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The Life and Work of Vera Stepen Pless

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Her life

Vera Stepen Pless was a groundbreaking mathematician, whose tenacity and grit helped open the door for women mathematicians. Her work in algebraic coding theory was foundational and her impact on the subject was enormous. Additionally, as an interesting and unique individual, people were almost as fascinated by her persona as they were with her work.

Vera was born March 5, 1931 and grew up in the Douglas Park neighborhood of Chicago. Her parents were Russian Jewish immigrants who came to Chicago early in the 20th century. She lived in a small two bedroom apartment above a pharmacy where one bedroom was her mother Helen Binder Stepen's dental practice. Vera herself slept in the dining room. Her mother had trained as a dentist in Russia and retrained in the United States in order to practice in Chicago. Many say that Vera inherited her ability to break through professional glass ceilings from her mother who was a highly educated professional at a time when immigrant women would rarely have been. Her father Lyman Stepen was brought to America by his sister, served in the U.S. army during World War I, and then trained as a jeweler. It was often said that from him Vera inherited a love of the visual and performing arts that persisted to the end of her life.



Young Vera Pless

Vera always had a strong academic drive. As a child she would often be seen reading books or deep in thought. She was intellectually mature beyond her age and had a great interest in music, the arts, and natural science. One of her favorite childhood books was the encyclopedic Book of Knowledge, which she used to read to her cousin to distract her from her “pain and anxiety” while her cousin was sitting in her mother’s novocaine-free dentist’s chair. Vera spent a great deal of time reading all sorts of books, including a passion for foreign dictionaries where she enjoyed looking up words. She showed her ingenuity as a child, when she and her cousin Estelle made fudge to sell to relatives and the fudge turned completely to liquid. Vera decided that they should sell fudge-ade instead.

As a young student, Vera also attended variously Hebrew school, Yiddish school and Russian school after regular school. She had a life long passion for languages, especially Hebrew. After she retired and until the very end of her life she had tutors come in for Hebrew and Yiddish lessons. Yiddish had been her parents’ language and she grew up with that language all around her, including frequent visits to the Yiddish theater. She noted that funny things were funnier in Yiddish and sad things were sadder in Yiddish.



Teenage Vera Pless

She left high school two years early and enrolled at the University of Chicago under a program for impoverished gifted students. The only thing that gave her a second thought about advancing so early was that she was in line to be first cello in her high school orchestra and going to college early meant she had to pass up that opportunity. She finished her undergraduate degree in only three years.



Student Vera Pless

She stayed on for a Masters degree, which she received in 1952. She then moved to Northwestern University where she obtained a Ph.D. under Alex F. T. W. Rosenberg in 1957 at the young age of 26. She defended her dissertation two weeks before her first child was born. Shortly before this she moved to Boston with her husband Irwin Pless who had accepted a position on the Physics faculty at the Massachusetts Institute of Technology and began her family.

Vera's first real academic position after some part-time teaching, was with the Air Force Cambridge Research Laboratory in Bedford, Massachusetts in 1963. She greatly enjoyed

this position because it gave her the freedom to pursue her academic interests and to raise her young family. During this time, she helped to found the organization *Women in Science and Engineering* which she later lead as president.



Professor Vera Pless

She worked at the Air Force Cambridge Research Laboratory until the laboratory was closed down due to a change in what the Defense Department was able to fund. Like many academic women during that period, she was unable to obtain a tenure track academic position. At the time, the chair of the Massachusetts Institute of Technology Mathematics department had said in public that there were no women anywhere in the world qualified to be on their faculty. Instead, in 1972, MIT hired her as a senior research associate in the electrical engineering department where she taught courses and published papers, but was not recognized as faculty. While she was at MIT, she was asked to be on the admissions committee to prevent bias against women. Vera noted that she was seeing more bias against Jews and that was where she ended up focusing. Throughout her academic career, Vera worked to give opportunity to those who were being denied. At one point, early in her career, Vera was at a math conference and being a woman they only gave her half the time for her talk as everyone else. She responded by giving only half a talk, namely, she stopped in the middle of the talk and sat down, leaving the audience to beg her to finish.

In 1975, she was able to secure a position at the University of Illinois at Chicago Circle. Because her publications supported it, Vera was offered a full professorship with tenure, albeit at a rather low salary, despite never having been an assistant or an associate professor. A couple of years after that, she got an offer for a professorship at another university at approximately twice her salary. The University of Illinois was able to match it, and so she

stayed there until she retired in 2006. In 2013, she was named a Fellow of the American Mathematical Society.

During her time at UIC, she directed 12 Ph.D. students. It was often noted that she was very motherly to her students. She would guide them through their early career and help them to find employment and opportunities.

Vera was passionate about the arts and music. In her last decade, each week she would take her trusted caregiver Sophie Katz to theater performances, to Chicago's Lyric opera, and to other cultural events.

Like other great mathematicians, people were fascinated not only with her mathematics but also with her as a person. Stories abound of this small woman, dressed in a sweater, looking like a kind grandmother, intimidating mathematicians all over the world.

She was very dedicated to her family and very proud of them as well. Vera had three very successful children Naomi, Ben and Daniel, and had four grandchildren, Lilah, Evie, Rebecca and Jesse.

Vera died three days short of her 89-th birthday, on March 2, 2020. True to her character, she died with a book in her hands.

Vera often told her daughter Naomi, that when she was a child, her mother told her that if there were people around that made her feel unsafe, then she should walk right down the middle of the street. She would often do this, carrying her cello home from lessons. This may be the paradigmatic example of Vera's character. Namely, when malevolent forces were around her, she did not cower in a corner, but rather picked up her work and walked bravely down the middle of the street.

Vera met our father, Irwin Pless, at the University of Chicago. He was a World War II Air Force veteran going to college on the GI Bill. She was attending under a program that admitted talented students after their second year of high school. There was a six year age gap between them. Vera recalled that her Russian immigrant parents were crazy about Irwin. It seems this was at least in part because he was set to rescue her from the spinsterhood they feared for her. They were relieved when at the advanced age (from their perspective) of 21, she was finally married. If they were also proud of her for obtaining her Master's degree that same year, it didn't impress itself enough on her to share with us later. Our parents moved to the Boston area when our father obtained a position in the MIT Physics department, and Vera commuted back to Chicago to finish her PhD. She took her PhD orals two weeks before Naomi's birth shortly after being required to move. The apartment they rented in Cambridge allowed them to have their beloved dog, Lucy, however babies were not permitted. As a result, they bought their first house, a small place in Waltham, and then moved again in 1963 to Lexington because they wanted a better school district for Naomi and

Ben. The University of Chicago, at the time Vera was there, had the Great Books Program, in which everything was learned from original sources. This included even scientific subjects such as physics. It was never clear to us whether she had ever really learned that light did not actually vibrate through a luminiferous ether. Thus when Naomi and Ben were in high school, and Vera was appointed to the MIT Electrical Engineering department in a senior research associate position, we were skeptical. Laughing, we asked if she even knew what Ohm's law was. She responded, "Of course I do", also with a laugh. "Be it ever so 'umble there's no place like Ohm!" As with the ether, it really wasn't clear whether she had truly never learned Ohm's law. Our father's office at MIT was in Tech Square. When Vera came to MIT, her office was also in Tech Square in the twin building across the courtyard from his. Vera spent three years at MIT working on Project MAC. With bemusement, she would occasionally mention an idiosyncratic student who did some programming for her. He would show up late at night and program until the early hours of the morning. She would find the results of his nocturnal efforts when she went in to work the next day. He was Richard Stallman, later to go on to found the GNU project and the Free Software Foundation. Stallman was not Vera's only programmer. When Dan was in elementary school, Vera would ask him to write programs she needed for her programmable TI-58 calculator. She paid him one dollar per program. Dan later went on to obtain a Ph.D. in computer science. It seemed to us that Vera traveled everywhere around the world and knew mathematicians wherever she went. As a teenager Ben traveled to Hungary with Vera to a mathematics conference. Vera always sought out whatever fun was to be had. When roving musicians serenaded them at a restaurant dinner leaning in close with their violins, Ben was embarrassed, but Vera loved it, dancing along in her seat to Ben's mortification. As was not unusual, she was the only woman mathematician at that conference. The organizers in their wisdom gave her only half the time for her talk that was allotted to the men. Despite having prepared a standard length talk, Vera did not seem upset. When her time was up halfway through her talk, she gathered up her papers, walked off the stage, and declined to take questions. Her only answer to inquiries was to point out that if people wanted to hear her whole talk they needed to give her a whole time slot. When Naomi and Ben were in college, Vera spent a sabbatical at Cambridge University in England. All three of us went to stay with her for a couple of cold English summer weeks (during which it never quite rained and never quite stopped raining). One of the memorable recollections of this visit was hosting John Conway for dinner. The rented house had a number of couch bolster pillows, and one of the highlights of the evening was a pillow fight between John and Dan and Ben. There was a crash when a pillow hit one of the framed pictures. Vera calmly took

it to a framing store a few days later and had the glass replaced. John and Ben stayed up until the wee hours of the morning playing game after game of backgammon which was John's passion at the time. Dan remembers that it was the passion of quite a few members of the math department at Cambridge. He recalls that at lunch in the common room, backgammon games were played at ferocious speed with players rolling the dice as soon as their opponents had relinquished them, with moves made as quickly. Vera loved to travel, and as Ben's wife once remarked about vacationing with her, Vera was always willing to try anything once (this in contrast to typical women of her generation). Vera may have passed that spirit of adventure on to Ben who became a biomedical entrepreneur. Was it different growing up with a mathematician mother? Probably. As kids the main difference seemed to be that neighbor moms (at that time) mostly didn't have careers. However the moms we knew well were the physicists Vera Kistiakowsky and Elizabeth Barranger. They were Vera's best friends and had kids close to our own ages. Differences we likely never noticed crystallized for Naomi when her husband-to-be asked a question about directions to get somewhere in Chicago. "Oh, it's very easy," Vera explained. "Chicago streets are laid out on Cartesian coordinates." He had no idea what she meant. This was a perfectly normal sentence in our family, and it forced Naomi to see that some of what we said, and even how we saw the world, was - well - different. Naomi remembers an argument Vera had with a medical receptionist who was being petty about a meaningless bit of bureaucracy. Finally agreeing to sign Vera up, the receptionist asked, "Is it Miss or Mrs.?" Vera's emphatic reply, "It's Doctor!" Now a physician, Naomi will admit to having on occasion answered the same question that same way. Vera was an email wizard! Years before anyone had email, Vera had email. In the days when if you made a mistake you had to erase back to where the mistake was and then retype the whole thing forward again, Vera had email. When Vera was on sabbatical at the Technion in Israel in 1989, Naomi and Vera would communicate with Naomi using Vera's University of Illinois email to reach Vera's Technion email. It would be another four years before Naomi qualified for an "experimental" email account through her home university. When Vera wanted to learn something new in mathematics, one of her techniques was to volunteer to teach it. As she explained to us, all she needed to do was to stay a couple of lessons ahead of the students. At a family gathering, defending a granddaughter complaining about calculus, Vera made it clear that in her opinion calculus did not even qualify as real mathematics, and said that she always had to relearn it when she was assigned to teach it. As far back as any of us can remember, Vera always had a small handle-less leather briefcase that she carried pretty much everywhere. In this were pads of graph paper marked

in her handwriting with arrays of ones and zeros. This briefcase was precious and was never left in a car. If she had a minute sitting in a restaurant, waiting in a waiting room or even lounging on the beach, its contents always provided Vera with something enjoyable to do. Ben remembers being at lunch on a trip to Israel with Vera. They were with a small crowd of (male) mathematicians. One of them posed a mathematics puzzle to be solved, and there ensued an enormous amount of loud talking, and talking over each other. Vera did not seem to be paying attention; she was quietly working away on one of her notepads as usual. During a break in the arguing she looked up and provided an answer to the puzzle and her reasoning for it. Everything froze for several moments while the other mathematicians considered what they had just heard. This was followed by a bit of appreciative mumbling and a sudden renewed interest in the food on the table. Early in 2020 Vera broke her hip. She died at home about a month later, just days before her 89th birthday. Unbeknown to us, Covid pandemic lockdowns would start in a few short weeks, but we were able to have (what was to become) the luxury of a real funeral with all of us, as well as all of Vera's grandchildren, in attendance. Having faced discrimination as a woman in mathematics, Vera always felt it her responsibility to make things easier for the women who came after her, and also for people who were from minority backgrounds or from other countries. She was devoted to music and art, and was generous in her contributions to the arts and to human rights causes. She loved music. Vera learned to play both the cello and the piano. If you played her a piece of classical music, she could identify the composer by ear. She was gifted in languages. She spoke Russian with her Russian immigrant dentist. She studied Hebrew and Yiddish literally until the day she died. She loved the arts, and enjoyed attending the performing arts as well as visiting art museums. It may be that for her math was simply another form of art, and it was her art form. Languages, art and math are all about patterns, and perhaps it was patterns that she loved. We would think of her as working steadily and consistently all the time with her pads of graph paper marked up with ones and zeros. That may have been true, but perhaps we were also seeing an artist devoted to her art. – Naomi Pless, Ben Pless and Dan Pless (Vera's children)

As someone who is functionally math-illiterate, I, Vera Pless' former son-in-law, had something of a unique relationship with Vera. I say, without pride, but truthfully, that the last math class I completed was in my sophomore year of high school. It was a business math class, covering the simplest mathematical concepts: basic operations as applied to fractions and that kind of thing. I failed it. I met Vera after courting (and eventually marrying) her daughter Naomi.

Vera and I first established a connection on a curious shared point of reference: the University of Chicago's Great Books Program. This program emerged from the University of Chicago in the 1940s, creating an undergraduate program that promoted seminar-based study of primary "Great Books" texts through Socratic dialogue. The program offered early college entrance to gifted students. Vera had been accepted at the tender age of 16. Vera would say her understanding of the physical universe came from Lucretius, whom she had read as so much poetry at the University of Chicago. I came to the "Great Books" program by way of a high school teacher. Perhaps as a way of helping a floundering student academically find himself, he suggested I consider enrolling in an undergraduate "Great Books" program. To that end I enrolled at Shimer College which had once been affiliated with the University of Chicago, and retained vestiges of the Great Books program. By the time I met Vera I also had very briefly attended St. John's College, another Great Books school, where I spent a scant few weeks lamely demonstrating mathematical proofs from Euclid's Elements - or trying to - before I dropped out, to everyone's relief. Not long after that I found unlikely love in San Francisco with Vera's daughter, Naomi. When in time I "was brought home" to meet Vera, I must have remarked I had trouble finding my way around the city. "Oh, it's very easy," Vera explained. "Chicago streets are laid out on Cartesian coordinates." She was clearly oblivious to the fact that she was not speaking standard English. If my relationship with Vera's daughter was unlikely, my relationship with Vera was more so. I can even now barely retain single-digit numbers in my head. And yet, Vera and I quickly formed a bond, one that strengthened over time. I'm fond of saying that Vera was the Jewish mother I never had, but always needed. While I loved my parents deeply, they failed to respond to my interest in music, or in any way encourage it. When Vera learned I loved music, she had her cello repaired and loaned it to me. The kindness and thoughtfulness of the gesture rather overwhelmed me at the time. I later learned that she had had difficulty choosing to attend the University of Chicago because it meant she had to give up the first cello position in her high school orchestra that she had been promised. That same high school cello was the one she refurbished for me. While I never learned to play the cello, I never forgot the faith Vera showed in me, and my musical talents were later to develop more successfully on other fronts, in part at least thanks to Vera. My relationship with Vera endured even through my divorce with her daughter. I stayed in touch, phoning her on Mother's Day and on her Birthday. I always kept my eye out for Yiddish and Hebrew folk song LPs and CDs that I'd send to her. She always responded with a gracious thank you note. Toward the end of her life, I was honored to be included in several of the family Passover Seders at Vera's house. During one of these

Seders, Vera looked up with a gleam in her eye, surveyed the assembled family, which included both me and Vera's own former husband and announced: "How funny?! There are more former spouses here than current spouses." She got a big laugh out of that. In my last conversation with Vera shortly before she died, I confided that she'd been like a mother to me. She replied simply that she'd always thought of me as a son. Given Vera's deep and abiding love of mathematics, the fact she could rate a math-illiterate like me in the "son" category shows Vera's heart was as formidable as her intellect. Vera viewed math as an art and a way of seeing life's beauty. That she also saw qualities in me my own mother failed to glimpse, and sought to help me find my way, forever endears me to Vera and her memory. – Nick Crews

Her work in mathematics

After her Ph.D. in 1957, which was titled "Quotient Rings of Continuous Transformation Rings." Her first publication was on the continuous transformation ring of biorthogonal bases spaces. After this Vera branched out and started to do research in algebraic coding theory. At the time, this area was very new and was just beginning to be studied in earnest mathematically. Her first paper in this area in 1963 was an extremely important one. She proved an expression for the first s moments of the Hamming weight enumerator of a linear code in terms of those of weight less than s in the orthogonal. This work was a compliment of the work done by Jesse MacWilliams in the same year giving the MacWilliams relations for linear codes, which is one of the foundational results of coding theory. The work of these two brilliant women would guide research to this present day in terms of applications of weight enumerators of codes.

Soon after in 1968, she proved another major result which was the uniqueness of the Golay codes. Specifically, she showed that any perfect codes with certain parameters must be equivalent to the Golay codes.

In the late 1960s and early 1970s, she published a series of papers studying maximal self-orthogonal codes and self-dual codes which are codes that are equal to their orthogonal. This study continues today and is a highly active area of research. The foundation of this study was laid by Vera and her coauthors of this time. She also used these results to obtain results in design theory. Over the next decades Vera produced numerous papers studying these codes, including studying the neighbors and children of self-dual codes. These results remain the foundation of the study of self-dual codes and are referenced continually.

The first time I heard Vera give a talk was 50 years ago, at the summer meeting at Penn State in 1971. She talked about her recent classification of binary self-dual codes up to length 18, presenting methods that would be used for a long time in a variety of settings. For several years she had been directing a project on

codes sponsored by the Air Force Cambridge Research Laboratories (AFCRL), in Bedford, Massachusetts. It was instrumental in bringing coding theory into the mathematical mainstream. Along with Vera, the main participants were Ed Assmus, Skip Mattson, and Richard Turyn. Andrew Gleason at Harvard was an advisor to the group; Jessie MacWilliams had been one of his students. I treasure the copies of some of the reports that project members kindly sent to me. Those reports contain preliminary versions of many classical coding-theory papers, notably “New 5-Designs”. My own entrance to the subject was a setting for quadratic-residue codes using the Weil representation [18, 19], inspired by the Gleason-Prange Theorem in “New 5-designs” and the AFCRL reports. I remember a later pleasant visit to Vera’s home near Route 128 north of Boston to talk about research directions. Our further work did not overlap much except for a happy collaboration with her and Neil Sloane on ternary self-dual codes [14]. Part of that study involved a nice connection with binary symplectic geometry (echoes of the Weil representation) that allowed us to draw on a classification by Jack McLaughlin [9]. Kindness seemed to be a hallmark of AMS special sessions on coding theory. Once in a session of another subject I was dismayed at the contentious atmosphere. How pleasant it was to return to “family” at the coding-theory session! Vera was involved in many of these meetings, giving splendid parties at her home in Oak Park, Illinois, when the meetings were in Evanston or Chicago. At her retirement AMS special session in Cincinnati in 2006 she overwhelmed the “family” participants by treating all of us to dinner!

– Harold (Thann) Ward



Young Mathematician Vera Pless

The seminal work of Vera Pless on a class of ternary self-dual codes that hold new 5-designs [10, 11], known nowadays in the coding theory literature as Pless

*symmetry codes, inspired my early interest in the interactions between error-correcting codes and combinatorial designs, at the time when I was a graduate student. I met Vera in person for the first time at the International Symposium on Information Theory in Ann Arbor in 1986, where I reported preliminary results from our first joint paper with Vera [15], which we wrote by communicating remotely via ordinary mail. In 1989, we met again at an Oberwolfach workshop in Germany, where we initiated a new joint project that involved also Willem Haemers and Chris Parker, on designs and a code invariant under the Conway simple group Co_3 [5]. I visited Vera at the University of Illinois at Chicago in the spring of 1991. This visit was the starting point of my academic career in the United States. During that time, together with Vera Pless and Jeff Leon, we discovered an extremal self-dual code whose existence was conjectured by Conway and Sloane [16]. A few years later, I was elated when Vera invited me to write a chapter on codes and designs [17] for the Handbook of Coding Theory [7] that she was editing together with Cary Huffman. Vera's book *Introduction to the Theory of Error-Correcting Codes* [12] and her joint book with Cary Huffman *Fundamentals of Error-Correcting Codes* [7] are my favorite textbooks for undergraduate and graduate level courses in coding theory that I have been teaching at Michigan Technological University. – Vladimir D. Tonchev*



Vera Pless with John Conway

As an example of the breadth of Vera's work was her paper in 1977 on encryption schemes for computer confidentiality, which was a very early work in cryptography, which was in its infancy at the time, and now is a very important area of research.

In 1982, Vera wrote the text *Introduction to the theory of error-correcting codes* which was published in three editions. It was an early text on coding theory and was the introduction to the subject for numerous mathematicians.

I first met Vera in the summer of 1988. At that time, I was a graduate student in the department of mathematics at the University of Wyoming, which was the

site of a summer school on combinatorics and coding theory. Vera taught coding theory and Richard Brualdi taught combinatorics. Vera's lectures were about covering radius of codes, which was a hot topic at the time. A young graduate student, with almost no background in coding theory, looking for directions for dissertation research, I was fascinated by Vera's lectures and enticed by the questions that she brought up. Coincidentally, earlier that year, for family reasons, I had applied to the graduate program at UIC (University of Illinois at Chicago) and I had received admission by the time I met Vera. I remember that one afternoon after the lecture, I went to Vera's summer school office. After a rather nervous self-introduction, I told Vera that I would go to UIC in the fall, and I would like to study with her. I do not remember her answer, but to this day, I still remember her smile.

My experience at UIC was one of the most rewarding in my life. While taking a maximum load of courses every semester, I started to work on covering radius of codes under Vera's supervision. I reported to her whenever I had anything new. Oftentimes, my primitive and fragmentary ideas were greeted by her generous encouragement and insightful suggestions. Under her influence, I became interested in the covering radius of the Reed-Muller code. In 1983, Patterson and Wiedemann discovered two cosets of the first order Reed-Muller code of length 2^{15} , which gave a new lower bound for the covering radius of that code. One day, I told Vera that I found that the two cosets did not have the weight distribution as claimed. Vera was a little surprised, but she trusted me. She contacted the authors and their correction appeared in 1990.

My daughter was born in 1990. I remember a very nice gift that Vera gave her. On the gift card, Vera's blessing was not addressed to the parents, but to the baby herself. The same year, I defended my thesis, graduated, and landed my first job in academia; all these would not have been possible without Vera's enormous help.

After I graduated from UIC, my research interests expanded to a few other areas of mathematics. However, Vera's influence on me lasted. A certain number of unsolved questions about the Reed-Muller code are always in the back of my mind and they rekindle my thoughts from time to time. Vera would have been pleased to know that a long-standing open question about the number of equivalence classes of cosets of the first order Reed-Muller code has been solved recently by one of her students [6].

In coding theory, there is a well-known family of identities called the Pless power moments; they are a mathematical legacy of Vera. As her student, I have the memory of many moments that I will always cherish. – Xiang-dong Hou

In the 1980s, she defined the class of duadic codes which are a class of binary, cyclic $[n, (n \pm 1)/2]$ and did extensive work on these. Vera also studied the putative $[72, 36, 16]$ Type II code and other work on self-dual codes. This work continued in the 1990s with work on neighbors, children, weight enumerators and the classification of self-dual codes.

Although Vera came from coding theory, I came from linear algebra and combinatorial matrix theory. Since a couple of years before 1986 I had taught a year-long graduate course in coding theory, I knew the language and some of the techniques of coding theory. Since Vera was mainly interested in linear codes, so subspaces of an n -dimensional vector space, we were able to communicate fairly well. This started a collaboration on coding theory for about 8 years that resulted in 11 papers and a collaboration on The Handbook of Coding Theory published in 1988.

Vera was in some regards a workaholic. When we were together, I often heard her say to me “so shall we get to work,” even though I may not have been quite ready to do so myself. She was very diligent and dedicated to research in coding theory. But she also loved movies, music, theater, travel, good food, and many intellectual activities. She loved the outdoors, walking and hiking, and to travel.

Our first paper (also with Janet Beissinger), published in 1988, gave another (more combinatorial) proof of the celebrated MacWilliams identities for linear codes and their duals. This was followed in 1989 in a paper (also with Richard M. Wilson) in which we introduced the notion of a short code and a related new function, the length function, which is the smallest length of a binary linear code of a given codimension and covering radius. These papers were followed by several other papers (one with Ning Cai) on so-called orphans of linear codes, specifically, the first order Reed-Muller codes, which arise naturally in the investigation of the covering radius.

Perhaps the joint paper with Vera that I like best is the one published in 1993 on greedy codes. These are binary codes defined by a greedy algorithm: Choose the next vector whose Hamming distance to each previously chosen vector is at least a specified number d . It turns out that these codes are actually linear and a parity check matrix for them is constructed.

While I began as one of the editors of the Handbook (with W. Cary Hoffman), I moved on to other things and was designated as an Assistant Editor of the two-volume tome. This handbook was a monumental project which resulted in over 2000 pages covering most aspects of coding theory and its applications. Vera worked tirelessly on this project and the resulting handbook shows this. – Richard Brualdi

I visited Vera Pless twice in UIUC, Chicago in 1997 and 1999. There I met Jon-Lark Kim and Cary Huffman for the first time. This visit was combined with some AMS sectional week end conference. I appreciated the clarity of her talks, always delivered with good mood, and peppered with small jokes. I still remember the way she described succinctly the difference between Type I and Type II codes by saying “You know, 2 is not 4!” with her midwest accent. I remember fondly the way she had to explain everything to anybody as if he/she were a child. Once, an expert in Langlands program was attending one of my seminar talks at UIUC where I had to use the group $PSL(2; p)$. Then, she turns back towards that person, and asks her bluntly: do you know this group? She also visited me in Sophia Antipolis, a science park near Antibes, France in early 1999. I was supposed to drive from Antibes to Toulon with Vera and Alex Vardy in my car to attend an HDR defense. As the reader can imagine, we started talking about codes, got an interesting conversation, and, distracted, I missed the right exit on the highway. We made it in time for the defense, but not in time for the preceding luncheon.

The two papers I wrote with Vera share the common feature of an appendix written by a number theorist. Doing research with Vera was moving from very concrete problems to nontrivial mathematical questions. Clarity and numerical examples never excluded depth in that approach. From my interaction with Vera, I take away this attitude towards research, and a lifelong interest in self-dual codes. – Patrick Solé

In the 2000s, Vera did work on \mathbb{Z}_4 codes and cyclic codes. Her work was some of the first examining codes over this ring. Of course, now codes over rings is a major area of study in coding theory.

I am Vera’s last Ph.D. student before she retired (she had one more after retirement). According to math genealogy, I am her eleventh student.

My first contact with her occurred sometime 1997 when I obtained my MS in Mathematics from Korea and I was looking for a graduated program in USA. I knew Vera from papers so I visited her homepage which showed her big smile. This gave me a courage to email her to ask if she could be my supervisor. Of course, I attached my master thesis and one preprint on self-dual codes. To my surprise, she emailed me back in a day saying that she could be my advisor. If she didn’t reply, my life would change.

My first meeting with her was in August 1998, when I arrived at Chicago and visited Math Department of UIC. I greeted her and she made a big smile. Soon, I attended the regular Coding Theory Seminar, where Vera, W. Cary Huffman,

Philippe Gaborit, Joe Fields, and a few more students attended. This opportunity trained me as a professional coding theorist. I also took all her graduate courses and then we moved to her office at least three times a week in order to discuss research problems. She was so energetic that I needed to prepare a lot. When I asked some questions in the early days, she used to say “You don’t know this?” and it was embarrassing. So I never asked her easy questions and instead I suggested new ideas. Then she cheered me up. This resulted in seven joint papers within five years.

Even though she was well-known as a coding theorist, she was also interested in Cryptography so that she wrote a book titled as “the Cryptoclub: Using Mathematics to Make and Break Secret Codes” with Janet Beissinger. She was a regular person. Come to school in the morning and go back to home before evening because she wanted to go home at Oak park by avoiding Chicago traffic. She used to invite people at her house after AMS meetings, where I could meet many famous mathematicians.

Vera visited Korea twice in early 2000’s for conferences I was involved in. When she started a talk in the second conference, the light of the room was out suddenly. Instead of waiting for the light back, she continued to explain her topic in a dark room. The audience focused on her voice and later the light came on. People told me later that they were very impressed her calmness.

I evaluated myself as a successor of Vera Pless in the sense that I wrote many papers on her favorite topics, self-dual codes and designs. In particular, her projection decoding algorithm for the binary Golay code of length 24, the most famous self-dual code, was extended to the (self-dual) Reed-Muller $R(2,5)$ code of length 32 [4]. Later, we found interesting designs arising from additive codes over $GF(4)$ for the first time [8], in which I showed her a Magma result, she made a conjecture, and we solved it!

Her teaching was very clear and example-oriented. She tried to give an intuition rather than a formalism. She was open with any kinds of ideas. I believe that I resembled her in these respects. My teaching and research follow her philosophy. I will also continue to teach and research until and after the retirement just as she did. I miss her voice telling me “You don’t know this?”. Yes, I will know it after all! – Jon-Lark Kim

At the AMS conference at Notre Dame University in April 2000 Vera Pless was sitting front and center at a talk. Despite her diminutive frame she posed an intimidating figure to all who spoke. It can be quite challenging to speak in front of one of the best researchers in a particular discipline. At the end of a talk on

codes over Frobenius rings, she gently raised her hand and asked the speaker in a soft voice “Why should we care about codes over Frobenius rings anyway?” The speaker was a bit flustered, after all, one of the giants of the field had just asked her why his talk was even relevant. Seven years later, at the AMS special session honoring the retirement of Than Ward at DePaul University in Chicago, Vera again sat front and center in the room. As I intended to answer the question as to why we should be interested in codes over Frobenius rings, I began my talk with the above quote from Vera Pless. Vera spoke loud enough for everyone in the room to hear, saying “Why does everyone remember every single thing I say?” For me the answer to this question is clear, the reason why everyone remembers every single thing she said is that every single thing she said was memorable, interesting, and important.

While I was at a conference in Atlanta, Vera had a heart attack. Jon-Lark Kim, Reshma Ramadurai and I went to visit her in the hospital. While we were in the room with her two doctors came in. She grilled them like it was their thesis defense and then got her daughter, who is a doctor, on the phone to go over all of the details with them. The medical staff kept coming in and talking to us like we were her children. I told Vera, “They think we, a Korean man, an Indian woman, and myself, are your children. That must be some mixed up gene pool.” But I think in a very real way, this is what Vera was to many mathematicians in coding theory. She was a strong mother, a good role model, and a caring mentor to any who sought her help. – Steven T. Dougherty



Vera Pless at work on her computer

Vera produced, with her coauthor Cary Huffman, two of the most important texts in coding theory. The first was *The Handbook of Coding Theory* in 1998. The second was in 2003 and is entitled *Fundamentals of error-correcting codes*. One cannot overestimate the importance of this text in the field. It has become the standard reference for classical coding theory.

In 1992 Vera was invited by Elsevier Science to edit a coding theory book. She invited me and Richard Brualdi to co-edit the book with her; we both accepted. I took a sabbatical at UIC in the fall of 1992 to work with Vera and Richard on an introductory chapter of the book. Richard helped extensively in the beginning of the project but changed his status to assistant editor due to his increasing responsibilities as chair of the mathematics department at the University of Wisconsin. The process began with brainstorming ideas for both topics and authors with suggestions from coding theory experts, many of whom eventually contributed chapters to the book. Until publication, both Vera and I were in frequent communication with authors, reading and making suggestions for their chapters. The book ended up with 25 chapters written by an international group of 33 authors. While the original publication date was scheduled for earlier, the two volume 2169 page Handbook of Coding Theory finally appeared in 1998. Several of the authors attended two AMS specials sessions on coding theory, in 1995 and 1998, organized by Vera and me. The second of these two was dedicated to the memory of Ed Assmus, one of the authors, who passed away shortly before the publication of the Handbook. – W. Cary Huffman



AMS Conference honoring Vera Pless

As an example of the breadth of her talent, Vera also published a book for middle school children, with her close friend Janet Beissinger, titled *The Cryptoclub: Using Mathematics to Make and Break Secret Codes*. This book describes the basics of cryptography in language that a middle school student can understand.

I first met Vera at the AMS Annual Meeting in 1981, when I was a candidate for an Assistant Professor job at the University of Illinois at Chicago (UIC). She introduced me to Bhama Srinivasan, also from UIC and Louise Hay, the head of UIC's Mathematics Department. I had already corresponded with Karen Uhlenbeck, the Chair of the Recruitment Committee, so I was pretty impressed – so many women in UIC's Math Department, I thought. But I later learned that, out of about 80 tenure/tenure-track faculty members in the department, only five were women and I had just met all but one of them. Nevertheless, though few in number, these pioneering women mathematicians were inspiring and supportive role models.

I didn't realize at that time that Vera, in particular, would have an important impact on me as a colleague, mentor, and friend for the next forty years. She gave me both professional and personal advice, ranging from important career moves to personal finance, and she encouraged and supported me all along the

way. Our fields overlapped a bit – mine was enumerative combinatorics – and we wrote one paper [3] together, but our biggest collaboration was on a project to teach cryptography to middle school students.

About 20 years ago, as I was writing a chapter about prime numbers for a fourth grade math textbook, I remembered from my graduate school days in the seventies the excitement when the development of public key cryptography was announced and I remembered that the security of some public key systems depends on the fact that large numbers are hard to factor. I called Vera for a refresher on those systems and to see if she thought the basics could be explained to fourth graders as an application of factorization. We decided that fourth grade was probably too young, but that middle school students might understand. Then, the more we thought about it, the more we realized there are many interesting ciphers that apply the kinds of mathematics middle school students learn. And, with the natural interest kids have surrounding secret messages, cryptography would be a great way for students to explore mathematics in a way that was different from school math.

We started meeting regularly to develop a draft of a book. We included shift ciphers, which involve addition and subtraction and a natural application of negative numbers. Another was the Vigenre Cipher, which was once believed to be unbreakable, but can actually be cracked by today's middle school students (as long as the key isn't too long) by finding repeating patterns in the message and then finding common factors of the distances between repetitions. Multiplicative and affine ciphers involve practice with simple arithmetic operations, followed by reducing modulo 26. Modular arithmetic is fundamental to cryptography and, while it is not typically covered in the middle school curriculum, it uses division with remainder, which is in the curriculum, and it reinforces the concepts students need to know about telling time on clocks. One way to crack affine ciphers is to solve systems of linear congruences – which reinforces what students are learning about how to solve linear equations. And, one of the fundamental tools used to crack messages in cryptography, frequency analysis, involves data collection, decimals, and percents – all important topics in the middle school. We also included the modern-day RSA cipher, which involves work with exponents. Its connection to prime numbers and factorization gives an inspiration for thinking about really large prime numbers, those much larger than students usually think about in school.

It was great fun to work with Vera to develop a story about the CryptoKids, fictional kids who formed a club to teach themselves about cryptography and mathematics. She had a playful sense about what might happen to real kids in school

and the conversations they might have. Some of the scenarios in the story were based on real events. For example, a teacher once intercepted a student's note and read it aloud to the class. The CryptoKids vowed to encrypt their messages in the future to avoid that embarrassment from happening again. Another time, an exhausted teacher told a student that "some things are always true: two plus two is always four, four plus four is always eight," and so on. "You just have to accept that." That was taken as a challenge to think of a counterexample: when telling time on clocks, the usual arithmetic isn't always true. On a twelve-hour clock, eight plus eight is not sixteen – it's four. That led the CryptoKids to explore modular arithmetic and to learn about ciphers that use it.

The conversations of the CryptoKids were often about some of the things kids who like to explore mathematics might wonder: When you are checking whether a number is prime, do you really have to test all the numbers less than that to look for divisors? They were happy to realize you only have to check up to the square root of the number. How many primes are there anyway? One of the CryptoKids wondered about that and another convinced her that there are infinitely many by informally describing what was essentially a proof of Euclid's theorem.

We used cryptography, not only as a vehicle for students to build mathematics skills, but also as an opportunity to showcase some of the problems research mathematicians are currently working on. For example, the fact that the security of RSA depends on the difficulty of factoring large numbers has spurred mathematicians to look for efficient methods for factoring. We wanted students to see mathematics as a vibrant, changing subject, with unanswered questions, rather than an old subject whose problems have all been figured out.

After a year or so of creating our story, our colleague, Phil Wagreich, suggested that we apply for a National Science Foundation grant so we could test the book with teachers and real kids. We got the grant and over the next three years we trained 19 pilot teachers in the Chicago area, then visited their classrooms to watch.

Vera had always been an excellent teacher – in fact, she had won a University-wide teaching award at least once – but I don't think she had visited middle school classrooms or worked with teachers in this way before and we both really enjoyed seeing how teachers used our materials with students at that level. We learned a lot from both the teachers and their students.

*The result of this work together was *The CryptoClub: Using Mathematics to Make and Break Secret Codes* [1], which has since been translated into both Korean and Chinese. It was interesting to see how the foreign language publishers*

adapted the book – they translated the instructional part but kept our original secret messages in English. That way students could learn about the ciphers and mathematics in their own language, while practicing their English skills as they encrypt and decrypt our messages.

It turned out that the topic of cryptography was very interesting to students, but the typical middle school teacher didn't have time during the regular school day to go all the way through the book to the deeper chapters on public key cryptography. They started teaching the material in after school programs and suggested modifications to better fit those settings. So after Vera retired, Bonnie Saunders and I continued the CryptoClub Project with new NSF grants to develop materials [2] for informal settings.

Although our original target audience was students in regular classrooms, we were surprised to realize there were other audiences for the book. One is pre-service teachers taking college number theory courses. As they learn more advanced topics in their courses, the book serves as a supplement that lets them experience how these topics can be taught in a playful way to their future students. The other audience is independent learners who want to explore cryptography and mathematics on their own outside of a regular classroom setting.

*Besides a book, our NSF grant funded us to develop an interactive cryptography website. What Vera and I initially had in mind was a site with simple tools for computing letter frequencies or doing other useful computations that would help to analyze encrypted messages, but when a graduate student, Rong Zeng, asked us to design a cryptography adventure game he could program in order to teach himself Flash programming, we took the bait. Vera and I had no experience with video games, but we did have an adventurous spirit, so we designed an animated game, *Stormy Night Treasure Hunt*, that took the player through a haunted castle to find a treasure at the end. That game has unfortunately gone the way of other Flash games and is no longer available, but the website and game were the precursor to the website, cryptoclub.org, that Bonnie Saunders and I later developed with the educational software company Eduweb and is still in use today.*

The cryptography-for-kids work we did showed a playful and creative side of Vera. And while we enjoyed working on it for several years, it was by no means the only thing she was doing at the time. During that period, she was also working on her two important coding theory texts [7, 12] with Cary Huffman, and she traveled to conferences and gave invited talks around the world.

Vera was in her late sixties when we started working on the book. She kept teaching until she was about 75, and continued to supervise graduate students

until she was 82. She was a role model to me in many ways, but as I approach my own impending retirement, I realize one of her biggest impacts on me was her example of how to age well. Even after health issues would have slowed most people down, she kept traveling, learning, and enjoying life. She was determined to live at home, even after she could not do it alone. She took language lessons, attended plays and concerts, and read an enormous amount. Her mathematics contributions will be remembered for their importance to the field, but I will also remember her lessons about how to live life to the fullest. – Janet Beissinger



Vera Pless had a remarkable career and a remarkable life. One could easily call it remarkable for the many doors she opened for women who followed her. One might also call it remarkable for the body of research she did on a wide variety of topics and for the important texts she wrote. Finally, one could also call it remarkable for the impact her life had on others, most especially her students and her collaborators. Whichever of these you find most important, we all owe a debt of gratitude for the life and work of Vera Stepen Pless.

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