LiQuOFETI: a FETI-inspired method for elliptic quadratic optimal control problems
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Problem statement

The goal is to solve, with some decomposed scheme, the linear quadratic optimal control
\[
\min_\mu \frac{1}{2} \int (y - y_{\text{target}})^2 + \frac{1}{2} \int f^2
\]
\[\text{s.t. } A \frac{\partial g}{\partial x} + \nabla (b(x)y) + c(x) + \mu y = f + f,\]
\[\forall \Omega,\]
\[y|_{\partial \Omega} = 0,\]
\[A \partial x |_{\partial \Omega} = 0,\]
\[\partial b(x)y |_{\partial \Omega} = 0,\]
\[\partial c(x) |_{\partial \Omega} = 0,\]
\[\partial \mu |_{\partial \Omega} = 0.\]

Here, we will assume that \( A \in L^\infty(\Omega, \mathbb{R}^{d_m \times d_m}), b \in W^{1, \infty}(\Omega, \mathbb{R}^m), c \in L^\infty(\Omega), F \in L^2(\Omega) \) and \( \mu > 0 \).

Algorithm: update of the multiplier

Data: \( \rho > 0, \omega^* < 1, \eta, \lambda > 1. \)

Choose an initial \( f^i, \rho^i, \lambda^i \), \( \forall i \). \( \forall i \), while \( \|\rho_i - \rho_{i-1}\|_2 \leq \eta \) or \( \lambda_i \) unchanged.

Choose \( \lambda^{i+1} = \lambda^i, \rho^{i+1} = \lambda^i + \rho^i (\|y^i - y^*\|) \).

Let \( \rho^{i+1} \) unchanged.

Decrease \( \omega_i : \omega_{i+1} = (\rho^{i+1})^{-1} \omega_i \).

Decrease \( \eta_i : \eta_{i+1} = (\rho^{i+1})^{-1} \eta_i \).

end

A Fourier analysis of \( \lambda^k \)

Using a Fourier Analysis of the necessary and sufficient conditions of optimality of (3) when \( F = 0 \), one may prove that the update of \( \lambda \) in the algorithm produces iterates \( \{\lambda^k\} \), such that
\[
R - \lambda^{k+1} = \left(1 - \|\rho^{k+1}\|^{-1} (D_s - D_\Sigma)^{-1}\right) R - \lambda^k
\]
where \( D_s = \sqrt{\pi a \omega^{1/2}} \Delta x^{1/2} \), and \( \lambda \) is the Fourier transform of \( \lambda \). This coefficient shows how fast \( \{\lambda^k\} \) converges, and it seems to converge fast!

Numerical example

We solve (1) with where \( \Omega = [-1, 1] \times [0, 1], y_{\text{target}}(x, z) = \sin(2\pi x) \sin(2\pi z) \) and \( G(x, z) = \pi^2 \sin(2\pi x) \sin(2\pi z) \) and \( n = 1 \). The optimal solution is \( f^* = 0, y^* = y_{\text{target}} \).

We solve this problem using our augmented lagrangian method. The problem is discretized using Q1 elements on a structured uniform mesh, and the interface is placed at \( \Gamma_i = \{ y = 0 \} \times [0, 1] \).

We retrieve a second order convergence of the solution with respect to the discretization stepsize.

References


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