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Consensus-based Formation Control of Multiple Nonholonomic Vehicles under Input Constraints

Emmanuel Nuño Antonio Loría Angel I. Paredes Tonatiuh Hernández

***Abstract*— We address the open problem of consensus-based formation control of nonholonomic multiagent vehicles with pre-imposed input constraints. That is the problem of stabilizing a group of second-order differential-drive nonholonomic robots, making them acquire a determined formation pattern around a non pre-specified point on the plane and a common non pre-specified orientation. This problem is also known as leaderless full consensus. Our controller is smooth and time-varying, and fully distributed. Its design is a natural modification of another controller proposed earlier, which relies on proportional feedback, damping injection, and a smooth time-varying term that injects persistency of excitation in the system to overcome the effects of nonholonomicity. It is assumed that the robots communicate over a network with an undirected-graph topology.**

***Index Terms*— Formation control, persistency of excitation, nonholonomic systems, autonomous vehicles.**

I. Introduction

THE consensus problem for multiagent systems, which in its simplest form consists in making the states of multiple dynamical systems converge to a common equilibrium point, is completely solved for linear autonomous systems under different assumptions regarding the network's topology [1], [2]. Some works focus on consensus under time-varying (switching) topology [3] or dynamic interconnections [4]. Now, while graph connectivity is known to be a necessary condition for consensus and some works focus on guaranteeing it [5], [6], it is certainly not sufficient in general. Indeed, consensus and synchronization strongly depend on the systems' dynamics and the above-mentioned works apply only to simple integrators or, more generally, to linear systems. Yet, in the case that the systems are nonlinear, many different behaviors may appear under the effect of interconnection, even for networks with static topologies [7].

Consensus-based formation control of nonholonomic vehicles [8] is particularly challenging even under the assumptions that they are interconnected over an undirected-static-graph topology because consensus is inherently a set-point stabilization problem, so it greatly differs from formation-tracking control [9], [10], which is otherwise a well-studied problem too. However, in general, formation tracking and other leader-follower control schemes do not apply to consensus-based formation control since nonholonomic systems are not stabilizable via smooth time-invariant feedback [11]. If time-invariant, controllers are necessarily non-smooth [12] and if time-varying—cf. [13], they may be smooth and typically rely on persistency of excitation [14], [15].

The consensus-based formation control goal may be *partial* or *full*. In the first case, only consensus in the Cartesian coordinates is of interest while controlling the orientation may be neglected [1, Ch. 11], [11]. In the second case, the objective is for all the vehicles to acquire common, non pre-specified, position and orientation [12], [13], [15]. One more important distinction to be made is between works that apply to velocity-controlled (first-order) models [12] and those that apply to more realistic force-controlled (second-order) models [16].

Now, as it is always the case with physical systems, in force-controlled vehicles the actuators are prone to saturation, which may seriously downgrade the overall performance and put the actuators at risk of thermal and mechanical failure saturation of the actuators must be avoided. An efficient recourse is to use saturated controllers to ensure that the inputs satisfy pre-imposed bounds [17]. Similar techniques have been also used for multiagent systems [18], [19], but more scarcely for networked nonholonomic vehicles. This is done, *e.g.*, in [1], but only partial consensus is addressed.

In this letter, we address the full consensus-formation control problem for second-order differential-drive vehicles, under input constraints. Our controller is smooth and time-varying. It is constituted of Proportional-plus-damping (P+d) control laws for the linear and angular-motion dynamics separately. In addition, the angular-motion controller is endowed with an independent persistently-exciting term that ensures global convergence to the consensus manifold. The control design follows the lines of our previous works [15], [20], [21], but differs from the latter in that here we assume velocity measurements to be available and the network to be delay-free. On the other hand, we design the controller to satisfy pre-imposed bounds.

In the next section we recall the multiagent unicycle's model and we formulate the control problem addressed here. In Section III we present our main result, which is illustrated via numerical simulations in Section V. Concluding remarks are provided in Section VI.

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II. Model and problem formulation

Consider N nonholonomic systems with kinematics

$$\dot{z}_i = \varphi(\theta_i)\, v_i, \quad \varphi(\theta_i) := [\cos(\theta_i)\ \ \sin(\theta_i)]^\top \tag{1a}$$

$$\dot{\theta}_i = \omega_i, \tag{1b}$$

where, for any $i \in \bar{N} := \{1, 2, ..., N\}$, $z_i = [x_i\ \ y_i]^\top \in \mathbb{R}^2$ denotes the Cartesian coordinates of the ith vehicle on the plane and $\theta_i \in \mathbb{R}$ denotes its orientation. In addition, let the vehicles' velocity dynamics be defined by

$$\begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \frac{1}{r_i} \begin{bmatrix} 1 & 1 \\ 2R_i & -2R_i \end{bmatrix} \begin{bmatrix} \tau_{li} \\ \tau_{ri} \end{bmatrix}, \tag{2}$$

where τ_{li} and τ_{ri} are the left and right wheel torques, respectively, I_i is the robot inertia, m_i is the mass, r_i is the wheel radius, and R_i is the wheel axle length. This model corresponds to that of differential-drive robots under a condition of non-slippage [22].

It is required that the vehicles meet in formation around a *non*-predefined rendezvous point on the plane, denoted $z_c := [x_c\ \ y_c]^\top$, and acquire a *non*-predefined common orientation, denoted θ_c, modulo a *given* offset $\delta_i = [\delta_{xi}\ \delta_{yi}]^\top$, with $i \in \bar{N}$, which determines the position of the ith vehicle relative to the unknown center of the formation. In other words, defining, $\bar{z}_i := z_i - \delta_i$ (correspondingly, $\bar{x}_i := x_i - \delta_{xi}$ and $\bar{y}_i := y_i - \delta_{yi}$) the control goal is to make

$$\lim_{t\to\infty} v_i(t) = 0, \quad \lim_{t\to\infty} \bar{z}_i(t) = z_c, \tag{3}$$

$$\lim_{t\to\infty} \omega_i(t) = 0, \quad \lim_{t\to\infty} \theta_i(t) = \theta_c \qquad \forall\, i \in \bar{N}, \tag{4}$$

for all initial conditions.

This is a leaderless consensus control problem, in which neither the coordinates (x_c, y_c) nor the angle θ_c are imposed a priori, but they depend on the initial postures, on the systems' *nonlinear* dynamics, and on network features. This problem has been successfully solved under different conditions; the originality of this letter resides in considering the following.

Constraint 1: Given $\bar{\tau}_{li} > 0$ and $\bar{\tau}_{li} > 0$, the left and right control input torques must satisfy $|\tau_{li}| \leq \bar{\tau}_{li}$ and $|\tau_{ri}| \leq \bar{\tau}_{ri}$. ◁

In order to solve the aforementioned problem, it is also assumed that each vehicle exchanges its relative position and its orientation with a set of neighbors, which we denote by $\mathcal{N}_i$. It is naturally assumed that once a communication is set between two vehicles i and $j \in \mathcal{N}_i$, the flow of information is bidirectional and is never lost. Whence the following.

Assumption 1: The network interconnection is static and it is modeled as an undirected and connected graph. ◁

Remark 1: That the graph is undirected and static may appear conservative if taken out of context. Even though there are many works in which Assumption 1 is relaxed, most often such results concern linear autonomous systems, some times nonlinear, but rarely nonholonomic. Tools of analysis, such as linear algebra, do not apply in the context of multiagent nonholonomic systems, specifically, controlled via time-varying feedback and Lyapunov functions for multiagent systems, even linear, under directed graphs is still under development. To the best of our knowledge, consensus control of nonholonomic systems under input constraints, even under Assumption 1 remains an open problem. ◁

To establish the proof of our result, it is convenient to define the Laplacian matrix $L := [\ell_{ij}] \in \mathbb{R}^{N\times N}$, where

$$\ell_{ij} = \begin{cases} \sum_{k\in\mathcal{N}_i} a_{ik} & i = j \\ -a_{ij} & i \neq j, \end{cases} \tag{5}$$

additionally $a_{ij} > 0$ if $j \in \mathcal{N}_i$ and $a_{ij} = 0$ otherwise. Defining $1_N := [1, \cdots, 1]^\top$, then by construction, $L1_N = 0$ and, after Assumption 1, L is symmetric, it has a unique zero-eigenvalue, and all of its other eigenvalues are strictly positive. Thus, $\text{rank}(L) = N - 1$.

III. Proposed Bounded Controller

The proposed scheme contains an inner control-loop that is designed as

$$\begin{bmatrix} \tau_{ri} \\ \tau_{li} \end{bmatrix} = \frac{r_i}{2} \begin{bmatrix} 1 & \frac{1}{2R_i} \\ 1 & -\frac{1}{2R_i} \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{\omega i} \end{bmatrix}. \tag{6}$$

After replacing (6) in (2), it yields

$$\dot{v}_i = \frac{1}{m_i} u_{vi}, \tag{7a}$$

$$\dot{\omega}_i = \frac{1}{I_i} u_{\omega i}. \tag{7b}$$

The control design exploits the fact that the model of the nonholonomic system consists in two interconnected second-order systems given by Eqs. (1a)-(7a) and (1b)-(7b). The former set of equations corresponds to the linear-motion dynamics while the latter corresponds to the angular-motion dynamics. Hence, we design a controller of each part of the plant separately.

The proposed scheme is composed of a bounded proportional to the error term plus a bounded damping injection part. The building block of this scheme is reported in [18] for Lagrangian systems that do not exhibit nonholonomic restrictions. The linear part of the controller is given by

$$u_{vi} = -d_{vi} \tanh(v_i) - p_{vi} \varphi(\theta_i)^\top e_{zi}, \tag{8}$$

where $p_{vi} > 0$ and $d_{vi} > 0$ are the proportional and the damping injection gains and the error e_{zi} is defined as

$$e_{zi} := \sum_{j\in\mathcal{N}_i} a_{ij} \tanh(\bar{z}_i - \bar{z}_j), \tag{9}$$

where $\tanh(\bar{z}_i - \bar{z}_j) := [\tanh(\bar{x}_i - \bar{x}_j) \quad \tanh(\bar{y}_i - \bar{y}_j)]^\top$.

The linear closed-loop equations are then given by

$$\Sigma_{vi}: \begin{cases} \dot{\bar{z}}_i = \varphi_i(\theta_i) v_i & \text{(10a)} \\ \dot{v}_i = -\frac{1}{m_i} \left[d_{vi} \tanh(v_i) + p_{vi} \varphi(\theta_i)^\top e_{zi} \right]. & \text{(10b)} \end{cases}$$

For the linear-motion dynamics, consider the following candidate Lyapunov function

$$V = \frac{1}{2} \sum_{i=1}^{N} \left[\frac{1}{p_{vi} m_i} v_i^2 + \sum_{j\in\mathcal{N}_i} a_{ij} \ln\left(\cosh(\bar{x}_i - \bar{x}_j) \right) \right] + \frac{1}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} a_{ij} \ln\left(\cosh(\bar{y}_i - \bar{y}_j) \right),$$

which is positive definite and radially unbounded with respect to v_i and $\bar{z}_i - \bar{z}_j$. Its derivative yields

$$\dot{V} = \sum_{i=1}^{N} \left[\frac{1}{p_{vi} m_i} v_i \dot{v}_i + \frac{1}{2} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\bar{z}}_i - \dot{\bar{z}}_j)^\top \tanh(\bar{z}_i - \bar{z}_j) \right].$$

Then, after Lemma 2.4 in [18] we have

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\bar{z}}_i - \dot{\bar{z}}_j)^\top \tanh(\bar{z}_i - \bar{z}_j) = \sum_{i=1}^{N} \dot{\bar{z}}_i^\top e_{zi},$$

so, using (10), we obtain

$$\dot{V} = - \sum_{i=1}^{N} \frac{d_{vi}}{p_{vi}} v_i \tanh(v_i) \leq 0. \tag{11}$$

Evaluating the latter along the trajectories and integrating on both sides of $\dot{V}(v(t), \bar{z}(t)) \leq 0$ we conclude that $|v_i(t)|$ and $|\bar{z}_i(t) - \bar{z}_j(t)|$ are uniformly bounded. Using the fact that $\ddot{V}$ is also uniformly bounded, by Barbalăt's Lemma, we have $\lim_{t \to \infty} \dot{V}(t) = 0$ and, in turn, $\lim_{t \to \infty} v_i(t) = 0$. Furthermore, in Section IV it is shown that, also, $\lim_{t \to \infty} \dot{v}_i(t) = 0$, so from Σ_{vi},

$$\lim_{t \to \infty} \varphi(\theta_i(t))^\top e_{zi}(t) = 0.$$

Notice, however, that this does not imply that e_{zi} converges to zero. In fact, the angular controller must be designed to excite all the modes in Σ_{vi}, via the term $\varphi(\theta_i(t))$. Therefore we design the controller for the angular-motion dynamics as

$$u_{\omega i} = - d_{\omega i} \tanh(\omega_i) - p_{\omega i} e_{\theta i} + \alpha_i(t, \theta_i, e_{zi}), \tag{12}$$

where $p_{\omega i} > 0$, $d_{\omega i} > 0$,

$$e_{\theta i} := \sum_{j \in \mathcal{N}_i} a_{ij} \tanh(\theta_i - \theta_j). \tag{13}$$

and α_i is the function

$$\alpha_i(t, \theta_i, e_{zi}) := k_{\alpha i} \psi_i(t) \varphi(\theta_i)^{\perp\top} e_{zi}, \tag{14}$$

where $k_{\alpha i} > 0$, $\varphi(\theta_i)^\perp := [-\sin(\theta_i) \quad \cos(\theta_i)]^\top$ is the annihilator of $\varphi(\theta_i)$ hence, $\varphi(\theta_i)^{\perp\top} \varphi(\theta_i) = \varphi(\theta_i)^\top \varphi(\theta_i)^\perp = 0$. The function ψ_i is twice differentiable, bounded, with bounded derivatives. Therefore, it holds that $|\psi_i(t)| \leq \bar{\psi}_i$, for $\bar{\psi}_i > 0$. Moreover, $\dot{\psi}_i$ is persistently exciting, that is, there exist T and $\mu > 0$ such that

$$\int_t^{t+T} \dot{\psi}_i(s)^2 ds \geq \mu, \qquad \forall\, t \geq 0. \tag{15}$$

The function α_i plays a fundamental role in the stabilization task. It prevents θ_i from stationing at a constant value θ_c as long as the consensus errors e_{zi} *persist* away from zero. During the transient, α_i acts as a uniformly bounded perturbation on the closed-loop angular-motion dynamics,

$$\Sigma_{\omega i} \begin{cases} \dot{\theta}_i = \omega_i, \\ \dot{\omega}_i = - \dfrac{1}{I_i} \left[d_{\omega i} \tanh(\omega_i) + p_{\omega i} e_{\theta i} - \alpha_i(t, \theta_i, e_{zi}) \right]. \end{cases} \tag{16}$$

In the next section we show that in view of (15), $\alpha_i \not\to 0$ unless $e_{zi} \to 0$.

Roughly speaking, α_i acts as perturbation that is δ-persistently exciting with respect to e_{zi} (namely, persistently exciting for any $\delta > 0$ and any $|e_{zi}| \geq \delta$—see [23]). Such excitation, when injected into the stable system $\Sigma_{\omega i}$, entails a similar behavior for $\theta_i(t)$ and, therefore, for $\varphi(\theta_i(t))$. The latter acts as the "external" time-varying signal needed in (10) to overcome the nonholonomicity and make $e_{zi} \to 0$. We do not prove formally that α_i is δ-persistently exciting, but these arguments serve as a rationale behind our main result.

Proposition 1 (Main Result): Consider the system modeled by (1) and (2) under Assumption 1 and Constraint 1 in closed loop with the controller defined by (6), (8), (12) and (14). Let the control gains $p_{vi}, p_{\omega i}, k_{\alpha i}$, d_{vi}, and $d_{\omega i}$ be positive constants such that, defining $\bar{u}_{vi} := d_{vi} + \sqrt{2} p_{vi} \ell_{ii}$ and $\bar{u}_{\omega i} := d_{\omega i} + \left(p_{\omega i} + \sqrt{2} k_{\alpha i} \bar{\psi}_i \right) \ell_{ii}$, satisfy

$$d_{\omega i} \geq \frac{\sqrt{2} k_{\alpha i} \bar{\psi}_i \ell_{ii}}{\tanh(1)}, \tag{17}$$

$$2 R_i \bar{u}_{vi} + \bar{u}_{\omega i} < \frac{4 R_i}{r_i} \min\{ \bar{\tau}_{ri},\ \bar{\tau}_{li} \}. \tag{18}$$

Then, (3) and (4) hold, that is, the consensus-formation objective is reached. □

IV. Proof of Proposition 1

We start by showing that the actuators do not saturate if (18) holds. Hence, the Constraint 1 is met. To that end, note that, from (8) and $|\tanh(\cdot)| < 1$, we have

$$|u_{vi}| \leq d_{vi} + \sqrt{2} p_{vi} \ell_{ii} =: \bar{u}_{vi},$$

and

$$|u_{\omega i}| \leq d_{\omega i} + \left(p_{\omega i} + \sqrt{2} k_{\alpha i} \bar{\psi}_i \right) \ell_{ii} =: \bar{u}_{\omega i}.$$

Therefore, from (6) we have

$$\max\{ |\tau_{ri}|,\ |\tau_{ri}| \} \leq \frac{r_i}{2} \left[\bar{u}_{vi} + \frac{1}{2 R_i} \bar{u}_{\omega i} \right]$$

and, in view of (18), it follows that $|\tau_{ri}| < \bar{\tau}_{ri}$ and $|\tau_{li}| < \bar{\tau}_{li}$.

The rest of the proof is divided in two parts, boundedness of the state trajectories and convergence of the synchronization errors.

Boundedness: we already established that v_i and $|\bar{z}_i - \bar{z}_j|$ and $\dot{v}_i \in \mathcal{L}_\infty$. For the orientations and the angular velocity, consider the Lyapunov function candidate,

$$W := \frac{1}{2} \sum_{i=1}^{N} \left[\frac{1}{p_{\omega i} I_i} \omega_i^2 + \sum_{j \in \mathcal{N}_i} a_{ij} \ln\left(\cosh\left(\theta_i - \theta_j \right) \right) \right],$$

which is positive definite and radially unbounded with respect to ω_i and $(\theta_i - \theta_j)$, for all $j \in \mathcal{N}_i$ and all $i \in \bar{N}$. The time-derivative of W along the trajectories of $\Sigma_{\omega i}$ in (16) satisfies

$$\dot{W} = - \sum_{i=1}^{N} \frac{1}{p_{\omega i}} \left[d_{\omega i} \omega_i \tanh(\omega_i) - \omega_i \alpha_i \right]. \tag{19}$$

This is obtained using [18, Lemma 2.4] to show that

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{\theta}_i - \dot{\theta}_j) \tanh\left(\theta_i - \theta_j \right) = \sum_{i=1}^{N} \dot{\theta}_i e_{\theta i}.$$

Now, by definition, α_i is uniformly bounded—see Eqs. (14) and (9) and recall that $|\psi_i(t)| \leq \bar{\psi}_i$. More precisely, $|\alpha_i| \leq \sqrt{2}k_{\alpha i}\bar{\psi}_i\ell_{ii}$, so

$$\dot{W} \leq -\sum_{i=1}^{N} \frac{1}{p_{\omega i}} \left[d_{\omega i}\omega_i \tanh(\omega_i) - \sqrt{2}k_{\alpha i}\bar{\psi}_i\ell_{ii}|\omega_i| \right].$$

Furthermore, since $\tanh(\cdot)$ is strictly increasing, for all $|\omega_i| \geq 1$, we have

$$\dot{W} \leq -\sum_{i=1}^{N} \frac{1}{p_{\omega i}} \left[d_{\omega i} \tanh(1) - \sqrt{2}k_{\alpha i}\bar{\psi}_i\ell_{ii} \right] |\omega_i|,$$

which, in view of (17) implies that $\dot{W} \leq 0$. Hence, for all $t \geq 0$ such that $|\omega_i(t)| \geq 1$ we have $\dot{W}(\omega(t), \theta(t)) \leq 0$ so $|\omega_i(t)|$ is bounded. For any other t, $|\omega_i(t)| \leq 1$. This implies that $\omega_i \in \mathcal{L}_\infty$. Also note that all terms on the right-hand side of (16) are uniformly bounded, so $\dot{\omega}_i \in \mathcal{L}_\infty$.

Convergence: Assume, for the time being, that $\alpha_i \equiv 0$ for all $i \in \bar{N}$. Then, a simple inspection of (19) shows that $\ddot{W} \in \mathcal{L}_\infty$ and, invoking Barbalăt's Lemma, we conclude that $\dot{W} \to 0$, which implies in turn that $\lim_{t\to\infty} \omega_i(t) = 0$. The same conclusion is drawn for $\dot{\omega}_i$, after differentiating on both sides of (16) and observing that $\ddot{\omega}_i$ is uniformly bounded. Hence, under the condition that $\alpha_i \equiv 0$, we see from (16) that $\dot{\omega}_i \to 0$ and $\omega_i \to 0$ imply that $\lim_{t\to\infty} e_{\theta i}(t) = 0$. Now, $e_{\theta i} = 0$ and the statement in [18, Lemma 2.4] imply that

$$\sum_{i=1}^{N} \theta_i e_{\theta i} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} a_{ij}(\theta_i - \theta_j) \tanh(\theta_i - \theta_j) = 0,$$

so, since $\theta_i - \theta_j$ has the same sign as $\tanh(\theta_i - \theta_j)$, then $e_{\theta i} = 0$ implies that $\theta_i - \theta_j = 0$, for all $j \in \mathcal{N}_i$ and $i \in \bar{N}$. Using the Laplacian matrix we obtain $L\theta = 0$, where $\theta = \mathrm{col}(\theta_i)$. Under Assumption 1, there exists $\theta_c \in \mathbb{R}$ such that $\theta = 1_N\theta_c$. The limits in (4) follow. Furthermore, because the closed-loop solutions are uniformly globally bounded under any bounded $\alpha_i \not\equiv 0$, the same conclusion follows using a cascades argument [24], provided that $\alpha_i \to 0$. The convergence of α_i is established next, using the persistency-of-excitation assumption on $\dot{\psi}_i$,

We already established that $\lim_{t\to\infty} v_i(t) = 0$—see below Ineq. (11). This, in turn, implies that

$$\lim_{t\to\infty} \int_0^t \dot{v}_i(t) = -v_i(0).$$

Hence, to prove that $\lim_{t\to\infty} \dot{v}_i(t) = 0$ by Barbălat's Lemma, we show that $\dot{v}_i$ is uniformly continuous. The latter follows from the fact that $\ddot{v}_i \in \mathcal{L}_\infty$. To better see this, using (10b), we write

$$\begin{aligned} \ddot{v}_i = &-\frac{1}{m_i}\left[d_{vi}\dot{v}_i \mathrm{sech}^2(v_i) + p_{vi}\omega_i\varphi(\theta_i)^{\top\perp} e_{zi} \right] \\ &+ \frac{1}{m_i} p_{vi}\varphi(\theta_i)^\top \dot{e}_{zi} \end{aligned} \tag{20}$$

and we observe that all the terms on the right-hand-side are bounded. Thus, $v_i \to 0$ and $\dot{v}_i \to 0$, so from (10b) we have

$$\lim_{t\to\infty} \varphi(\theta_i(t))^\top e_{zi}(t) = 0. \tag{21}$$

Invoking Barbalăt's Lemma, it also follows that $\lim_{t\to\infty} \ddot{v}_i(t) = 0$. From $\dot{v}_i \to 0$, (9), and (10a), we have $\dot{e}_{zi} \to 0$. Thus, the first and last terms on the right-hand side of (20), as well as $\ddot{v}_i$, vanish individually. Consequently,

$$\lim_{t\to\infty} \omega_i(t)\varphi(\theta_i(t))^{\top\perp} e_{zi}(t) = 0. \tag{22}$$

Now, because φ and $\varphi^\perp$ take values in orthogonal spaces, if (21) and (22) hold simultaneously whereas $\omega_i \not\to 0$, then $\lim_{t\to\infty} e_{zi}(t) = 0$. If, alternatively, (21) and (22) hold simultaneously because $\lim_{t\to\infty} \omega_i(t) = 0$, then

$$\lim_{t\to\infty} \int_0^t \dot{\omega}_i(\sigma)d\sigma = -\omega_i(0).$$

Moreover,

$$\ddot{\omega}_i = -\frac{1}{I_i}\left[d_{\omega i}\dot{\omega}_i \mathrm{sech}^2(\omega_i) + p_{\omega i}\dot{e}_{\theta i} - \dot{\alpha}_i \right], \tag{23}$$

where

$$\begin{aligned} \dot{\alpha}_i = &\, k_{\alpha i}\dot{\psi}_i(t)\varphi(\theta_i)^{\perp\top} e_{zi} - k_{\alpha i}\omega_i\psi_i(t)\varphi(\theta_i)^\top e_{zi} \\ &+ k_{\alpha i}\psi_i(t)\varphi(\theta_i)^{\perp\top}\dot{e}_{zi} \end{aligned} \tag{24}$$

is bounded because so are all the terms on the respective right-hand sides of (24) and (23). Hence $\lim_{t\to\infty} \dot{\omega}_i(t) = 0$ and $\lim_{t\to\infty} \int_0^t \ddot{\omega}_i(\sigma)d\sigma = -\dot{\omega}_i(0)$. A direct similar computation shows that, also, $\omega_i^{(3)} \in \mathcal{L}_\infty$, so, by Barbalăt's Lemma, it follows that $\lim_{t\to\infty} \ddot{\omega}_i(t) = 0$. From this and (23) it follows that $\dot{\alpha}_i \to 0$, so from (24) we conclude that $\lim_{t\to\infty} \dot{\psi}_i(t)\varphi(\theta_i(t))^{\perp\top} e_{zi}(t) = 0$. However, since $\dot{\psi}_i(t)$ satisfies (15), $\dot{\psi}_i(t) \not\to 0$, so the last limit holds only if $\lim_{t\to\infty} \varphi(\theta_i(t))^{\perp\top} e_{zi}(t) = 0$, which together with (21), implies that $\lim_{t\to\infty} e_{zi}(t) = 0$.

Finally, invoking Lemma 2.4 in [18], note that if $e_{zi} = 0$

$$\sum_{i=1}^{N} \bar{z}_i^\top e_{zi} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} a_{ij}(\bar{z}_i - \bar{z}_j)^\top \tanh(\bar{z}_i - \bar{z}_j) = 0,$$

so it follows that $\bar{z}_i - \bar{z}_j = 0$, for all $j \in \mathcal{N}_i$ and $i \in \bar{N}$. Using the Laplacian matrix we obtain $(L \otimes I_2)\bar{z} = 0$, where $\bar{z} = \mathrm{col}(\bar{z}_i)$. Under Assumption 1 there exists $\bar{z}_c \in \mathbb{R}$ such that $\bar{z} = 1_N \otimes \bar{z}_c$. We conclude that $\lim_{t\to\infty} e_{zi}(t) = 0$ implies the second limit in (3).

V. Simulation results

To illustrate the performance of the controller proposed above, in this section we present some comparative numerical simulations using a network of six differential drive mobile robots. In addition, to illustrate the controller's robustness, we use a robot model with a center of mass located off the axis connecting the two wheels. Such misplacement entails Coriolis terms of second order in the velocities on the right-hand side of Equations (7)—*cf.* [22], which are neglected in the design of the controller in Proposition 1.

We compare the controller of Proposition 1 with one in which $\tanh(s)$ is replaced by s everywhere. If delays are neglected, this is the controller proposed in [15]. We used

TABLE I

PHYSICAL PARAMETERS AND ACTUATORS BOUNDS

index	m_i [Kg]	I_i [Kg m^2]	R_i [m]	r_i [m]	$\bar{\tau}_i$ [N]
$1,2$	10	3	0.3	0.05	105
$3,4$	36	15.625	0.15	0.15	480
$5,6$	23	9.3125	0.225	0.1	230

three pairs of robots with different physical parameters for each pair and actuator capacity—see Table I.

The initial conditions for each robot, as well as the offsets that define a triangular formation, are given in Table II below.

TABLE II

INITIAL CONDITIONS

index	$x_i(0)$	$y_i(0)$	$\theta_i(0)$	δ_{xi}	δ_{yi}
1	-7.5	0	π	5	5
2	-7.5	5	$\pi/2$	7.5	10
3	7.5	5	$\pi/2$	10	15
4	7.5	0	0	12.5	10
5	2.5	0	$-\pi/4$	15	5
6	-2.5	10	$-3\pi/4$	10	5

The control gains for each robot are set to satisfy the conditions (17) and (18), as follows: $\mathbf{p}_v = [600\ \ 1000\ \ 600\ \ 600\ \ 600\ \ 600]^\top$, $\mathbf{d}_v = [200\ \ 100\ \ 200\ \ 300\ \ 300\ \ 300]^\top$, $\mathbf{d}_\omega = [400\ \ 400\ \ 600\ \ 600\ \ 400\ \ 400]^\top$, $\mathbf{k}_{\alpha i} = [20\ \ 20\ \ 30\ \ 30\ \ 20\ \ 20]^\top$, and $p_{\omega i} = 50$, and we use the multi-periodic function

$$\psi_i(t) \ := \ 1.25 + \sum_{k\in\{1,3,5,7,9\}} \frac{4}{k\pi} \sin\left(\frac{kt}{2}\right),$$

but periodicity is not necessary for persistency of excitation.

For a fair comparison we used identical initial conditions, gains and persistence of excitation functions for both control algorithms. Finally, the network interconnection topology is illustrated in Fig. 1 below.

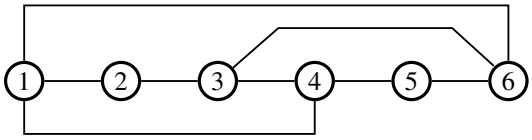

Fig. 1. Undirected-graph topology used in the numerical simulations

The simulation results are illustrated in Figures 2–6. As it is appreciated from Figures 2 and 3 full consensus, *i.e.*, in position and orientation, is achieved under either control scheme and with a fairly similar transient performance—see also Figures 4 and 5. Figure 6 contains a matrix of plots corresponding to the input torques. Each row corresponds to each subset of robots according to their physical parameters, while the input torques for each controller, using saturation and not, are depicted side to side in two columns. The plots on the right correspond to the presaturated input torques, which remain well within the respective limits for each pair of robots: $\sup_{t\geq 0}|\tau_i| \in \{100.2\ \ 99.8\ \ 471.4\ \ 478.9\ \ 170.8\ \ 226.5\}$ for $\{i\} = \{1, 2, \ldots\}$—cf. Table I.

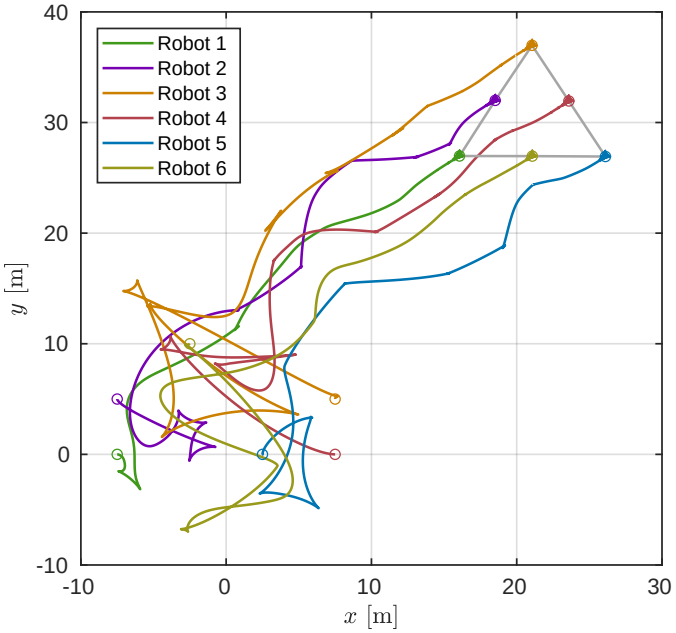

Fig. 2. Paths followed by the agents under the unbounded scheme (orientation of each agent is represented by an arrow.

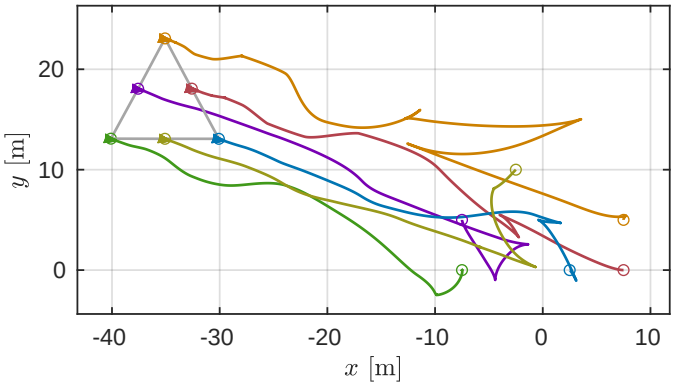

Fig. 3. Paths followed by the agents under the bounded scheme (orientation of each agent is represented by an arrow.

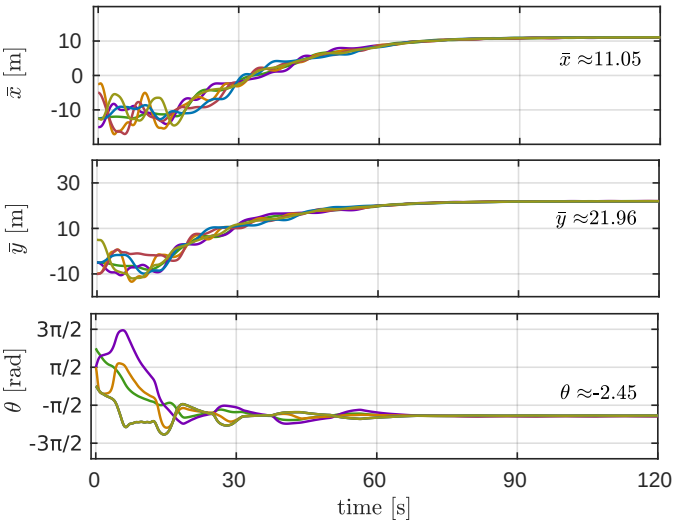

Fig. 4. Pose consensus for the unbounded scheme.

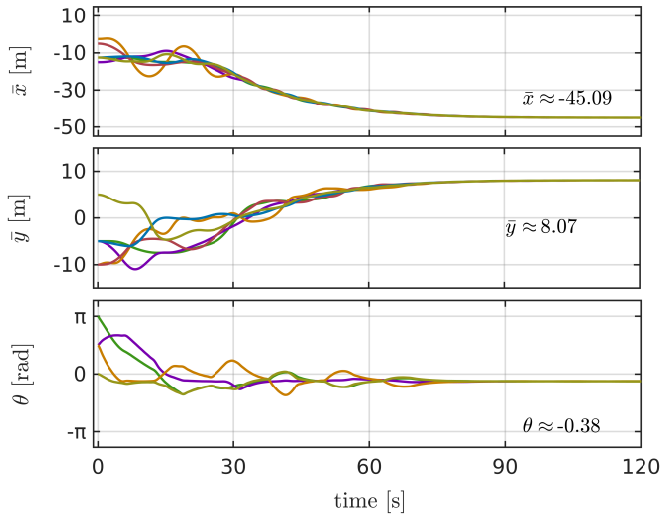

Fig. 5. Pose consensus for the bounded scheme.

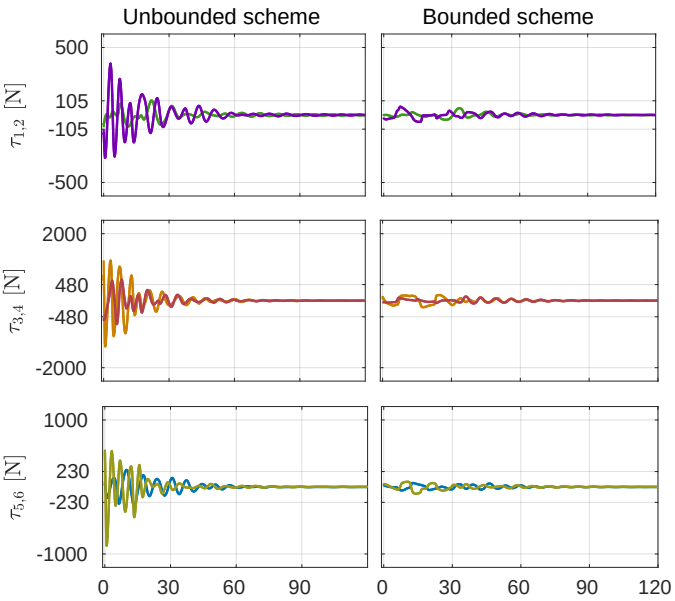

Fig. 6. Torque results for the unbounded scheme.

VI. Conclusions

The results presented in this letter extend previous works on consensus-formation control of nonholonomic systems, to the important and unaddressed case in which the control inputs are guaranteed to respect bounds imposed *a priori*. The analysis of the closed-loop system guarantees that the consensus errors converge to zero asymptotically. Constructing a strict Lyapunov function to guarantee uniform global asymptotic stability of the consensus manifold is still a significant open problem, whose solution may form a solid basis to relax the assumption that the graph is static and bidirectional.

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