



**HAL**  
open science

# A physical baseline to model photovoltaic development and estimate its net product

Aymeric Lamorlette

► **To cite this version:**

Aymeric Lamorlette. A physical baseline to model photovoltaic development and estimate its net product. 2022. hal-03785232v1

**HAL Id: hal-03785232**

**<https://hal.science/hal-03785232v1>**

Preprint submitted on 23 Sep 2022 (v1), last revised 16 Nov 2022 (v4)

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## **Disclaimer**

I would like to apologise to the reader for the crudeness of this version. References are sparse and results lack discussion. Please keep in mind this is a work in progress.

# A physical baseline to model photovoltaic development and estimate its net product

A. Lamorlette

*Aix-Marseille Univ, CNRS, IUSTI UMR 7343  
5 rue Enrico Fermi, 13 013 Marseille Cedex 13, France  
Correspondence to [aymeric.lamorlette@univ-amu.fr](mailto:aymeric.lamorlette@univ-amu.fr)  
Tel (33) 491 113 811 ; Fax (33) 491 118 502*

---

## Abstract

Photovoltaic production dynamic is usually modeled using a sigmoid or S-curve function, without any physical bases. This paper aims at developing a physical model based on energy conservation and ERoEI definition, that could reproduce a S-curve function and allow to investigate the net energy delivered by the photovoltaic industry. This approach gives some new insights on the discussion on energy delivered by renewable and low carbon energy. It also allows to investigate energy policies that could lead to a smooth energy transition.

*Keywords:* PV production, ERoEI, dynamic model, net energy, energy transition

---

## 1 Introduction

2 In the frame of fossil fuel depletion and climate change, development of  
3 renewable, or at least low carbon energy source is a major issue for mod-  
4 ern societies. In particular, photovoltaic (PV) system development is often  
5 model by a logistic (sigmoid or S-curve) function (Cherp et al. (2021)), the  
6 solution of the Verhulst equation. Despite the ability of the S-curve to fit PV  
7 development in most countries, as in Cherp et al. (2021), there is no physical  
8 bases for such model, hence the analysis of its parameters does not provide  
9 insight on the PV development capability. In this article, a physical model  
10 is suggested for PV system development, based on energy conservation and  
11 ERoEI, as previously done for fossil liquid fuel in Lamorlette (2022). This  
12 model is discussed to see how it can provide a S-curve, in accordance with

13 Cherp et al. (2021). It is also applied to global PV system development  
 14 to investigate the re-investment trend of the PV industry. This allows to  
 15 estimate the net energy provided by PV systems to society, which depends  
 16 indirectly on EROEI, contrary to the common assumption that net energy  
 17 on gross energy is equal to  $(EROEI - 1)/EROEI$ . This point is discussed  
 18 in details, providing insights on the controversy on PV net energy discussed  
 19 in Ferroni and Hopkirk (2016) and Raugei et al. (2017). Finally, investment  
 20 for energy source development in the frame of energy transition is discussed,  
 21 based on recent work on fossil fuel depletion, to suggest adequate policies  
 22 that could ensure a smooth energy transition.

## 23 **1. Development of the PV industry**

24 As proposed in Lamorlette (2022) for the development of the oil industry,  
 25 based on energy conservation and EROEI definition, a model is here developed  
 26 to describe the development of the PV industry. It is firstly derived based  
 27 on yearly (discrete) production, then the model is written using a continuous  
 28 formalism, what leads to an ODE.

### 29 *1.1. Discrete approach*

30 The growth and decay is here considered over a one year time laps ( $\Delta t = 1$   
 31 year). The growth is due to the industry expansion coming from energy  
 32 re-investment. The decay is due to the closing of production sites, which  
 33 are designed to last for a period  $\tau$ , according to there EROEI calculation,  
 34 with  $\tau \sim 20 - 30$  years (a PV unit lifespan). PV unit efficiency decrease is  
 35 indirectly taken into account in the growing term, as explained in the next  
 36 section.

#### 37 *1.1.1. Growing*

38 Let us consider a yearly production of PV energy  $Q$  (in GW), available  
 39 for the consumer. A fraction  $k$  of this production (dimensionless, with  $k$   
 40 in  $[0; 1]$ ) is re-invested to develop the PV industry on a year  $n$ . Energy  
 41 conservation and EROEI definition allow to write the increase in production  
 42  $\Delta Q^+$  on a year  $n$ , considering an equivalent homogeneous production over  
 43 the whole lifespan  $\tau$  of a PV unit (i.e. smoothing the efficiency decrease of  
 44 a PV unit over its whole lifespan). With an energy investment  $Q_n k$ , one get  
 45 an energy  $Q_n k EROEI_n$  during  $\tau$  years after the investment is done, based on

46 ERoEI definition. The energy increase per unit time reads  $Q_n k ERoEI_n / \tau$ .

47 It leads to:

$$\Delta Q_n^+ = Q_n k_n \frac{ERoEI_n}{\tau} \Delta t. \quad (1)$$

### 48 1.1.2. Decay

49 The decay on a year  $n$ ,  $\Delta Q^-$ , is equal to the increase on year  $n - \tau$ , as it  
50 is due to the closing of sites developed  $\tau$  years before. Hence one get:

$$\Delta Q_n^- = \Delta Q_{n-\tau}^+ = Q_{n-\tau} k_{n-\tau} \frac{ERoEI_{n-\tau}}{\tau} \Delta t. \quad (2)$$

51 Using Eq.(1) and Eq.(2), the increase in production on a year  $n$ ,  $Q_{n+1} - Q_n$   
52 can be written:

$$Q_{n+1} - Q_n = \Delta Q_n^+ - \Delta Q_n^- = \frac{1}{\tau} (Q_n k_n ERoEI_n - Q_{n-\tau} k_{n-\tau} ERoEI_{n-\tau}) \Delta t. \quad (3)$$

### 53 1.2. Continuous equivalence

54 Eq.(3) can be written as an ODE, taking  $\Delta t \rightarrow 0$ . It leads to  $(Q_{n+1} -$   
55  $Q_n) / \Delta t = \dot{Q}$  and therefore:

$$\dot{Q} = \frac{1}{\tau} (Q(t) k(t) ERoEI(t) - Q(t - \tau) k(t - \tau) ERoEI(t - \tau)). \quad (4)$$

56 This equation shows a convolution term which could make it complicated or  
57 even impossible to solve analytically. Nevertheless, as discussed later, the  
58 term  $\Delta Q^-$  can be neglected at short or medium time scale, which would lead  
59 to analytical solutions for Eq.(4).

## 60 2. Solution for a linear investment policy at medium time scale, 61 recovering the S-curve

62 Let us first discuss the relative contribution of decay in the production  
63 dynamic, that is  $\Delta Q^- / \Delta Q^+$ . At a time scale smaller than  $\tau$  (i.e. short time  
64 scale), this contribution is strictly zero. During the development phase of a  
65 PV industry (at local scale (a country), or at global scale), according to all  
66 the measured values presented in Cherp et al. (2021),  $Q$  evolves at a time  
67 scale way smaller than  $\tau$ . It means that practically,  $\Delta Q^-(t) \ll \Delta Q^+(t)$  and

68 then  $\Delta Q^-$  can be neglected at medium time scale, the time scale at which  
 69 PV is developing. It leads to:

$$\dot{Q} = \frac{1}{\tau} Q(t) k(t) ERoEI(t). \quad (5)$$

70 Let us now consider a linearly evolving investment over a period on which a  
 71 PV industry plans to develop from  $Q(t_0) = Q_0$  (an initial, external energy  
 72 input) to  $Q(t_f) = Q_{goal}$  (a goal of production), where  $t_0$  is the time at the  
 73 beginning of the investment and  $t_f$  the time at the end of the investment.  
 74 Then  $k = 1$  at the beginning of the investment ( $k(t_0) = 1$ ),  $k = 0$  when the  
 75 industry is fully developed (i.e.  $k(t_f) = 0$ ) and  $k$  evolves as  $1 - Q/Q_{goal}$  in  
 76 between (i.e.  $k$  proportional to  $-Q$ ). This leads to:

$$\dot{Q} = \frac{1}{\tau} Q(t) \left( 1 - \frac{Q(t)}{Q_{goal}} ERoEI(t) \right). \quad (6)$$

77 Eq.(6) is strictly a Verhulst equation, with  $Q_{goal}$  the carrying capacity. With  
 78 constants  $\tau$  and  $ERoEI$ , it leads to a S-curve solution, which is in line with  
 79 the work of Cherp et al. (2021).

80 This approach does not directly take into account the potential lack of  
 81 material to produce PV units or the lack of space to due to geophysical  
 82 constraints, but it actually takes it into account via a future decrease of PV  
 83  $ERoEI$ . However, this aspect is not developed in this study as it considers  
 84 only constant  $ERoEI$  as a baseline, in order to investigate the different  
 85 analytical solution one can get from Eq.(3) or Eq.(4).

### 86 **3. Short time scale analysis of global PV production**

87 Contrary to many countries where the saturation (or “market size”) has  
 88 been reached (as Portugal, Germany, Belgium or Greece, according to Cherp  
 89 et al. (2021)), the global PV production seems to be in its development phase.  
 90 Indeed, according to IEA data on PV (IEA (2018)), the PV product follows  
 91 an exponential growth, with a characteristic time  $t_{exp} \sim 2.82$  years (this fit  
 92 provides a determination coefficient higher than 0.995 on the period 1992-  
 93 2018). At short or medium time scale, Eq.(3) or Eq.(4) can be solved at  
 94 constants  $\tau$  and  $ERoEI$ , leading to an exponential solution. This solution  
 95 exhibits the following characteristic time:

$$t_{exp} = 1/\ln(1 + kERoEI/\tau). \quad (7)$$

96 Identification can then leads to an estimation of the PV industry investment.

97 *3.1. Estimation of PV industry investment*

98 The previous remark suggests a way to estimate the re-investment of the  
 99 PV industry on itself in order to ensure a self-growing business. This as-  
 100 sumption seems reasonable, since PV industry can hardly expect external  
 101 energy for its development, as global energy consumption ensure the global  
 102 GNP (as suggested by the work of Tim Garrett), and our economy requires  
 103 GNP to keep increasing. Therefore, without external energy input, the pre-  
 104 vious relationships allow to estimate the investment, as long as averaged,  
 105 effective values can be evaluated for  $\tau$  and  $ERoEI$ .  $\tau$  is taken equal to 20  
 106 to 30 years, based on a PV unit lifespan. As remarked in section 1.1.1, the  
 107 production  $Q$  that is considered is taken at the consumer, which means that  
 108  $ERoEI$  should be evaluated at the most extreme boundary. This would refer  
 109 to an extended ERoEI ( $ERoEI_{EXT}$  to follow the nomenclature of Raugei  
 110 et al. (2017)), where  $ERoEI_{EXT} \sim 7 - 8$ . Please note that in the following,  
 111  $ERoEI$  will always refer to an extended ERoEI,  $ERoEI_{EXT}$ .

112 An identification of  $k$  is then performed based on  $t_{exp}$  global value and  
 113 theoretical expression (Eq.(7)). The following behaviour is observed: For  $\tau =$   
 114 20 years, an  $ERoEI_{EXT}$  of 8 or lower would lead to  $k \geq 1$ . For  $\tau = 30$  years,  
 115 an  $ERoEI_{EXT}$  of 12 or lower would lead to  $k \geq 1$  which seems economically  
 116 impossible, as it would mean that PV industry is an energy sink. This  
 117 emphasises the result of Raugei et al. (2017) on the minimum possible value  
 118 of PV extended ERoEI. It also suggests that global PV development is close  
 119 to its maximum with  $k(t) = 1$  from the beginning of PV development.

120 *3.2. Estimation of the net production*

121 Contrary to what the development of Eq.(4) could suggest, based on the  
 122 energy available for the consumer, it does not strictly provides the net energy  
 123 from PV systems. Indeed, a fraction  $k$  is taken for re-investment, hence the  
 124 net product  $Q_N$  reads:

$$Q_N(t) = (1 - k(t))Q(t) . \quad (8)$$

125 According to the previous remark on the averaged, effective, value of  $k$ ,  
 126 it suggests that net energy from PV was and is still close to zero. This  
 127 could be a reason that motivated the study of Ferroni and Hopkirk (2016)  
 128 which claims that PV is not producing any net energy and could even be an  
 129 energy sink. The answer of Raugei et al. (2017) to the study of Ferroni and  
 130 Hopkirk (2016) seems right in term of  $ERoEI$  values, however the study of

131 Ferroni and Hopkirk (2016) seems right in term of net energy delivery. This  
 132 whole result on net energy from PV systems is nevertheless very far from  
 133 the classical relationship presented in Murphy (2014) for oil (for instance),  
 134 where net energy is supposed to depend directly on gross energy and ERoEI  
 135 through the relation:

$$Q_N = Q \frac{ERoEI - 1}{ERoEI}. \quad (9)$$

136 It seems that the confusion might come from the original derivation of this  
 137 relationship, which was done for oil. Indeed, for oil, this relationship almost  
 138 applies according to energy conservation, with two additional assumptions.  
 139 Firstly, as highlighted in Lamorlette (2022), it should read:

$$Q_N = Q \frac{ERoEI_{EXT} - 1}{ERoEI}. \quad (10)$$

140 The paper of Hall et al. (2009) contains everything in the text to develop  
 141 Eq.(10), even a numerical application in Table 3. The paper of Lamorlette  
 142 (2022) merely presents a more complete mathematical formulation of the  
 143 concepts presented in Hall et al. (2009). Secondly, as discussed in Lamorlette  
 144 (2022),  $Q_N$  in Eq.(10) still misses the role of energy re-investment. Hence  
 145 a  $(1 - k)$  (with  $k$  being the investment of the oil industry) correction was  
 146 added. As  $k$  for oil is historically lower than 1% (see Lamorlette (2022)),  
 147 Eq.(10) is practically applicable to oil with accurate results.

148 This whole rational however breaks down for PV systems, as PV industry  
 149 development depends straightforwardly on  $ERoEI$  but its net energy does  
 150 not, mainly because re-investment is close to unity. As the same energy  
 151 development modeling can be applied to wind, nuclear or other low carbon  
 152 energy, net energy from these sources should be carefully evaluated, taking  
 153 into account the cost of the industry development.

#### 154 4. Toward a smooth energy transition

155 According to the study of Lamorlette (2022), it is suggested that the peak  
 156 oil might arrive as soon as 2028 (based on the results obtained with the most  
 157 reliable ERoEI data for oil). It can be extrapolated to estimate the peak  
 158 gas around 2036 (based on the results obtained with the ERoEI data for oil  
 159 and gas). As the peak might slow down the whole economy and therefore  
 160 all investment dynamic, it seems safer to ensure the production of the total  
 161 amount of PV energy required to “replace” oil and gas before the peak, since



162  $k$  will have to be reduced to compensate the lack of energy as soon as the peak  
163 occurs. Based on the previous results, it is suggested that PV production  
164 can merely replace the energy from oil around 2028. Indeed, in 2020, PV  
165 represents about 2% of the global energy mix, where oil represents about 31%  
166 BP (2020). According to Eq.(7) and the global value of  $t_{exp}$ , PV production  
167 can be multiplied by 15 from 2022 to 2028. It means that when the peak  
168 gas will be reached, it is most likely that  $k$  will already be too small to keep  
169 the PV production growing to compensate for the lack of gas. Moreover, if  
170 one wants to get rid of coal in the energy mix, it is clearly impossible before  
171 getting  $k = 0$  (which leads to the end of energy production increase and the  
172 beginning of an energy steady state).

173 It means another energy source, of higher ERoEI, is required to ensure  
174 a smooth transition, using an external energy input that has to be small  
175 enough for our economy to cope with this take from the global energy mix.  
176 As nuclear energy shows  $ERoEI_{Nu}$  in the range [40; 60] (WNA (2018)), it  
177 might be an adequate source. Problem is that since the construction of a  
178 nuclear power plant is way longer than the construction of PV units, this  
179 construction time  $t_{invest}$  must be taken into account for nuclear, contrary to  
180 PV, then Eq.(3) must be modified the following way as it must be used over  
181 a time laps  $\Delta t = t_{invest}$ , using the same rationale as previously (subscript  $Nu$   
182 relates to nuclear):

$$Q_{Nu_{n+1}} - Q_{Nu_n} = \frac{t_{invest}}{\tau_{Nu}} (Q_{Nu_n} k_{Nu_n} ERoEI_{Nu_n} - Q_{Nu_{n-\tau_{Nu}}} k_{Nu_{n-\tau_{Nu}}} ERoEI_{Nu_{n-\tau_{Nu}}}). \quad (11)$$

183 In order to evaluate the ability of nuclear to grow, based on Eq.(11), one  
184 can calculate the growth rate over an investment time step (here equal to  
185  $t_{invest} = 8$  years, according to EDF (2018)) and integrate it over the time  
186 remaining until transition ( $N_{trans} = 15$  years). Based on Eq.(11), the growth  
187 rate  $g_{Nu}$  is equal to  $1 + t_{invest} ERoEI_{Nu} / \tau_{Nu}$ . This growth rate is applied  
188 over  $N_{trans} / t_{invest} \sim 2$  investment time steps. It means that the increase in  
189 energy  $I_{Nu}$ , starting with a yearly amount  $Q_0$  can be estimated the following  
190 way:

$$I_{Nu} = Q_0 t_{invest} g_{Nu}^{N_{trans}/t_{invest}}. \quad (12)$$

191 The calculation of  $I_{PV}$  can be done straightforwardly from Eq.(3). The PV  
192 growth rate  $g_{PV}$  is equal to  $1 + ERoEI / \tau$ , then

$$I_{PV} = Q_0 g_{PV}^{N_{trans}}. \quad (13)$$

193 The quantity  $I/Q_0$  represents the ability of a given technology to grow. For  
 194 the transition scenario presented here,  $I_{PV}/Q_0 \sim 260$ . Taking  $\tau_{Nu} = 40$   
 195 years (EDF (2018)),  $I_{Nu}/Q_0 \sim 490 - 980$  (depending on the value used for  
 196  $ERoEI_{Nu}$ , 40 or 60), suggesting that nuclear is a safer investment to cope  
 197 with the end of fossil fuels. To cover the lack of energy due to the end of  
 198 gas and coal use (i.e. roughly 50% of the global energy mix), based on the  
 199 most pessimistic  $ERoEI_{Nu}$  value, an initial yearly investment representing  
 200 0.1% of the global annual energy budget would be enough to ensure a smooth  
 201 transition toward a steady-state of energy around 2035-2040, with a contin-  
 202 uous increase until that point, whereas PV would require a 0.2% investment  
 203 to do so. Besides, these calculations apply only if ERoEI does not decrease  
 204 during the transition, what might happen due to material or geophysical  
 205 constraints, as stated earlier. It seems it has more chance to happen with  
 206 PV than with nuclear, due to the size of facilities and amount of material  
 207 needed to produce a unit of energy. It also suggests that the more we wait to  
 208 do this investment, the more difficult it would be to cope with the decrease  
 209 of available energy that will happen around 2035-2040.

210 It is worth noting here that this rational has been conducted at global  
 211 scale, but it could also be applied at local scale (i.e. a country) to determine  
 212 a specific, efficient investment policy, based on the resources and energy mix  
 213 of a given country.

## 214 Conclusion and opening

215 In this article, a model is presented to study PV production dynamic.  
 216 This model is able to reproduce the behavior of the previous S-curve function  
 217 model. It also shows the following features:

- 218 • The model development, based on energy conservation and ERoEI def-  
 219 inition, is applied to PV but can be applied also to wind and nuclear.
- 220 • It allows to explain why the usual relationship between gross and net  
 221 energy and ERoEI ( $Q_N/Q = 1 - 1/ERoEI$ ) does not apply to PV.
- 222 • It provides insights on the PV net energy discussion.
- 223 • It suggests an energy policy that ensure a smooth energy transition,  
 224 which could only lead to a steady-state production of energy.

225 Finally, it seems important to notice that, according to Barrau (2022),  
226 climate change is “the tree that hides the forest”. The collapse of ecosys-  
227 tems is a deeper and more important problem our societies are facing. The  
228 steady-state presented before is very unlikely low enough to ensure the bal-  
229 ance of earth ecosystems. Therefore, a sequel to this work could be to ask:  
230 “What steady-state (i.e. global human energy per unit time) could allow the  
231 ecosystems to recover a regime made of quasi-static transformations, max-  
232 imising the lifespan of the earth system as we know it, by minimising entropy  
233 creation ?”.

## 234 **References**

235 A. Cherp, V. Vinichenko, J. Tosun, J. A. Gordon, J. Jewell, National growth  
236 dynamics of wind and solar power compared to the growth required for  
237 global climate targets, *Nature Energy* 6 (2021) 742–754.

238 A. Lamorlette, A dynamic model for liquid fossil fuel production based on  
239 gross product/eroei coupling, *Energy* 260 (2022) 124693.

240 F. Ferroni, R. J. Hopkirk, Energy return on energy invested (eroei) for  
241 photovoltaic solar systems in regions of moderate insolation, *Energy Policy*  
242 94 (2016) 336–344.

243 M. Raugei, S. Sgouridis, D. Murphy, V. Fthenakis, R. Frischknecht,  
244 C. Breyer, U. Bardi, C. Barnhart, A. Buckley, M. Carbajales-Dale, et al.,  
245 Energy return on energy invested (eroei) for photovoltaic solar systems in  
246 regions of moderate insolation: A comprehensive response, *Energy Policy*  
247 102 (2017) 377–384.

248 Annual Energy Review, International Energy Agency, World Energy Invest-  
249 ment (2018).

250 D. Murphy, The implication of the declining energy return on investment  
251 for oil production, *Philos. Trans. R. Soc. Lond. A: Math. Phys. Eng. Sci.*  
252 (2014).

253 A. Hall, S. Balogh, D. Murphy, What is the minimum EROI that a sustain-  
254 able society must have?, *Energies* (2009) 25–47.

255 Statistical Review of World Energy, British Petroleum (2020).

- 256 World Nuclear Association, 2018. [https://world-nuclear.org/information-](https://world-nuclear.org/information-library/energy-and-the-environment/energy-return-on-investment.aspx)  
257 [library/energy-and-the-environment/energy-return-on-investment.aspx](https://world-nuclear.org/information-library/energy-and-the-environment/energy-return-on-investment.aspx).
- 258 EDF, 2018.
- 259 A. Barrau, Il faut une révolution politique, poétique et philosophique, 2022.