-loop hybrid system. In addition, under an appropriate hybrid persistence of excitation condition, convergence of the tracking plus estimation errors is ensured. After justifying the lack of uniformity as well as the absence of well-posedness of the resulting closed-loop system, we refine the design of the discrete adaptation law using set-valued maps. Hence, we guarantee well posedness of the resulting closed-loop system and uniform global asymptotic stability of the subset where the tracking-plus-estimation errors are null.

The remainder of this paper is organized as follows. In Section II, we recall some definitions and notations on hybrid systems as per in [10]. In Section III, the problem at hand is formulated. Our main results are presented in Section IV and some concluding remarks are provided in Section V.

II. PRELIMINARIES

Notation. Given a vector x of dimension m_x , and a nonempty set $K \subset \mathbb{R}^{m_x}$, $|x|_K := \inf_{y \in K} |x - y|$ denotes the distance between x and the set K and $U(K)$ denotes an open neighborhood around K . By \mathbb{B} , we denote the closed unit ball in \mathbb{R}^{m_x} centered at the origin. For a map $\phi : \mathbb{R}_{\geq 0} \times \mathbb{N} \rightarrow \mathbb{R}^{m_x} \cup \{\emptyset\}$, $\text{dom } \phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ denotes the domain of definition of ϕ , namely, $\phi(t, j) \neq \emptyset$ if and only if $(t, j) \in \text{dom } \phi$. Moreover, $|\phi|_{\infty} := \sup\{|\phi(t, j)| : (t, j) \in \text{dom } \phi\}$. A continuous function $\beta : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a class \mathcal{K}^{∞} function if it is strictly increasing, $\beta(0) = 0$, and $\lim_{v \rightarrow \infty} \beta(v) = \infty$. A continuous function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a class \mathcal{KL} function if $\beta(t, \cdot) \in \mathcal{K}^{\infty}$ for all $t \geq 0$ and $\lim_{t \rightarrow \infty} \beta(t, v) = 0$ for all $v \geq 0$. For a symmetric semi-positive definite matrices $\Gamma_1, \Gamma_2 \in \mathbb{R}^{m_x \times m_x}$, $\lambda_{\min}(\Gamma_1)$ and $|\Gamma_1| := \lambda_{\max}(\Gamma_1)$ denote the smallest and the largest eigenvalues of Γ_1 , respectively. Furthermore, we write $\Gamma_1 \geq 0$ when $\lambda_{\min}(\Gamma_1) \geq 0$ and $\Gamma_1 > 0$ when $\lambda_{\min}(\Gamma_1) > 0$. Also, we say that $\Gamma_1 \geq \Gamma_2$ if $\Gamma_1 - \Gamma_2 \geq 0$ and $\Gamma_1 > \Gamma_2$ if $\Gamma_1 - \Gamma_2 > 0$. Finally, $F : \mathbb{R}^{m_x} \rightrightarrows \mathbb{R}^{m_x}$ denotes a set-valued map associating each element $x \in \mathbb{R}^n$ to a subset $F(x) \subset \mathbb{R}^{m_x}$. Finally, given a hybrid arc $\Phi : \text{dom } \Phi \rightarrow \mathbb{R}^{m_{\theta} \times m_{\theta}}$, $m_{\theta} \in \{1, 2, \dots\}$ (see Part I [1, Definition 1]), given a positive constant $K \in \mathbb{R}_{>0} \cup \{+\infty\}$ and $(t, j) \in \text{dom } \Phi$, we use $E_{t,j,K}^{\Phi} \subset \text{dom } \Phi$ to denote the shortest hybrid time domain, starting from (t, j) and contained in $\text{dom } \Phi$, of length larger or equal than K . Let m_K be the maximum amount of jumps achieved on $E_{t,j,K}^{\Phi}$; namely, $E_{t,j,K}^{\Phi} := [t_j, t_{j+1}] \times \{j\} \cup \dots \cup [t_{m_K}, t_{m_K+1}] \times \{m_K\} \subset \text{dom } \Phi$.

A. Continuity Properties in Set-Valued Maps

Consider a set-valued map $F : K \rightrightarrows \mathbb{R}^n$, where $K \subset \mathbb{R}^m$.

- The map F is said to be *outer semicontinuous* at $x \in K$ if, for every sequence $\{x_i\}_{i=0}^\infty \subset K$ and for every sequence $\{y_i\}_{i=0}^\infty \subset \mathbb{R}^n$ with $\lim_{i \rightarrow \infty} x_i = x$, $\lim_{i \rightarrow \infty} y_i = y \in \mathbb{R}^n$, and $y_i \in F(x_i)$ for all $i \in \mathbb{N}$, we have $y \in F(x)$; see [10, Definition 5.9].
- The map F is said to be *lower semicontinuous* (or, equivalently, *inner semicontinuous*) at $x \in K$ if for each $\epsilon > 0$ and $y_x \in F(x)$, there exists $U(x)$ satisfying the following property: for each $z \in U(x) \cap K$, there exists $y_z \in F(z)$ such that $|y_z - y_x| \leq \epsilon$.
- The map F is said to be *upper semicontinuous* at $x \in K$ if, for each $\epsilon > 0$, there exists $U(x)$ such that for each $y \in U(x) \cap K$, $F(y) \subset F(x) + \epsilon \mathbb{B}$; see [11, Definition 1.4.1].
- The map F is said to be *continuous* at $x \in K$ if it is both upper and lower semicontinuous at x .

Furthermore, the map F is said to be upper, lower, outer semicontinuous, or continuous if, respectively, it is upper, lower, outer semicontinuous, or continuous for all $x \in K$.

B. Hybrid Inclusions

In this paper, we consider hybrid systems of the form

$$\mathcal{H} : \begin{cases} \dot{x} = F(x) & x \in C \\ x^+ \in G(x) & x \in D, \end{cases} \quad (1)$$

with the state variable $x \in \mathcal{X} \subset \mathbb{R}^{m_x}$, the flow set $C \subset \mathcal{X}$, the jump set $D \subset \mathcal{X}$, the flow and jump maps $F : C \rightarrow \mathbb{R}^{m_x}$ and $G : D \rightrightarrows \mathbb{R}^{m_x}$, respectively.

Definition 1 (Concept of solution to \mathcal{H}): A hybrid arc $\phi : \text{dom } \phi \rightarrow \mathbb{R}^{m_\phi}$ is a *solution* to \mathcal{H} if

(S0) $\phi(0, 0) \in \text{cl}(C) \cup D$;

(S1) for all $j \in \mathbb{N}$ such that $I^j := \{t : (t, j) \in \text{dom } \phi\}$ has nonempty interior, $t \mapsto \phi(t, j)$ is locally absolutely continuous and

$$\begin{aligned} \phi(t, j) &\in C && \text{for all } t \in \text{int}(I^j), \\ \dot{\phi}(t, j) &= F(\phi(t, j)) && \text{for almost all } t \in I^j; \end{aligned}$$

(S2) for all $(t, j) \in \text{dom } \phi$ such that $(t, j+1) \in \text{dom } \phi$, $\phi(t, j) \in D$, $\phi(t, j+1) \in G(\phi(t, j))$.

A hybrid inclusion $\mathcal{H} = (C, F, D, G)$ is well-posed if the following *hybrid basic conditions* are satisfied.

(A1) The sets C and D are closed.

(A2) The flow map $F : C \rightarrow \mathbb{R}^n$ is continuous.

(A3) The jump map $G : D \rightrightarrows \mathbb{R}^n$ is outer semicontinuous, locally bounded, and has nonempty images.

We recall that, in well-posed hybrid systems [10, Definition 6.2], the solutions enjoy very useful robustness properties with respect to perturbations affecting the initial condition [10, Chapter 6].

C. Stability Notions

Inspired by [12], we introduce the global stability (GS) notion for hybrid systems.

Definition 2 (GS): A closed subset $\mathcal{A} \subset \mathcal{X}$ is said to be globally stable (GS) for \mathcal{H} if, for each $\epsilon > 0$, there exists a neighborhood $U(\mathcal{A})$ around \mathcal{A} such that, for each ϕ solution to \mathcal{H} starting from $U(\mathcal{A})$ at $(t_o, j_o) \in \text{dom } \phi$, we have $|\phi(t, j)|_{\mathcal{A}} \leq \epsilon$ for all $(t, j) \in \text{dom } \phi$.

Next, we introduce the global asymptotic stability (GAS) notion for hybrid systems.

Definition 3 (GAS): A closed subset $\mathcal{A} \subset \mathcal{X}$ is said to be globally asymptotically stable (GAS) for \mathcal{H} if the set \mathcal{A} is GS for \mathcal{H} and, for each $\epsilon > 0$, for each complete solution ϕ to \mathcal{H} , and for each $(t_o, j_o) \in \text{dom } \phi$, there exists $(t, j) \in \text{dom } \phi$ such that $t + j \geq t_o + j_o$ and $|\phi(t, j)|_{\mathcal{A}} \leq \epsilon$.

Finally, we refer the reader to Part I [1] for the definitions of uniform global stability (UGS) and uniform global asymptotic stability (UGAS).

III. PROBLEM FORMULATION

As in Part I [1], we introduce the following class of hybrid systems

$$\mathcal{H}_{e\theta} : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \\ \dot{t} \\ \dot{j} \end{bmatrix} = \begin{bmatrix} A_c e + \psi_c(t, j, e) \tilde{\theta} \\ \mathcal{A}_c(t, j, e) \\ 1 \\ 0 \end{bmatrix} & (t, j, e, \tilde{\theta}) \in C_{e\theta} \\ \begin{bmatrix} e^+ \\ \tilde{\theta}^+ \\ t^+ \\ j^+ \end{bmatrix} = \begin{bmatrix} A_d e + \psi_d(t, j, e) \tilde{\theta} \\ \tilde{\theta} + \mathcal{A}_d(t, j, e, e^+) \\ t \\ j + 1 \end{bmatrix} & (t, j, e, \tilde{\theta}) \in D_{e\theta}, \end{cases}$$

where $(t, j, e, \tilde{\theta}) \in \mathcal{X}_{e\theta}$, $\mathcal{X}_{e\theta} := \text{dom } \psi_c (= \text{dom } \psi_d) \times \mathbb{R}^{m_\theta}$, $(A_c, A_d) \in \mathbb{R}^{m_e \times m_e} \times \mathbb{R}^{m_e \times m_e}$, the sets $(C_{e\theta}, D_{e\theta}) \subset \mathcal{X}_{e\theta} \times \mathcal{X}_{e\theta}$ are the flow and the jump sets, respectively, the maps $\mathcal{A}_c : \text{dom } \psi_c \rightarrow \mathbb{R}^{m_\theta}$ and $\mathcal{A}_d : \text{dom } \psi_d \times \mathbb{R}^{m_e} \rightarrow \mathbb{R}^{m_\theta}$ are, respectively, the continuous and the discrete-time adaptation laws to be designed, finally, the hybrid arcs $\psi_c, \psi_d : \text{dom } \psi_d (= \text{dom } \psi_c) \rightarrow \mathbb{R}^{m_e \times m_\theta}$ are, respectively, called *the continuous- and the discrete-time regressor functions*.

Our goal in this paper is to solve the following problem.

Problem 1: Design the adaptation laws \mathcal{A}_c and \mathcal{A}_d such that the resulting hybrid system $\mathcal{H}_{e\theta}$ is well posed and the set $\mathcal{A} := \{(e, \tilde{\theta}, t, j) : e = \tilde{\theta} = 0\}$ is UGAS.

Note that Problem 1 is motivated and compared to related literature in Part I [1]. Furthermore, Problem 1 is solved therein under the assumption that ψ_d is globally bounded. In this paper, we relax such a requirement by allowing ψ_d to have a linear growth rate with respect to e . More precisely, we consider the following assumption.

Assumption 1: There exist $c_1, c_2 \geq 0$ such that

$$|\psi_d(t, j, e)| \leq c_1 + c_2|e| \quad \forall (t, j, e, \tilde{\theta}) \in D_{e\theta}.$$

Remark 1: Assumption 1 is also used in the discrete-time adaptive scheme proposed in [4]. However, only (non-uniform) global stability of \mathcal{A} plus (non-uniform) convergence of e are established. The same result is obtained in [3] without restricting the growth of ψ_d . However, the adaptation law used therein is, relatively, of higher dimension and the stability conclusions are weaker than those established in the paper.

The following additional assumptions are used in both the current paper and its companion [1]. Furthermore, they are thoroughly discussed and motivated in the aforementioned reference, but we repeat them here for convenience.

Assumption 2: There exist $P, Q \in \mathbb{R}^{m_e \times m_e}$ symmetric and positive definite such that

$$\begin{aligned} A_c^\top P + PA_c &\leq -Q, \\ A_d^\top PA_d - P &\leq -Q. \end{aligned} \quad (2)$$

Assumption 3 (Regularity of ψ_c and ψ_d): The maps ψ_c and ψ_d are continuous.

Next, we specify the following functions

$$\begin{aligned} \phi_c(t, j) &:= \psi_c(t, j, 0), \\ \phi_d(t, j) &:= \psi_d(t, j, 0), \\ \Delta_c(t, j, e) &:= \psi_c(t, j, e) - \psi_c(t, j, 0), \\ \Delta_d(t, j, e) &:= \psi_d(t, j, e) - \psi_d(t, j, 0). \end{aligned}$$

Assumption 4 (Regularity of ϕ_c): For each $j \in \mathbb{N}$, the map $t \mapsto \phi_c(t, j)$ is absolutely continuous on

$$I^j := \{t : (t, j) \in \text{dom } \phi_c\}.$$

Furthermore, there exists $\bar{\phi} > 0$ such that

$$\begin{aligned} \sup_{(t, j) \in \text{dom } \phi_c} \{|\phi_c(t, j)|\} &\leq \bar{\phi} \\ \text{ess sup}_{t \in I^j} \{|\dot{\phi}_c(t, j)|\} &\leq \bar{\phi}. \end{aligned}$$

Assumption 5 (Growth rate of Δ_c and Δ_d): There exists a positive non-decreasing function $\kappa_o : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ such that, for each $(t, j, e) \in \text{dom } \psi_c = \text{dom } \psi_d$, we have

$$\max\{|\Delta_c(t, j, e)|, |\Delta_d(t, j, e)|\} \leq \kappa_o(|e|)|e|.$$

Assumption 6 (HPE of (ϕ_c, ϕ_d)): There exist $K, \mu > 0$ such that, for each hybrid time domain

$$\bigcup_{j=j_o}^J ([t_j, t_{j+1}] \times \{j\}) \subset \text{dom } \phi_c$$

such that $(t_{J+1} - t_o) + (J - j_o) \geq K$, we have

$$\begin{aligned} \sum_{j=j_o}^J \int_{t_j}^{t_{j+1}} \phi_c(s, j)^\top \phi_c(s, j) ds + \sum_{j=j_o}^J \phi_d(t_{j+1}, j)^\top \phi_d(t_{j+1}, j) \\ \geq \mu I_{m_\theta}. \end{aligned}$$

IV. MAIN RESULTS

We contribute with two adaptive controllers that solve Problem 1. The interest of the first is that it consists in a rather natural design, inspired from classical adaptive-control algorithms and in particular from [4], but it is tailored for hybrid systems. However, it leads to non-uniform stability properties. The second is a refinement of the first, for which UGAS is guaranteed.

A. Primary Solution

Given $\gamma_o > 0$, we introduce the set

$$\mathcal{G}_o := \{\Gamma \in \mathbb{R}^{m_\theta \times m_\theta} : \Gamma = \Gamma^\top > 0, |\Gamma| \leq \gamma_o\}.$$

Now, we introduce the adaptation laws

$$\begin{aligned} \mathcal{A}_c(t, j, e) &:= -\frac{\mu}{c} \Gamma \frac{\psi_c(t, j, e)^\top}{1 + \mu e^\top P e} P e \\ \mathcal{A}_d(t, j, e, e^+) &:= -G_{o\Gamma}(t, j, e, \Gamma) \psi_d(t, j, e)^\top (e^+ - A_d e), \end{aligned} \quad (3)$$

where

$$G_{o\Gamma}(t, j, e, \Gamma) := \Gamma - \Gamma \psi_d^\top [I_{m_e} + \psi_d \Gamma \psi_d^\top]^- \psi_d \Gamma, \quad (4)$$

and $\Gamma \in \mathcal{G}_o$ is a state variable governed by

$$\begin{cases} \dot{\Gamma} = 0 & \text{if } (t, j, e, \tilde{\theta}) \in C_{e\theta} \\ \Gamma^+ = G_{o\Gamma}(t, j, e, \Gamma) & \text{if } (t, j, e, \tilde{\theta}) \in D_{e\theta}. \end{cases} \quad (5)$$

Remark 2: Note that, for each $(t, j, e, \Gamma) \in \text{dom } \psi_d \times \mathcal{G}_o$, we have that

$$0 < G_{o\Gamma}(t, j, e, \Gamma) \leq \Gamma.$$

Indeed, while the right inequality is straightforward, to deal with the left inequality, we use the Woodbury matrix identity [13] to obtain

$$G_{o\Gamma}(t, j, e, \Gamma)^- = \Gamma^- + \psi_d^\top \psi_d.$$

Hence, by definition of the set \mathcal{G}_o , we conclude that $G_{o\Gamma}(t, j, e, \Gamma) \in \mathcal{G}_o$, for all $\Gamma \in \mathcal{G}_o$. Furthermore, under Assumption 3, we conclude that $G_{o\Gamma}$ is continuous on $\text{dom } \psi_d \times \mathcal{G}_o$.

Remark 3: The proposed discrete adaptation \mathcal{A}_d is designed as in [4]. We design \mathcal{A}_c so that we can find a Lyapunov function that is non-increasing along the jumps as well as the flows of $\mathcal{H}_{e\theta}$ in spite of the relaxed Assumption; see Proposition 1.

The resulting closed-loop system is governed by the hybrid dynamics

$$\mathcal{H}_{e\theta}^a : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \\ \dot{\Gamma} \\ \dot{t} \\ \dot{j} \end{bmatrix} = \begin{bmatrix} A_c e + \psi_c(t, j, e) \tilde{\theta} \\ -\frac{\mu}{c} \Gamma \frac{\psi_c(t, j, e)^\top}{1 + \mu e^\top P e} P e \\ 0 \\ 1 \\ 0 \end{bmatrix} & (t, j, e, \tilde{\theta}, \Gamma) \in C_{e\theta}^a \\ \begin{bmatrix} e^+ \\ \tilde{\theta}^+ \\ \Gamma^+ \\ t^+ \\ j^+ \end{bmatrix} = \begin{bmatrix} A_d e + \psi_d(t, j, e) \tilde{\theta} \\ \tilde{\theta} - G_{o\Gamma}(t, j, e, \Gamma) \psi_d(t, j, e)^\top (e^+ - A_d e) \\ G_{o\Gamma}(t, j, e, \Gamma) \\ t \\ j + 1 \end{bmatrix} & (t, j, e, \tilde{\theta}, \Gamma) \in D_{e\theta}^a \end{cases}$$

where $C_{e\theta}^a := C_{e\theta} \times \mathcal{G}_o$ and $D_{e\theta}^a := D_{e\theta} \times \mathcal{G}_o$.

In the sequel, we propose a Lyapunov-based approach to analyze stability of the set \mathcal{A} given by

$$\mathcal{A} := \{(t, j, e, \tilde{\theta}, \Gamma) \in \mathcal{X}_{e\theta} \times \mathcal{G}_o : e = \tilde{\theta} = 0\}. \quad (6)$$

Theorem 1: Consider system $\mathcal{H}_{e\theta}^a$ such that Assumptions 1-6 hold. Let the parameters (c, μ) be such that

$$\mu \geq \frac{2c_2^2 \gamma_o}{|P| (1 + 2\gamma_o c_1^2)} \quad (7a)$$

$$c \geq 2(1 + 2\gamma_o c_1^2) \left(\mu + 2 \left| Q^{-1/2} A_d^\top P^{1/2} \right|^2 \right) |P|, \quad (7b)$$

where $((c_1, c_2), (P, Q))$ come from Assumption 1 and Assumption 2, respectively. Then, the set \mathcal{A} is GAS for $\mathcal{H}_{e\theta}^a$.

B. Refining the Primary Solution

The solution proposed in the previous section has two drawbacks. First, the stability guarantees in Theorem 1 are not uniform. Second, the hybrid system $\mathcal{H}_{e\theta}^a$ is not well posed. In particular, the sets $C_{e\theta}^a$ and $D_{e\theta}^a$ are not closed since the set \mathcal{G}_o is not so. To handle the latter two issues, we propose to redefine the set \mathcal{G}_o as a closed subset that we denote by \mathcal{G}_1 . Furthermore, we modify the update law of Γ to guarantee that it remains in the new closed subset \mathcal{G}_1 after the jumps. That is, for some constants $\gamma_o, \gamma_1 > 0$, we introduce the set

$$\mathcal{G}_1 := \{\Gamma \in \mathcal{G}_o : \Gamma \geq \gamma_1 I_{m_\theta}\}.$$

Furthermore, we introduce the set-valued map $G_{1\Gamma} : \text{dom } \psi_d \times \mathcal{G}_1 \rightrightarrows \mathcal{G}_1$ given by

$$G_{1\Gamma}(t, j, e, \Gamma) := \{M \in \mathcal{G}_1 : M \geq G_{o\Gamma}(t, j, e, \Gamma)\},$$

where $G_{o\Gamma}(t, j, e, \Gamma)$ is given in (4). The resulting closed-loop system is governed by the hybrid dynamics

$$\mathcal{H}_{e\theta}^b : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \\ \dot{\Gamma} \\ \dot{t} \\ \dot{j} \end{bmatrix} = \begin{bmatrix} A_c e + \psi_c(t, j, e) \tilde{\theta} \\ -\frac{\mu}{c} \Gamma \frac{\psi_c(t, j, e)^\top}{1 + \mu e^\top P e} P e \\ 0 \\ 1 \\ 0 \end{bmatrix} & (t, j, e, \tilde{\theta}, \Gamma) \in C_{e\theta}^b \\ \begin{bmatrix} e^+ \\ \tilde{\theta}^+ \\ \Gamma^+ \\ t^+ \\ j^+ \end{bmatrix} \in \begin{bmatrix} A_d e + \psi_d(t, j, e) \tilde{\theta} \\ \tilde{\theta} - G_{o\Gamma}(t, j, e, \Gamma) \psi_d(t, j, e)^\top (e^+ - A_d e) \\ G_{1\Gamma}(t, j, e, \Gamma) \\ t \\ j + 1 \end{bmatrix} & (t, j, e, \tilde{\theta}, \Gamma) \in D_{e\theta}^b \end{cases}$$

where $C_{e\theta}^b := C_{e\theta} \times \mathcal{G}_1$ and $D_{e\theta}^b := D_{e\theta} \times \mathcal{G}_1$.

Lemma 1: Under Assumption 3, $G_{1\Gamma}$ is outer semicontinuous, locally bounded, and has nonempty images.

Theorem 2: Consider system $\mathcal{H}_{e\theta}^b$ under Assumptions 1-6 hold. Let the parameters (c, μ) satisfy (7). Then, $\mathcal{H}_{e\theta}^b$ is well posed and the set \mathcal{A} in (6) is UGAS.

C. Proof of Theorem 1

To avoid heavy notations, when the arguments in the functions (ψ_c, ψ_d) , (ϕ_c, ϕ_d) are omitted, it means that they are evaluated at (t, j, e) and (t, j) , respectively.

1) *Global Stability:* To establish GS of the set \mathcal{A} for $\mathcal{H}_{e\theta}^a$, we use the following intermediate result.

Proposition 1: Consider system $\mathcal{H}_{e\theta}^a$ such that Assumptions 1 and 2 hold. Consider the Lyapunov function candidate

$$V(e, \tilde{\theta}, \Gamma) := \ln(1 + \mu e^\top P e) + c \tilde{\theta}^\top \Gamma^- \tilde{\theta}, \quad (8)$$

where the parameters μ and c satisfy (7). Then, along each solution ϕ to $\mathcal{H}_{e\theta}^a$, we have

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have

$$\dot{V}(e, \tilde{\theta}, \Gamma) \leq -\frac{\mu e^\top Q e}{1 + \mu e^\top P e} \quad \text{for almost all } t \in I^j.$$

- For each $((t, j), (t, j+1)) \in \text{dom } \phi \times \text{dom } \phi$, we have

$$\begin{aligned} \Delta V &:= V(e^+, \tilde{\theta}^+, \Gamma^+) - V(e, \tilde{\theta}, \Gamma) \\ &\leq -\frac{\mu}{2} \frac{e^\top Q e}{1 + \mu e^\top P e} - \frac{c}{2} \tilde{\theta}^\top \psi_d^\top [I_{m_e} + \psi_d \Gamma \psi_d^\top]^- \psi_d \tilde{\theta}. \end{aligned}$$

According to Proposition 1 and along each solution ϕ to $\mathcal{H}_{e\theta}^a$, starting from $\phi_o := (t_o, j_o, e_o, \tilde{\theta}_o, \Gamma_o)$, we have

$$\begin{aligned} V(e(t, j), \tilde{\theta}(t, j), \Gamma(t, j)) &\leq V(e_o, \tilde{\theta}_o, \Gamma_o) \\ &\leq \ln(1 + \mu e_o^\top P e_o) + c \tilde{\theta}_o^\top \Gamma_o^- \tilde{\theta}_o \end{aligned}$$

for all $(t, j) \in \text{dom } \phi$. At the same time, the function V

satisfies the inequality

$$V(e, \tilde{\theta}, \Gamma) \geq \ln(1 + \mu e^\top P e) + \frac{c}{\gamma_o} \tilde{\theta}^\top \tilde{\theta}.$$

Hence, there exists a class \mathcal{K}^∞ function κ , such that

$$V(e, \tilde{\theta}, \Gamma) \geq \kappa\left(|t, j, e, \tilde{\theta}, \Gamma|_{\mathcal{A}}^2\right) = \kappa\left(|e, \tilde{\theta}|^2\right). \quad (9)$$

As a result, for all $(t, j) \in \text{dom } \phi$, we have

$$|(e(t, j), \tilde{\theta}(t, j))|^2 \leq \kappa^{-1}\left(V(e_o, \tilde{\theta}_o, \Gamma_o)\right). \quad (10)$$

The latter implies that, for all $(t, j) \in \text{dom } \phi$, we have

$$\begin{aligned} |(e(t, j), \tilde{\theta}(t, j))|^2 &\leq \kappa^{-1}\left(\ln(1 + \mu e_o^\top P e_o) + c \tilde{\theta}_o^\top \Gamma_o^{-1} \tilde{\theta}_o\right) \\ &\leq \kappa^{-1}\left(\left[\mu |P| + \frac{c}{\lambda_{\min}(\Gamma_o)}\right] |(e_o, \tilde{\theta}_o)|^2\right). \end{aligned}$$

The latter inequality is enough to conclude global stability of the set \mathcal{A} according to Definition 2.

Remark 4: Note that we cannot show UGS since we cannot upper bound the term $\frac{c}{\lambda_{\min}(\Gamma_o)}$ by a positive constant.

2) Global Asymptotic Attractivity: In this section, we establish that, for each $\epsilon > 0$, the complete solutions to $\mathcal{H}_{e\theta}^a$ reach the set $\mathcal{A} + \epsilon\mathbb{B}$. For this, given $V_o > 0$, we introduce the Lyapunov-like function given by

$$\begin{aligned} Z(t, j, e, \tilde{\theta}, \Gamma) &:= \rho_1(V_o)V(e, \tilde{\theta}, \Gamma) - \rho_2(V_o)e^\top \phi_c(t, j)\tilde{\theta} \\ &\quad - \tilde{\theta}^\top Q_\Phi(t, j, \infty)\tilde{\theta}, \end{aligned}$$

where $\rho_1, \rho_2 : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{> 0}$ are continuous non-decreasing functions to be specified later, the hybrid arc Q_Φ is given by

$$\begin{aligned} Q_\Phi(t, j, \infty) &:= \sum_{i=j}^{m_\infty} \int_{t_i}^{t_{i+1}} \exp^{(t+j)-(s+i)} \Phi(s, i) ds \\ &\quad + \frac{1}{2} \sum_{i=j}^{m_\infty} \left[\int_{t_{i+1}}^{t_{i+2}} \exp^{(t+j)-(s+i)} ds \right] \Phi(t_{i+1}, i), \end{aligned} \quad (11)$$

$$\Phi := \phi_{cd} \phi_{cd}^\top, \text{ and } \phi_{cd}(t, j) := \begin{cases} \phi_c(t, j) & \text{if } t \in \text{int}(I_j) \\ \phi_d(t, j) & \text{otherwise,} \end{cases}$$

where m_∞ is the number of jumps of the hybrid arc Φ . Note that key properties of the arc $Q_\Phi(\cdot, \infty)$ are in [1]. Furthermore, we rely on the following key result.

Proposition 2: Consider system $\mathcal{H}_{e\theta}^a$ such that Assumptions 1-6 and (7) hold. Then, given $V_o > 0$, there exist positive constants (λ_c, λ_d) such that, along every solution ϕ to $\mathcal{H}_{e\theta}^a$ starting from $(e_o, \tilde{\theta}_o, \Gamma_o, t_o, j_o)$ with $V(e_o, \tilde{\theta}_o, \Gamma_o) \leq V_o$, it holds that:

- for each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have

$$\dot{Z}(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_c |e, \tilde{\theta}|^2 \quad \text{for almost all } t \in I^j,$$

- for each $((t, j), (t, j+1)) \in \text{dom } \phi \times \text{dom } \phi$, we have

$$Z(t, j+1, e^+, \tilde{\theta}^+, \Gamma^+) - Z(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_d |e, \tilde{\theta}|^2.$$

At this point, we introduce the function

$$\bar{Z}(t, j, e, \tilde{\theta}, \Gamma) := Z(t, j, e, \tilde{\theta}, \Gamma) + \rho_3(V_o)V(e, \tilde{\theta}, \Gamma),$$

where

$$\begin{aligned} \rho_3(V_o) &:= \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left(\frac{\exp^{V_o}}{\mu} + \frac{1}{c} \right) \rho_2(V_o) \\ &\quad + \gamma_o |Q_\Phi(\cdot, \infty)|_\infty. \end{aligned} \quad (12)$$

Remark 5: Using Propositions 1 and 2, we can easily see that along every solution ϕ to $\mathcal{H}_{e\theta}^a$ starting from $(e_o, \tilde{\theta}_o, \Gamma_o, t_o, j_o)$ with $V(e_o, \tilde{\theta}_o, \Gamma_o) \leq V_o$, we have

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have

$$\dot{\bar{Z}}(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_c |(e, \tilde{\theta})|^2 \quad \text{for almost all } t \in I^j.$$

- For each $((t, j), (t, j+1)) \in \text{dom } \phi \times \text{dom } \phi$, we have

$$\bar{Z}(t, j^+, e^+, \tilde{\theta}^+, \Gamma^+) - \bar{Z}(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_d |(e, \tilde{\theta})|^2.$$

Next, we show the existence of $\underline{\rho}, \bar{\rho} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{> 0}$ continuous and non-decreasing functions such that

$$\underline{\rho}(V_o)V \leq \bar{Z} \leq \bar{\rho}(V_o)V. \quad (13)$$

Indeed, using Young's inequality, we note that

$$\begin{aligned} \bar{Z} &\leq (\rho_1(V_o) + \rho_3(V_o))V - \rho_2(V_o)e^\top \phi_c \tilde{\theta} \\ &\leq (\rho_1(V_o) + \rho_3(V_o))V \\ &\quad + \rho_2(V_o) \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left[e^\top P e + \tilde{\theta}^\top \Gamma^{-1} \tilde{\theta} \right]. \end{aligned}$$

Next, when $V(e, \tilde{\theta}, \Gamma) \leq V_o$, and using the fact that $x \leq (1+x)\ln(1+x)$ for all $x \geq 0$, we conclude that $e^\top P e \leq \frac{\exp^{V_o}}{\mu} \ln(1 + \mu e^\top P e)$. As result, we have that

$$\begin{aligned} \bar{Z} &\leq (\rho_1(V_o) + \rho_3(V_o))V - \rho_2(V_o)e^\top \phi_c \tilde{\theta} \\ &\leq (\rho_1(V_o) + \rho_3(V_o))V + \rho_2(V_o) \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \\ &\quad \times \left[\frac{\exp^{V_o}}{\mu} \ln(1 + \mu e^\top P e) + \tilde{\theta}^\top \Gamma^{-1} \tilde{\theta} \right] \\ &\leq (\rho_1(V_o) + \rho_3(V_o))V \\ &\quad + \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left(\frac{\exp^{V_o}}{\mu} + \frac{1}{c} \right) \rho_2(V_o)V. \end{aligned} \quad (14)$$

Hence, we can choose $\bar{\rho}$ as

$$\begin{aligned} \bar{\rho}(V_o) &:= \rho_1(V_o) + \rho_3(V_o) \\ &\quad + \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left(\frac{\exp^{V_o}}{\mu} + \frac{1}{c} \right) \rho_2(V_o). \end{aligned}$$

On the other hand, we note that

$$\begin{aligned} \bar{Z} &\geq (\rho_1(V_o) + \rho_3(V_o))V - \rho_2(V_o)e^\top \phi_c \tilde{\theta} \\ &\quad - \tilde{\theta}^\top Q_\Phi(t, j, \infty)\tilde{\theta} \\ &\geq (\rho_1(V_o) + \rho_3(V_o))V - \gamma_o |Q_\Phi(t, j, \infty)| \tilde{\theta}^\top \Gamma^{-1} \tilde{\theta} \\ &\quad - \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left(\frac{\exp^{V_o}}{\mu} + \frac{1}{c} \right) \rho_2(V_o)V \\ &\geq (\rho_1(V_o) + \rho_3(V_o))V \\ &\quad - \bar{\phi} \gamma_o^{1/2} |P^{-1/2}| \left(\frac{\exp^{V_o}}{\mu} + \frac{1}{c} \right) \rho_2(V_o)V, \end{aligned}$$

where the second inequality comes from (14). Hence, by taking ρ_3 as in (12), we conclude that $\bar{Z} \geq \rho_1(V_o)V =$

$\rho(V_o)V$. To complete the proof, we use a contradiction argument. That is, we assume the existence of a complete solution $\phi := (e, \tilde{\theta}, \Gamma, t, j)$ to $\mathcal{H}_{e\tilde{\theta}}^a$ starting from $\phi_o := (e_o, \tilde{\theta}_o, \Gamma_o, t_o, j_o)$ with $V(e_o, \tilde{\theta}_o, \Gamma_o) \leq V_o$ and $\epsilon > 0$ such that

$$|\phi(t, j)|_{\mathcal{A}}^2 = |(e, \tilde{\theta})|^2 \geq \epsilon \quad \forall (t, j) \in \text{dom } \phi. \quad (15)$$

Now, combining (15) and Remark 5, we conclude that, along the solution ϕ , the following properties hold.

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have

$$\dot{\bar{Z}}(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_c \epsilon \quad \text{for a.a. } t \in I^j.$$

- For each $((t, j), (t, j + 1)) \in \text{dom } \phi \times \text{dom } \phi$, we have

$$\bar{Z}(t, j + 1, e^+, \tilde{\theta}^+, \Gamma^+) - \bar{Z}(t, j, e, \tilde{\theta}, \Gamma) \leq -\lambda_d \epsilon.$$

The latter two items imply that \bar{Z} must reach zero in finite (hybrid) time. Moreover, using (13), we conclude that V along the solution ϕ must also reach zero in finite (hybrid) time. Finally, using (9), we conclude that $|\phi|_{\mathcal{A}}$ must also reach the origin in finite (hybrid) time, which contradicts (15).

D. Proof of Theorem 2

Under Assumption 3 and using Lemma 1, we conclude that $\mathcal{H}_{e\tilde{\theta}}^b$ is well posed.

Next, to prove UGS, we note that the only difference between $\mathcal{H}_{e\tilde{\theta}}^b$ and $\mathcal{H}_{e\tilde{\theta}}^a$ is along the jumps; more precisely, in the discrete update of Γ . As a result, to find an upper-bound to ΔV along the jumps, we have to analyze the difference

$$\tilde{\theta}^{+\top}(\Gamma^+) - \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta} = \tilde{\theta}^{+\top} M^- \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta}, \quad (16)$$

where $M \in G_{1\Gamma}(t, j, e, \tilde{\theta}, \Gamma)$. Note that, in the proof of Proposition 1 and along the jumps of $\mathcal{H}_{e\tilde{\theta}}^a$, we had

$$\tilde{\theta}^{+\top}(\Gamma^+) - \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta} = \tilde{\theta}^{+\top}(G_{o\Gamma}(\cdot))^- \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta}.$$

Now, if we re-express (16) as

$$\begin{aligned} \tilde{\theta}^{+\top}(\Gamma^+) - \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta} &= \tilde{\theta}^{+\top} M^- \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta} \\ &= \tilde{\theta}^{+\top}(M^- - (G_{o\Gamma}(\cdot))^-) \tilde{\theta}^+ + \tilde{\theta}^{+\top}(G_{o\Gamma}(\cdot))^- \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta}, \end{aligned}$$

and we use the Loewner order of symmetric positive definite matrices [14], we conclude that $(M^- - (G_{o\Gamma}(\cdot))^-) \leq 0$. Hence, along the jumps of $\mathcal{H}_{e\tilde{\theta}}^b$, we have

$$\tilde{\theta}^{+\top}(\Gamma^+) - \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta} \leq \tilde{\theta}^{+\top}(G_{o\Gamma}(\cdot))^- \tilde{\theta}^+ - \tilde{\theta}^\top \Gamma^- \tilde{\theta}.$$

As a result, the statement of Proposition 1 remains valid if we replace $\mathcal{H}_{e\tilde{\theta}}^a$ therein by $\mathcal{H}_{e\tilde{\theta}}^b$. Furthermore, we note, in view of Remark 4, that the Lyapunov function candidate V in (8) is now proper; namely, we can find class \mathcal{K}^∞ functions $\bar{\lambda}$ and $\underline{\lambda}$ such that

$$\underline{\lambda}(|e, \tilde{\theta}|^2) \geq V(e, \tilde{\theta}) \leq \bar{\lambda}(|e, \tilde{\theta}|^2).$$

Hence, UGS follows using [1, Theorem 1].

Finally, using the exact same computations, we can show that Proposition 2 remains valid if we replace $\mathcal{H}_{e\tilde{\theta}}^a$ therein by $\mathcal{H}_{e\tilde{\theta}}^b$, which is enough to prove UGAS of the set \mathcal{A} . ■

V. CONCLUSION

In this work, we revisited the hybrid adaptive-control framework proposed in Part I [1]. More specifically, we relaxed the boundedness of ψ_d by allowing a linear growth with respect to e . As a result, a high-order adaptation law that is set valued is proposed. The resulting adaptive-control scheme guarantees attraction of the closed-loop tracking-plus-estimation errors, under a suitable hybrid persistence of excitation condition.

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