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Adaptive Control/Identification for Hybrid Systems Part I: with Bounded Discrete Regressor

Mohamed Maghenem Adnane Saoud Antonio Loría

***Abstract*— In this and the companion paper [1], we propose a direct-adaptive-control framework for hybrid dynamical systems with unknown parameters. The approach addresses both the tracking-control and the parameter-estimation problems and relies on Lyapunov theory for hybrid systems. In this paper, we address these problems for systems with bounded discrete regressor. This assumption may appear restrictive at first sight, but its interest lies in that it yields a fairly simple static discrete adaptation law. Furthermore, parameter-estimation convergence is guaranteed under an original property of persistency of excitation tailored for hybrid systems, which is introduced here. The main results rely on Lyapunov theory for hybrid systems and establish uniform global asymptotic stability for the resulting closed-loop hybrid system. In the companion paper [1] the boundedness assumption on the regressor is relaxed and a high-order discrete adaptation law is proposed.**

I. Introduction

Direct adaptive control is a well-understood and popular method used when stabilizing a plant whose constant parameters are unknown. The related literature in both continuous and discrete-time systems is very rich and different stability and convergence statements are available [2], [3]. Roughly speaking, given a nonlinear function $f : \mathbb{R}^{m_x} \times \mathbb{R}^{m_u} \times \mathbb{R}^{m_\theta} \to \mathbb{R}^{m_x}$ representing the dynamics of a continuous or a discrete-time system. The function f involves the system's state vector $x \in \mathbb{R}^{m_x}$, the input $u \in \mathbb{R}^{m_u}$, and the unknown constant parameters $\theta \in \mathbb{R}^{m_\theta}$. Given a reference trajectory, $r : \operatorname{dom} r \to \mathbb{R}^{m_x}$, where $\operatorname{dom} r := \mathbb{N}$ in the discrete-time case and $\operatorname{dom} r := \mathbb{R}_{\geq 0}$ in the continuous-time case, it is usually assumed that one can design a feedback law $\kappa : \operatorname{dom} r \times \mathbb{R}^{m_y} \times \mathbb{R}^{m_\theta} \to \mathbb{R}^{m_u}$, function of the reference r, the system's output $y \in \mathbb{R}^{m_y}$, and the parameters' estimate $\hat{\theta} \in \mathbb{R}^{m_\theta}$ such that, when substituting $u = \kappa$ in f and using the tracking and estimation errors $e := x - r$ and $\tilde{\theta} := \hat{\theta} - \theta$, respectively, we obtain

$$f(x, \kappa(t, y, \hat{\theta}), \theta) := \dot{r} + Ae + \psi(t, e)\tilde{\theta} \tag{1}$$

in continuous time and

$$f(x, \kappa(j, y, \hat{\theta}), \theta) := r(j+1) + Ae + \psi(j, e)\tilde{\theta} \tag{2}$$

in discrete time. The matrix A is usually Hurwitz and the function ψ defines what is called a *regressor function*. Note that in some works the forms in (1) and (2) are expressed in terms of Lyapunov inequalities [4], [5], which allows f to have a more general structure in closed loop. The objective of direct adaptive-control is to design an update law for $\hat{\theta}$ called *adaptation law* such that the resulting closed-loop system is stable and either e, $\tilde{\theta}$, or both converge to the origin.

This problem is solved for discrete- and continuous-time systems in [4] and [6], respectively, where the tracking-plus-estimation problem is considered. In [4], uniform global asymptotic stability of the set $\{(e, \tilde{\theta}, t) : e = \tilde{\theta} = 0\}$ is established using Lyapunov's 2nd method. However, only stability and convergence to the set $\{(e, \tilde{\theta}, j) : e = \tilde{\theta} = 0\}$ are guaranteed in [6], which relies on a trajectory-based approach. In general, for discrete-time systems, the existing adaptation laws are designed based on recursive least-squares and least-mean-squares algorithms with focus on either state convergence only or parameter estimation only; see for example [5], [7], [8], [9]. Hence, the stability guarantees are usually weaker than those established for continuous-time systems. For example, in [8], only (non-uniform) stability plus (non-uniform) convergence of e are established; see also [7].

On the other hand, to address the tracking-plus-estimation problem, the conditions on the regressor ψ are usually stronger for discrete-time systems compared to the continuous-time case [7]. For example, in [6] boundedness of the regressor ψ needs to be verified. In [8], boundedness of ψ is relaxed by allowing a linear growth with respect to e. It is important to note that relaxing the regularities of ψ, in the discrete-time case, usually yield to a high-dimension adaptation law [7].

To the best of our knowledge, the only available adaptive control algorithm for hybrid systems is provided in [10], where the impulsive-systems formalism is used. However, only the tracking problem is considered in the latter reference. In this paper, we propose a direct adaptive-control framework for a class of uncertain hybrid systems. That is, given a hybrid plant and a hybrid reference trajectory, we assume that the continuous and the discrete-time right-hand sides of the plant can be expressed in the form of (1) and (2), respectively, using an appropriate control action. Inspired by existing adaptive-control literature, we design a continuous and a discrete-time adaption laws for $\hat{\theta}$ during the plant's continuous and discrete-time evolution, respectively. Furthermore, we derive sufficient conditions to guarantee uniform global asymptotic stability of the set $\{(e, \tilde{\theta}, t, j) : e = \tilde{\theta} = 0\}$ for the resulting closed-loop system. In particular, to address the parameter-estimation task, an appropriate hybrid persistence of excitation condition is introduced. Boundedness of the discrete regressor is assumed, which allows us to propose a static discrete adaptation law. A relaxation of the latter requirement is proposed in Part II, at the price of using a higher-order discrete adaptation.

The remainder of this paper is organized as follows. In Section II, we recall some definitions and notations on hybrid

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systems as per in [11]. In Section III, we present some motivational examples for the problem at hand, which is formulated in Section IV. Our main results are presented in Section V and some concluding remarks are provided in Section VI.

II. Preliminaries

Notation. For a nonempty set we define $K \subset \mathbb{R}^{m_x}$, $|x|_K := \inf_{y\in K} |x-y|$ defines the distance between x and the set K, $\text{int}(K)$ denotes the interior of K. For a subset $O \subset \mathbb{R}^n$, $K\backslash O$ denotes the subset of elements of K that are not in O. For a map $\phi : \mathbb{R}_{\geq 0} \times \mathbb{N} \to \mathbb{R}^{m_x} \cup \{\emptyset\}$, $\text{dom}\,\phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ denotes the domain of definition of ϕ, namely, $\phi(t,j) \neq \emptyset$ if and only if $(t,j) \in \text{dom}\,\phi$. Furthermore, $|\phi|_\infty := \sup\{\phi(t,j) : (t,j) \in \text{dom}\,\phi\}$. For a symmetric semi-positive definite matrix $\Gamma \in \mathbb{R}^{m_x \times m_x}$, $\lambda_{min}(\Gamma)$ and $|\Gamma| := \lambda_{max}(\Gamma)$ denote the smallest and the largest eigenvalues of Γ, respectively.

A. Hybrid Dynamical Systems

Following [11], we view a hybrid dynamical system, denoted $\mathcal{H} := (C, F, D, G)$, as a combination of a constrained differential and a constrained difference equations given by

$$\mathcal{H} : \begin{cases} \dot{x} = F(x) & x \in C \\ x^+ = G(x) & x \in D, \end{cases} \tag{3}$$

with the state variable $x \in \mathcal{X} \subset \mathbb{R}^{m_x}$, the flow set $C \subset \mathcal{X}$, the jump set $D \subset \mathcal{X}$, the flow and jump maps $F : C \to \mathbb{R}^{m_x}$ and $G : D \to \mathbb{R}^{m_x}$, respectively.

Next, we introduce the concept of a *hybrid arc*.

Definition 1 (Hybrid arc): A map $\phi : \text{dom}\,\phi \to \mathbb{R}^{m_x}$ with $\text{dom}\,\phi \subset \mathbb{R}_{\geq 0} \times \mathbb{N}$ is a hybrid arc if ϕ is parameterized by an ordinary time variable $t \in \mathbb{R}_{\geq 0}$ and a discrete jump variable $j \in \mathbb{N}$ and $\text{dom}\,\phi$ is such that for each $(T, J) \in \text{dom}\,\phi$, $\text{dom}\,\phi \cap ([0,T] \times \{0,1,\ldots,J\}) = \cup_{j=0}^{J} ([t_j, t_{j+1}] \times \{j\})$ for a sequence $\{t_j\}_{j=0}^{J+1}$, such that $t_{j+1} \geq t_j$, $t_0 = 0$, and $t_{j+1} = T$.

Having the notion of hybrid arcs, we define the concept of solution to $\mathcal{H}$.

Definition 2 (Concept of solution to $\mathcal{H}$): A hybrid arc $\phi : \text{dom}\,\phi \to \mathbb{R}^{m_x}$ is a *solution* to $\mathcal{H}$ if

(S0) $\phi(0,0) \in \text{cl}(C) \cup D$;

(S1) for all $j \in \mathbb{N}$ such that $I^j := \{t : (t,j) \in \text{dom}\,\phi\}$ has nonempty interior, $t \mapsto \phi(t,j)$ is locally absolutely continuous and

$$\begin{aligned} &\phi(t,j) \in C && \text{for all} && t \in \text{int}(I^j), \\ &\dot{\phi}(t,j) = F(\phi(t,j)) && \text{for almost all} && t \in I^j; \end{aligned}$$

(S2) for all $(t,j) \in \text{dom}\,\phi$ such that $(t, j+1) \in \text{dom}\,\phi$,

$$\phi(t,j) \in D, \qquad \phi(t,j+1) = G(\phi(t,j)).$$

Well-posed hybrid systems [11, Definition 6.2] refer to a class of hybrid systems where the solutions enjoy very useful structural properties [11, Chapter 6]. A hybrid system $\mathcal{H} = (C, F, D, G)$ is well-posed if the following conditions, known as the hybrid basic conditions, are satisfied; see [11, Assumption 6.5] and [11, Theorem. 6.8] for more details.

(A1) The sets C and D are closed.

(A2) The maps F and G are continuous.

For a control hybrid system of the form

$$\mathcal{H}_u : \begin{cases} \dot{x} = F(x, u_c) & x \in C \\ x^+ = G(x, u_d) & x \in D, \end{cases} \tag{4}$$

with $u := (u_c, u_d) \in \mathbb{R}^{m_{u_c}} \times \mathbb{R}^{m_{u_d}}$, we introduce the notion of a solution pair (x, u) to $\mathcal{H}_u$.

Definition 3 (Concept of solution pair to $\mathcal{H}_u$): The hybrid arcs $\phi : \text{dom}\,\phi \to \mathbb{R}^{m_x}$ and $u : \text{dom}\,u \to \mathbb{R}^{m_u}$ form a *solution pair* to $\mathcal{H}_u$ if $\text{dom}\,u = \text{dom}\,\phi$ and

(Sp0) $\phi(0,0) \in \text{cl}(C) \cup D$;

(Sp1) for all $j \in \mathbb{N}$ such that $I^j := \{t : (t,j) \in \text{dom}\,\phi\}$ has nonempty interior, $t \mapsto \phi(t,j)$ is locally absolutely continuous and

$$\begin{aligned} &\phi(t,j) \in C && \text{for all } t \in \text{int}(I^j), \\ &\dot{\phi}(t,j) = F(\phi(t,j), u_c(t,j)) && \text{for almost all } t \in I^j; \end{aligned}$$

(Sp2) for all $(t,j) \in \text{dom}\,\phi$ such that $(t, j+1) \in \text{dom}\,\phi$,

$$\phi(t,j) \in D, \;\; \phi(t,j+1) = G(\phi(t,j), u_d(t,j)).$$

B. Stability Notions in Hybrid Systems

In this section, we recall some stability notions; see [11].

Definition 4 (UGS): A closed subset $\mathcal{A} \subset \mathcal{X}$ is said to be uniformly globally stable (UGS) for $\mathcal{H}$ if there exists $\beta \in \mathcal{K}^\infty$ such that, for each ϕ solution to $\mathcal{H}$ starting at $(t_o, j_o) \in \text{dom}\,\phi$, we have

$$|\phi(t,j)|_{\mathcal{A}} \leq \beta(|\phi(t_o, j_o)|_{\mathcal{A}}) \qquad \forall (t,j) \in \text{dom}\,\phi.$$

Definition 5 (UGAS): A closed subset $\mathcal{A} \subset \mathcal{X}$ is said to be uniformly globally asymptotically stable (UGAS) for $\mathcal{H}$ if there exists $\beta \in \mathcal{KL}$ such that, for each ϕ solution to $\mathcal{H}$ starting at $(t_o, j_o) \in \text{dom}\,\phi$ we have

$$|\phi(t,j)|_{\mathcal{A}} \leq \beta(t - t_o + j - j_o, |\phi(t_o, j_o)|_{\mathcal{A}}) \quad \forall (t,j) \in \text{dom}\,\phi.$$

Definition 6 (UGES): A close set $\mathcal{A} \subset \mathcal{X}$ is said to be uniformly globally exponentially stable (UGES) for $\mathcal{H}$ if there exist $\kappa > 0$ and $\lambda > 0$ such that, for each ϕ solution to $\mathcal{H}$ starting at (t_o, j_o) and for each $(t,j) \in \text{dom}\,\phi$, we have

$$|\phi(t,j)|_{\mathcal{A}} \leq \kappa \exp^{-\lambda(t+j-t_o-j_o)} |\phi(t_o, j_o)|_{\mathcal{A}}.$$

In the following theorems, we recall characterizations of UGS and UGES for hybrid systems using Lyapunov functions.

Theorem 1 (Lyapunov characterization of UGS): A closed subset $\mathcal{A} \subset \mathcal{X}$ is UGS for $\mathcal{H}$ if there exists a continuously-differentiable function $V : \mathcal{X} \to \mathbb{R}_{\geq 0}$ and class $\mathcal{K}^\infty$ functions $(\bar{\lambda}, \underline{\lambda})$, such that

$$\underline{\lambda}\left(|x|_{\mathcal{A}}^2\right) \leq V(x) \leq \bar{\lambda}\left(|x|_{\mathcal{A}}^2\right),$$

and along each solution ϕ to $\mathcal{H}$, we have

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have $\dot{V}(\phi(t,j)) \leq 0$ for a.a. $t \in I^j$.
- For each $((t,j),(t,j+1)) \in \text{dom}\,\phi \times \text{dom}\,\phi$,

$$V(\phi(t,j+1)) - V(\phi(t,j)) \leq 0.$$

Theorem 2 (Lyapunov characterization of UGES): A closed subset $\mathcal{A} \subset \mathcal{X}$ is UGES for $\mathcal{H}$ if there exists a function $V : \mathbb{R}_{\geq 0} \times \mathbb{N} \times \mathcal{X} \to \mathbb{R}_{\geq 0}$ and positive constants $(\bar{\lambda}, \underline{\lambda}, \lambda_c, \lambda_d)$ such that $\underline{\lambda}|x|_{\mathcal{A}}^2 \leq V(t,j,x) \leq \bar{\lambda}|x|_{\mathcal{A}}^2$, and, along each solution ϕ to $\mathcal{H}$, it holds that:

- for each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, $t \mapsto V(t, j, \phi(t,j))$ is absolutely continuous on I^j and

$$\dot{V}(t, j, \phi(t,j)) \leq -\lambda_c V(t, j, \phi(t,j)) \text{ for a.a. } t \in I^j,$$

- for each $((t,j),(t,j+1)) \in \operatorname{dom}\phi \times \operatorname{dom}\phi$,

$$V(t,j+1,\phi(t,j+1)) - V(t,j,\phi(t,j)) \leq -\lambda_d V(t,j,\phi(t,j)).$$

III. Motivation : A Direct-Adaptive-Control Framework for Hybrid Systems

Consider the class of uncertain control hybrid systems

$$\mathcal{H}_u : \begin{cases} \dot{x} = f_c(\theta,x) + g_c(x)u_c & x \in C \\ x^+ = f_d(\theta,x) + g_d(x)u_d & x \in D, \end{cases} \tag{5}$$

where $\theta \in \mathbb{R}^{m_\theta}$ is a vector of constant unknown parameters, $x \in \mathbb{R}^{m_x}$ is the state, $u := (u_c, u_d) \in \mathbb{R}^{m_{u_c}} \times \mathbb{R}^{m_{u_d}}$ includes the continuous and the discrete inputs, $f_c, f_d : \mathbb{R}^{m_x} \to \mathbb{R}^{m_x}$ are the flow and the jump drifts, respectively, $g_c, g_d : \mathbb{R}^{m_x} \to \mathbb{R}^{m_x \times m_{u_c}}$ are the continuous and the discrete input matrices, respectively, finally, $(C,D) \in \mathbb{R}^{m_x} \times \mathbb{R}^{m_x}$ are the flow and the jump sets, respectively. Without loss of generality, we assume that the sets C and D are closed.

Assume that we want to find an input $u^\star$ such that, each solution ϕ to the closed-loop system $\mathcal{H}_{u^\star}$ tracks a solution to $\dot{\phi}_r = A_c^\star \phi_r$ along the flows and tracks a solution to $\phi_r^+ = A_d^\star \phi_r$ along the jumps. The matrices $(A_c^\star, A_d^\star) \in \mathbb{R}^{m_x \times m_x} \times \mathbb{R}^{m_x \times m_x}$ are given. In particular, given a solution pair $(\phi, u) : \operatorname{dom}\phi (= \operatorname{dom} u) \to \mathbb{R}^{m_x} \times \mathbb{R}^{m_u}$ to $\mathcal{H}_u$, we introduce the corresponding hybrid reference model given by

$$\mathcal{H}_r : \begin{cases} \begin{cases} \dot{\phi}_r = A_c^\star \phi_r \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t,j,\phi_r) \in C_r \\ \begin{cases} \phi_r^+ = A_d^\star \phi_r \\ t^+ = t, \quad j^+ = j+1, \end{cases} & (t,j,\phi_r) \in D_r, \end{cases}$$

where

$$C_r := \{(t,j,\phi_r) : (t,j) \in \operatorname{dom}\phi \text{ and } (t,j+1) \notin \operatorname{dom}\phi\},$$
$$D_r := \{(t,j,\phi_r) : (t,j) \in \operatorname{dom}\phi \text{ and } (t,j+1) \in \operatorname{dom}\phi\}.$$

Furthermore, we let $\phi_r : \operatorname{dom}\phi_r (= \operatorname{dom}\phi) \to \mathbb{R}^{m_x}$ be a resulting solution to $\mathcal{H}_r$ and we define the tracking error $e := \phi - \phi_r$. The goal is to find $u^\star$, under which, the error e is governed by the hybrid dynamics:

$$\mathcal{H}_e : \begin{cases} \begin{cases} \dot{e} = A_c e \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t,j,e) \in C_e, \\ \begin{cases} e^+ = A_d e \\ t^+ = t, \quad j^+ = j+1, \end{cases} & (t,j,e) \in D_e, \end{cases}$$

where

$$C_e := \{(t,j,e) : (t,j) \in \operatorname{dom}\phi \text{ and } (t,j+1) \notin \operatorname{dom}\phi\},$$
$$D_e := \{(t,j,e) : (t,j) \in \operatorname{dom}\phi \text{ and } (t,j+1) \in \operatorname{dom}\phi\},$$

and $(A_c, A_d) \in \mathbb{R}^{m_x \times m_x} \times \mathbb{R}^{m_x \times m_x}$ are design matrices.

By designing $u^\star := (u_c^\star, u_d^\star)$ as a feedback law satisfying

$$\begin{aligned} g_c(x)u_c^\star &= A_c^\star \phi_r - f_c(\theta,x) + A_c e \\ g_d(x)u_d^\star &= A_d^\star \phi_r - f_d(\theta,x) + A_d e, \end{aligned} \tag{8}$$

we conclude that the error between ϕ and ϕ_r is governed by $\mathcal{H}_e$. Now, we assume that the input $u^\star$ satisfying (8) can be expressed as follows:

$$\begin{cases} u_c := \kappa_c(\theta,x,\phi_r) := \Psi_c(x)\theta + G_c(x,\phi_r,e) \\ u_d := \kappa_d(\theta,x,\phi_r) := \Psi_d(x)\theta + G_d(x,\phi_r,e), \end{cases} \tag{9}$$

where $\Psi_c : \mathbb{R}^{m_x} \to \mathbb{R}^{m_{u_c} \times m_\theta}$, $\Psi_d : \mathbb{R}^{m_x} \to \mathbb{R}^{m_{u_d} \times m_\theta}$, $G_c : \mathbb{R}^{m_x} \times \mathbb{R}^{m_x} \times \mathbb{R}^{m_x} \to \mathbb{R}^{m_{u_c}}$, and $G_d : \mathbb{R}^{m_x} \times \mathbb{R}^{m_x} \times \mathbb{R}^{m_x} \to \mathbb{R}^{m_{u_d}}$ are known functions satisfying:

$$\begin{aligned} g_c(x)\Psi_c(x)\theta &= -f_c(x,\theta), \\ g_d(x)\Psi_d(x)\theta &= -f_d(x,\theta), \\ g_c(x)G_c(x,\phi_r,e) &= A_c^\star \phi_r + A_c e, \\ g_d(x)G_d(x,\phi_r,e) &= A_d^\star \phi_r + A_d e. \end{aligned}$$

However, since θ is unknown, instead of applying the control law in (9), we apply

$$u_c^\star := \kappa_c(\hat{\theta},x,\phi_r) \quad \text{and} \quad u_d^\star := \kappa_d(\hat{\theta},x,\phi_r), \tag{10}$$

where $\hat{\theta}$ is an estimate of θ. Hence, the tracking error e is governed by

$$\mathcal{H}_e : \begin{cases} \begin{cases} \dot{e} = A_c e + g_c(x)\Psi_c(x)\tilde{\theta} \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t,j,e) \in C_e, \\ \begin{cases} e^+ = A_d e + g_d(x)\Psi_d(x)\tilde{\theta} \\ t^+ = t, \quad j^+ = j+1, \end{cases} & (t,j,e) \in D_e, \end{cases}$$

where $\tilde{\theta} := \hat{\theta} - \theta$ is the estimation error. Then, for each $(t,j) \in \operatorname{dom}\phi_r$, we let

$$\begin{aligned} \psi_c(t,j,e) &:= g_c(e+\phi_r(t,j))\Psi_c(e+\phi_r(t,j)), \\ \psi_d(t,j,e) &:= g_d(e+\phi_r(t,j))\Psi_d(e+\phi_r(t,j)), \end{aligned}$$

so the system $\mathcal{H}_e$ can be expressed as

$$\mathcal{H}_e : \begin{cases} \begin{cases} \dot{e} = A_c e + \psi_c(t,j,e)\tilde{\theta} \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t,j,e) \in C_e \\ \begin{cases} e^+ = A_d e + \psi_d(t,j,e)\tilde{\theta} \\ t^+ = t, \quad j^+ = j+1, \end{cases} & (t,j,e) \in D_e \end{cases}$$

The objective, in this case, is to solve the following problem:

Problem 1: Design an update law for $\hat{\theta}$ along the flows, of the form $\dot{\hat{\theta}} := \mathcal{A}_c(t,j,e)$, and an update law along the jumps of the form $\hat{\theta}^+ := \hat{\theta} + \mathcal{A}_d(t,j,e,e^+)$ such that, for the resulting closed-loop system, we have UGAS of the set

$$\mathcal{A} := \{(e,\tilde{\theta},t,j) : e = \tilde{\theta} = 0\} \tag{13}$$

Remark 1: Solving Problem 1, on the one hand, extends the existing results in the continuous-time case to the more general context of hybrid systems, and, on the other hand, improves the stability guarantees established in the existing literature on discrete time systems; see Remark 7 below for further detail.

IV. Problem Formulation

The adaptive control problem described in Section III motivates the study of a class of systems that comprise those appearing in model-reference adaptive control for hybrid systems. This more general kind of systems are given by the equations

$$\mathcal{H}_{e\theta} : \begin{cases} \begin{cases} \dot{e} = A_c e + \psi_c(t,j,e)\tilde{\theta} \\ \dot{\tilde{\theta}} = \mathcal{A}_c(t,j,e) \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t,j,e,\tilde{\theta}) \in C_{e\theta}, \\ \begin{cases} e^+ = A_d e + \psi_d(t,j,e)\tilde{\theta} \\ \tilde{\theta}^+ = \tilde{\theta} + \mathcal{A}_d(t,j,e,e^+) \\ t^+ = t, \quad j^+ = j+1, \end{cases} & (t,j,e,\tilde{\theta}) \in D_{e\theta}, \end{cases}$$

where $(t, j, e, \tilde{\theta}) \in \mathcal{X}_{e\theta}$, $\mathcal{X}_{e\theta} := \operatorname{dom}\psi_c (= \operatorname{dom}\psi_d) \times \mathbb{R}^{m_\theta}$, $(A_c, A_d) \in \mathbb{R}^{m_e \times m_e} \times \mathbb{R}^{m_e \times m_e}$, the sets $(C_{e\theta}, D_{e\theta}) \subset \mathcal{X}_{e\theta} \times \mathcal{X}_{e\theta}$ are the flow and the jump sets, respectively, the maps $\mathcal{A}_c : \operatorname{dom}\psi_c \to \mathbb{R}^{m_\theta}$ and $\mathcal{A}_d : \operatorname{dom}\psi_d \times \mathbb{R}^{m_e} \to \mathbb{R}^{m_\theta}$ are, respectively, the continuous and the discrete-time adaptation laws to be designed, finally, the hybrid arcs $\psi_c,\ \psi_d : \operatorname{dom}\psi_d (= \operatorname{dom}\psi_c) \to \mathbb{R}^{m_e \times m_\theta}$ are, respectively, the continuous and the discrete *regressor* functions. Our goal is to solve the following problem for $\mathcal{H}_{e\theta}$.

Problem 2: Design the adaptation laws $\mathcal{A}_c$ and $\mathcal{A}_d$ to guarantee well-posedness of the resulting hybrid system $\mathcal{H}_{e\theta}$ as well as UGAS of the set $\mathcal{A}$ in (13).

Remark 2: Adaptive control of various classes of uncertain systems to achieve different control tasks can be transformed into solving Problem 2 for a dynamical system in the form of $\mathcal{H}_{e\theta}$; see [9], [10], [12], [13]. In particular, in [12] and [13], multi-layered neural networks are used to approximate a general unknown nonlinear dynamics. The unknown parameters of the neural network, that is the best to approximate the dynamics, are estimated via a direct adaptive-control scheme leading to a structure, in the error coordinates, that is similar to the one in $\mathcal{H}_{e\theta}$.

In this paper, we solve Problem 2 under the following assumption on ψ_d.

Assumption 1: There exists $\bar{\psi}_d > 0$ such that

$$|\psi_d(t, j, e)| \leq \bar{\psi}_d \qquad \forall (t, j, e, \tilde{\theta}) \in D_{e\theta}.$$

Remark 3: When ψ_d is bounded, the adaptation law $\mathcal{A}_d$ we propose is static and relatively simple. Note that, Problem 2 is addressed in Part II [1] for systems with ψ_d of linear growth in e, but this is done using a higher order adaptation law.

A. Additional Assumptions

Before addressing Problem 2, we introduce a set of key assumptions on the hybrid system $\mathcal{H}_{e\theta}$.

Assumption 2: There exist $P, Q \in \mathbb{R}^{m_e \times m_e}$ symmetric and positive definite such that

$$\begin{aligned} {A_c}^\top P + P A_c &\leq -Q \\ {A_d}^\top P A_d - P &\leq -Q. \end{aligned} \tag{15}$$

Remark 4: Assumption 2 allows to build a common quadratic Lyapunov function that is non-increasing along the flows as well as along the jumps of $\mathcal{H}_{e\theta}$. A similar assumption is used in [10] for impulsive systems.

Next, we consider the following regularity assumption on the continuous- and the discrete-time regressors (ψ_c, ψ_d).

Assumption 3: The maps ψ_c and ψ_d are continuous.

Remark 5: As it can be seen from (A2), Assumption 3 is necessary to guarantee well-posedness of $\mathcal{H}_{e\theta}$.

Next, we specify the following functions

$$\begin{aligned} \phi_c(t, j) &:= \psi_c(t, j, 0), \\ \phi_d(t, j) &:= \psi_d(t, j, 0), \\ \Delta_c(t, j, e) &:= \psi_c(t, j, e) - \psi_c(t, j, 0), \\ \Delta_d(t, j, e) &:= \psi_d(t, j, e) - \psi_d(t, j, 0). \end{aligned}$$

Assumption 4 (Regularity of ϕ_c): For each $j \in \mathbb{N}$, the map $t \mapsto \phi_c(t, j)$ is absolutely continuous on $I^j := \{t : (t, j) \in \operatorname{dom}\phi_c\}$, and there exists $\bar{\phi} > 0$ such that $|\phi_c(t, j)|_\infty \leq \bar{\phi}$, and $\operatorname{ess\,sup}\{|\dot{\phi}_c(t, j)| : t \in I^j\} \leq \bar{\phi}$.

Remark 6: Note that the first inequality in Assumption 4 is crucial to guarantee parameter estimation for continuous-time adaptive control systems; see [14], [15]. However, the second inequality allows us to construct a Lyapunov function that involves the term ϕ_c.

Assumption 5 (Growth rate of Δ_c and Δ_d): There exists a positive non-decreasing function $\kappa_o : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that, for each $(t, j, e) \in \operatorname{dom}\psi_c = \operatorname{dom}\psi_d$, we have $\max\{|\Delta_c(t, j, e)|, |\Delta_d(t, j, e)|\} \leq \kappa_o(|e|)|e|$.

Finally, we introduce the following hybrid persistence of excitation (HPE) condition on the pair (ϕ_c, ϕ_d).

Assumption 6 (HPE of (ϕ_c, ϕ_d)): There exist $K,\ \mu > 0$ such that, for each hybrid time domain

$$\bigcup_{j=j_o}^{J} ([t_j, t_{j+1}] \times \{j\}) \subset \operatorname{dom}\phi_c$$

such that $(t_{J+1} - t_o) + (J - j_o) \geq K$, we have

$$\sum_{j=j_o}^{J} \int_{t_j}^{t_{j+1}} \phi_c(s, j)^\top \phi_c(s, j) ds + \sum_{j=j_o}^{J} \phi_d(t_{j+1}, j)^\top \phi_d(t_{j+1}, j) \geq \mu I_{m_\theta}.$$

Assumption 6 is a generalization of the classical continuous and discrete-time persistence of excitation (PE) conditions to the proposed hybrid framework. In particular, when the arcs ϕ_c and ϕ_d are eventually continuous, Assumption 6 reduces to the classical continuous-time PE condition [16]. Similarly, when the arcs ϕ_c and ϕ_d are eventually discrete or Zeno, Assumption 6 reduces to the classical discrete-time PE condition [17]. To the best of our knowledge, HPE was introduced for the first time in [18] to study the hybrid gradient algorithm. Finally, we stress that PE is a necessary condition to guarantee UGAS of the set $\mathcal{A}$ in direct adaptive-control schemes [14].

V. Main Result

In this paper, we study the continuous and the discrete-time adaptation laws given by

$$\begin{aligned} \mathcal{A}_c(t, j, e) &:= -\Gamma \psi_c(t, j, e)^\top P e \\ \mathcal{A}_d(t, j, e, e^+) &:= -\Gamma \psi_d(t, j, e)^\top P e^+, \end{aligned} \tag{16}$$

where $\Gamma \in \mathbb{R}^{m_\theta \times m_\theta}$ is a symmetric and positive-definite design matrix and $P \in \mathbb{R}^{m_e \times m_e}$ comes from Assumption 2.

Remark 7: The choice of the continuous-time regressor $\mathcal{A}_c$ in (16) is commonly used in continuous-time adaptive-control literature; see [4], [19]. Furthermore, the discrete-time regressor $\mathcal{A}_d$ in (16) is inspired from [20]. According to the latter reference, quantifying the convergence properties using this choice of $\mathcal{A}_d$ is not available in the literature of discrete-time systems. This is one of the contributions of this paper.

Using (16), the system $\mathcal{H}_{e\theta}$ can be expressed as

$$\mathcal{H}_{e\theta} : \begin{cases} \begin{cases} \dot{e} = A_c e + \psi_c(t, j, e)\tilde{\theta} \\ \dot{\tilde{\theta}} = -\Gamma \psi_c(t, j, e)^\top P e \\ \dot{t} = 1, \quad \dot{j} = 0, \end{cases} & (t, j, e, \tilde{\theta}) \in C_{e\theta} \\ \begin{cases} e^+ = A_d e + \psi_d(t, j, e)\tilde{\theta} \\ \tilde{\theta}^+ = \tilde{\theta} - \Gamma \psi_d(t, j, e)^\top P e^+ \\ t^+ = t, \quad j^+ = j + 1, \end{cases} & (t, j, e, \tilde{\theta}) \in D_{e\theta} \end{cases}$$

Theorem 3: Consider system $\mathcal{H}_{e\theta}$ such that Assumptions 1-6 hold. Let $\Gamma \in \mathbb{R}^{m_\theta \times m_\theta}$ be a symmetric and positive definite

matrix satisfying the following inequalities

$$\begin{aligned} &|\Gamma| \leq \frac{1}{2\bar{\psi}_d^2|P|}, \\ &8|\Gamma|^2|P|\bar{\psi}_d^4 + 2|\Gamma|\bar{\psi}_d^2 \leq \frac{1}{\left|Q^{-1/2}A_d^\top P^2 A_d Q^{-1/2}\right|}, \end{aligned} \tag{18}$$

where (P, Q) come from Assumption 2 and $\bar{\psi}_d$ comes from Assumption 1. Then, $\mathcal{H}_{e\theta}$ satisfies the hybrid basic conditions (A1)-(A2) and the set $\mathcal{A}$ in (13) is UGAS.

A. Proof of Theorem 3

The hybrid basic conditions are trivially satisfied under Assumption 3 and since the sets $C_{e\theta}$ and $D_{e\theta}$ are assumed to be closed.

UGAS is established in two main steps: first, to recognize that the origin is uniform globally stabile, which implies uniform global boundedness of the hybrid solutions and, second, to follow output injection arguments, to establish uniform global attractivity. Below, we present the main guidelines.

1) Uniform Global Stability: We establish UGS of the set $\mathcal{A}$ for $\mathcal{H}_{e\theta}$ using Theorem 1. To do so, we introduce the Lyapunov function candidate

$$V(e, \tilde{\theta}) := e^\top P e + \tilde{\theta}^\top \Gamma^- \tilde{\theta}, \tag{19}$$

where $\Gamma = \Gamma^\top \in \mathbb{R}^{m_\theta \times m_\theta}$ is positive definite and (P, Q) come from Assumption 2.

Proposition 1: Consider system $\mathcal{H}_{e\theta}$ such that Assumption 1, Assumption 2, and (18) hold. Then, the set $\mathcal{A}$ is UGS for $\mathcal{H}_{e\theta}$. In particular, along each solution $\phi := (e, \tilde{\theta}, t, j)$ to $\mathcal{H}_{e\theta}$, the following properties hold:

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have
$$\dot{V}(e, \tilde{\theta}) \leq -e^\top Q e \qquad \text{for almost all} \quad t \in I^j.$$
- For each $((t, j), (t, j+1)) \in \text{dom}\, \phi \times \text{dom}\, \phi$, we have
$$V(e^+, \tilde{\theta}^+) - V(e, \tilde{\theta}) \leq -\frac{1}{2} e^\top Q e - \frac{1}{4}\tilde{\theta}^\top \psi_d^\top P \psi_d \tilde{\theta}.$$

2) Output-Injection-Based Decomposition: Following the approach of [21], we re-express system $\mathcal{H}_{e\theta}$ as a (hybrid) system that is linear in the variables e and $\tilde{\theta}$, denoted $\mathcal{H}^n_{e\theta}$, subject to an additive nonlinear term that vanishes when $e = 0$. To do so, we note that

$$\mathcal{H}_{e\theta} : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} A_c e + \phi_c(t, j)\tilde{\theta} \\ -\Gamma \phi_c(t, j)^\top P e \end{bmatrix} + K_c(t, j, e, \tilde{\theta}) \\ \quad \dot{t} = 1, \quad \dot{j} = 0, \qquad (t, j, e, \tilde{\theta}) \in C_{e\theta} \\ \begin{bmatrix} e^+ \\ \tilde{\theta}^+ \end{bmatrix} = \begin{bmatrix} A_d e + \phi_d(t, j)\tilde{\theta} \\ \tilde{\theta} - \Gamma \phi_d(t, j)^\top P e^+ \end{bmatrix} + K_d(t, j, e, \tilde{\theta}, e^+) \\ \quad t^+ = t, \quad j^+ = j + 1, \qquad (t, j, e, \tilde{\theta}) \in D_{e\theta}, \end{cases}$$

where

$$K_c(\cdot) := \begin{bmatrix} \Delta_c(\cdot)\tilde{\theta} \\ -\Gamma \Delta_c(\cdot)^\top P e \end{bmatrix} \quad K_d(\cdot) := \begin{bmatrix} \Delta_d(\cdot)\tilde{\theta} \\ -\Gamma \Delta_d(\cdot)^\top P e^+ \end{bmatrix}.$$

Note that, under Assumptions 1 and 5, we can find a positive non-decreasing function $\kappa : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that

$$\begin{aligned} \sup\{|K_c(t, j, e, \tilde{\theta})|, |K_d(t, j, e, \tilde{\theta}, e^+)| : (t, j) \in \text{dom}\, \phi_c\} \\ \leq \kappa(|(e, \tilde{\theta})|)|e|. \end{aligned}$$

3) Uniform Exponential Stability When $K_c = K_d = 0$: We use Theorem 2 to show UES of the set $\mathcal{A}$ for the hybrid system:

$$\mathcal{H}^n_{e\theta} : \begin{cases} \begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} A_c e + \phi_c(t, j)\tilde{\theta} \\ -\Gamma \phi_c(t, j)^\top P e \end{bmatrix} \\ \quad \dot{t} = 1, \quad \dot{j} = 0, \qquad (t, j, e, \tilde{\theta}) \in C_{e\theta} \\ \begin{bmatrix} e^+ \\ \tilde{\theta}^+ \end{bmatrix} = \begin{bmatrix} A_d e + \phi_d(t, j)\tilde{\theta} \\ \tilde{\theta} - \Gamma \phi_d(t, j)^\top P e^+ \end{bmatrix} \\ \quad t^+ = t, \quad j^+ = j + 1, \qquad (t, j, e, \tilde{\theta}) \in D_{e\theta}. \end{cases}$$

To do so, we introduce the Lyapunov function candidate

$$\begin{aligned} W(t, j, e, \tilde{\theta}) &:= aV(e, \tilde{\theta}) - be^\top \phi_c(t, j)\left[\tilde{\theta} + \Gamma \hat{\phi}_d(t, j)^\top P e\right] \\ &- \left[\tilde{\theta} + \Gamma \hat{\phi}_d(t, j)^\top P e\right]^\top Q_\Phi(t, j, \infty)\left[\tilde{\theta} + \Gamma \hat{\phi}_d(t, j)^\top P e\right]. \end{aligned} \tag{22}$$

where $a, b > 0$ are constants specified in the rest of the proof, V is introduced in (19), the arc $\hat{\phi}_d : \text{dom}\, \phi_d \to \mathbb{R}^{m_e \times m_\theta}$ is given by

$$\hat{\phi}_d(t, j) := \begin{cases} \phi_d(t_j, j-1) & \text{if } (t, j) \in (t_{j+1}, j) \cup \text{int}(I_j) \\ \phi_d(t_{j+1}, j) & \text{if } (t, j) = (t_{j+1}, j+1), \end{cases}$$

the arc $\Phi : \text{dom}\, \phi_c (= \text{dom}\, \phi_d) \to \mathbb{R}^{m_\theta \times m_\theta}$ is given by

$$\Phi(t, j) := \phi_{cd}(t, j)^\top \phi_{cd}(t, j)$$

with $\phi_{cd} : \text{dom}\, \phi_c (= \text{dom}\, \phi_d) \to \mathbb{R}^{m_e \times m_\theta}$ satisfies

$$\phi_{cd}(t, j) := \begin{cases} \phi_c(t, j) & \text{if } t \in \text{int}(I_j) \\ \phi_d(t, j) & \text{otherwise,} \end{cases} \tag{23}$$

finally, the arc $Q_\Phi(\cdot, \infty) : \text{dom}\, \phi_c (= \text{dom}\, \phi_d) \to \mathbb{R}^{m_\theta \times m_\theta}$ is given by

$$\begin{aligned} Q_\Phi(t, j, \infty) &:= \sum_{i=j}^{m_\infty} \int_{t_i}^{t_{i+1}} \exp^{(t+j)-(s+i)} \Phi(s, i) ds \\ &+ \frac{1}{2} \sum_{i=j}^{m_\infty} \left[\int_{t_{i+1}}^{t_{i+2}} \exp^{(t+j)-(s+i)} ds\right] \Phi(t_{i+1}, i), \end{aligned} \tag{24}$$

where $m_\infty \in \mathbb{N} \cup \infty$ is the maximum number of jumps of Φ and $\{t_i\}_{i=j}^{m_\infty}$ is the corresponding sequence of jump instants.

Proposition 2: Consider system $\mathcal{H}^n_{e\theta}$ such that Assumptions 1, 2, 4, 6 and the inequalities in (18) hold. Then, the set $\mathcal{A}$ in (13) is UES for $\mathcal{H}^n_{e\theta}$. In particular, for an appropriate choice of the positive constants (a, b), we can find positive constants $\left(\bar{\lambda}, \bar{\lambda}_1, \underline{\lambda}, \lambda_c, \lambda_d\right)$ such that the Lyapunov function candidate W in (22) satisfies the following properties:

1) For each $(t, j) \in \text{dom}\, \phi_d$, we have

$$\begin{aligned} \underline{\lambda}|(e, \tilde{\theta})|^2 \leq W \leq \bar{\lambda}|(e, \tilde{\theta})|^2, \\ \left|\nabla_{(e,\tilde{\theta})} W\right| \leq \bar{\lambda}|(e, \tilde{\theta})|, \end{aligned}$$

and, for each $k \in \mathbb{R}^2$ and $(t, j) \in \text{dom}\, \phi_c$, we have

$$\begin{aligned} W(t, j, e^+ + k_1, \tilde{\theta}^+ + k_2) - W(t, j, e^+, \tilde{\theta}^+) \\ \leq \bar{\lambda}_1 |k|(|e, \tilde{\theta}| + |k|). \end{aligned}$$

2) Along each solution $\phi := (e, \tilde{\theta}, t, j)$, we have

- For each $((t,j),(t,j+1)) \in \operatorname{dom}\phi \times \operatorname{dom}\phi$,

$$W(t,j+1,e^+,\tilde{\theta}^+) - W(t,j,e,\tilde{\theta}) \leq -\lambda_d W(t,j,e,\tilde{\theta}).$$

- For each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$, we have

$$\dot{W}(t,j,e,\tilde{\theta}) \leq -\lambda_c W(t,j,e,\tilde{\theta}) \quad \text{for a.a. } t \in I^j.$$

Remark 8: It is important to note that the proof of Proposition 2 provides an explicit approach to design the parameters a and b so that the two items in Proposition 2 hold. Thus, our approach allows for an explicit estimation of the convergence rate [22].

4) UGAS for $\mathcal{H}_{e\theta}$: To prove UGAS of the set $\mathcal{A}$ for $\mathcal{H}_{e\theta}$, we consider a solution $\phi := (e,\tilde{\theta},t,j)$ to $\mathcal{H}_{e\theta}$ starting from $(e_o,\tilde{\theta}_o,t_o,j_o) \in \mathcal{X}_{e\theta}$. Furthermore, we introduce the Lyapunov-like function

$$Z(t,j,e,\tilde{\theta}) := W(t,j,e,\tilde{\theta}) + \rho(|(e_o,\tilde{\theta}_o)|)V(e,\tilde{\theta}), \quad (25)$$

where $\rho : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a continuous and non-decreasing function that we specify later, and the functions (V,W) are defined in (19) and (22), respectively.

From Proposition 1 it follows that along every solution ϕ to $\mathcal{H}_{e\theta}$ starting from $(e_o,\tilde{\theta}_o,t_o,j_o)$, we have

$$|(e(t,j),\tilde{\theta}(t,j))| \leq R_o|(e_o,\tilde{\theta}_o)| \quad \forall (t,j) \in \operatorname{dom}\phi.$$

On the other hand, from Proposition 2 it follows that, along every solution ϕ to $\mathcal{H}_{e\theta}$ starting from $(e_o,\tilde{\theta}_o,t_o,j_o)$, and for each $j \in \mathbb{N}$ such that $\text{int}(I^j) \neq \emptyset$ and for almost all $t \in I^j$, we have

$$\begin{aligned}\dot{Z}(t,j,e,\tilde{\theta}) \leq & -\lambda_c \underline{\lambda}|(e,\tilde{\theta})|^2 + \bar{\lambda}\kappa\left(R_o|(e_o,\tilde{\theta}_o)|\right)|(e,\tilde{\theta})||e| \\ & - \rho(|(e_o,\tilde{\theta}_o)|)e^\top Q e,\end{aligned}$$

and for an appropriate choice of $\rho(\cdot)$ the sum of the terms on right-hand-side of the previous inequality is negative definite.

Now, during jumps note that, for each $((t,j),(t,j+1)) \in \operatorname{dom}\phi \times \operatorname{dom}\phi$, we have

$$\begin{aligned}&Z(t,j+1,e^+,\tilde{\theta}^+) - Z(t,j,e,\tilde{\theta}) \\ &\leq W(t,j+1,e^+,\tilde{\theta}^+) - W(t,j+1,e^+ - K_{d1},\tilde{\theta}^+ - K_{d2}) \\ &\quad + W(t,j+1,e^+ - K_{d1},\tilde{\theta}^+ - K_{d2}) - W(t,j,e,\tilde{\theta}) \\ &\quad - \rho(|e_o,\tilde{\theta}_o|)e^\top Q e.\end{aligned}$$

In addition, from Proposition 2, it holds that

$$\begin{aligned}Z(t,j+1,&e^+,\tilde{\theta}^+) - Z(t,j,e,\tilde{\theta}) \leq \\ &- \lambda_d \underline{\lambda}|(e,\tilde{\theta})|^2 + \kappa_2(R_o|(e_o,\tilde{\theta}_o)|)|e||(e,\tilde{\theta})| \\ &- \rho(|e_o,\tilde{\theta}_o|)e^\top Q e,\end{aligned}$$

where $\kappa_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a non-decreasing function. Again, the right-hand side of the inequality above is negative definite for an appropriate choice of $\rho(\cdot)$. Hence, the result follows.

VI. Conclusion

In this paper, we proposed a direct adaptive-control framework for uncertain hybrid systems. The proposed framework is Lyapunov-based and establishes UGAS of the set $\mathcal{A}$ for the resulting closed-loop hybrid system. Future work aims at relaxing the structure of $\mathcal{H}_{e\theta}$ by considering right-hand sides that are nonlinear in both e and $\tilde{\theta}$, then imposing the needed properties for UGAS via a set of Lyapunov-based inequalities.

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