

Uncertainty of Key Performance Indicators for Industry 4.0: a methodology based on the theory of belief functions

Amel Souifi^{a,b,*}; Zohra Cherfi Boulanger^c, Marc Zolghadri^{a,d}, Maher Barkallah^b,
Mohamed Haddar^b

^aQuartz laboratory, SUPMECA, 3 Rue Fernand Hainaut, 93407, Saint Ouen.

^bLA2MP Ecole Nationale d'Ingénieurs de Sfax, Université de Sfax, Route Soukra Km 3.5, 3038 Sfax, Tunisia.

^cRoberval laboratory, Université de Technologie de Compiègne, 60203 Compiègne cedex, France.

^dLAAS-CNRS, 7 Avenue du Colonel Roche, 31400 Toulouse, France

*Corresponding author: amel.souifi@supemeca.fr

Abstract

1 For the past few years, we have been hearing about Industry 4.0 (or the fourth industrial revolu-
2 tion), which promises to improve productivity, flexibility, quality, customer satisfaction and employee well-
3 being. To assess whether these goals are achieved, it is necessary to implement a performance
4 management system (PMS). However, a PMS must take into account the various challenges associ-ated
5 with Industry 4.0, including the availability of large amounts of data. While it represents an
6 opportunity for companies to improve performance, big data does not necessarily mean good data. It can
7 be uncertain, imprecise, ambiguous, etc. Uncertainty is one of the major challenges and it is essential
8 to take it into account when computing performance indicators to increase confidence in decision making.

9 To address this issue, we propose a method to model uncertainty in key performance indicators
10 (KPIs). Our work allows associating with each indicator an uncertainty noted m , computed on the
11 basis of the theory of belief functions. The KPI and its associated uncertainty form a pair
12 (KPI, m). The method developed allows calculating this uncertainty m for the input data of the
13 performance management system. We show how these modeled uncertainties should be propagated to
14 the KPIs. For these KPI uncertainties, we have defined rules to support decision-making. The method
15 developed, based on the theory of belief functions, is part of a methodology we propose to define and
16 extract smart data from massive data.

17 To our knowledge, this is the first attempt to use this theory to model uncertain performance
18 indicators. Our work has shown its effectiveness and its applicability to a case of bottle filling line
19 simulation. In addition to these results, this work opens up new perspectives, particularly for taking
20 uncertainty into account in expert opinions and in industrial risk assessment.

21
22
23 **Keywords**— Industry 4.0, Performance management, Decision support, Big Data, Uncertainty modeling

24 1 Introduction

25 Recent technological developments as well as the decreasing costs of the resulting technologies have allowed
26 companies to consider a transition to the Industry 4.0. This transformation will be carried out on the technological,
27 organizational and human level in order to meet the needs of customers in terms of personalized and more complex
28 products. Industry 4.0 refers to the fourth industrial revolution characterized by the implementation of several
29 promising technologies such as Cyber Physical Systems, the Internet of things, Big Data, Cloud Computing, digital
30 twin, cobotics, augmented and virtual reality, etc. These technological developments related to digitalization,
31 the implementation of increasingly disruptive technological innovations and the use of artificial intelligence make
32 process management more complex and subject to uncertainty (Magruk et al. 2016). For instance, Cyber-Physical
33 Systems (CPS) integrate computational and physical capabilities that can interact with humans (Baheti and Gill
34 2011). This makes CPS increasingly complex, hence the need to manage uncertainty to ensure the reliability of
35 these systems (Tao et al. 2020). Another challenge of the digital era is related to the availability of large amounts of
36 data. Despite the emphasis placed on Big Data, there is a lack of approaches to address the challenges associated
37 with massive data, including data quality and more specifically data uncertainty for decision making. Indeed,
38 with the Internet of Things, sensors everywhere and smartphones, data is becoming more and more available, but
39 accompanied by doubts about its veracity and value. The number of data sources is becoming very large and
40 issues of uncertainty cannot be neglected (Rodriguez et al. 2009). Therefore, this revolution brings about several
41 changes in the organization, practices and management of work. In particular, performance management systems
42 (PMS) must be re-examined to address the challenges of Industry 4.0, namely data uncertainty.

43 Performance is expressed in terms of results obtained in relation to a previously set objective. To enhance
44 performance and ensure the effective and efficient management of any business, performance measurement and
45 management systems (PMMS) are used (Melnyk et al. 2014). Performance management has evolved from per-
46 formance measurement (i.e., what to measure, how to measure it, and how to report the results) to performance
47 management (i.e., how to use the measures to manage an organization's performance) (Umit S Bititci et al. 2015).
48 Performance measurement involves activities such as setting goals, developing metrics, collecting, analyzing and

49 reporting performance information, and interpreting and evaluating performance gaps (Smith and Umit Sezer
50 Bititci 2017). Performance measurement systems consist of a set of performance expressions that must be orga-
51 nized in a coherent way with respect to the company’s objectives. Since they are numerous, it is necessary to
52 reduce the dimensionality of these performance measures. This is achieved by aggregating the basic measures
53 into performance indicators that provide a global expression of performance. One of the problems encountered
54 during this dimensionality reduction is the decomposition of the global objective in order to define the elementary
55 performance expressions and the aggregation rules needed to measure the global performance (Berrah, Mauris,
56 and Montmain 2008). Performance management consists in ensuring that the organization’s objectives have been
57 achieved in a consistent, effective and efficient manner (Suhardi 2015). It ”should provide management with
58 insight into how well the organization is performing and the extent to which organizational goals are being met”
59 (Baird, Schoch, and Chen 2012). It also allows identifying successes, determining if customer needs are being
60 met, understanding business processes, making factual decisions and identifying bottlenecks and waste (Balfaqih
61 et al. 2016).

62 Key Performance Indicators (KPIs) are aggregated indicators used to measure the degree of achievement of
63 objectives in a performance measurement system. KPIs are defined in the standard ISO 22400 as ”quantifiable
64 and strategic measures that reflect the critical success factors of an organization.” According to the same standard,
65 KPIs must be quantifiable: the value of the KPI can be specified numerically without being penalized by the
66 presence of uncertainty, provided that this uncertainty can also be quantified. Therefore, KPIs are challenged to
67 incorporate a notion of uncertainty in their expression. The need to model the uncertainty associated with KPIs
68 can be justified by three arguments:

- 69 • Decision-makers can assess not only the values of the performance indicators presented to them, but also
70 the confidence they have in those values.
- 71 • If the uncertainty associated with an indicator is significant, it is necessary to investigate the causes in
72 order to reduce it.
- 73 • The credibility of the information is measured before implementing actions that could be costly.

74 Restricting a study to certain data leads to focusing on a small part of the problem (Parsons 1996). In
75 this perspective, we are interested in quantifying the uncertainty of performance indicators to support decision
76 making. The rest of the paper is organized as follows. We start with the state of the art on uncertain performance
77 indicators and explain the choice of the theory of belief functions for uncertainty modeling. Next, we present
78 certain basic elements of the Dempster-Shafer theory. Then, we propose a methodology for using this theory
79 for modeling, combining, and propagating uncertainty in performance indicators to support decision making.
80 We develop a new method to calculate the uncertainty of a new data vector based on historical data, and we
81 propose decision rules adapted to the case of uncertain performance measures. Finally, we apply the proposed
82 methodology in the case of a filling line simulation model.

83 2 State of the art

84 ”A statement is uncertain if one cannot assess its truth or falsity given the available data ”(Smets 1990). In
85 the literature, two types of uncertainty are recognized: random uncertainty and epistemic uncertainty. Random
86 uncertainty describes the inherent variation associated with the system or environment of interest (Oberkampf,
87 DeLand, et al. 2002). It is also referred to as variability, or irreducible, observable, inherent or stochastic un-
88 certainty. Probability theory is commonly used to represent random uncertainty when experimental data are
89 available (Oberkampf, Helton, et al. 2004). Epistemic uncertainty is described as reducible, subjective and cogni-
90 tive. It is related to a lack of information about a phenomenon or a lack of confidence and results from scientific
91 ignorance, measurement uncertainty, unobservability, censorship, or other lack of knowledge (Ferson et al. 2015).
92 Possibility theory , Dempster-Shafer theory (also called evidence theory and theory of belief functions), interval
93 analysis and probability theory can be used to model epistemic uncertainty.

94 In the context of performance management, probability theory, fuzzy logic and confidence intervals are gener-
95 ally used to model uncertainty. In supply chain management, fuzzy sets are used to model uncertain data (Petrovic
96 2001). In (Sonmez, M. C. Testik, and O. M. Testik 2018) the authors considered overall equipment effectiveness
97 (OEE) to measure the performance of an item of manufacturing equipment. They modeled uncertainty related to
98 production speed and stoppage duration measurements used to calculate OEE components. The methods used
99 to handle uncertainty depend on its source: if uncertainty is due to the use of linguistic terms or some minor
100 stoppages, idling, or speed losses being ignored, fuzzy arithmetic is used. However, in the case of poor accuracy,
101 arithmetic intervals are suggested. Another attempt to model uncertainty for OEE was carried out in (Zammori,
102 Braglia, and Frosolini 2011). The authors considered OEE as a stochastic random variable and generated its
103 probability density function (pdf) by aggregating the pdf of the causes of basic waste. In (Mauro et al. 2021),
104 authors investigated the influence of the uncertainty arising from sampling errors on a composite index (CI),
105 which is the result of the aggregation of different indicators in the context of the environmental performance
106 of Italian regions. They proposed a methodology based on the parametric bootstrap technique to estimate the
107 standard error of a CI when one or more of its indicators are affected by a sampling error. They showed that
108 non-negligible sampling errors could affect the reliability of the composite indicators. In (Yang, Kornas, and Daub

109 2021), a Bayesian approach was applied to the estimation of process capability indices (PCIs) for the data-sparse
110 early prototype production phase. In particular, the credible interval, i.e, the Bayesian analogy of the confidence
111 interval for Cpk was calculated.

112 In this work, we seek to model epistemic uncertainty for performance indicators in the context of Industry 4.0.
113 In addition to the presence of large masses of data, Industry 4.0 can face the challenge of sparse data, especially
114 since some production systems do not yet exist. Therefore, we need to rely on experts opinions to have the
115 necessary data to evaluate their performance. Hence the need for a framework for modeling uncertainty that is
116 suitable for massive or sparse statistical data and subjective data.

117 In probability theory, we can distinguish two interpretations of the probability of an event A , noted $P(A)$: the
118 frequentist interpretation, which considers the value $P(A)$ as the limit of the frequency of occurrence of an event A
119 assuming a sufficient number of repetitions of the experiment, and the subjectivist interpretation, in which $P(A)$
120 is a subjective value characterizing an agent's beliefs about the occurrence of A (Le Duy 2011). The probability
121 theory is based on the axiom of additivity: by knowing $P(A)$, we can determine $P(\bar{A}) = 1 - P(A)$ which makes
122 this theory unsuitable for uncertainty modeling when the knowledge about an event and its opposite event is very
123 limited (Le Duy 2011). To model epistemic uncertainty, the theory of probability is based on the assumption
124 of a probabilistic distribution such as the normal distribution, which is interpreted as the subjective probability
125 (degree of belief or a confidence degree) of the possible values of the parameter or indicator. This distribution
126 cannot always be known with precision and there is no reason that a subjective confidence level should be a
127 probability (Raufaste, Silva Neves, and Mariné 2003). According to (Shafer 1976), if we do not know the chances
128 (objective probabilities), it requires an extraordinary coincidence for our confidence levels to be probabilities.

129 Due to the limitations of probability theory in modeling epistemic uncertainty, non-probabilistic theories can
130 be used, namely possibility theory and the Dempster-Shafer theory. Based on two functions of possibility and
131 necessity, the theory of possibility was first introduced by (Zadeh 1978) and represents an extension of his theory
132 of fuzzy sets. These two theories offer the opportunity to model both imprecision and uncertainty and to handle
133 in a more natural way the state of belief in an assertion (Dubois and Prade 2012). When modeling uncertainty
134 by probability theory, the condition of additivity linking our state of belief in favor of the realization of the event
135 and the belief in its opposite event seems to be rigid (Dubois and Prade 2012). Thus, modeling is simplified
136 via the possibility and necessity functions of the possibility theory, and the plausibility and belief functions of
137 the Dempster-Shafer theory. Another contribution compared to the probability theory is the fact that these
138 approaches solve the problem related to the choice of the probabilistic distribution by using belief functions and
139 possibility measures. In addition, both theories provide rules for the fusion of uncertain data, making them a
140 powerful tool for managing uncertainty in decision making (Dubois and Prade 1994)(Du and Zhong 2021). **The
141 main differences between the three uncertainty modeling theories are described in the annex. They mainly concern
142 the interpretation of the uncertainty measure in each theory, the way of treating uncertainty with statistical data
143 and expert opinions, the link between the uncertainty of an event and its opposite and the uncertainty modeling
144 of imprecise data. Each comparison criterion is followed by a comment indicating the difference between the
145 theories.**

146 As a result of the comparison carried out, we notice that the two theories of possibility and belief functions are
147 adapted to the modeling of data available in the digital era: statistical data in sufficient quantity or sparse and
148 subjective data. As the possibility theory can be considered as a special case of the theory of evidence (Ayyub
149 2001), we have chosen to model the uncertain performance indicators using the Dempster-Shafer theory. This
150 choice is aimed at demonstrating the suitability of this theory to the context of performance management. The
151 flexibility of modeling data uncertainty, the development of the formalism and its strong fundamentals, as well as
152 the interesting framework for data fusion, support the choice of the Dempster-Shafer theory. It is also suitable
153 for the representation of partial knowledge and ignorance of an A proposition. This is possible by assigning a low
154 degree of belief to A and its negation \bar{A} in order to show that there is weak evidence on this proposition (Shafer
155 1976).

156 3 Dempster-Shafer Theory

157 The Dempster-Shafer Theory (DST), also known as the theory of belief functions, is considered as a generalisation
158 of the Bayesian theory of subjective probability (Xiao 2020). It is also seen as a generalization of the probability
159 theory as it allows assigning probabilities to sets as opposed to mutually exclusive singletons (Ullah, Youn, and
160 Han 2021). The DST is based on three functions: the basic probability assignment function also known as the
161 mass function (m), the Belief function (Bel) and the Plausibility function (Pl). In order to make this theoretical
162 section useful later, we start by introducing the concept of mass function in belief theory, then we address the
163 data fusion rules and we end with the propagation of uncertainty. To clarify our statements, we will propose
164 illustrations related to the example of the OEE developed in the case study (section 5). For a more detailed
165 discussion of this theory, readers may refer to (Shafer 1976).

3.1 Mass function

Let Ω be a set of N exclusive and exhaustive propositions, $\{H_i\}$, $i = 1, \dots, N$, called frame of discernment. Let us consider $P(\Omega)$, the power set composed with the 2^N propositions of Ω .

$$P(\Omega) = \{\emptyset, \{H_1\}, \{H_2\}, \dots, \{H_1 \cup H_2\}, \dots, \Omega\} \quad (1)$$

The mass function is defined as the mapping of the power set $P(\Omega)$ onto a number between 0 and 1.

$$m : P(\Omega) \rightarrow [0, 1] \quad (2)$$

The mass function m satisfies the following conditions:

$$m(\emptyset) = 0 \text{ and } \sum_{A \in P(\Omega)} m(A) = 1 \quad (3)$$

Any subset A of Ω such that $m(A) > 0$ is called a focal element. The value of the mass function for a given set A , $m(A)$ expresses the fraction of the mass of belief placed strictly in A , but to no particular subset of A . The mass attributed to a simple proposition expresses the degree of confidence of one proposition with respect to the others, and the mass of a compound hypothesis expresses the confusion associated with the lack of information to decide between one hypothesis and another (Le Duy 2011). The mass function m and its associated focal element are called a belief structure, making it possible to represent a piece of evidence or information about an element x drawn from the set Ω (Yager 1987).

Illustrative example. The actual production time (APT) (time of value-added tasks) is a data used in the calculation of the overall equipment effectiveness. Suppose a decision maker asks an operator and the production line manager for its value. The values they give are more or less certain. For example, the operator says that the APT during one hour is 45 minutes whereas the line manager says it is 40 minutes. The set $\{40, 45\}$ is the frame of discernment. Let us now associate with the data given by the operator an uncertainty of 0.4 and to that of the line manager 0.6. So, 45 and 40 are focal elements. $(45, 0.4)$ and $(40, 0.6)$ are belief structures.

3.2 Evidence combination

combination rules are used to aggregate data coming from multiple sources (Ullah, Youn, and Han 2021). If we have two belief functions m_1 and m_2 associated with two sources of evidence, combining the evidence entails finding a combined belief structure m over the frame of discernment. During data fusion, the uncertainty on the resulting data is calculated using different combination rules. In order to choose methods used in the uncertainty combination, the following criteria have to be considered :

- Independence of information sources. If one of the measurements of the same quantity is taken by two employees of the same company, the credibility of one is not necessarily independent of that of the other. Indeed, they may be trained in the same way, use the same equipment and be under the same conditions.
- There may be partial, total or no conflict between sources of evidence.
- Source reliability: either all sources are reliable or at least one reliable source exists.

For instance, Dempster's rule is used in the case of distinct sources of information which are not totally conflicting. The resulting operator from the combination using Dempster's rule is noted the orthogonal sum \oplus . The result of the combination is given by:

$$m_{1 \oplus 2(A)} = \frac{1}{1 - K} \sum_{B \cap C = A} m(B)m(C), \quad \forall \emptyset \neq A, B, C \subset \Omega \quad (4)$$

where $K = m(\emptyset) = \sum_{B \cap C = \emptyset} m(B)m(C)$. This factor refers to the degree of conflict between the two sources (Jing and Y. Tang 2021). When there is no intersection between the combined masses, K is equal to 1 and function $m_{1 \oplus 2(A)}$ is undefined (Murphy 2000).

Illustrative example. Let us again consider the example of the actual production time (APT). Suppose that two different operators have recorded it for the same period of time. The values are not the same for each measurement. The first operator found a production time between 50 and 52 minutes 5 times and a production time between 53 and 55 minutes twice. The second operator recorded an APT between 50 and 53 minutes 3 times and a production time between 54 and 55 minutes 4 times. For each operator we take a mass function equal to the number of times the interval occurred divided by the number of records. Considering each operator i as a data source S_i , the intervals for each source as well as the mass functions are summarized in the table 1. This table shows also the results of the Dempster's rule of combination while merging the uncertain data provided by the two operators.

Table 1: Combination of data sources using the Dempster’s rule.

Source of data	Operator 2	
Operator 1	$([50, 52], 0.71)$	$([53, 55], 0.28)$
$([50, 53], 0.42)$	$([50, 52], 0.52)$	$(\{53\}, 0.205)$
$([54, 55], 0.57)$	Total conflict	$([54, 55], 0.278)$

By merging the data, we know that the actual production time can belong to the interval $[50, 52]$ with a confidence value of 0.52 or it belongs to the interval $[54, 55]$ with a mass function equal to 0.278. Also it can be equal to 53 with a mass function of 0.205. We notice that there may be no intersection between the intervals, indicating a total conflict between the two sources. Then, we can take into account the information provided by both operators using the disjunctive rule that results in the union of the two data sets. However, it produces less accurate results compared to the data provided by the sources.

3.3 Uncertainty propagation

The propagation of uncertainty in the Dempster-Shafer theory consists of the propagation of the belief functions of input variables into the output variable. Let Y be an output variable with values in \mathbb{R} . Let us consider X_1, X_2, \dots, X_n a set of independent input variables. Each input variable is modeled by a mass function given by $m^{X_i} : 2^{\Omega^{X_i}} \rightarrow [0, 1], \Omega^{X_i} \subseteq \mathbb{R}$ is the definition field of X_i .

If $y = f(X_1, X_2, \dots, X_n)$, the propagation of uncertainty is performed through function f to obtain the uncertainty on Y . To propagate belief function from X_1, X_2, \dots, X_n to $Y = f(X_1, X_2, \dots, X_n)$, we can use the Cartesian product (Marques, Ynuhui, and Marrel 2018). We first define the Cartesian Product of the input variables: $\Omega^{X_1} * \Omega^{X_2} * \dots * \Omega^{X_n} = \{(x_1, x_2, \dots, x_n) : x_1 \in \Omega^{X_1} \wedge x_2 \in \Omega^{X_2} \dots \wedge x_n \in \Omega^{X_n}\}$.

Let s be a subset from the Cartesian product of focal elements of X_1, X_2, \dots, X_n . Suppose that f is a continuous function, then the focal elements of Y are given by:

$$[\min(f(x), \max(f(x)) \forall x \in s \subseteq \Omega^{X_1} * \Omega^{X_2} * \dots * \Omega^{X_n} \quad (5)$$

Illustrative example. The quality ratio is given by good quantity (GQ) divided by produced quantity(PQ). Suppose that $GQ = 60$ with a mass function equal to $m_1 = 0.9$ and the $PQ = 65$ with $m_2 = 0.95$. Then the quality ratio is equal to $60/65 = 0.92$ with a mass function $m = m_1 * m_2 = 0.9 * 0.95 = 0.855$.

The Dempster-Shafer theory offers a simple and intuitive model of the uncertainty of key performance indicators. It also gives the possibility to merge data from multiple sources and combine their uncertainty. In the next section, we propose an approach to quantify uncertainty in key performance indicators.

4 Proposed Approach

In this section, we present our approach for quantifying the uncertainty of a performance indicator. It can be summarized as follows: Key performance indicator modeling, uncertainty modeling and decision making. This approach can be used for indicators based on uncertain data. This uncertainty can be due to a defect in a measuring device, to the non-credibility of the operator who provided the production data, or to a lack of knowledge on the part of the expert who has been asked to assess such a situation. The complexity of the production systems during the transition to Industry 4.0 can also generate uncertainty: when implementing new systems, the factory staff is not yet familiar with its functioning and may not be able to judge the quality of the data generated. Another significant cause of data uncertainty is the non-reporting of certain production stoppages considered minor (Sonmez, M. C. Testik, and O. M. Testik 2018), the approximate measurement of time such as equipment failure, which may begin, for example, at the time of the maintenance department’s intervention although the failure could have occurred much earlier. Also, the multitude of data sources is an origin of uncertainty because we cannot decide which one is more reliable.

4.1 Key performance indicators modeling

The first step in expressing uncertainty for performance indicators is the definition of a hierarchy structure for performance measurements. In this work, we classify performance measures into key performance indicators (KPI), performance indicators (PI) and data. KPIs (for example overall equipment effectiveness) are generally the result of aggregating performance indicators. They synthesize many aspects of the business and allow assessing the overall performance (Kang et al. 2016). Performance indicators are generally ratios calculated from elementary data. They reflect one performance aspect. Quality ratio, throughput rate and availability are examples of performance indicators. Data include direct measurement from sensors, time elements, production statistics, like quantity produced, number of failures, etc.

4.2 Uncertainty modeling

We propose to express uncertainty associated with key performance indicators by a pair (KPI, m) with $m \in [0, 1]$. m is given by the mass function from the Dempster-Shafer theory associated with a focal element KPI . The value of m is calculated by propagating the data uncertainty to the performance indicators and then to key performance indicators.

Data uncertainty is also modeled using the theory of belief functions. We associate with each datum a mass function m_i . This value is not always easy to determine. It can be provided by experts having a knowledge of the system or of the phenomenon studied or calculated from statistical data. We propose in the following a method to calculate data mass function based on historical data. If a piece of data comes from different sources with different mass functions, it is possible to opt for data fusion and an uncertainty combination. Once the mass function of the elementary data is known, it can be propagated to performance indicators. These steps are illustrated in the figure 1.

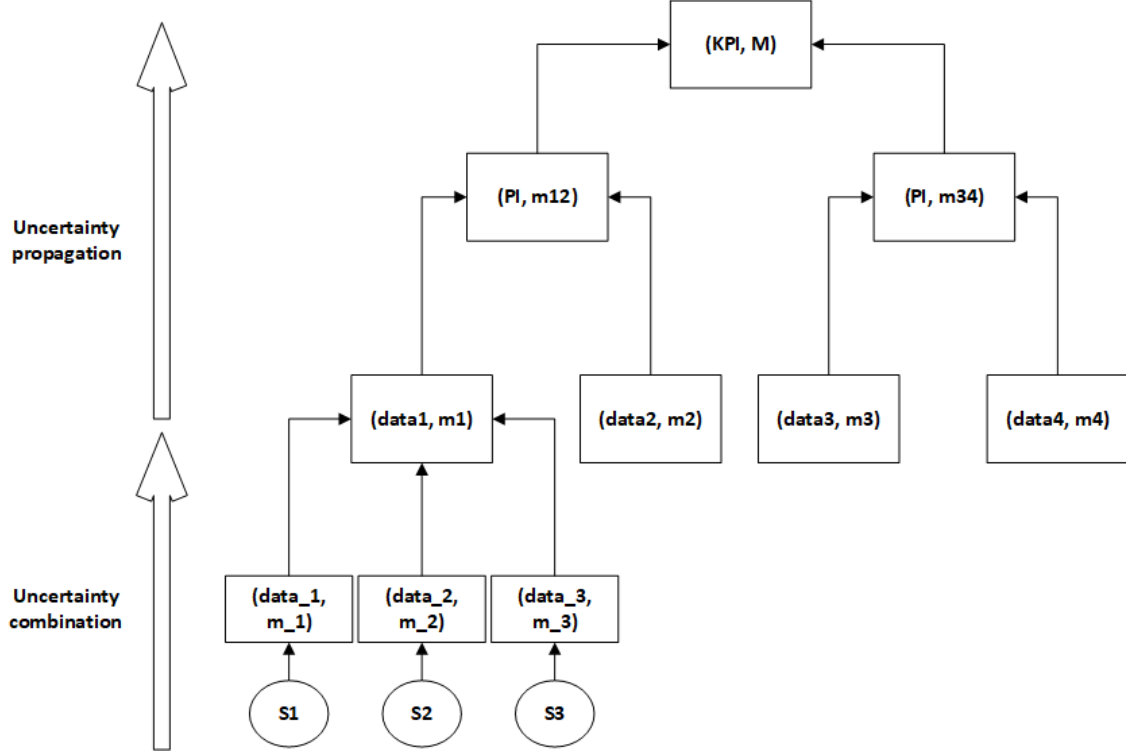


Figure 1: Proposed approach for modeling uncertainty in KPI

• Mass function determination

The determination of the belief function is a fundamental problem in the theory of belief functions. A method for modeling belief functions inspired by the k-nearest neighbors (k-NN) algorithm is proposed in (Denoeux 2008). The objective is to use the Dempster-Shafer framework for the classification of a pattern x on the basis of its k nearest neighbors. Let $\Omega = \{C_1, C_2, \dots, C_c\}$ a set of c classes and $\Gamma = (x_1, L_1), (x_2, L_2), \dots, (x_n, L_n)$ a training set of n p-dimensional patterns x_i with $L_i \in 1, 2, \dots, c$ their label indicating the pattern's membership to one class of Ω . Let x_s be a vector to be classified based on information contained in the training set Γ . If we note Φ^s the set of k-nearest neighbors of x_s according to the Euclidean distance, each pattern x_i from Φ^s with label $L_i = q$ provides a piece of evidence regarding the membership of x_s to C_q . If x_i is close to x_s (the Euclidean distance between x_i and x_s is small), then we tend to believe that x_i and x_s belong to the same class C_i . Otherwise, if x_s is far from x_i according to the Euclidean distance, then we can assume that x_i provides very little information about the label C_q of x_s . Thus, each vector close to x_s constitutes a source of information represented by a mass function $m^{s,i}$ whose formula is given by:

$$m^{s,i} \{Cq\} = \alpha 0 \Phi_q(d^{s,i}) \quad (6)$$

$$m^{s,i} \{\Omega\} = 1 - \alpha 0 \Phi_q(d^{s,i}) \quad (7)$$

$$m^{s,i} \{A\} = 0 \quad \forall A \in 2^\Omega \setminus \{\Omega, Cq\} \quad (8)$$

where C_q is the class of x_s , $d^{s,i} = d(x_s, x_i)$ the Euclidean distance between x_s and x_i , α_0 is a constant equal to 0.95 and Φ_q a decreasing function given by:

$$\Phi_q(d) = \exp(-\gamma_q d^2)$$

γ_q is a positive parameter associated with class C_q .

Once the mass functions are computed for each neighbor of x_s , they are combined using Dempster's combination rule to obtain a global mass function m representing our belief about the class of x_s . m is given by:

$$m = \oplus_{i=1}^k m(\cdot; x_i) \quad (9)$$

As mentioned in the introduction 1, the data can be statistical or subjective. The strategy for determining the mass functions depends on the type of data. To calculate the confidence values associated with the value of a variable X in the presence of historical data, we propose the following method (figure 2) based on the approach proposed in (Denoeux 2008) for classification. Let us assume that the collected data are multivariate, i.e. they are represented by a matrix $X_{n,p}$ where n is the number of observations and p is the number of monitored variables. We note x_{ij} the value of the variable p for the individual i and m_{ij} its confidence value. Each value x_{ij} is obtained from a source S_{ij} which can be reliable or not. That is why we assign to each source a value $w_{ij} \in [0, 1]$ characterizing its reliability.

The determination of m_{ij} is seen as a classification problem. Indeed, a system can be in several macro-states: nominal, degraded, in failure, etc. Each macro-state corresponds to possible data values. These macro-states can be identified through a study of the system's history. The number of macro-states must be small in order to allow easy decision making. When observing the system at a given time, if we are able to determine whether the instantaneous state belongs to one of these macro-states, we will then be able to determine the state of the system and propose the necessary monitoring actions. This approach can then be assimilated to a classification approach. The macro-states are classes determined thanks to the historical data.

Therefore, we can proceed as follows (figure 2) :

- Step1: Identification of Macro-states or classes. These classes (macro-states) are initially determined via unsupervised learning making it possible to discover patterns in the data. The K-means algorithm was chosen in order to constitute homogeneous groups of data. Each group of data corresponds to a state of the system under study (e.g. nominal operating state, degraded operation, system in failure).
- Step2: Determination of mass function of macro-states. Once the classes are defined and the historical data are divided into groups, an expert assigns a mass function m^c to each group while taking into account the functioning of the system and the realizability of these variables. This mass function reflects the confidence that the expert has in the representation of a system state by data group. For example, if the group C_1 contains x_1, x_2, x_3 and it represents a degradation state of the system, then we give them a confidence value equal to 0.7 showing that we are about 70% sure that these values represent the degraded state of the system.
- Step 3: Classification of a new vector of observations and calculation of its mass function. For a new vector of observations x_{n+1} , we seek to classify it into one of the data groups C_k . Then, we calculate the distance separating the vector from its k nearest neighbors in the group (k should remain small, for example $k = 3$ or $k = 4$) to find the mass function m by the formula given in equation 9. Once this is done, the confidence value of x_{n+1} m_{n+1} is given by $m_{n+1} = m * m^k$.
- Step 4: Determination of mass function for the observation x_{n+1}^j . If the observations constituting the data vector come from various sources that are more or less reliable, we proceed to the calculation of the confidence value of each observation separately. As the confidence value associated with the data vector x_{n+1} is known, the mass function for each observation x_{n+1}^j is given by $m_{n+1}^j = w_{(n+1),j} * m_{n+1}$.

• Uncertainty combination

If the data used in the calculation of an indicator come from different sources, of varying degrees of credibility and which may be conflicting, we can merge them using the combination rules.

• Uncertainty propagation

Once we have quantified the uncertainty in the data, we can propagate it to the performance indicators via the function taking the data as input and the performance indicator as output.

4.3 Decision making

Defining the level of confidence in the performance measure informs the decision maker about our belief in the veracity of the information. To make decisions based on uncertain performance indicators, the following approach is proposed (figure 3):

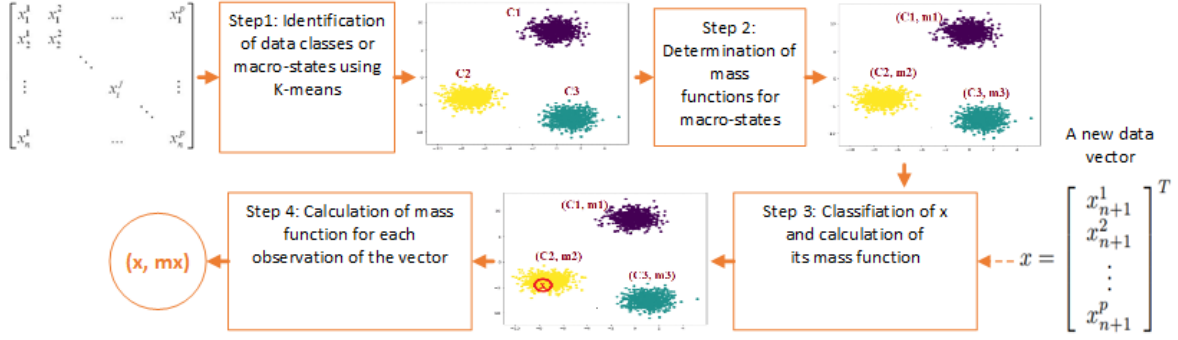


Figure 2: Proposed methodology to determine mass function based on historical data

- 333 1. Two confidence values are defined: the "target confidence" and the "uncertainty acceptability limit" such
334 that the uncertainty acceptability limit \leq the target confidence.
- 335 2. We define the "gravity" as the impact of a bad decision. Its value is based on a scale of 1 to 9. For a gravity
336 of 1, a bad decision has almost no impact on the system. If the gravity is equal to 9, an inappropriate
337 decision could have catastrophic consequences on the production system (an explosion for example). An
338 acceptable level of gravity called "critical gravity" should then be set.
- 339 3. Decision-making zones are then defined with regard to the confidence value and the gravity of a bad decision:
 - 340 • Ignorance zone, *confidence value* \leq *uncertainty acceptability limit*. It represents situations where the
341 confidence in the available data is lower than the minimum limit. Therefore, the ignorance is too
342 great; additional information has to be acquired before any decision-making.
 - 343 • Credible zone, *confidence value* \geq *target confidence*. In this zone, the level of confidence is such that
344 it is quite possible to make decisions, regardless of the potential consequences of the decisions.
 - 345 • Precaution zone, *uncertainty acceptability limit* \leq *confidence value* \leq *target confidence* and *gravity* \leq
346 *critical gravity*. The level of confidence lies between the target confidence and the acceptability limit.
347 In this case, decisions can be made as long as the consequences are assumed.
 - 348 • Critical zone, *uncertainty acceptability limit* \leq *confidence value* \leq *target confidence* and *gravity* \geq
349 *critical gravity*. No decision should be made regarding the potential gravity of the consequences.
- 350 4. When decision making is possible (credible or precaution zones), the decision maker uses the values of the
351 performance indicators to correct or improve the performance of the production system.

352 5 Case study

353 Several companies are considering a transition to Industry 4.0. For many, Industry 4.0 deployment involves
354 managing and exploiting massive data from existing systems and acquiring additional data by installing sensors
355 everywhere. However, this approach is questionable since data, although considered as the oil of the 21st century,
356 is not always relevant. Indeed, data has to be in sufficient quantity, of good quality and with an acceptable cost.
357 Therefore, we are interested in modeling data uncertainty, an aspect of data imperfection, to show its impact
358 on decision making. Since there was no possibility of applying the methodology to a real case, we proceeded by
359 simulation. The simulation model is that of a perfume filling line strongly inspired by a real case. The company
360 under consideration fills perfume bottles for different perfumers. The line is automated but has a low score of
361 overall equipment effectiveness (OEE) which led us to consider the OEE as the indicator of interest. Thus, to
362 show an application of the proposed approach for quantifying KPI uncertainty, we use simulation data to calculate
363 the uncertain OEE and support decision making.

364 5.1 Overall equipment effectiveness (OEE)

OEE is a popular key performance indicator used for measuring the productivity of an individual item of equip-
ment in a factory (Muchiri and Pintelon 2008). It allows improving performance through better information on
production, losses identification and quality improvement (ISO 2014). OEE integrates the availability, effective-
ness and quality of an equipment. Its computation formula is given by :

$$OEE = Availability \times Quality \times Effectiveness$$

Availability is the ratio between actual production time (APT) of an item of equipment and planned busy
time (PBT).

$$Availability = \frac{APT}{PBT}$$

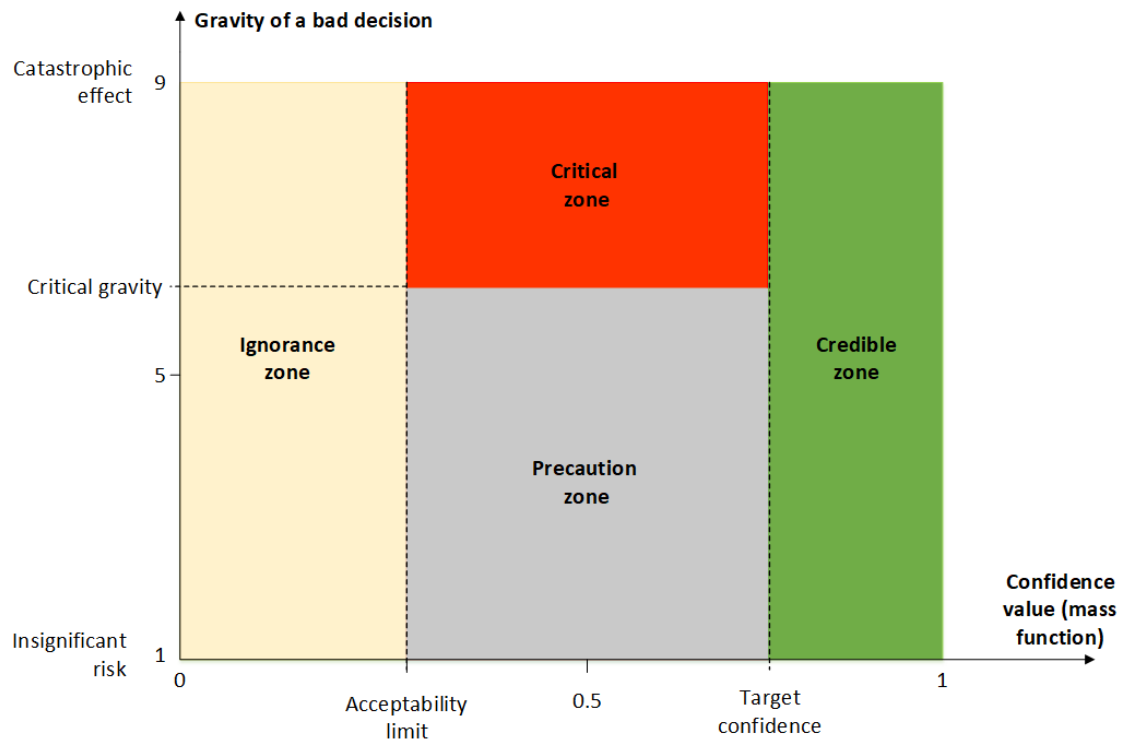


Figure 3: Zones of decision making

The quality ratio defines the relationship between the good quantity (GQ) and the produced quantity (PQ).

$$Quality\ ratio = \frac{GQ}{PQ}$$

Finally, effectiveness is given by the ratio between the target cycle (planned run time per item (PRI) multiplied by produced quantity (PQ)) and the actual cycle represented by (APT).

$$Effectiveness = \frac{PRI \times PQ}{APT}$$

³⁶⁵ The definitions of the quantities used in the formulas of the above indicators are provided in table 2. These
³⁶⁶ definitions are quoted from (ISO 2014).

Table 2: Terms and definitions

Term	Definition
Actual production time (APT)	The actual production time is the actual time during which a working unit is producing. It includes only the value-adding functions.
Planned busy time (PBT)	The planned busy time represents the planned operation time minus the planned downtime. The planned operation time is the planned time in which a work unit can be used.
Good quantity(GQ)	The good quantity is defined as the produced quantity that meets quality requirements.
Produced quantity (PQ)	The produced quantity is the quantity that a work unit has produced in relation to the production order.
Planned run time per item (PRI)	The planned run time per item is the planned time for producing one quantity unit.

367 OEE is a key performance indicator; availability, effectiveness and quality ratio are performance indicators.
 368 Produced quantity PQ and actual production time APT, for example, are elementary data. Its hierarchy is given
 369 by figure 4.

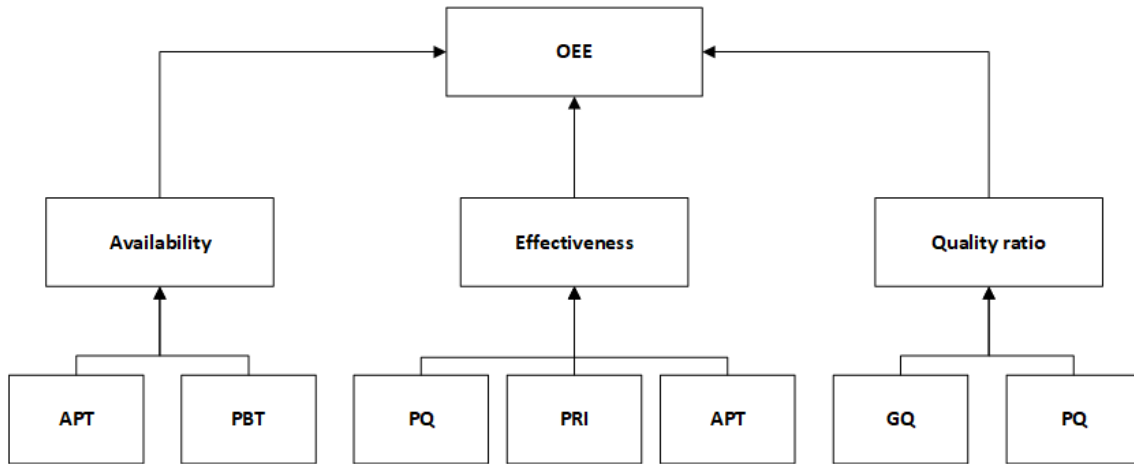


Figure 4: Hierarchy of OEE

370 5.2 Simulation model

371 The simulation model of the filling line (figure 5) has nine workstations consisting of: a post that places empty
 372 bottles in buckets, a filling machine, a bottle capping workstation, a cartoner, a carton sealing machine, a
 373 workstation to separate bottles from buckets, a cellophane wrapping machine, a quality control station, and
 374 packaging. Although the model simplifies reality, it must take into account the variability of the real system
 375 that can be modeled. Therefore, we have introduced an uncertainty on the model parameters in the form of
 376 probability distributions for production rates, failure laws and production quality. This uncertainty, although
 377 artificial, helps to mimic the sometimes unpredictable behavior of the system and introduces some variability.
 378 Since the production line has several workstations, we will choose a single workstation for which the overall

379 equipment effectiveness is monitored. To do so, we have decided to focus on the bottleneck machine whose
 380 identification is carried out in the following step.

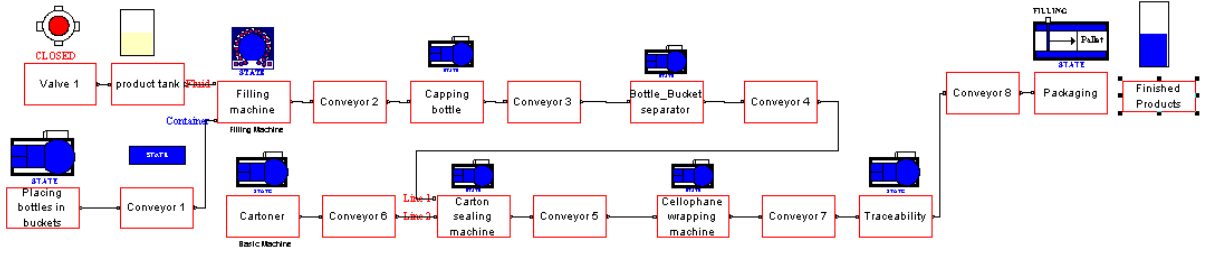


Figure 5: Simulation model

381 5.3 Bottleneck machine identification

382 To find the bottleneck machine in the filling line, we use the approach proposed in (H. Tang 2019). This approach
 383 is aimed at calculating **standalone OEE** based on standalone availability, standalone effectiveness and standalone
 384 quality: the equipment of interest is considered as if it works alone in the production line.

385 The standalone availability A_{sa} of an item of equipment is given by:

$$A_{sa} = \frac{\text{Actual production time}}{(\text{planned busytime} - (\text{starving time} + \text{blocking time}))} \quad (10)$$

386 The standalone effectiveness E_{sa} is given by:

$$E_{sa} = \frac{\text{Actual production rate}}{(\text{designed process rate} - \text{speed loss due to upstream operations})} \quad (11)$$

387 And the standalone quality Q_{sa} is given by:

$$Q_{sa} = \frac{\text{number of good units produced}}{\text{number of units produced} - \text{number of bad quality units due to upstream operations}} \quad (12)$$

388 Then the standalone overall effectiveness equipment OEE_{sa} is given by:

$$OEE_{sa} = A_{sa} * E_{sa} * Q_{sa} \quad (13)$$

389 These indicators allow focusing only on the subsystem faults considered to ensure an effective improvement.
 390 Indeed, a subsystem can be starved due to its upstream subsystem or blocked by its downstream one. The
 391 determination of E_{sa} and Q_{sa} is not always easy and a simplified version of OEE_{sa} can be calculated taking into
 392 account only A_{sa} (H. Tang 2019). It is given by:

$$OEE_{sa-simplified} = A_{sa} * E * Q \quad (14)$$

393 5.3.1 OEE and standalone OEE calculation

394 Based on simulation data, we calculate availability, effectiveness, quality and standalone availability for each work-
 395 station in the line every hour for a one-month simulation. Then, we calculate the OEE and the $OEE_{sa-simplified}$
 396 which we note simply OEE_{sa} . The following tables show the weekly average of OEE and OEE_{sa} for each work-
 397 station during one month. The results are given by the following tables and figures.

398

Table 3: Average of OEE and OEE_{sa} for each workstation during the first week

Workstation	OEE	OEE_{sa}
Placing empty bottles in buckets	44	89
Filling machine	55	94
Separating bottles from buckets	44	90
Capping bottles workstation	43	79
Cartoner	42	90
Carton sealing machine	41	75
Cellophane wrapping machine	40	75

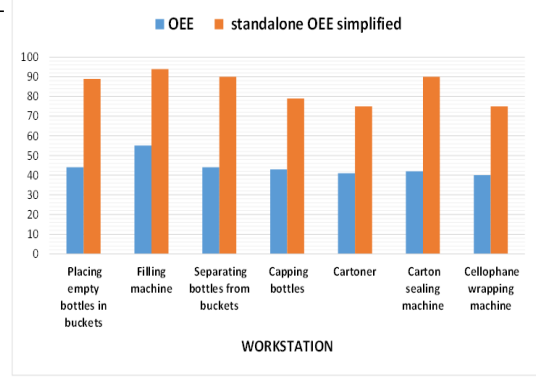


Figure 6: Comparison between OEE and standalone OEE during the first week

Table 4: Average of OEE and OEE_{sa} for each workstation during the second week

Workstation	OEE	OEE_{sa}
Placing empty bottles in buckets	43	85
Filling machine	52	93
Separating bottles from buckets	48	90
Capping bottles workstation	42	79
Cartoner	40	89
Carton sealing machine	40	77
Cellophane wrapping machine	39	77

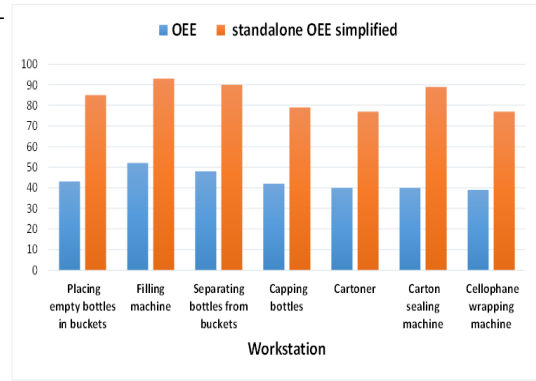


Figure 7: Comparison between OEE and standalone OEE during the second week

Table 5: Average of OEE and OEE_{sa} for each workstation during the third week

Workstation	OEE	OEE_{sa}
Placing empty bottles in buckets	45	91
Filling machine	56	93
Separating bottles from buckets	51	90
Capping bottles	44	81
Cartoner	42	90
Carton sealing machine	42	76
Cellophane wrapping machine	41	75

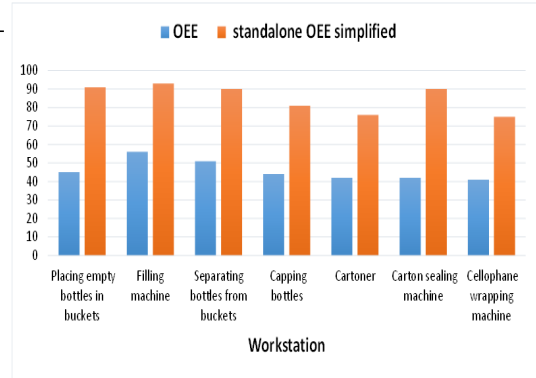


Figure 8: Comparison between OEE and standalone OEE during the third week

Table 6: Average of OEE and OEE_{sa} for each workstation during the fourth week

Workstation	OEE	OEE_{sa}
Placing empty bottles in buckets	41	87
Filling machine	50	91
Separating bottles from buckets	46	86
Capping bottles workstation	40	78
Cartoner	38	87
Carton sealing machine	38	75
Cellophane wrapping machine	37	76

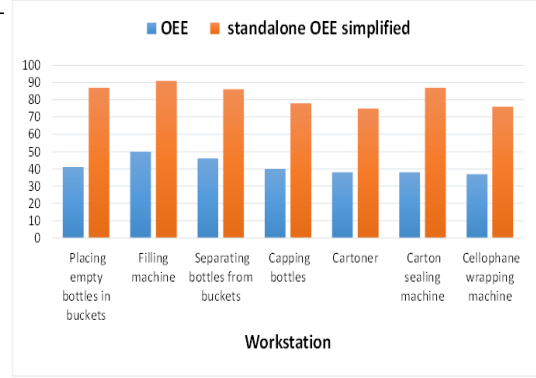


Figure 9: Comparison between OEE and standalone OEE during the fourth week

5.3.2 Identifying the bottleneck machine

We compared the average standalone OEE values during each week to identify the bottleneck station. This weekly study is justified by the fact that the bottleneck station may move from station i to station j during the study period. During the first week, the cellophane wrapping machine and the carton sealing machine are bottlenecks as they have the lowest values of the standalone OEE (75%). Although these machines have the lowest OEE values (40% and 41%), OEE is not suitable for this study. During the second week, the same machines are bottlenecks. The cellophane wrapping machine is the bottleneck during the third week and the carton sealing machine is the bottleneck during the fourth week.

Since most of the time the cellophane wrapping machine is a bottleneck and is the last machine in the line, we chose to focus our study on it.

5.4 Uncertainty modeling

To quantify the uncertainty of OEE, we start by quantifying the uncertainty of the elementary data used to calculate it. As mentioned in the section 4.1, we need to know the actual production time (APT), the planned busy time (PBT), the produced quantity (PQ), the good quantity (GQ), the planned runtime per item (PRI) and the good quantity to calculate the OEE. These data include constants covering PBT and PRI and variables including APT, PQ and GQ. For the constants, we assume that we have certain values. Furthermore, to simplify the model, we assume that the quality rate is constant during the simulation. Then, we express the uncertainty associated only with the variables mainly the APT and the PQ.

5.4.1 System macro-states definition

To have a history of the operation of the cellophane machine, the object of study, we collected simulation data each hour, including the number of units produced, the actual production time, the failure time, and the planned stopped time for one month simulation. This data is used to determine the different states of the cellophane wrapping machine. To achieve this, we use clustering as an unsupervised method to define groups of data corresponding to the different states of the system. We applied the k-means algorithm to the data and obtained 6 groups of data illustrated in figure 10. We can interpret the different groups as follows:

- First group: It corresponds to machine starvation during 65% of the time. This information is confirmed by simulation data concerning the time of machine starvation by hour.
- Second group: This group corresponds to the nominal operation of the machine. Indeed, the production time is on average 55 minutes per hour and the number of parts produced per hour is 1489.
- Third group: When it comes to this group, the machine is starved for 50% of the time per hour.
- Fourth group: This data set corresponds to a starvation for 30% of the time.
- Fifth group: It corresponds to the faulty state of the system characterized by an average failure time of 30 minutes per hour.
- Sixth group: This last group of data corresponds to a starving state with the presence of planned stops.

5.4.2 Uncertainty calculation

The next step is to calculate the uncertainty for new observations. Let us assume that we have observations in the form of multivariate data involving APT, PQ, the time of failure and the planned stopped time per hour. We aim at calculating the uncertainty of this data vector. We start by classifying the data vector to determine the state of the system and then we calculate the uncertainty based on the distance between it and its k-nearest

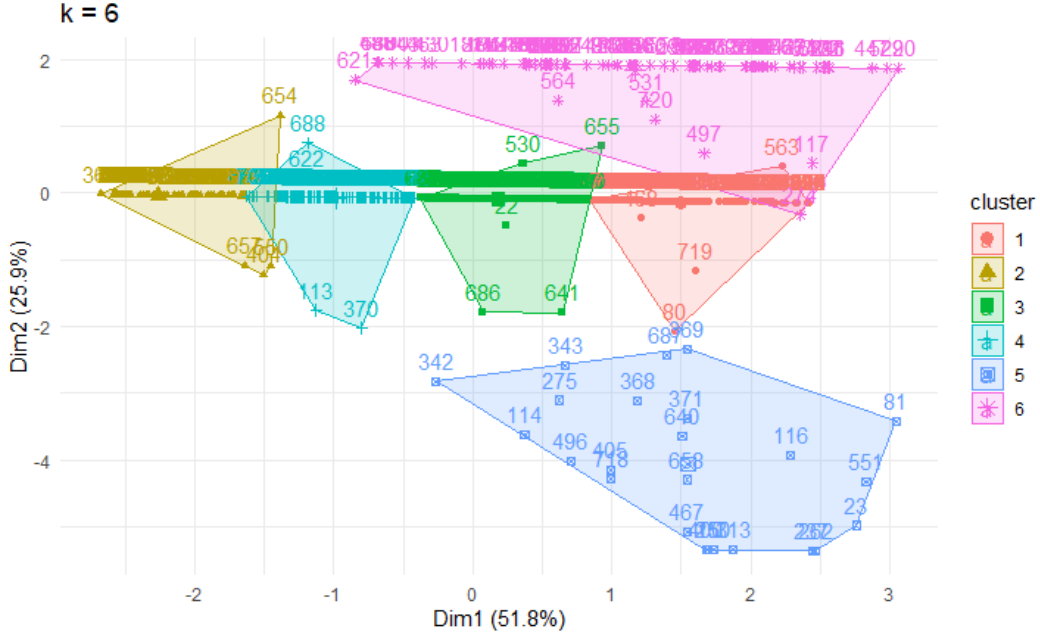


Figure 10: System macro-states represented by data groups

445 neighbors. We apply this method to observations generated artificially by Generative adversarial networks (GAN).
 446 This generation of artificial data allows on the one hand the generation of uncertainties and, on the other hand,
 447 overcoming the problem of having sufficient data describing all the possible states of the system.

In table 7, we present observations generated by the neural networks, its classes and mass functions.

Table 7: Observations generated artificially by GAN networks with its mass functions.

<i>Id</i>	<i>Units produced</i>	<i>APT</i>	<i>Time failed</i>	<i>Time stopped</i>	<i>Class</i>	<i>Mass function</i>
1	808	25.6	0.44	1.27	4	0.99711
2	1192	37.79	0.45	5.16	3	0.16531
3	604	19.2	0.2	1.12	1	0.98767
4	167	5.25	0.12	0.24	1	0.99759

448 According to the classification results, the first vector belongs to class 4, i.e. the cellophane wrapper starved for
 449 30% of the time. As the mass function based on the distance separating this vector from its 10 nearest neighbors
 450 is equal to 0.99711, the information on the state of the cellophane machine is more than 99% credible. The
 451 second vector belongs to class 3 (the machine starved for 50% of the time).However, the mass function is equal
 452 to 0.16531. This value is very low and therefore we cannot make any decision based on the second observation.
 453 In this case, more investigation is required to reduce the uncertainty level.
 454

455 5.4.3 Uncertainty propagation

456 We expressed the uncertainty of *APT* and *PQ*, necessary for the calculation of availability, effectiveness and
 457 therefore of the *OEE* (the quality rate is assumed to be constant). We start by propagating the uncertainty to
 458 the performance indicators: availability and effectiveness (quality is supposed constant) and then to *OEE*.

459 In the determination of the mass function, we considered multivariate data and we assumed that the data
 460 sources are reliable. Thus, the uncertainty value is the same for all variables. In this case, we will not use
 461 the Cartesian product to propagate the uncertainty. We simply assume that the uncertainty of availability and
 462 effectiveness is the same as the uncertainty of the data (actual production time and produced quantity). Thus,
 463 the same uncertainty is associated with the *OEE*. Otherwise, if each datum is collected separately with more or
 464 less reliable sources, we proceed to the propagation by Cartesian product. The values of *OEE* with their mass
 465 functions are given by table 8.

466 5.5 Decision making

467 Based on the uncertain values of *OEE*, the decision maker may or may not make a decision. For example, for
 468 the first value of *OEE* equal to 39%, the mass function is 0.99711. Then this value of *OEE* is almost certain

Table 8: Values of *OEE* with their mass functions

<i>OEE</i>	<i>Mass function</i>
39	0.99711
61	0.16531
29	0.98767
8	0.99759

and it is possible to make a decision without worrying about the quality of the information. And based on the classification of observations into macro-states, we know that the system is in a starved state. So, an investigation of the performance of the upstream machine (in our case, the carton sealing machine) is necessary to fix this problem. This indication helps us to understand the causes of such a low value of the overall equipment efficiency. On the other hand, for the second value of *OEE*, the decision maker cannot make any decision because the mass function value (0.16531) is very low, meaning that they do not know the real value of the overall equipment effectiveness.

Thus, the contribution of the uncertainty calculation method is twofold: it reassures us of the veracity of the information before implementing an action and it provides us with insights into the causes of a loss in performance.

The uncertainty quantification approach is intended to assist decision making. In the presence of uncertain data, the decision maker can make a decision or request more information in the case where the uncertainty is very high. The classification of the data vector to a system state helps to diagnose the production system while giving an idea of the data quality level. This avoids making critical or costly decisions with a high risk of being wrong.

6 Conclusion and perspectives

The ever-increasing amount of data available in companies creates new challenges for decision support. As quantity is no guarantee of quality, data can be imprecise, uncertain, incomplete, etc. It is therefore necessary to deploy techniques to model these data imperfections. Our work is performed in the context of Industry 4.0 and decision support in the presence of massive and uncertain data. We used the theory of belief functions to provide some answers to this problem. This theory has the ability to represent the degree of confidence that a user can associate with statistical data or expert knowledge. We then proposed a method allowing: (i) the quantification of these uncertainties by a scalar m associated with each elementary datum, and (ii) propagating them to the KPIs. Furthermore, a method to compute the uncertainty of a new vector of observations from historical data was developed and decision rules to follow in the case of uncertain performance measures were proposed. In order to show the interest of the proposed approach in decision making, we used data from a simulation model of a perfume filling line. We proceeded to the quantification of the uncertainty related to the *OEE*.

The modeling of this uncertainty associated with the KPIs allows making relevant decisions by separating the following situations:

1. Zone of ignorance, low confidence independently of the risk associated with a decision: no decision should be made.
2. Zone of credibility, high confidence independently of the risk associated with a decision: easy decision.
3. Precaution zone, medium confidence and low risk: decision possible provided that the consequences are assumed.
4. Critical zone, medium confidence and high risk: decision is not advisable.

This makes it possible to prevent the risks associated with inappropriate decisions and additional costs, for example.

Several perspectives can be considered. Despite the interest of the results obtained, our method should be extended, in particular by improving the techniques of confidence degree propagation and the control of the number of performance indicators. Naturally, one of our goals is to apply the method to real cases, which will help us to refine the approach.

7 Annex

Table 9: Comparison of theories for epistemic uncertainty modeling

Criteria	Probability theory	Dempster-Shafer theory	Possibility theory
<p>Uncertainty measure and its interpretation</p>	<p>Probability $P(A1)$: the probability of an event is interpreted as the limit of the frequency of occurrence of the event when the experiment is repeated many times (objectivist view) or the belief that an event will occur (subjectivist view) (Dubois 2006). $A1$ is an individual elementary event from Ω such that $\Omega = \{A1, A2, \dots, An\}$, a finite set of mutually exclusive and exhaustive propositions about some problem domain, called the frame of discernment (Denoeux 2008).</p> <p>Example: a coin is tossed three times and we are interested in the random variable X given by the number of heads obtained. The probability that X is equal to 2 is noted by $P(X = 2) = \frac{1}{4}$.</p> <p>Now consider the subjective probability. We are interested in the probability that a dam will fail following an extreme flooding condition. Since this event is rare, it is not possible to calculate the probability of it happening based on the frequency of occurrence.</p> <p>An approach to determine this uncertainty consists in modeling the system, its performances and its uncertainties in order to build up a state of knowledge on the occurrence of the event under consideration (Ayyub 2001).</p>	<p>Mass function $m(A)$: $m(A)$ is interpreted as a measure of the belief that one commits exactly to A, and not to any subset of A (Baraldi and Zio 2010), due to lack of knowledge (Dubois and Prade 2009). A is a set of the power set of the frame of discernment $\Omega : P(\Omega) = \{\emptyset, \{A1\}, \{A2\}, \{A1 \cup A2\}, \dots, \Omega\}$.</p> <p>Example: a murder has been committed and there are 3 suspects: John, Jack and Sophie. The frame of discernment is $\Omega = \{John, Jack, Sophie\}$</p> <p>A witness saw the murderer run away, but he is nearsighted and can only attest that it was a man. We know that the witness is drunk 20% of the time. The information about the suspect is uncertain and can be represented by a mass function m on Ω such that : $m(\{John, Jack\}) = 0.8$ and $m(\Omega) = 0.2$. The mass 0.2 is not assigned to Sophie because the testimony does not accuse Sophie at all (Denœux 2010).</p>	<p>Possibility $\pi(A)$: possibility is a subjective measure that expresses the degree to which a person considers that an event can occur. It relates to our perception of the degree of feasibility or ease of attainment.</p> <p>Example: we take the example of (Zadeh 1978) on Hans' breakfast. The variable of interest that we will note X is the number of eggs that Hans will eat tomorrow. The possibility and probability values for X are assumed to be known.</p> <p>For example, $\pi(X = 1) = 1$ and $P(X = 1) = 0.1$, $\pi(X = 8) = 0$ and $P(X = 8) = 0$. $\pi(X = 1)$ is interpreted as the degree of ease with which Hans can eat one egg. $P(X = 1)$ is the probability that Hans eats one egg for breakfast. We notice that it is totally possible that Hans eats but that it is unlikely. So a high degree of possibility does not imply a high degree of probability and a low degree of probability does not imply a low degree of possibility. However, if an event is impossible, it is necessarily improbable.</p>
<p>Comment: The interpretation of uncertainty as a probability is questioned if it is not possible to repeat the experiment a sufficient number of times to obtain the probability distribution. Indeed, if we know the probabilities, it is possible that we adopt them as degrees of belief. However, if we do not know them, it is an extraordinary coincidence that our degrees of belief in a proposition are equal to the probability of occurrence (Shafer 1976)</p>			

<p>Uncertainty modeling for : 1. statistical data and 2. expert opinions</p>	<p>A probability distribution can be identified if the amount of data is sufficient. In the case of the elicitation of an expert opinion on the value of a parameter X, they can provide for example the 5% and 95% fractiles of the probability density function and a mean, median or mode of the distribution. To construct the probability density based on these data, a choice of parametrized family of distribution function is usually made (Dubois and Prade 1994).</p>	<p>As in probability theory, if we have enough data, we can construct possibility and belief distributions. Otherwise, the degrees of confidence provided by the expert are taken as measures of uncertainty without the need for a prior definition of distribution functions.</p>	
<p>Comment: If we have sufficient amounts of data, measures of probability, possibility, and belief have in common that all three can be characterized by a distribution on the elements of the reference set. In the case of subjective data, there is no reason that a subjective belief should be probability. In fact, the determination of a probability distribution requires more information than an expert is able to provide. And the a priori choice of a specific probability distribution is unfaithful to the modeling of expert opinion because it assumes that the analyst collecting expert opinions has an idea of the value of the parameter however ignorant this analyst may be (Dubois and Prade 1994).</p>			
<p>Link between the uncertainties related to A and \bar{A}</p>	<p>$P(A) + P(\bar{A}) = 1$. This equation shows that the knowledge of the probability of A completely defines that of its opposite event.</p>	<p>If we do not have enough information about the realization of an event and its opposite, we can allocate weak mass functions to each of them ($m(A)$ and $m(\bar{A})$ have low values).</p>	<p>$\max(\pi(A), \pi(\bar{A})) = 1$. This equation means that of two contrary events, at least one is possible, the possibility of one not implying the impossibility of the other (Masson 2005). Then the possibility of an event and that of its contrary are weakly linked.</p>
<p>Comment: Because of the axiom of additivity, probabilistic modeling is not suitable when the knowledge about an event and its opposite is very limited. Thus, this theory does not distinguish between a lack of belief and non-belief in the realization of an event.</p>			

<p style="text-align: center;">Modeling imprecision AND uncertainty simultaneously</p>	<p>The probability $p(A)$ is allocated to a singleton A.</p> <p>Example: we consider the variable of interest given by Peter's age. We know that he is between 20 and 25 years old.</p> <p style="text-align: center;">$\Omega =$ $\{20, 21, 22, 23, 24, 25\}$.</p> <p>We can therefore assign the probabilities to each singleton of Ω: $P(\{20\}, P(\{21\})$, etc.</p>	<p>The degree of belief $m(A)$ can be associated with a singleton, a subset A of the frame of discernment or the total frame of discernment which allows expressing uncertainty and imprecision in a natural way.</p> <p>Example: We consider Peter's age again. One of his friends told us that he is over 21 years old but we are 80% confident in his piece of information. Thus the information on Peter's age is imprecise and may belong to the set $\{22, 23, 24, 25\}$ and uncertain because the source is not totally reliable. The mass functions are given by: $m(\{22, 23, 24, 25\}) = 0.8$ and $m(\Omega) = 0.2$.</p>	<p>Based on the fuzzy sets theory, the possibility theory allows representing information that is subject to both uncertainty and imprecision: we can express a degree of possibility for a subset of propositions: $\pi(\{A1, A2\})$.</p> <p>Example: for Peter's age, we know that he will be awarded his bachelor degree soon. So, it is possible that he is between 22 and 25 years old. This information is imprecise because we don't know Peter's exact age but also uncertain. It can be represented by: $\pi(X \in \{22, 23, 24, 25\}) = 1$.</p>
<p>Comment: Available information is generally incomplete. For instance, imprecision is a form of incompleteness since imprecise information is not sufficient for an agent to answer a question in a given context (Dubois and Prade 2009). Uncertainty and imprecision are two closely related imperfections: in many situations, the more precision is required in the statement of an assertion, the less certain the assertion is (Bouchon-Meunier 2007). In this case, a probability distribution is not adapted to model epistemic uncertainty because it hides the imprecision in the data. A probability distribution may provide too much information to reflect poor expert data or unreliable sensors (Dubois and Prade 1994). On the other hand, the theory of belief functions and the possibility theory have the advantage of modeling both imprecision and uncertainty in data.</p>			

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