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Abstract:
A simple model is investigated according to which the deformation of the vacuum corresponding to the localization of a particle is described by particular solutions of an appropriately rescaled Wheeler - De Witt (WdW) equation. The role of the usual gravitational constant is in fact played by a different constant, defined by a suitable geometric representation of the Higgs mechanism. The particle is then described in terms of a micro-universe of size $\sim 10^{-13}$ cm, without contradiction with the point-like nature of the charges. Different definitions of the temporal variable give rise to different solutions which, suitably connected, lead to a temporal evolution of the particle that is wider than that normally contemplated. For example, it is possible to dynamically describe quantum jumps and obtain a decoherence mechanism compatible with the phenomenon of flavor mixing. Some cosmological consequences and the possibilities of experimental verification are briefly discussed.

Keywords: Wheeler de Witt equation; micro-cosmology; third quantization; Higgs mechanism;

1. Introduction

There is a general consensus that the Standard Model (SM) provides a sufficient reference for the accurate description of the dynamics of elementary particles and their interactions. However, the high number of free parameters of the model, the mystery of the multiplication of fundamental fermions (quarks and leptons) in three families and other factors easily convince that a lot of work must be done to gain an adequate understanding of the deep physical reasons justifying such a theoretical construction. In particular, the basic states of the model (quarks and leptons) are assumed by experience and there is not any their derivation from first principles, accepted in a shared way by the community. There is therefore room for the research of a mathematical modeling of elementary fermions (and hadrons) that can integrate the Lagrangian of the Standard Model and illuminate at least some aspects of its physics.

In this article we propose a description of these physical entities in terms of a Wheeler - De Witt (WdW) equation. Normally this formalism, which integrates General Relativity and Quantum Field Theory, is used in the context of quantum cosmology. We propose an adaptation to the problem of elementary particles. In a certain sense, our path is the opposite of the one that led to the use of quantum wave equations in the study of dark matter at the galactic level [1-4] or in the modeling of the cosmological evolution *tout court* [5,6].

Our line of reasoning starts from the observation that an elementary particle is always a charge-carrying entity: gravitational charge, weak charge and possibly electric charge and/or strong charge. The spacetime localization of a particle is in fact the localization of the charges it carries. From the high-energy scattering experiments we know that the charge centers of elementary fermions are, at least with a very good approximation, point-like.

We postulate a direct relationship between charges and geometry, assuming that the localization of a charge $+q$ at a point O of spacetime corresponds to the creation of a five-dimensional single-sheet hyperboloid (de Sitter space), tangent to spacetime in O. The energy cost (incurred by the field to which the charge is coupled) for the creation of this de Sitter space is given by the ratio of the square of the charge and the radius of the hyperboloid (de Sitter radius). On the other hand, the mass of the particle is determined, according to the SM, by the coupling constant to certain scalar fields: in the case of elementary fermions, the Higgs field. In this approach, therefore, the coupling with these fields has the geometric meaning of sizing the de Sitter radius of the particle, obtained starting from a universal radius of curvature connected to the scalar fields.

The de Sitter space associated with the particle is a quantum micro-universe, and it is for this reason that we choose to describe it by means of a WdW equation. However, in this equation the gravitational constant must be scaled with respect to the conventional one. In fact, it does not describe, in this context, the intensity of the gravitational force; instead, it fixes the negative pressure exerted by the scalar fields which opposes the collapse of the de Sitter space. It is the presence of this pressure that determines a finite value of the de Sitter radius, that is, a finite value of the mass of the particle.

The elementary particles must then be solutions of the "rescaled" WdW equation. As we will show, the Lorentzian solutions are the ordinary de Broglie harmonic phase factors which, after Lorentz transformation and superposition, provide the wave function of the particle, which satisfies the ordinary wave equation. These solutions correspond to a definite value of the de Sitter radius, that is to a definite mass of the particle, in accordance with the usual dispersion law.

Euclidean solutions, which have no equivalent in SM, describe the collapse of the de Sitter space of the particle (instantaneous disappearance of charges, decoupling from scalar fields) or its creation (instantaneous appearance of charges, coupling with scalar fields). Since the charges of various types are generally conserved, these solutions must always be combined in order to guarantee such conservation. Suitable combinations of Euclidean solutions can be used to dynamically describe quantum jumps, without the need to introduce any modification of the standard wave equations. A dynamic mechanism of decoherence on a microscopic scale is then provided (*with selection of the outgoing states*), directly correlated to the single elementary interactions between particles. This mechanism conforms to the current theory and to the projection postulate.

The independence of this mechanism from statistical average processes on "environmental" degrees of freedom makes it applicable even in highly symmetrical situations where such processes are ineffective. We will discuss this aspect by taking, as a particular example, that of the nucleation of the inhomogeneities of the density of matter at the end of the inflationary era. However, the argument is more general and the illustrated mechanism could provide a simple key to explain the emergence of classicality.

The ordering of the various Sections is different from that followed in this Introduction to illustrate the topics. In Section 2 the essential ideas underlying the third quantization and the WdW equation are briefly recalled, and the formal analogies between particles and "universes" are underlined. The approach is illustrated in Section 3. In particular, in Section 3.1 the elementary solutions are obtained; Section 3.2 illustrates the various interpretations of these solutions as a function of the choice of the time variable (a reflection of the well-known "time problem" involved in the WdW equation); Section 3.3 describes the combinations of solutions associ-

ated with quantum jumps. The size of the de Sitter space of the particle is discussed in Section 3.4, while Section 3.5 describes the connections with the Higgs mechanism.
Section 4 illustrates the connections with several open problems. Section 4.1 illustrates a possible link, currently unexplored, with the mixing of quarkic and leptonic flavors; the possibility of experimental verifications of the proposed model by means of measurements of decoherence rapidity is also underlined. Section 4.2 is dedicated to the example of post-inflationary nucleation and it is emphasized that only decoherence mechanisms such as the one described here are able to solve the problem of origination of the seeds of the structures. Finally, Section 4.3 illustrates the connections with the Planck scale and the Bekenstein relation, and the possible ramifications with the cosmological constant. A last section dedicated to conclusions closes the article.

2. Third quantization

The third quantization formalism is a quantum field theory applied to a system of many universes in a superspace. We speak of "third quantization" because what is quantized is the wave function of the universe, and this in turn depends on the material fields within that universe, which have already been subjected to the second quantization. The starting point is the Wheeler-De Witt equation. One of the variables on which the wave function, solution of the equation, depends is assumed as a time variable and on this basis the same procedure used in the second quantization is applied. This approach is applied to quantum cosmology problems related to the tunneling of universes and the nucleation and reabsorption of baby universes within a parent universe. We will not describe it here in general terms, referring the reader to the relevant bibliography [7,8,9]; instead we will try to summarize the main ideas by taking as an example a particular case close to our topic.
We therefore consider a Friedmann-Robertson-Lemaitre-Walker space with closed spatial sections, filled with a homogeneous and isotropic fluid whose pressure is p and whose density is η, these quantities being connected by the equation of state $p = w\eta$ (we use here units $c = 1$). In addition is assumed the presence of a massless scalar field ξ. The relevant Wheeler-De Witt equation can be written in the form [10]:

$$\left(a^2 \frac{\partial^2}{\partial a^2} + a \frac{\partial}{\partial a} + \frac{\omega_0^2 a^{2q} - a^4}{\hbar^2} - \frac{\partial^2}{\partial \xi^2} \right) \Psi(a, \xi) = 0 \qquad (1)$$

where a is the scale factor and ω_0 is an integration constant dependent on the energy density on a defined hyper-surface Σ. The parameter $q = 3(1 - w)/2$ defines the matter scheme: radiation for $w = 1/3$, dust for $w = 0$ and pure cosmological constant for $w = -1$. Assuming the latter case, we have $q = 3$ and $\omega_0^2 = \Lambda$. Therefore, the Eq. (1) becomes:

$$\left[\hbar^2 \frac{\partial^2}{\partial a^2} + \frac{\hbar^2}{a} \frac{\partial}{\partial a} - \frac{\hbar^2}{a^2} \frac{\partial^2}{\partial \xi^2} + \left(\Lambda a^4 - a^2 \right) \right] \Psi(a, \xi) = 0 \qquad (2)$$

At this point the wave function Ψ is promoted to operator [7], and the possibility of its decomposition in normal modes is admitted:

$$\hat{\Psi}(a,\xi) = \int dk[e^{ik\xi}A_k(a)\hat{c}_k^+ + e^{-ik\xi}A_k^*(a)\hat{c}_k] \qquad (3)$$

The amplitudes A_k then satisfy the equation of the damped harmonic oscillator:

$$\left(\hbar^2\frac{\partial^2}{\partial a^2} + \frac{\hbar^2}{a}\frac{\partial}{\partial a} + \omega_k^2(a)\right)A_k(a) = 0 \qquad (4)$$

where:

$$\omega_k(a) = \sqrt{\Lambda a^4 - a^2 + \frac{\hbar^2 k^2}{a^2}} \qquad (5)$$

The creation and annihilation operators of mode k of the universe are expressed by:

$$\hat{c}^+ = \sqrt{\frac{\omega_{0k}}{2\hbar}}\left(\hat{\Psi} - \frac{i}{\omega_{0k}}\hat{p}_\Psi\right) \; ; \qquad \hat{c} = \sqrt{\frac{\omega_{0k}}{2\hbar}}\left(\hat{\Psi} + \frac{i}{\omega_{0k}}\hat{p}_\Psi\right) \qquad (6)$$

where ω_{0k} is the value derived from (5) on Σ. It should be noted that the values of $\omega_k(a)$ given by (5) can be as well as real than complex. The transition from the real to the complex domain represents the transition from the Lorentzian to the Euclidean region of the universe. For example, in the case $k = 0$ one has a de Sitter space for $a > \Lambda^{-1/2}$, while for $a < \Lambda^{-1/2}$ one has a de Sitter instanton collapsing on Euclidean time.

An essential point is that having defined the creation and annihilation operators, it is possible to connect a state of the universe to the void in the same way that, in second quantization, a state of the field is connected to the vacuum [7]:

$$\left|\hat{\Psi}(a)\right\rangle = \prod_k e^{\nu_k c_k^+ - \nu_k^* c_k}\left|0\right\rangle_k \qquad (7)$$

Here we use the term "void", typical of the third quantization formalism, to distinguish this condition of absence of universes from the more familiar "vacuum" of quantized fields spread over a space whose persistence is guaranteed [7].

An expression such as (7) can describe, for example, the creation of pairs of universes or their annihilation, the merging of universes or the stemming of universes from parent universes and other processes quite similar to those known in elementary particle physics [7]. However, while these concepts are usually applied to the study of quantum cosmology and therefore to situations in which, roughly speaking, the universe is "small" and behaves like a particle, we ask ourselves instead whether, on the contrary, particles can not be described as micro-universes. In fact, it seems to us that this language is adequate for a non-singular description of the particles and their interactions.

3. Particles as micro-universes

3.1 Micro-cosmology

We now come to articulate the main ideas of our proposal of application of the third quantization to the study of elementary particles. A premise seems necessary to avoid possible misunderstandings. When we talk about cosmology in a non-quantum context, we usually take General Relativity as the reference theory. It is known that General Relativity is a theory of gravitation, even if in fact it should be said that it is a theory of gravitation and *inertia* (the two aspects are locally indistinguishable by virtue of the Principle of Equivalence, which constitutes the basis of theory). However, Einstein's equations are not just the gravity-inertia equations; they are the canonical equations that describe any conceivable coupling between matter-energy and spatial curvature that satisfies the general covariance criteria. They can therefore be used in a more general context, taking care to replace the gravitational constant appearing in them with a coupling constant pertinent to the case under investigation [11,12]. It is in this more general sense that we use them here to describe the particle micro-universes which, of course, are not governed by gravitation; what we instead propose in the following Section 3.5 is, accordingly to the Standard Model, a connection between the inertia of the particle (its proper mass) and a curvature associated with suitable scalar fields (Higgs).
Since we are dealing here with particles and therefore we are within the quantum realm, we actually consider the quantum version of Einstein's cosmological equations, namely the Wheeler-De Witt equation. This equation can be written in many forms. We start from the version proposed by Kiefer and collaborators [13], relating to a Friedmann universe with scale factor a and cosmological constant Λ, containing matter in the form of a scalar field ξ with potential $V(\xi)$:

$$\left[\frac{\hbar^2\sigma^2}{12}\frac{\partial^2}{\partial\varepsilon^2}-\frac{\hbar^2}{2}\frac{\partial^2}{\partial\xi^2}+e^{6\varepsilon}\left(V(\xi)+\frac{\Lambda}{\sigma^2}\right)-3e^{4\varepsilon}\frac{\kappa}{\sigma^2}\right]\Psi(\varepsilon,\xi)\;=\;0 \qquad (8)$$

In Eq. (8) is $\varepsilon = \ln(a/a_0)$, where a_0 is the reference scale factor and κ is the curvature index of the spatial section of the universe: κ = -1 for a hyperbolic section, 0 for a flat section and 1 for a closed space. The symbol Ψ denotes the wave function of the universe, which expresses its amplitude of probability as a function of internal variables ε, ξ. σ is the coupling constant, which in the original theory is related to the Newtonian constant of gravitation G by the relation $\sigma^2 = 8\pi G$. As we said, our case is different and for it we have [14]:

$$\sigma^2\;=\;\frac{12khc}{\left(Mc^2\right)^2} \qquad (9)$$

where M is the rest mass of the particle to which the micro-universe is associated and is k = 1 for hadrons, k = α (fine structure constant) for charged leptons. For reasons that will be detailed later, we are interested in flat solutions (κ = 0) for which the term in round brackets in Eq. (8) is null. With these positions the Eq. (8) becomes:

$$\left(\frac{\sigma^2}{6}\frac{\partial^2}{\partial\varepsilon^2}-\frac{\partial^2}{\partial\xi^2}\right)\Psi(\varepsilon,\xi)=0 \qquad (10)$$

In many quantum gravity applications it is customary to choose the units of measurement so that $\sigma^2 = 6$. We prefer first to multiply the Eq. (10) by $6/\sigma^2$, then incorporate a factor $\sigma/6^{1/2}$ in ξ. With this choice ξ can be interpreted as a radius of curvature, consistent with what will be proposed in Section 3.5. In any case, a D'Alembert equation is obtained:

$$\left(\frac{\partial^2}{\partial\varepsilon^2} - \frac{\partial^2}{\partial\xi^2}\right)\Psi(\varepsilon,\xi) = 0 \qquad (11)$$

The Wheeler-De Witt equation does not contain time. This fact constitutes the origin of the well-known "problem of time" [13,15], which we do not delve into here. But what if the quantity ε, which appears in Eq. (11) as a delocalized quantum variable, is taken as a time label? In this case it becomes an external parameter which, being dimensionless, can be written in the form $\varepsilon = \tau/\theta$, where the variable τ and the constant θ are time intervals. The scale factor then takes the form $a = a_0\exp(\tau/\theta)$. This is in fact the expression of the scale distance in a de Sitter space with cosmic time τ and a cosmological constant $\Lambda = 3/\theta^2$, solution of the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \qquad (12)$$

in which the point denotes the derivation with respect to τ. In this sense, our micro-universes are de Sitter spaces.

Let us now consider evanescent solutions of Eq.(11). We look for separable solutions of the form $\Psi(\varepsilon,\xi) = \Phi(\varepsilon)\Gamma(\xi)$ that are evanescent. With simple calculations we obtain:

$$\Phi(\varepsilon) = \exp(-\varepsilon) = \exp\left(-\frac{\tau}{\theta}\right); \qquad \Gamma(\xi) = \exp(-\xi) \qquad (13)$$

The Eqs.(13) give a physical meaning to the parameter θ as an indicator of the rapidity of evanescence: comparing the evanescences associated with different particles, there will be more or less rapid ones depending on the corresponding value of θ.

Moreover, Eq.(11) also admits separable harmonic solutions which can be identified with the usual de Broglie phase factor, associated with a persistent micro-universe, by placing $\varepsilon = t/\theta$:

$$\Phi(\varepsilon) = \exp(\pm ik\varepsilon) = \exp\left(\pm\frac{ikt}{\theta}\right) = \exp\left(\pm\frac{iMc^2t}{\hbar}\right); \qquad \Gamma(\xi) = \exp(\pm ik\xi) \qquad (14)$$

It is therefore natural to identify the real time variable *t* with the *external* time of the laboratory (proper time of the particle), and instead identify the imaginary time τ as an internal time of the micro-universe. The *k* factor is the same as that which appears in (9).

As we will see in the following Sections, the evolution parameter can be alternatively identified with the phase angle φ, instead of with an external or internal time. It is possible to do this by replacing, in Eqs.(13), τ/θ with $|\varphi|$; alternatively, the replacement $\pm kt/\theta \rightarrow \varphi$ can be made in Eqs.(14), and we will return to this point later.

3.2 Micro-universes and spacetime

It is now necessary to illustrate the connection of the de Sitter space of an elementary particle with spacetime. For this purpose, let's imagine the usual spacetime as immersed in a five-dimensional environment-space in which also the particle de Sitter micro-universe, of radius $c\theta \leq c\theta_0$, is present. Here we denote with c the limit speed and with θ_0 a positive constant, having the dimensions of a time, the meaning of which we will return to later. The de Sitter micro-universe is tangent to spacetime at an event-point O corresponding to the spacetime position of the particle; for example the position of the center of charge of an electron [16,17]. Within the usual, four-dimensional spacetime it is therefore possible, in the rest frame of reference of the particle, to represent O as ($\boldsymbol{x}$, t), where t is the proper time of the particle. In quantum terms, $\boldsymbol{x}$ is an eigenstate of the position operator of the particle at instant t. Generally, this position is not defined; it is instead delocalized according to the spatial dependence of the wave function of the particle [18].

The particle micro-universe is a quantum entity described by a wave function Ψ, solution of Eq.(11) [19,20]. The square modulus of this wave function represents the probability of the state of the micro-universe. The de Sitter quantum micro-universe is defined by a parameter which is its radius $c\theta \leq c\theta_0$, where θ is the so called de Sitter time [21], function of the type of particle; the maximum value θ_0 of θ defines a spatial scale of the micro-universes associated with elementary particles. We define the amplitude of probability of the *existence* of the micro-universe as $\Psi = (\rho/\theta_0)\exp[\pm i(\varphi - \varphi_0)]$, where φ is the particle phase angle and ρ, with $0 \leq \rho \leq \theta_0$, has the dimensions of a time.

The de Broglie oscillation corresponds to the propagation in t of the particle when the state of the micro-universe associated with it is persistent, that is $\rho = \theta_0$ (the probability of existence is constant and unitary). In this circumstance $\Psi = \exp[\pm i(\varphi(t) - \varphi_0)]$. For a free particle of renormalized mass M is $\varphi(t) = Mc^2t/\hbar$ [22]; under a Lorentz transformation $(ict,0,0,0) \rightarrow (x_0, x_1, x_2, x_3)$, the Ψ function becomes a plane wave in the ordinary Minkowski spacetime. If $\boldsymbol{n}$ is the versor along the axis t and $y = ict$, then the vector decomposition relation $\boldsymbol{n}\,\partial_y\Psi = \gamma^\mu\partial_\mu\Psi$, involving the Dirac operators γ_μ ($\mu = 0, 1, 2, 3$), holds. From the scalar product $(\boldsymbol{n}\,\partial_y)\bullet(\boldsymbol{n}\,\partial_y\Psi) = \partial_y^2\Psi = -(Mc/\hbar)^2\Psi = (\gamma^\nu\partial_\nu)\bullet(\gamma^\mu\partial_\mu\Psi) = \partial^\mu\partial_\mu\Psi$ the ordinary Klein Gordon equation thus follows. This equation is linear. Therefore the generic component of the wave function of the particle will be given by a linear superposition of these plane waves, each of which represents –in accordance with the here discussed hypothesis- the propagation of a persistent particle micro-universe. The de Broglie oscillation of a specific plane wave thus corresponds to a persistent (Lorentzian) solution of the Wheeler-De Witt equation in which the phase angle φ depends on t. See ref. [23] for a different demonstration.

If the micro-universe is not persistent, but evanescent or nucleating from a vacuum, its probability of existence is not identically 1. If the variation of ρ occurs for a constant value of φ and of t, then it corresponds to a single instant in the domain of the laboratory time t; in this case we can pose $\rho/\theta_0 = \exp(-|\tau|/\theta)$ and assume τ as a time variable. In this expression there is no loss of generality: the value of the "internal" time variable τ is only a substitute for ρ/θ_0, the two quantities being biunivocally connected by a monotone function. As we have seen, these wave functions are imaginary time (Euclidean) solutions of the Wheeler-De Witt equation. In the next Section 3.3 we will use this possibility to dynamically describe quantum jumps.

Finally, we can have both Lorentzian and Euclidean solutions in which the evolution parameter is φ, considered independent of time parameters such as t or τ. We hypothesize that these solutions can describe, in terms that we will detail later, the coupling of a particle with gauge bosons.

3.3 Time structure

As is known, the temporal evolution of the de Broglie phase of a particle (and more generally of its wave function) undergoes a discontinuity in correspondence with an instantaneous elementary interaction event. This event, which sets the new initial condition of this evolution, is called a "quantum jump". The notion of quantum jump was first introduced by Bohr in his atomic model of 1913 [24], precisely in relation to the radiative processes involving an atomic electron. The quantum jump is therefore the definition of a value φ_0 starting from which the phase angle φ of the considered particle is defined. This is the value φ_0 which appears in the expression $\Psi = \exp[\pm i(\varphi - \varphi_0)]$, valid under the persistence condition $(\rho/\theta_0)^2 = 1$.
The propagation of the persistent micro-universe continues up to the phase angle φ_f at which the particle is involved in a new quantum jump (lower horizontal solid lines in Figures 1,2); this jump determines the selection of a new angle φ_1 which replaces φ_0 in the expression of Ψ. The same time instant t_1 is associated both with φ_1 and φ_f, while the phase undergoes a jump determined by the difference $\varphi_1 - \varphi_f$ (lower horizontal broken lines in Figures 1,2). The succession of angles φ_0, φ_1, ... is specific to the history of the single particle, and this fact leads to the phenomenon of decoherence. Indeed, while the term ωt of the de Broglie oscillation remains unchanged to it is added, at the instant t_1 of the jump, the phase variation $\Delta\varphi = \varphi_1 - \varphi_f$. Decoherence is induced by the fact that the succession of phase changes $\Delta\varphi$ to which the de Broglie oscillation undergoes, in concomitance with the successive quantum jumps to which the particle is subject, is specific to the single particle.
In a quantum jump, ρ undergoes a variation from θ_0 to 0 for φ fixed to φ_f, (ascending vertical solid lines in Figure 1) followed by a variation from 0 to θ_0 for φ fixed to φ_1 (descending vertical solid lines in Figure 1). The first variation corresponds to the passage from a state having a probability of existence $(\rho/\theta_0)^2 = 1$ to another state having probability of existence $(\rho/\theta_0)^2 = 0$; we can consider it as the destruction of the state entering the quantum jump. The second variation corresponds to the transition from a state of probability $(\rho/\theta_0)^2 = 0$ to a state of probability $(\rho/\theta_0)^2 = 1$; we can consider it as the creation of a new initial state of the de Broglie phase factor. These two variations appear, in the domain of ordinary laboratory time, as limited to a single instant t_1 with no duration. The two variations correspond to two wave functions $\rho/\theta_0 = \exp(-|\tau|/\theta)$ at $\varphi = \varphi_f$, $\varphi = \varphi_1$ respectively ("exponential tails"). The first tail (τ going from 0 to $+\infty$) leads the old particle micro-universe into a condition of "void", *ie* the absence of micro-universes. The second tail (τ going from $-\infty$ to 0) produces the new particle micro-universe starting from this absence. The propagation of these waves does not occur in ordinary time t, but in unobservable "time" τ [14].

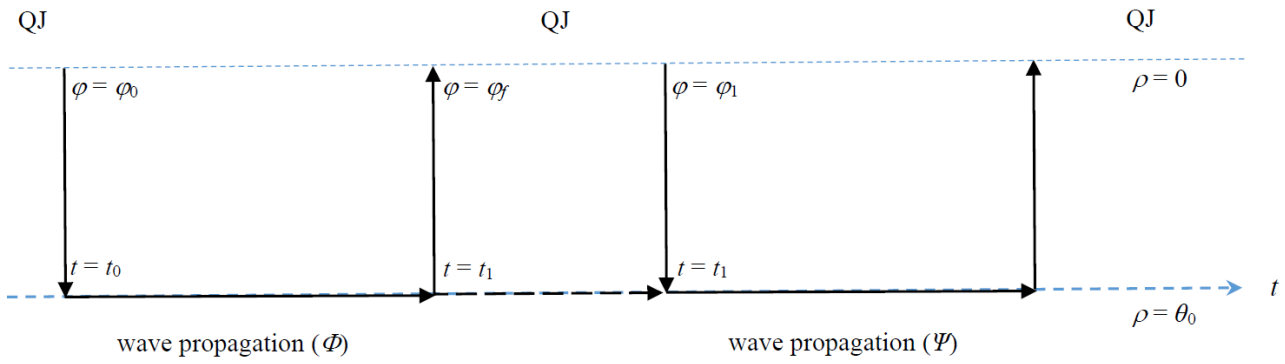

Figure 1; Retarded propagation and the $\Phi \rightarrow \Psi$ jump. Vertical solid lines represent the exponential tails in τ (Euclidean time). Lower horizontal solid lines represent the propagation in the ordinary (Lorentzian) time t. Lower horizontal broken lines represent the propagation of the intermediate micro-universe in Lorentzian or Euclidean "time" φ. QJ = quantum jump.

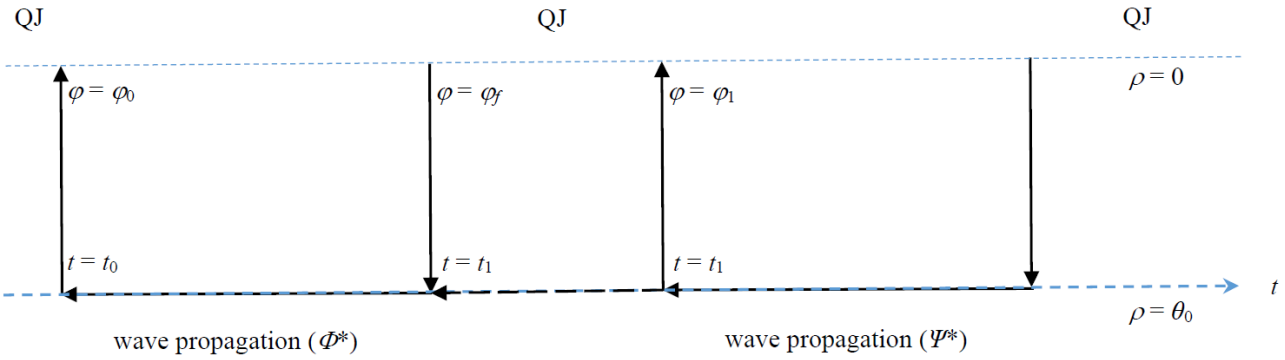

Figure 2; Advanced propagation and the $\Psi^* \rightarrow \Phi^*$ jump. Vertical solid lines represent the exponential tails in τ (Euclidean time). Lower horizontal solid lines represent the propagation in the ordinary (Lorentzian) time t. Lower horizontal broken lines represent the propagation of the intermediate micro-universe in Lorentzian or Euclidean "time" φ. The creation of Ψ matches the annihilation of Ψ^*. QJ = quantum jump.

Summarizing, at $t = t_1$ the particle wave function becomes, from Lorentzian, Euclidean and therefore associated with an evanescent particle micro-universe; from the perspective of the time domain t, it dissolves instantaneously. A second wave function, Euclidean, emerges from the void as a fluctuation and becomes Lorentzian in the form of the new outgoing wave function; this emergence is instantaneous from the perspective of the time domain t. The two Euclidean wave functions are limited, from the perspective of the domain t, to the instant t_1 only. However they are separated by the values of the phase angle φ intermediate between the final angle φ_f and the new initial angle φ_1. These two extreme values of φ are connected by a micro-universe that evolves in "time" φ, crossing the intermediate values between φ_f and φ_1 (lower horizontal broken lines in Figures 1,2). Also this passage is, from the usual perspective of time t, enclosed in a single instant t_1 which is that of the jump. This "intermediate" micro-universe conveys the charges and other quantities conserved from the old micro-universe entering the jump (which became evanescent after having

transferred them to it) to the Euclidean fluctuation emerging from the void, which thus becomes the new micro-universe emerging from the jump. In the case in which the particle undergoing the quantum jump is an elementary fermion of the Standard Model (lepton or quark), the corresponding intermediate micro-universe evolving in φ represents the absorption or emission of the gauge boson that mediates the interaction inducing the jump (photon, $W^{\pm}$, Z^0, H^0, gluons or graviton). The conserved quantum numbers are transferred to the outgoing state by this third micro-universe, which acts as a vehicle for them.
Finally, we observe that micro-universes are quantum entities and can therefore be entangled. If, in the entering state, the particle is in entanglement relationship with other particles, the outgoing state selected by the quantum jump includes the states of the other particles, which have not undergone effective interaction, correlated by the entanglement relationship with the outgoing state of the particle that interacted.

3.4 Size of the micro-universe

We now focus on the relationship between the de Sitter time of a particle micro-universe and the mass M of the particle. We postulate that the coupling energy between the charge q_0 and the de Sitter radius r is $-q_0^2/r$. The transition from $r = \infty$ to a finite value of r corresponds to the creation of a particle carrying that charge, and the corresponding energy variation is the rest energy of the particle:

$$-Mc^2 = \left(-\frac{q_0^2}{r}\right) - \left(-\frac{q_0^2}{\infty}\right) = -\frac{q_0^2}{r} \qquad (15)$$

where q_0 is the renormalized charge, M is the renormalized particle mass and $r = c\theta$ is the de Sitter radius. The Eq. (15) is the energy released by the field coupled to the charge to create the particle. The inverse passage from a finite value of r to $r = \infty$ corresponds to the annihilation of a particle carrying that charge, and the corresponding energy variation (opposite to (15)) is the energy released to the field by the annihilation of the particle.
It is evident that compressing the radius reduces the coupling energy $-q_0^2/r$, and this suggests the existence of a force that opposes this compression. The finite value assumed by r must derive from a balance between the action of the field and that, opposite, of this force. The nature of this force will be discussed in the next Section.
Substituting to q_0 the values e (elementary electric charge) in the case of charged leptons and $(\hbar c)^{1/2}$ in the case of hadrons (constant of strong coupling of hadrons in the hadrodynamical approximation [25]), the expression (15) gives respectively $Mc^2 = e^2/c\theta$ for charged leptons and $Mc^2 = \hbar c/c\theta$ for hadrons. Therefore, $r = c\theta$ coincides respectively with the classical radius of the charged lepton and with the Compton length of the hadron. This result is exactly the relation $\theta = \hbar k/Mc^2$ derived from the identification of Eq.(14) with the de Broglie phase factor. The maximum value θ_0 of θ thus remains experimentally identified (restricting our attention to only charged leptons and hadrons) as r_{cl}/c, where r_{cl} is the classical radius of the electron.
It should be emphasized that the finite radius r is not the radius of an extended charge distribution; the physical meaning of this radius is different. The vertical projection of de Sitter space, tangent in O to spacetime, determines a spatial region enclosed within a horizon (de Sitter horizon). It is within this region that the mass M of the particle can be thought to be distributed. It is within this region that the scalar fields act, stabilizing the

horizon and thus allowing the mass to acquire a finite value. The spatial section enclosed in this horizon is Euclidean; this is the reason why we restricted ourselves, in the consideration of Eq.(8), to flat solutions only.

3.5 If not gravitation, what else? A hypothesis on the Higgs mechanism

The curvature of the particle micro-universe can be expressed as the product of a constant, dependent on the particle (hadronic state or single quark or lepton), for a universal curvature. The constant is the coupling factor of that particle to this curvature. The universal curvature can be connected to the expectation value in vacuum of a suitable scalar field: the field of sigma models in the hadronic case [26,27], the Higgs field in the case of elementary fermions of the Standard Model. Here we limit ourselves to considering the Higgs field, referring the reader to the works of other authors for the discussion of the hadronic case [28,29]. However, the line of reasoning is the same in both cases.
The Higgs field is introduced in the usual way, as an iso-doublet Ω of scalar fields defined on the space SU(2) of the weak isospin. The fields will depend on the spacetime position. We denote by ε (having the dimensions of an energy) the expectation value of the vacuum (VEV), defined in the usual way. Let's consider the energy f defined by the relation:

$$\left|\Omega\right|^2 = \Omega^+\Omega = \varepsilon f \tag{16}$$

The symmetry breaking, that is the passage from $f = 0$ (the "false vacuum") to $f = \varepsilon$ (the "true vacuum"), corresponds in the SU(2) space to the choice of a specific direction of the spinor Ω of norm ε as the new vacuum state. $|\Omega| = \varepsilon$ must be a stable point of the potential of the free Higgs field. Therefore, this potential must be even in $\varepsilon(f - \varepsilon)$ and then be expressed by a sum of even powers of $(|\Omega|^2 - \varepsilon^2)$. The only possibility compatible with renormalization is that of a completely harmonic dynamics of the variable $\varepsilon(f - \varepsilon)$, in accordance with the usual choice of a potential proportional to $\varepsilon^2(f - \varepsilon)^2$, that is $(|\Omega|^2 - \varepsilon^2)^2$.
It is possible to associate the Higgs field to a de Sitter space whose curvature is the universal curvature $-R_0$, rewriting its internal potential in the form [30]:

$$V(\Omega) = \frac{\sqrt{Det(g_{\mu\nu})}}{4\pi}\left[\frac{R_0}{6}\Omega^+\Omega - \frac{\chi_0 R_0}{36}\left(\Omega^+\Omega\right)^2\right] \tag{17}$$

where the determinant of the metric matrix of the de Sitter space appears and we pose:

$$R_0 = 6\mu^2 \tag{18}$$

while χ_0 is the internal "gravitational constant" of the Higgs micro-universe.
The relationship between the value of the vacuum expectation ε and the rest mass M_W of the W boson is that predicted by the conventional mechanism:

$$\varepsilon = \frac{\sqrt{2}\,c^2}{g} M_W \tag{19}$$

where g represents the electroweak coupling constant. Expanding around ε in the usual way we then have:

$$\varepsilon = c^2\left(\frac{3}{\chi_0}\right)^{1/2} \quad \Rightarrow \quad \chi_0 = \frac{3}{2}\frac{g^2}{M_W^2} \tag{20}$$

$$M_H = \frac{\hbar}{c}\left(-\frac{R_0}{3}\right)^{1/2} \quad \Rightarrow \quad R_0 = -\frac{3M_H^2c^2}{\hbar^2} \tag{21}$$

$$V(\Omega) = \mu^2\,\Omega^+\Omega + \lambda\left(\Omega^+\Omega\right)^2 \tag{22}$$

where $\lambda = -\chi_0 R_0/36$, M_H is the Higgs boson mass and:

$$\mu^2 = -\frac{1}{2}\frac{M_H^2c^2}{\hbar^2} \tag{23}$$

The existence of a de Sitter space associated with the Higgs field in accordance with (17)-(23) represents our specific hypothesis relating to the Higgs field. On the other hand, it is known from the Standard Model that the mass of an elementary fermion (quark or lepton) is given by the product of ε for a specific coupling constant of that fermion. As can be seen from (20), this is equivalent to rescaling the gravitational constant, which passes from the value χ_0 to a new value χ specific to the fermion.

If V is the volume of the projection, on the Minkowski spacetime, of the new de Sitter space associated with the fermion and p is the pressure within this space, the rest mass M must satisfy the relation $pdV = c^2dM$. Based on what has been previously argued, an increase of V must result in a decrease of M; this is congruent with a negative p. Therefore $dM/dV = -\eta$, where η is the matter density inside the horizon; that is $p + \eta c^2 = 0$.

The micro-space of scalar curvature R and cosmological constant Λ_0 satisfies the Einstein equations:

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} + \Lambda_0 g_{\mu\nu} = \chi T_{\mu\nu} \tag{24}$$

where μ, $\nu = 0,1,2,3$ and the symbols have the usual meaning. Here the metric tensor $g_{\mu\nu}$ is that experienced by the centers of charge within the particle, while Einstein's "gravitational" constant χ actually measures the coupling with the VEV. By schematizing the internal vacuum as a perfect fluid, the stress tensor takes the form:

$$T_{\mu\nu} = \left(\eta + \frac{p}{c^2}\right)u_\mu u_\nu + pg_{\mu\nu} \tag{25}$$

where the four-velocity field of matter u_μ is introduced. Since $p + \eta c^2 = 0$, the (24) becomes:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + (\Lambda_0 + \chi\eta c^2) g_{\mu\nu} = 0 \qquad (26)$$

which is the equation of a de Sitter space with an "effective" cosmological constant $\Lambda = \Lambda_0 + \chi\eta c^2$. Neglecting the native (cosmological) constant Λ_0, we have $\Lambda = \chi\eta c^2 = R/4$. The χ rescaling therefore coincides with a rescaling of R_0 in R.

In other words, Eq. (20) provides an expression for the expectation value ε of the Higgs field in the vacuum where the "gravitational" constant χ_0 of the micro-universe corresponding to this vacuum appears. The Standard Model defines the mass M of a particle as the product of ε by a suitable dimensionless coupling constant dependent on the particle. In terms of the present model, this implies the rescaling $\varepsilon \to f\varepsilon$ where:

$$f\varepsilon = c^2 \left(\frac{3}{\chi} \right)^{1/2} \qquad (27)$$

This rescaling leads to a new micro-universe (that of the particle) with a "gravitational" constant χ. From Eqs. (20), (27) we have:

$$f^2 = \frac{3}{\chi} \frac{\chi_0}{3} = \frac{\chi_0}{\chi} \qquad (28)$$

From the relation $\Lambda = \chi\eta c^2$ between the cosmological constant Λ of the new micro-universe, its mass density η and the “gravitational” constant χ we obtain:

$$\frac{1}{\chi} \propto \frac{\eta c^2}{\Lambda} \propto M^{-2} \frac{M}{(M^{-1})^3} = M^2 \qquad (29)$$

since $\Lambda = 3/(c\theta)^2$ and $c\theta$ is inversely proportional to M. This result conforms to Eq. (9). We therefore have:

$$f^2 \propto \frac{1}{\chi} \propto M^2 \qquad (30)$$

That is $M \propto f$, according to the conventional Higgs mechanism and the Standard Model.

Finally, we note that, setting $p = V(\xi)$, the relation $p + \eta c^2 = 0$ for the rescaled particle micro-universe precisely represents the cancellation of the expression within round brackets in Eq. (8). This result therefore justifies the restriction we applied to the solutions of that equation.

4. Open questions and experimental control

4.1 Decoherence and flavor mixing

In a quantum jump induced by a fundamental interaction, the intermediate micro-universe evolving in the phase angle φ can be described by an evanescent amplitude $\Phi = \exp(- \mid \varphi \mid)$ or by a harmonic amplitude $\Phi = \exp(\pm i\varphi)$. In the first case, the phase plays the role of a Euclidean time; in the second of a Lorentzian time. In both cases, the existence of such a micro-universe is reduced, in the temporal domain t in which we operate, to a single instant. The first possibility prefers smaller phase jumps, while the second makes all phase variations $\Delta\varphi$ equally probable. The latter possibility is the one normally considered since the early days of quantum mechanics (see for example Heisenberg [31]). A direct experimental control would consist in measuring the phase of the wave function before and after the quantum jump, and in determining the difference $\Delta\varphi$ of the two phases. By constructing a frequency histogram of the various values of this difference, one could go back to the correct determination of the probability $P(\Delta\varphi)$. However, such an approach is impossible, because notoriously the global phase of a wave function is not measurable. Even more: the wave function is defined up to a global phase, so such a proposal has no logical basis.

It is therefore necessary to approach the problem indirectly. Let us imagine to send a beam of bosonic particles, perfectly coherent in phase, on a medium where they undergo quantum jumps through interactions with microscopic components of the medium itself. It is evident that in the evanescent case decoherence occurs more slowly than in the "Lorentzian" case, because smaller phase jumps are preferred. Therefore it will take more jumps to obtain the same degree of decoherence obtainable in the alternative "Lorentzian" case. From a theoretical point of view, the degree of decoherence can be estimated, if Φ is known, by averaging on the histories of single particles. Or, for example, by studying the behaviour of the off-diagonal terms of the density matrix after repeated quantum jumps. The theoretical prediction of the evolution of decoherence as a function of the number of interactions can then be compared with the experimental result. We do not develop here the relative considerations, which would require a separate paper, but what is interesting, even beyond the case in point, is that such a methodology could provide experimental indications on Φ and therefore, from our point of view, on the nature of intermediate micro-universe. The relevant point here is that if our argument is correct, the hypothesized internal dynamics of a quantum jump is - at least partially - experimentally investigable.

An interesting detail is that in the case of an evanescent intermediate micro-universe, the dependence of the wave function [Eq. (13)] from the scalar field is of decreasing exponential type. If we apply such a model to the electron-photon interaction ($e^- \rightarrow e^- + \gamma$), the values of the scalar field at the junction with the micro-universes of the two electrons (incoming and outgoing) must be equal. These micro-universes must in fact have the same curvature because they correspond to particles of identical mass and therefore, if the considerations made in Section 3.5 are correct, the values of the scalar field that induces this curvature must be identical.

However, this is no longer true in processes in which the gauge boson mediates an interaction between elementary fermions of different mass, which occurs only with the W bosons. In this circumstance the values ξ_1, ξ_2 of the scalar field at the junction with the two fermionic micro-universes will be different. It is then possible to hypothesize an exchange amplitude of the type $A(\xi_1,\xi_2)\exp(- |\xi_1-\xi_2|)$ with ξ_1,ξ_2 depending on the masses of the two fermions. This amplitude is unitary if A is seen as a propagator associated with the boson; from the perspective of the time domain t, this transition amplitude appears as a multiplicative factor of the coupling constant. This could perhaps be a way to describe the mixing of quarkic or leptonic flavors (CKM and PMNS matrices respectively) in weak interactions mediated by charged current.

4.2 Cosmological implications

The authors of an interesting paper [32] ask themselves what may be, at a cosmological level, the cause of the breaking of the homogeneity and isotropy of the distribution of matter in the phases of cosmic evolution immediately following the end of the inflationary era. In fact, it is generally believed that the "quantum fluctuations" originating during inflation are the seeds from which density inhomogeneities develop. The subsequent amplification of these inhomogeneities caused by gravitation leads to the emergence of structures, such as clusters and galaxies and, within the latter, stars and planets. The important point is that while amplification processes are essentially classical, in the sense that quantum aspects do not seem to play a significant role in them, the origination of seeds is due to the conversion of quantum fluctuations and therefore falls within the realm of quantum processes. The quantum fields that govern the inflationary era notoriously represent possibilities: the possibility of events. What leads to the formation of seeds is the actualization of these possibilities in effective inhomogeneities of density over spacetime.
The authors examine the question by considering the prescriptions on the collapse of the wave function derived from all the main classes of quantum formalism interpretations and arrive at the conclusion that none of them is able to generate, from the initial perfect symmetry, the asymmetry necessary for the structure formation. They conclude that a symmetry breaking such as that required can only be obtained by admitting an objective collapse of the wave function at a microphysical level.
Our approach, however, sees precisely the quantum jump as a collapse of an objective nature, originating entirely from single, fundamental interactions between elementary particles. This form of collapse can therefore be considered a natural candidate as a solution to the problem. The relevant point here is that the representation of elementary particles as a connection of different quantum micro-universes that evolve in different forms of time, only one of which corresponds to the usual notion of time, allows segments of non-unitary time evolution. The fundamental interactions therefore take on, in addition to their usual unitary aspect, also a non-unitary aspect that is present since the beginning in the representation adopted.

4.3 Relationship with the cosmological constant and the Planck scale

The existence of a cosmological constant $\Lambda = 3/t_0^2$, independent of cosmic time, is known. This suggests a scale transformation of the gravitational constant G of the form:

$$Gt_0 = G'\theta_0 \tag{31}$$

The maximum de Sitter time of a particle micro-space, θ_0, can be thought to derive from Planck time:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{32}$$

by replacing G with G'. This leads to the relation:

$$\theta_0 = t_P \sqrt{\frac{t_0}{\theta_0}} \tag{33}$$

By substituting the experimental values of t_P and t_0 (the latter derived from the observational value of Λ) we obtain that $c\theta_0$ is in the order of the classical radius of the electron. This result is consistent with our previous choice to identify $c\theta_0$ with this radius. Eq.(33) can be written under two alternative forms, the first of which is:

$$G\left(\frac{\hbar}{c^2\theta_0}\right)^2 \cdot \frac{1}{c\theta_0} = \frac{\hbar}{t_0} \tag{34}$$

The meaning of Eq.(34) is transparent. In order to localize, in spacetime, the mass distribution of a particle of de Sitter radius equating the maximum value $c\theta_0$ it is necessary, by virtue of the uncertainty principle, to concentrate an energy equal to $\hbar/\theta_0$ in that spacetime region. The left hand term of Eq.(34) represents the self-gravitation energy of this energetic concentration. The contribute of self-gravitation alone to the temporal localization of the micro-space, in terms of the uncertainty principle, is expressed by the right hand term of Eq.(34): t_0 is the uncertainty about the localization. Energy concentrations of value $\hbar/\theta$ on spatial scales $c\theta$, with $\theta \leq \theta_0$, increase the left hand term and consequently correspond to intervals of temporal uncertainty less than t_0. Since it is not possible to concentrate energy avoiding its gravitational self-coupling, it is not possible to localize the particle mass distribution in spacetime without having an uncertainty time interval $\leq t_0$, determined by this self-coupling. It is for this reason that a cosmological time horizon of extension t_0 appears. This reasoning brings the value of the cosmological constant Λ back to the localization of elementary particles, as a sort of ineluctable infrared limit, freeing it from the energy density of the vacuum. The difficulty originating from the observed disagreement between Λ and the value of this density is therefore avoided.
The second alternative form of Eq.(3) can be obtained by dividing the area of the de Sitter horizon of the micro-space of radius $c\theta_0$ into "Planck cells" of extension ct_P. The number of such cells is:

$$N_{cells} \approx \left(\frac{\theta_0}{t_P}\right)^2 = \frac{t_0}{\theta_0} \approx N_D \tag{35}$$

where N_D is the Dirac number ($\sim 10^{41}$) obtained by substituting the numerical values of the constants. If we consider, on the five-dimensional de Sitter space of the particle, two distinct points of tangency on the ordinary spacetime, no interaction can distinguish these points if they fall within the same cell. In fact, a virtual quantum exchanged between points of the same cell should have a mass greater than that of Planck, and would in fact be a non-exchangeable black hole. Eq.(35) is a form of the well-known Bekenstein relation $(t_0/t_P)^2 \sim 10^{123}$ and this result seems to indicate a connection with the holographic theme.

5. Conclusions

In this paper we have proposed a description of elementary particles as particular solutions of the Wheeler - De Witt equation. These solutions are associated with the spacetime localization of the charge center of the particle. Geometrically, they represent micro-spaces with constant positive curvature, tangent to spacetime at a point corresponding to the position of the charge center. The Lorentzian solutions describe the micro-universes related to the de Broglie phase factors, from the quantum superposition of which the particle wave function

understood in the usual sense is obtained. These solutions can be connected to the void by Euclidean micro-universes, and in this way it is possible to describe non-unitary quantum jumps. These jumps are induced by the usual fundamental interactions, and the coupling with the relevant gauge boson is described by the exchange of an intermediate micro-universe in the interaction vertex. In principle, the nature of this intermediate micro-universe can, at least in part, be investigated experimentally through measurements of decoherence rapidity. Furthermore, specific exchange mechanisms of this micro-universe could form a basis for the explanation of the mixing of flavors in the weak interaction mediated by charged bosons.
It therefore becomes possible to apply the language of the third quantization to the description of the particle. The third quantization was developed in the late 1980s to study the fluctuations of space-time, and proved to be a very powerful and clear framework describing the multiverse and various aspects of its conceptual constellations such as virtual black holes, wormholes and baby universes [33 -36]. It develops from the Wheeler-de Witt equation, in which the time variable is not fixed a priori; the possible solutions are therefore incomputable [37]. The choice of the time variable thus implies the selection of an evolutionary mechanism.
The well known approach of multiverse and particle-like universes [38-40] is used here by replacing gravitation with scalar fields, as the Higgs field, which size the particle micro-universe. The particle, in its "corpuscular" aspect, is seen as an event rather than an object. In this sense, no ontological role of a classical type is assigned to the particle micro-universe, unlike previous similar theoretical elaborations [41]. This "event based" perspective of elementary processes finally offers simple mechanisms for breaking the homogeneity of the distribution of matter at the end of the inflationary era and for generating the seeds of structures destined to evolve into galaxies and clusters.

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