A Matheuristic for the Commodity Constrained Split Delivery VRP

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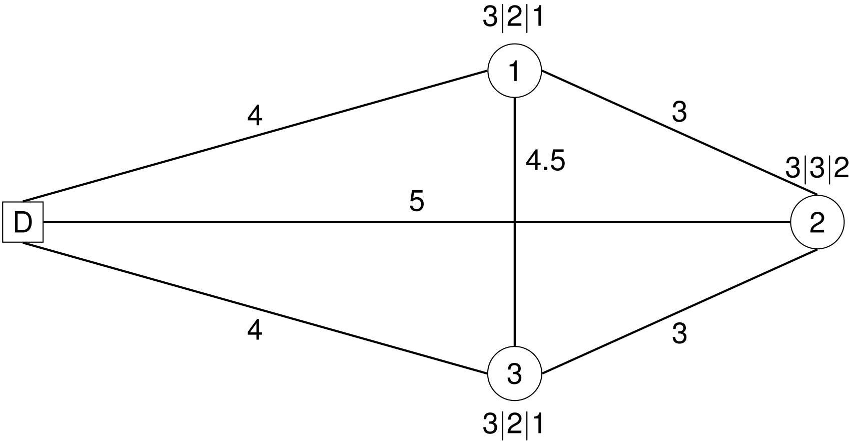
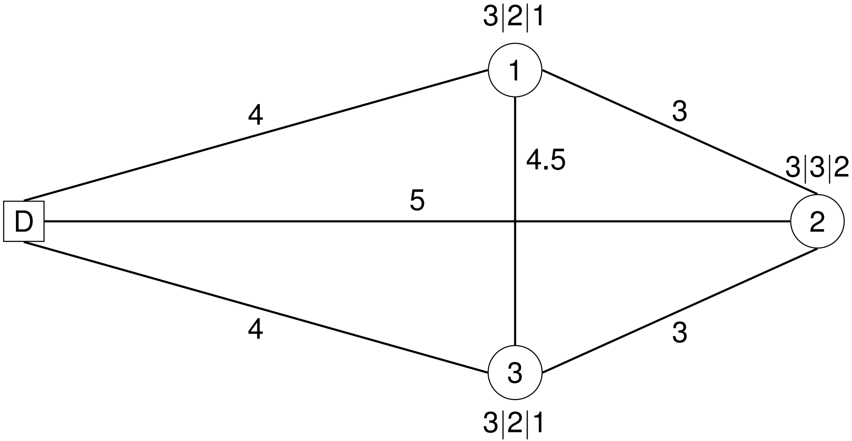
INTRODUCTION

EXAMPLE [Archetti et al., 2016]

Separate routing strategy



Commodity constrained split delivery mixed routing strategy



Context and related literature

The C-SDVRP:

- was first introduced in [Archetti et al., 2016]
- exact algorithms : Branch-and Cut algorithm [Archetti et al., 2016]; Branch-price-and-cut algorithms [Archetti et al., 2015],[Gschwind et al., 2019]
- heuristics: VRP based heuristic with node replications [Archetti et al., 2016]; ALNS [Gu et al., 2019]
- has an application in the short and local fresh food supply chain [Gu et al., 2022]

AN EXTENDED FORMULATION

Notation

- $\mathcal{R}$ set of **feasible routes**
- C_r: **cost of route** $r \in \mathcal{R}$
- $\mathcal{C}$ set of customers.
- $\mathcal{K}$ set of commodities.
- $R_{jk} \geq 0$ demand of customer $j \in \mathcal{C}$ for commodity $k \in \mathcal{K}$.
- a^r_{jk}: **binary parameter** $= 1$ if route $r \in \mathcal{R}$ delivers $k \in \mathcal{K}_j$ to customer $j \in \mathcal{N}$, $= 0$ otherwise
- For all routes $r \in \mathcal{R}$ a **binary variable**

$$\lambda_r = \begin{cases} 1 & \text{if route } r \text{ is in the solution,} \\ 0 & \text{otherwise.} \end{cases}$$

Formulation

$$\min \sum_{r \in \mathcal{R}} C_r \lambda_r \tag{1}$$

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$$\sum_{r \in \mathcal{R}} a^r_{jk} \lambda_r \geq 1 \qquad \forall j \in \mathcal{C}, \forall k \in \mathcal{K} \text{ s.t. } R_{jk} > 0 \qquad (2)$$

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$$\lambda_r \in \{0, 1\} \quad \forall r \in \mathcal{R} \tag{4}$$

A MATHEURISTIC

General setting

- We price λ_r variables, $r \in \mathcal{R}_o$, $o \in \mathcal{D}$
- The pricing problem $\rho_o := \min\{\bar{C}_r \mid r \in \mathcal{R}_o\}$, $o \in \mathcal{D}$ can be formulated as an **Elementary Shortest Path Problem with Resource Constraints (ESPPRC)** [Archetti et al., 2015]
- We tackle the ESPPRC by a **label setting dynamic programming technique** [Feillet et al., 2004]

Classical features

Our matheuristic incorporates

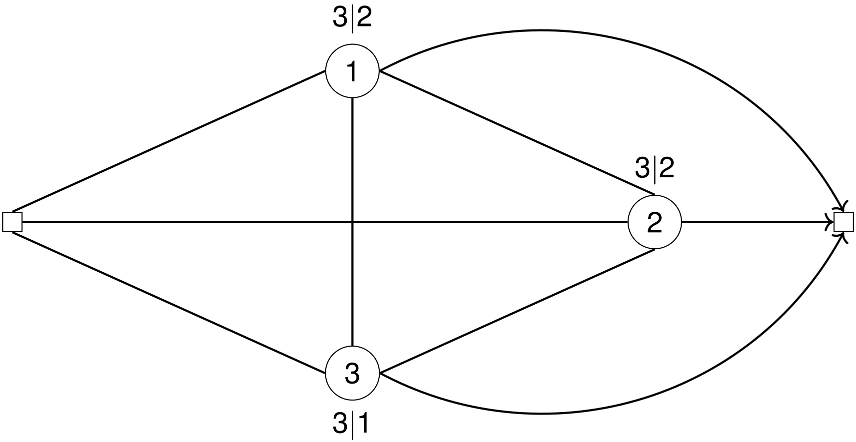
- the **ng-path** relaxation [Baldacci et al., 2011] and the **bidirectional labeling search** [Righini and Salani, 2006]
- **heuristic approaches** to solve the ESPPRC
- automatic dual pricing smoothing **stabilization** [Pessoa et al., 2020]
- **capacity cuts** separation [Ralphs et al., 2003]
- separation of valid inequalities for the set covering polytope and the number partitioning polytope

Heuristic Column Generator (HCG)

A three phase procedure:

- **First phase**: **define a subgraph** by using a clustering technique on the set of customers
- **Second phase**: **compute a lower bound** on the optimal value of the pricing problem by solving an ESPPRC on a modified graph
- **Third phase**: **determine all the negative reduced cost feasible routes** by solving the ESPPRC in **acyclic graphs**

Second phase



Third phase

Valid Inequalities

Valid Inequalities

- **Set covering polytope** [Balas and Ng, 1989]:
  - for all customer $j \in \mathcal{C}$ and all subsets $\hat{\mathcal{K}} \subseteq \mathcal{K}_j$

    $\mathcal{R}_j^{\hat{\mathcal{K}}}$: set of routes delivering to j all the commodities in $\hat{\mathcal{K}}$

    $\bar{\mathcal{R}}_j^{\hat{\mathcal{K}}}$: set of routes delivering to j some (but not all) commodities in $\hat{\mathcal{K}}$

$$2 \sum_{r \in \mathcal{R}_j^{\hat{\mathcal{K}}}} \lambda_r + \sum_{r \in \bar{\mathcal{R}}_j^{\hat{\mathcal{K}}}} \lambda_r \geq 2, \quad \forall j \in \mathcal{C}, \forall \hat{\mathcal{K}} \subseteq \mathcal{K}_j$$

Valid Inequalities

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- **Number partitioning problem polytope**:
  - for all customer $j \in \mathcal{C}$ and all $k = 1, \ldots, |\mathcal{K}_j|$
    x_k^j: number of routes which deliver k commodities to j

$$\sum_{k=1}^{|\mathcal{K}_j|} k x_k^j \leq |\mathcal{K}_j|, \quad \forall j \in \mathcal{C}$$

COMPUTATIONAL RESULTS

Instances

Instances from [Archetti et al., 2016] built from the R101 and C101 Solomon instances for the VRP with Time Windows.

- $|\mathcal{C}| \in \{15, 20, 40, 60, 80, 100\}$
- $|\mathcal{K}| \in \{2, 3\}$

Each commodity required with a probability *p* of 0.6 or 1
Quantity of each commodity within $[1, 100]$ or $[40, 60]$.

Summary of results on small sized instances.

instances						Matheuristic vs ALNS		
	n	M	p	$avg.n_{cc}$	$nbIns$	$avg.gap$	$avg.time$	$\%time.improv.$
C101	15	2	0.6/1	22/30	16	0.36	0.36	-96.4
C101	15	3	0.6/1	28/45	16	0.02	0.22	-97.6
R101	15	2	0.6/1	22/30	16	0.17	3.62	-81.3
R101	15	3	0.6/1	28/45	16	0.16	1.20	-93.7

Preliminary results on large sized instances.

instances						Matheuristic vs ALNS		
	n	M	p	$avg.n_{cc}$	$nbIns$	$avg.gap$	$avg.time$	$\%time.improv.$
C101	100	2	0.6	136.4	10	0.67	74.45	-71.7
C101	100	3	1	300	5	0.86	633.5	-44.5
R101	100	2	0.6	136.4	10	0.72	271.3	-24.0
R101	100	3	1	300	5	-0.28	82.6	-90.8

FUTURE WORK

Future work

- Perform extensive **computational experiments**
- Develop a Branch&Price&Cut algorithm

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THANK YOU!