

# A domain decomposition approach for a Topology Optimisation problem

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# Model

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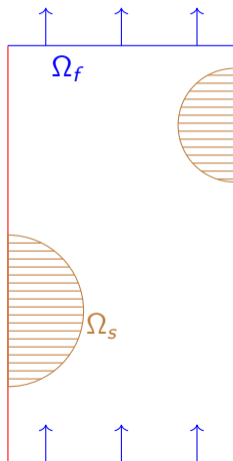
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We would like to solve:

$$\min_{\alpha, \phi} \mathcal{J}(u, \theta, p)$$

such that:

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega$$

$$(\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \text{Re}^{-1} \Delta \mathbf{u} - \text{Ri} \theta \mathbf{e}_y + \eta h_\tau(\alpha) \mathbf{u} = 0 \quad \text{in } \Omega$$

$$\nabla \cdot (\mathbf{u} \theta) - \nabla \cdot (\text{Re}^{-1} \text{Pr}^{-1} \mathbf{k}_\tau(\alpha, \phi) \nabla \theta) = 0 \quad \text{in } \Omega$$

$\alpha$  defines the distribution of solid,  $\phi$  the thermal conductivity.

# Restriction: temperature evolution only

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We focus only on the heat diffusion part of the equations for now. The goal is to solve:

$$\begin{aligned} \min \quad & \|\theta - \theta_{\text{target}}\|_{L^2(\Omega)}^2, \\ \text{s.t.} \quad & \nabla \cdot (u\theta) - \nabla \cdot (k\nabla\theta) = f \text{ in } \Omega, \\ & \theta|_{\partial\Omega} = \theta_0, \\ & k \in [a, b], \end{aligned}$$

with  $f, u$  given s.t.  $\nabla \cdot u = 0$ , and using the control  $k$ .

# How to accelerate the resolution?

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- Faster computation of the gradient through domain decomposition with optimized transmission conditions : indirect method.
- Relaxed optimization problems by subdomain, weakly coupled between subdomains : direct method.

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# Optimality conditions

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## Gradient expression

Denote  $\theta(k)$  the solution of:

$$\begin{aligned} -\nabla \cdot (k\nabla\theta - u\theta) &= f, \\ \theta|_{\partial\Omega} &= \theta_0, \end{aligned}$$

and  $J(k) = \|\theta(k) - \theta_{\text{target}}\|_{L^2(\Omega)}^2$ . Then:

$$\partial_k J(k) = \nabla\theta \cdot \nabla\lambda$$

where  $\lambda$  solves the adjoint equation

$$\begin{aligned} \nabla \cdot (k\nabla\lambda - u\lambda) &= \theta - \theta_{\text{target}}, \\ \lambda|_{\partial\Omega} &= 0, \end{aligned}$$



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Therefore, in order to compute the gradient of the cost, one needs to solve for fixed  $k$ :

$$\begin{cases} -\nabla \cdot (k\nabla\theta - u\theta) = f, \\ \nabla \cdot (k\nabla\lambda + u\lambda) = \theta - \theta_{\text{target}}, \\ \theta|_{\partial\Omega} = \theta_0, \\ \lambda|_{\partial\Omega} = 0. \end{cases}$$

Fast resolution of this system  $\implies$  fast computation of the gradient, used in a descent method.

# Domain decomposition

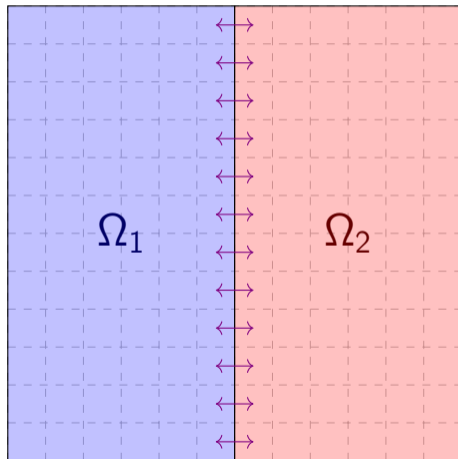
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$$\nabla \cdot (u\theta) - \nabla \cdot (k\nabla\theta) = f \text{ on } \Omega$$

$$\nabla \cdot (k\nabla\lambda + u\lambda) = \theta - \theta_{\text{target}},$$

B.C. on  $\partial\Omega$

We replace that with:

$$\nabla \cdot (u\theta_1^n) - \nabla \cdot (k\nabla\theta_1^n) = 0 \text{ on } \Omega_1$$

$$\nabla \cdot (k\nabla\lambda_1^n + u\lambda_1^n) = \theta_1^n - \theta_{\text{target}} \text{ on } \Omega_1,$$

$$\nabla \cdot (u\theta_2^n) - \nabla \cdot (k\nabla\theta_2^n) = 0 \text{ on } \Omega_2$$

$$\nabla \cdot (k\nabla\lambda_2^n + u\lambda_2^n) = \theta_2^n - \theta_{\text{target}} \text{ on } \Omega_2,$$

B.C. on  $\partial\Omega_1 \cap \partial\Omega_2$

B.C. on  $\partial\Omega$

# Domain decomposition

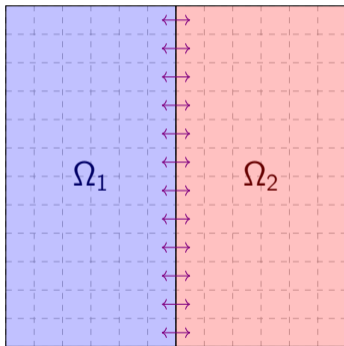
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At the interface, Robin conditions :

$$\begin{aligned} k\partial_x \begin{pmatrix} \theta_1^n \\ \lambda_1^n \end{pmatrix} - \frac{u_1}{2} \begin{pmatrix} \theta_1^n \\ \lambda_1^n \end{pmatrix} + S_1 \begin{pmatrix} \theta_1^n \\ \lambda_1^n \end{pmatrix} \Big|_{\Gamma_n} &= \\ &= \partial_x \begin{pmatrix} \theta_2^{n-1} \\ \lambda_2^{n-1} \end{pmatrix} - \frac{u_1}{2} \begin{pmatrix} \theta_2^{n-1} \\ \lambda_2^{n-1} \end{pmatrix} + S_1 \begin{pmatrix} \theta_2^{n-1} \\ \lambda_2^{n-1} \end{pmatrix} \Big|_{\Gamma_n}, \\ k\partial_x \begin{pmatrix} \theta_2^n \\ \lambda_2^n \end{pmatrix} - \frac{u_1}{2} \begin{pmatrix} \theta_2^n \\ \lambda_2^n \end{pmatrix} + S_2 \begin{pmatrix} \theta_2^n \\ \lambda_2^n \end{pmatrix} \Big|_{\Gamma_n} &= \\ &= \partial_x \begin{pmatrix} \theta_1^{n-1} \\ \lambda_1^{n-1} \end{pmatrix} - \frac{u_1}{2} \begin{pmatrix} \theta_1^{n-1} \\ \lambda_1^{n-1} \end{pmatrix} + S_2 \begin{pmatrix} \theta_1^{n-1} \\ \lambda_1^{n-1} \end{pmatrix} \Big|_{\Gamma_n}. \end{aligned}$$

What choice of operator  $S_i$  ?

# Subdomain gradient computation

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Framework for Fourier analysis : suppose  $\Omega = \mathbb{R}^2$ , and the interface is  $\Gamma_\cap = \{0\} \times \mathbb{R}$ . Suppose  $k$  is constant on  $\Omega$  and  $f = \theta_{\text{target}} = 0$ . There exist functions  $A_\theta^n(\omega), A_\lambda^n(\omega), B_\theta^n(\omega), B_\lambda^n(\omega)$  such that:

$$\hat{\theta}_1^n(x, \omega) = A_\theta^n(\omega) e^{r_+^\theta(\omega)(x-L)},$$

$$\hat{\theta}_2^n(x, \omega) = B_\theta^n(\omega) e^{r_-^\theta(\omega)x},$$

$$\hat{\lambda}_1^n(x, \omega) = A_\lambda^n(\omega) e^{r_+^\lambda(\omega)(x-L)} + A_{\lambda\theta}(\omega) \hat{\theta}_1^n(x, \omega),$$

$$\hat{\lambda}_2^n(x, \omega) = B_\lambda^n(\omega) e^{r_-^\lambda(\omega)x} + B_{\lambda\theta}(\omega) \hat{\theta}_2^n(x, \omega),$$

where  $A_{\lambda\theta}(\omega)$  and  $B_{\lambda\theta}(\omega)$  are known and

$$r_\pm^\theta(\omega) = \frac{u_1 \pm \sqrt{u_1^2 + 4k^2\omega^2 - 4iku_2\omega}}{2k}, \quad r_\pm^\lambda(\omega) = \frac{-u_1 \pm \sqrt{u_1^2 + 4k^2\omega^2 + 4iku_2\omega}}{2k}$$

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We analyse now the transmission conditions. We suppose that:

$$\mathcal{F}_y(\mathcal{S}_i(\theta, \lambda))(x, \omega) = \sigma_i(\omega)(\hat{\theta}, \hat{\lambda})$$

where  $\sigma_i$  is a  $2 \times 2$  complex matrix. The transmission conditions then read:

$$k\partial_x \begin{pmatrix} \hat{\theta}_1^n \\ \hat{\lambda}_1^n \end{pmatrix} (0, \omega) - \frac{u_1}{2} \begin{pmatrix} \hat{\theta}_1^n \\ -\hat{\lambda}_1^n \end{pmatrix} (0, \omega) + \sigma_j(\omega) \begin{pmatrix} \hat{\theta}_1^n \\ \hat{\lambda}_1^n \end{pmatrix} (0, \omega) = \\ (M_r^+(0, \omega) + \sigma_j(\omega)M_0^+(0, \omega)) \begin{pmatrix} A_\theta^n(\omega) \\ A_\lambda^n(\omega) \end{pmatrix},$$

where  $M_r^+$  and  $M_0^+$  are lower triangular matrices. (A similar expression holds for  $\theta_2, \lambda_2$ ).

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Using the iterative process, one proves the following recurrence:

$$\begin{pmatrix} A_{\theta}^n(\omega) \\ A_{\lambda}^n(\omega) \end{pmatrix} = \underbrace{[M_r^+(0, \omega) + \sigma_1(\omega)M_0^+(0, \omega)]^{-1} [M_r^-(0, \omega) + \sigma_1(\omega)M_0^-(0, \omega)]}_{M_1(\omega)} \underbrace{[M_r^-(0, \omega) - \sigma_2(\omega)M_0^-(0, \omega)]^{-1} [M_r^+(0, \omega) - \sigma_2(\omega)M_0^+(0, \omega)]}_{M_2(\omega)} \begin{pmatrix} A_{\theta}^{n-2}(\omega) \\ A_{\lambda}^{n-2}(\omega) \end{pmatrix}$$

Note:  $M_1$  and  $M_2$  are both lower triangular.

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Therefore, the optimal choice of  $\sigma_i$  reads:

$$\begin{aligned}\sigma_1^{\text{opt}}(\omega) &= -M_r^-(0, \omega) (M_0^-(0, \omega))^{-1} \\ &= \begin{pmatrix} -kr_-^\theta(\omega) + \frac{u_1}{2} & 0 \\ kB_{\lambda\theta}(\omega)[r_-^\lambda(\omega) - r_-^\theta(\omega)] & -kr_-^\lambda(\omega) - \frac{u_1}{2} \end{pmatrix},\end{aligned}$$

where

$$r_\pm^\theta(\omega) = \frac{u_1 \pm \sqrt{u_1^2 + 4k^2\omega^2 - 4iku_2\omega}}{2k}, \quad r_\pm^\lambda(\omega) = \frac{-u_1 \pm \sqrt{u_1^2 + 4k^2\omega^2 + 4iku_2\omega}}{2k}$$

If we apply an inverse Fourier transform: non-local operator  $S_i$ .

# Optimized operator

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$$\begin{aligned}\nabla \cdot (u\theta) - \nabla \cdot (k\nabla\theta) &= 0 \text{ on } \Omega \\ -\nabla \cdot (u\lambda) - \nabla \cdot (k\nabla\lambda) &= F(\theta) \text{ on } \Omega \\ \text{B.C. on } \partial\Omega\end{aligned}$$

Instead of using the optimal (non-local) operator, search for the optimal lower-triangular matrix  $P_i$ , constant, for:

$$\begin{aligned}k\partial_x \begin{pmatrix} \theta_1^n \\ \lambda_1^n \end{pmatrix} (0, y) - \frac{u_1}{2} \begin{pmatrix} \theta_1^n \\ -\lambda_1^n \end{pmatrix} (0, y) + P_1 \begin{pmatrix} \theta_1^n \\ \lambda_1^n \end{pmatrix} (0, y) = \\ k\partial_x \begin{pmatrix} \theta_2^{n-1} \\ \lambda_2^{n-1} \end{pmatrix} (0, y) - \frac{u_1}{2} \begin{pmatrix} \theta_2^{n-1} \\ -\lambda_2^{n-1} \end{pmatrix} (0, y) + P_1 \begin{pmatrix} \theta_2^{n-1} \\ \lambda_2^{n-1} \end{pmatrix} (0, y)\end{aligned}$$



# Optimized operator

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Optimal in what sense? Denote  $P_i = \begin{pmatrix} p_{i1} & 0 \\ p_{i3} & p_{i4} \end{pmatrix}$ . We still have the recurrence

$$\begin{pmatrix} A_{\theta}^n(\omega) \\ A_{\lambda}^n(\omega) \end{pmatrix} = \tilde{M}_1(\omega)\tilde{M}_2(\omega) \begin{pmatrix} A_{\theta}^{n-2}(\omega) \\ A_{\lambda}^{n-2}(\omega) \end{pmatrix}$$

We want to solve the following problem:

$$\min_{P_{ij}} \max_{[\omega_1, \omega_2]} \|\tilde{M}_1(\omega)\tilde{M}_2(\omega)\|$$

for some matrix norm  $\|\cdot\|$ .

# Optimized operator

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If one uses the spectral radius, one sees that only the diagonal elements appear. It simplifies the problem to  $\min_{p_{11}, p_{21}} \max_{\omega \in [\omega_1, \omega_2]} R(p, \omega)$ , where

$$R(p, \omega) = \left| \frac{\left(-\sqrt{C(\omega)} + 2p_{11}\right) \left(-\sqrt{C(\omega)} + 2p_{21}\right)}{\left(\sqrt{C(\omega)} + 2p_{11}\right) \left(\sqrt{C(\omega)} + 2p_{21}\right)} \right|$$

For this min-max problem, we already have a solution<sup>1</sup>. But what about the off diagonal element  $p_{i3}$ ? It can't be found using the spectral radius!

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<sup>1</sup>O. Dubois. Optimized Schwarz methods for the advection-diffusion equation and for problems with discontinuous coefficients. PhD thesis, McGill University PhD, 2007

# Subdomain gradient computation

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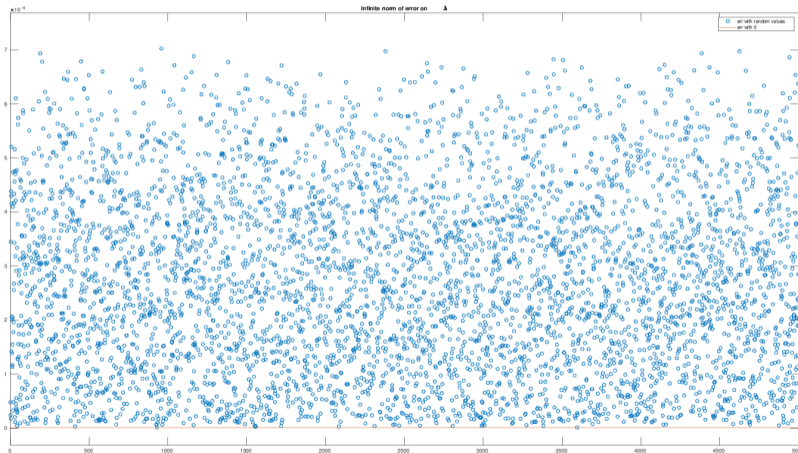


Figure: Error after 20 iterations. Blue dots: random values in the off-diagonal matrix element. Red line: zero in the off-diagonal element.

# Perspective: parallel optimization

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However, computing the gradient in a parallel way may not be efficient:

- It creates a parallel execution part for computing the gradient, but it becomes sequential for the optimization part.
- All the analysis was carried with a constant  $k$ , while it should be a control function.
- Each change of the control  $k$  implies a new parallel computation

⇒ What if we solve the optimization problem on each subdomain instead?

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# Decomposition in the constraints

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$$\begin{aligned} \min \quad & \|\theta - \theta_{\text{target}}\|_{L^2(\Omega)}^2, \\ \text{s.t.} \quad & \begin{cases} \nabla \cdot (u\theta) - \nabla \cdot (k\nabla\theta) = 0 \text{ in } \Omega, \\ \theta|_{\partial\Omega} = \theta_0 \\ k \in [a, b] \text{ a.e. in } \Omega. \end{cases} \iff \end{aligned}$$

$$\begin{aligned} \min \quad & \sum_{i=1}^2 \|\theta_i - \theta_{\text{target}}\|_{L^2(\Omega_i)}^2, \\ \text{s.t.} \quad & \begin{cases} \nabla \cdot (u\theta_i) - \nabla \cdot (k_i\nabla\theta_i) = 0 \text{ in } \Omega_i, \\ \theta_i|_{\partial\Omega} = \theta_0, \quad k_i \in [a, b] \text{ a.e. in } \Omega_i \\ \theta_1|_{\Gamma_n} = \theta_2|_{\Gamma_n}, \quad k_1|_{\Gamma_n} = k_2|_{\Gamma_n} \\ k_1\partial_{n_1}\theta_1 + a(u \cdot n_1)\theta_1 + p\theta_1 = -k_2\partial_{n_2}\theta_2 - a(u \cdot n_2)\theta_2 + p\theta_2. \end{cases} \end{aligned}$$

# Decomposition in the constraints

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How to handle the continuity constraints? Add a virtual control on  $\Gamma_\cap$ , and penalize.

$$\begin{aligned} \min \quad & \sum_{i=1}^2 \|\theta_i - \theta_{\text{target}}\|_{L^2(\Omega_i)}^2 + \int_{\Gamma_\cap} \lambda(\theta_1 - \theta_2) + \frac{\rho}{2} \int_{\Gamma_\cap} (\theta_1 - \theta_2)^2, \\ \text{s.t.} \quad & \begin{cases} \nabla \cdot (u\theta_i) - \nabla \cdot (k_i \nabla \theta_i) = 0 \text{ in } \Omega_i, \\ \theta_i|_{\partial\Omega} = \theta_0, \quad k_i \in [a, b] \text{ a.e. in } \Omega_i \\ k_1|_{\Gamma_\cap} = k_2|_{\Gamma_\cap} \\ k_1 \partial_{n_1} \theta_1 + a(u \cdot n_1) \theta_1 + p \theta_1 = -k_2 \partial_{n_2} \theta_2 - a(u \cdot n_2) \theta_2 + p \theta_2 = g \text{ on } \Gamma_\cap. \end{cases} \end{aligned}$$

Note : once discretized (with FV), the constraint  $k_1 = k_2$  is handled through a (geometric) mean of the values on each side of the interface.

## Decomposition in the constraints

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$$\begin{aligned} \min \quad & \sum_{i=1}^2 \|\theta_i - \theta_{\text{target}}\|_{L^2(\Omega_i)}^2 + \int_{\Gamma_\cap} \lambda^n (\theta_1 - \theta_2) + \frac{\rho^n}{2} \int_{\Gamma_\cap} (\theta_1 - \theta_2)^2, \\ \text{s.t.} \quad & \begin{cases} \nabla \cdot (u\theta_i) - \nabla \cdot (k_i \nabla \theta_i) = 0 \text{ in } \Omega_i, \\ \theta_i|_{\partial\Omega} = \theta_0, \quad k_i \in [a, b] \text{ a.e. in } \Omega_i \\ k_1|_{\Gamma_\cap} = k_2|_{\Gamma_\cap} \\ k_1 \partial_{n_1} \theta_1 + a(u \cdot n_1) \theta_1 + \rho \theta_1 = -k_2 \partial_{n_2} \theta_2 - a(u \cdot n_2) \theta_2 + \rho \theta_2 = g \text{ on } \Gamma_\cap. \end{cases} \end{aligned}$$

This is an augmented lagrangian method. Given  $\lambda^n$  and  $\rho^n$ , solve the optimization problem, giving  $k_i^n$ ,  $\theta_i^n$ . Based on the value of  $\|\theta_1^n - \theta_2^n\|_{L^2(\Gamma_\cap)}$ :

- increase  $\rho$  (the error is still too big),
- update  $\lambda$  :  $\lambda^{n+1} = \lambda^n + \rho^n(\theta_1^n - \theta_2^n)$ .



# 1D simplified example

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$$\begin{aligned} \min \quad & \|\theta - \theta_{\text{target}}\|_{L^2(\Omega)}^2, \\ \text{s.t.} \quad & \begin{cases} -\frac{d}{dx} \left( k \frac{d\theta}{dx} \right) = 0 \text{ in } \Omega = [-1, 1], \\ \theta(-1) = \theta(1) = 0, \\ k \in [1, 11] \text{ a.e. in } \Omega \end{cases} \end{aligned}$$

$\theta_{\text{target}}$  solution of  $\frac{d}{dx} \left( k_{\text{target}} \frac{d\theta_{\text{target}}}{dx} \right) = 0$ , where  $k_{\text{target}}(x) = \begin{cases} 3 & \text{if } x < 0.38 \\ 9 & \text{otherwise} \end{cases}$ .

Discretization using a finite volume scheme with a mesh respecting the interface boundary  $\Gamma_{\cap} = \{0.4\}$ .

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$$\begin{aligned} \min \quad & \sum_{i=1}^2 \|\theta_i^n - \theta_{\text{target}}\|_{L^2(\Omega)}^2 + \int_{\Gamma_{\cap}} \lambda^n (\theta_1^n - \theta_2^n) + \frac{\rho^n}{2} (\theta_1^n - \theta_2^n)^2, \\ \text{s.t.} \quad & \begin{cases} -\frac{d}{dx} \left( k_1 \frac{d\theta_1^n}{dx} \right) = 0 \text{ in } \Omega_1 = [-1, 0.4], \\ -\frac{d}{dx} \left( k_2 \frac{d\theta_2^n}{dx} \right) = 0 \text{ in } \Omega_2 = [0.4, 1], \\ \theta_1(-1) = \theta_2(1) = 0, \\ k_1 \frac{d\theta_1}{dx}(0.4) = k_2 \frac{d\theta_2}{dx}(0.4) = g, \\ k_i \in [1, 11] \text{ a.e. in } \Omega_i. \end{cases} \end{aligned}$$

We know that  $\lambda^n$  should converge to  $\lambda^* = 0$ .

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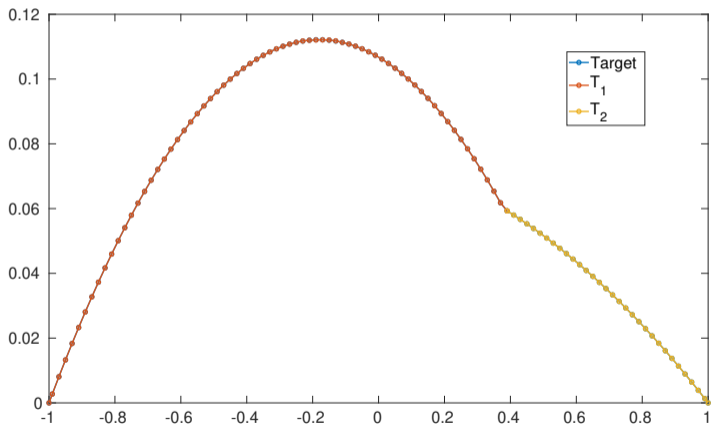


Figure: Solution of the optimal control problem computed with split domains:  $\theta$

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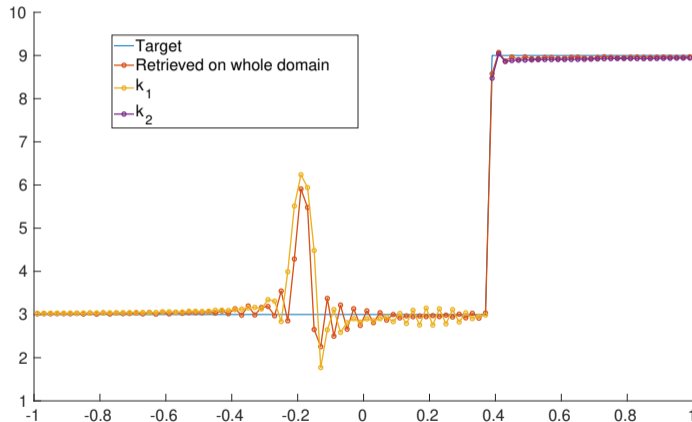


Figure: Solution of the optimal control problem computed with split domains:  $k$

# Conclusion

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## Take-home message

- Optimize then decompose (indirect approach):
  - Optimized operator found, with a lot of restrictions.
  - Still unclear how to use it in an optimization algorithm without breaking the parallel process
- Decompose then optimize (direct approach):
  - Easily generalized
  - Analysis not finished
  - Seems to work numerically

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Question time!