

# Theoretical (and numerical) study of pad roll instability in a three fluid-layers system

Antoine Simon, Marie-Charlotte Renoult, Jorge César Brändle de Motta  
and Christophe Dumouchel

Normandie Univ, UNIROUEN, INSA Rouen, CNRS, CORIA

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## Issue:

Need for a reliable energy storage system for a stationary application

## Possibility:

Liquid Metal Battery

## Composition:

A molten salt (the electrolyte E) between 2 Liquid Metals (the anode LMA and the cathode LMC)

## Objective:

Stability analysis of a LMB model to prevent short circuits

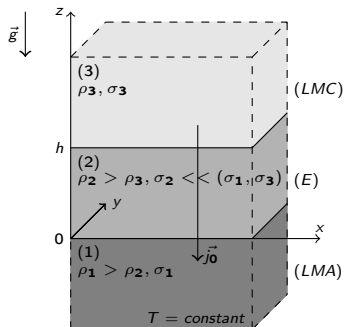
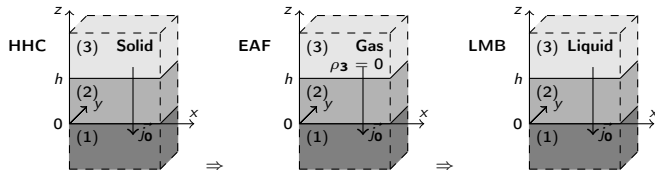


Figure 1: Sketch of the LMB model

# Literature review

Sneyd, A. D., J. Fluid Mech. (1985):

Linear stability analysis of two systems close to LMB (Hall-Héroult Cell, HHC and Electric Arc Furnace, EAF) taking into account the far magnetic field.



HHC: the local magnetic field has only a stabilizing effect.

EAF: the local magnetic field can destabilize the system.

LMB: Our study.

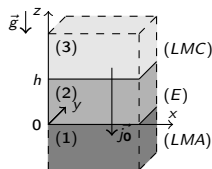
Herreman, W., J. & al. Fluid Mech. (2019):

Metal pad roll instability with a perturbative method taking into account only the top two layers.

# Assumptions and method

## For the fluids:

- Incompressible fluids
- Inviscid fluids
- Infinite depth for extreme layers
- Isothermal conditions



## For the electric and the magnetic fields:

- Slowly variable regime (quasistatic approximation)
- Coulomb force disregarded
- Magnetic field not bounded at the interfaces

## Method:

- Solve base state and disturbed state (linearized) for volume equations and their jump conditions

# Volume equations

Mass and momentum conservation:

$$\begin{aligned}\vec{\nabla} \cdot \vec{u} &= 0 \\ \rho_i \left( \vec{u}_{,t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right) &= -\vec{\nabla} p + \vec{f}_L + \rho \vec{g}\end{aligned}$$

With  $\vec{f}_L = \vec{j} \times \vec{B}$ , the Lorentz force

Maxwell equations and Ohm's law:

$$\begin{aligned}\vec{\nabla} \times \vec{B} &= \mu_0 \vec{j} \\ \vec{\nabla} \times \vec{E} &= \vec{0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{j} &= \sigma \vec{E}\end{aligned}$$

Equations valid for all the layers

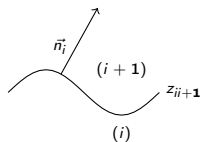
No coupling between  $\vec{u}$ ,  $\vec{B}$ ,  $\vec{E}$

# Jump conditions

For mass and momentum :

$$\vec{u}_i \cdot \vec{n}_i = z_{ii+1,t} = \vec{u}_{i+1} \cdot \vec{n}_i$$

$$p_{i+1} - p_i = 0$$



For the electric and the magnetic fields :

$$\vec{B}_i \cdot \vec{n}_i = \vec{B}_{i+1} \cdot \vec{n}_i$$

$$\vec{B}_i \times \vec{n}_i = \vec{B}_{i+1} \times \vec{n}_i$$

$$\vec{j}_i \cdot \vec{n}_i = \vec{j}_{i+1} \cdot \vec{n}_i$$

$$\vec{E}_i \times \vec{n}_i = \vec{E}_{i+1} \times \vec{n}_i$$

With  $i = \{1; 2\}$  and  $\vec{n}_i = {}^t(-z_{ii+1,x}, -z_{ii+1,y}, 1)$

Conditions valid at  $z = z_{ii+1}$

# Base state

Assumptions :

$$\vec{u}_i^* = \vec{0}$$

$$z_{12}^* = 0$$

$$z_{23}^* = h$$

Equations:

$$-\vec{\nabla}(p_i^* + \rho_i g z) + j_0 \vec{e}_z \times \vec{B}_{i0} = \vec{0}$$

$$p_2^* - p_1^* = 0 \text{ at } z = 0$$

$$p_3^* - p_2^* = 0 \text{ at } z = h$$

Admissible fields:

$\vec{B}_i^* = \vec{B}_{C_i} + \alpha_{ij} x_j$  with

$$\alpha = \mu_0 j_0 \begin{pmatrix} Q & R - \frac{1}{2} & 0 \\ R + \frac{1}{2} & -Q & 0 \\ 0 & 0 & 0 \end{pmatrix} = \alpha_{local} + \alpha_{far}$$

Q and R refers to the far magnetic field

$z_{ii+1} = \eta_{ii+1}(x, y, z, t)$  with  $\eta_{ii+1}$  perturbation of the interfaces

$\vec{u}_i = \vec{u}_i'(x, y, z, t)$  : with  $\vec{u}_i'$  perturbation of the velocities

$p_i = p_i^*(z) + p_i'(x, y, z, t)$  : with  $p_i'$  perturbation of the pressures

$\vec{E}_i = \vec{E}_i^* + \vec{E}_i'(x, y, z, t)$  : with  $\vec{E}_i'$  perturbation of the electric field

$\vec{B}_i = \vec{B}_i^*(x, y, z) + \vec{B}_i'(x, y, z, t)$  : with  $\vec{B}_i'$  perturbation of the magnetic field

Linearized volume equations :

$$\vec{\nabla} \cdot \vec{u}_i' = 0$$

$$\rho_i \frac{\partial \vec{u}_i'}{\partial t} = -\vec{\nabla} p_i' + \vec{f}_{Li}'$$

With  $\vec{f}_{Li}' = j_o \vec{e}_z \times \vec{B}_i' + \vec{j}_i' \times \vec{B}_i^*$ , the disturbed Lorentz force

With  $i \in \llbracket 1; 3 \rrbracket$



# Linearized jump conditions

For mass and momentum:

$$u_{iz} = z_{ii+1,t} = u_{i+1z}$$
$$p'_{i+1} - p'_i - (\rho_{i+1} - \rho_i)gz_{ii+1} = 0$$

For the electric and the magnetic fields :

$$\vec{B}'_i \cdot \vec{n}_i = \vec{B}'_{i+1} \cdot \vec{n}_i$$
$$\vec{B}'_i \times \vec{n}_i = \vec{B}'_{i+1} \times \vec{n}_i$$
$$\vec{J}'_i \cdot \vec{n}_i = \vec{J}'_{i+1} \cdot \vec{n}_i$$
$$\vec{E}'_i \times \vec{n}_i = \vec{E}'_{i+1} \times \vec{n}_i$$

With  $\vec{n}_i = {}^t(0, 0, 1)$

Conditions valid at  $z = (i - 1)h$

# Dispersion relationship

Analysing the disturbance into normal modes  $\Rightarrow$

$$\Omega^4 f_4(K, \rho_{r12}, \rho_{r13}) + \Omega^2 f_2(K, \rho_{r12}, \rho_{r13}, J) + f_0(K, \rho_{r12}, \rho_{r13}, J) = 0$$

With:

$$f_4 = \rho_{r12}(\rho_{r12} \sinh(K) + \cosh(K)) + \rho_{r13}(\rho_{r12} \cosh(K) + \sinh(K))$$

$$f_2 = \rho_{r12}(1 - \rho_{r13})(\sinh(K) + \cosh(K)) - (1 + \rho_{r13})J \sinh(K)^{-1}$$

$$f_0 = (\rho_{r12} + \rho_{r13})(1 - \rho_{r12}) \sinh(K) - (1 + \rho_{r13})J \sinh(K)^{-1} - J^2 \sinh(K)^{-1}$$

Dimensionless numbers:

- $\Omega = \omega / \sqrt{gk}$  : pulsation
- $K = kh$  : wave number
- $\rho_{r12} = \rho_2 / \rho_1$  and  $\rho_{r13} = \rho_3 / \rho_1$  : densities
- $J = J_{local} + J_{far}$  : magnetic field

With  $J_{local} = -\mu_0 j_0^2 / \rho_1 g$  and  $J_{far} = J_{far}(Q, R)$

# Conditions of stability

For  $\rho_{r12} = 0.27$  and  $\rho_{r13} = \{0; 0.25\}$  (values of Herreman & al.)

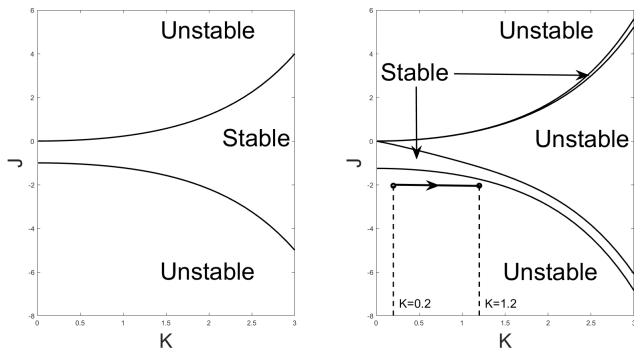


Figure 2: Stability diagram for an EAF model where  $\rho_{r13} = 0$  (left) and the present LMB model (right)

An unstable region appears in the middle of the stable one  $\Rightarrow$  making difficult the conception of such a battery.

# Visualization of the interfaces

For the 2 cases of Figure 2 right:

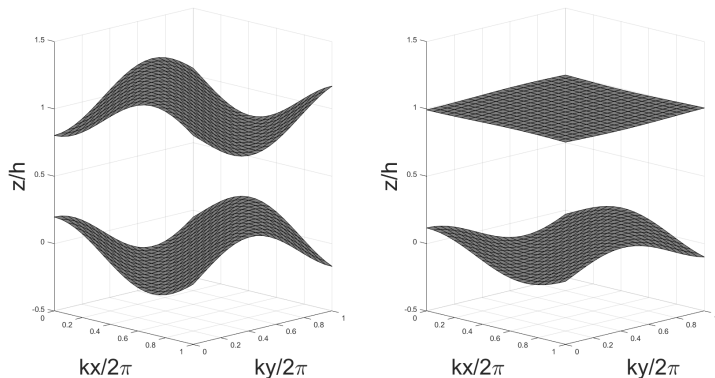


Figure 3: Coupling between the interfaces for  $K = 0.2$  (left) and  $K = 1.2$  (right)

When  $K$  increases the interfaces become uncoupled.

# Conclusion and perspectives

## Conclusion :

- Inviscid model for liquid metal batteries taking into account the coupling between the two interfaces.
- Find a liquid metal as light as possible for the third layer.

## Perspectives :

- Cross-validate the theoretical results with OpenFoam.
- Viscid model for LMB.
- Develop a weakly non linear theory to extend the time of validity of the model.