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Distributed Finite-time Coverage Control of Multi-quadrotor Systems

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Abstract—This paper investigates the distributed coverage control algorithm of robotic sensor networks consisting of multiple quadcopters which guarantees its finite-time convergence. The proposed technique alters the objective function originated from the locational optimisation problem to accommodate the consensus constraint, and solves the problem within a given time limit. The coverage problem is solved by sending angular-rate and thrust commands to the quadcopters. By exploiting the finite-time stability theory, we ensure that the rotational and translational controllers of the quadcopters are finite-time stable, able to be implemented distributedly, and able to collaboratively drive the quadcopters towards the desired position and velocity of the Voronoi centroid independent of their initial states.

Index Terms—Coverage control, finite-time stability, motion control, robotic sensor network, flying robots

I. INTRODUCTION

Coverage control problem of the robotic sensor network (RSN) has attracted attention from the robotics community. One of the main tasks of an RSN is to maximise the coverage of the deployed sensors, which would lead to the best measurement data of the corresponding environment. The locational optimisation has been proposed to find the best locations of agents given an interest function; and centroidal Voronoi Tessellation has become a well-known tool to tackle this problem [1], and [2]. By adopting the locational optimisation problem, a simple proportional controller was initially developed [3], [2]. This algorithm is improved to tackle time-varying density coverage on a group of nonholonomic mobile robots in [4]. Various coverage control strategies to relax the assumptions about unlimited, isotropic and homogenous sensing range, and convex environment have been addressed in [5], [6], [7] and [8]. Adaptive coverage control to estimate the information density function has been studied in [9], [10]. Regarding the communication topology, the result in [11] has included a dynamically routing communication algorithm while optimising the coverage control problem. However, the coverage algorithm from the control perspective that guarantees timely convergence in a finite time has not been investigated among the existing strategies. Timeliness has become a crucial requirement in many applications, such as in a post-disaster evacuation and nuclear decommissioning, to prevent worsening situations [12]. In control theory, timeliness relates to the settling or convergence time of an autonomous system

from initial values to the origin. The study reported in [13] has initiated the finite-time stability analysis in the control system by showing that the convergence time depends on the initial states. By utilising the proposed finite-time strategy, a finite-time consensus of a team of agents with single-integrator dynamics, double-integrator dynamics and nonholonomic dynamics were presented in [14], [15]. The finite-time based controller has also found applications in the pose synchronisation of spacecraft whose poses are represented by dual-quaternion as reported in [16]. However, since those results depend on the initial values, the system requires more time to converge to a formation if the agents are initially separated with a large distance. To overcome this problem, authors in [17] proposed a finite-time consensus controller that guarantees the convergence within a specified boundary of settling time regardless of the initial states. Subsequently, work in [18] has extended the result in [17] to consensus of multi-agent systems with double-integrator. Furthermore, this approach has also been applied to design a finite-time consensus controller with the presence of time delays in a networked system in [19]. This paper presents a distributed coverage control algorithm of a multi-quadcopter sensor network that guarantees finite-time stability; whose contributions of this work are highlighted as follows. First, different from the existing approaches, we aim to solve the locational optimisation and consensus problems simultaneously. This scheme is to maintain the position and velocity of the Voronoi centroid and the formation shape of the agents. Second, the proposed strategy exploits the finite-time stability theory to ensure that the desired position and velocity, i.e., the Voronoi centroids, and the desired attitude are timely reached. Third, since we consider quadcopters as the agents, the algorithm also considers the nonlinear dynamics.

The remainder of this paper is organised as follows. Section 2 briefly reviews the notions of graph theory, locational optimisation and quaternion. Section 3 states the main problem addressed in this paper. Afterwards, the main algorithms for finite-time coverage control of the quadcopter formation are presented in Section 4. Finally, numerical simulations validating the proposed algorithm are provided in Section 5, followed by concluding remarks in Section 6.

II. PRELIMINARIES

A. Graph Theory

A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a collection of n vertices $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ connected by a collection of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. If there exists an edge $(v_i, v_j) \in \mathcal{E}$, vertex v_i is able to receive information from vertex j . If, for $(v_i, v_j) \in \mathcal{E}$, there exists $(v_j, v_i) \in \mathcal{E}$, the graph is called undirected. We refer $v_j \in N_i \subset \mathcal{V}$, for $v_j \neq v_i$, to the neighbor of vertex v_i if $(v_i, v_j) \in \mathcal{E}$.

B. Locational Optimisation

Consider n robots deployed in an environment denoted by a convex set $\mathcal{Q} \subset \mathbb{R}^d$. The set containing the position of all robots is denoted by $\mathcal{P} = \{p_i\}_{i=1}^n \subset \mathcal{Q}$ with p_i is the position of robot i . Sensing unreliability function $g : \mathcal{Q} \times \mathcal{Q} \rightarrow \mathbb{R}_+ : (x, p_i) \mapsto g(x, p_i)$, is a function that provides the quantitative information of the sensing performance at point $x \in \mathcal{Q}$ measured by agent i at p_i . In our discussion, we assume the sensing unreliability function to be isotropic, increasing and convex. A function is isotropic if the value is independent on its direction. Hence, the function $g(x, p_i)$ can be re-parametrised to a norm-valued function $f : \mathbb{R} \rightarrow \mathbb{R}_+$ such that $g(x, p_i) = f(\|x - p_i\|)$, for $i \in \{1, 2, \dots, n\}$.

The distribution of information in the environment is represented by a density function, or information distribution function, and denoted by $\phi : \mathcal{Q} \rightarrow \mathbb{R}_+ : x \mapsto \phi(x)$. This density function indicates the importance of a quantity to measure at particular point q .

Generated by the sensor positions at time t , \mathcal{P} , we are able to use the Voronoi tessellation of \mathcal{Q} given by

$$V_i(p_i) = \{x \in \mathcal{Q} : \|x - p_i\| \leq \|x - p_j\|, \forall p_j \in \mathcal{P}, j \neq i\}. \quad (1)$$

With this Voronoi partitions, the objective function of the locational optimisation is formulated as ¹

$$H(\mathcal{P}) = \sum_{i=1}^n \int_{V_i} g(x, p_i) \phi(q) dx. \quad (2)$$

The following lemma states the convexity of the objective function of the locational optimization.

Lemma 1 (Sensing Unreliability Function). *Assume that the sensing unreliability function is isotropic, increasing, and convex in $p_i \in \mathcal{P}$, for all $i \in \{1, 2, \dots, n\}$. Then, for a positive density function, the cost function H in 2 is convex.*

In our study, we will utilise the quadratic sensing unreliability function as, $f(\|x - p_i\|) = \|x - p_i\|^2$. With the quadratic function, we may borrow some notions of rigid body motion. Consider the mass, moment of inertia and centroid of i -th Voronoi region expressed as

$$M_{V_i} = \int_{V_i} \phi(x) dx, \quad \mathcal{I}_{V_i} = \int_{V_i} x \phi(x) dx, \quad \text{and} \quad C_{V_i} = \frac{\mathcal{I}_{V_i}}{M_{V_i}}, \quad (3)$$

¹In the following discussion, we use V_i conveniently to refer to $V_i(p_i)$.

respectively. Therefore, applying the parallel-axis theorem of rigid-body motion [20] to the cost function (2) leads to an equivalent expression given by

$$H(p) = \sum_{i=1}^n \mathcal{I}_{V_i} + \sum_{i=1}^n M_{V_i} \|p_i - C_{V_i}\|^2, \quad (4)$$

where $p = [p_1^\top, \dots, p_n^\top]^\top \in \mathbb{R}^{nd}$ denotes the vectorised positions of the robots. The coverage control problem could be considered as a problem that aims to designing control inputs of robots that are capable of driving them towards the optimal positions such that the objective function of the locational optimisation is minimised.

C. Quaternion-based Rotation

To avoid singularity of the Euler-angle-based rotation, the rotational motions of a rigid body are parameterised using quaternions, whose set is denoted by $\mathbb{H} = \{q \in \mathbb{R}^4 | q^\top q = 1\}$. Rotation from frame \mathcal{W}^b to frame \mathcal{W}^a can be represented by quaternion $q_1 \in \mathbb{H}$. The element-wise expression of this quaternion is given by $q_1 = [\eta_1 \quad \bar{q}_1^\top]^\top = [\cos \frac{\vartheta_1}{2} \quad k_1^\top \sin \frac{\vartheta_1}{2}]^\top$, where ϑ_1 is the rotation angle around the unit vector k_1 . In this paper, dot operator represents the quaternion multiplication of quaternions, for example,

$$q_3 = q_1 \cdot q_2, \quad \text{for } q_1, q_2 \in \mathbb{H}. \quad (5)$$

A function $T : \mathbb{H} \rightarrow \mathbb{R}^{4 \times 4}$ is defined as

$$T(q_1) = \begin{bmatrix} \eta_1 & -\bar{q}_1^\top \\ \bar{q}_1 & \eta_1 I + S(\bar{q}_1) \end{bmatrix}, \quad (6)$$

where the cross product between two vectors $v_1, v_2 \in \mathbb{R}^3$ is represented using a skew-symmetric matrix operator $S \in \mathbb{R}^{3 \times 3}$ such that $v_1 \times v_2 = S(v_1)v_2$. Utilising this function, expression (5) becomes

$$q_3 = T(q_1)q_2. \quad (7)$$

The angular velocity of frame \mathcal{W}^a relative to frame \mathcal{W}^b referenced in frame \mathcal{W}^b is defined using $\omega_1 \in \mathbb{R}^3$. The relationship between the time derivative of quaternion q_1 and the angular velocity ω_1 is given by

$$\dot{q}_1 = \frac{1}{2} q_1 \cdot \begin{bmatrix} 0 \\ \omega_1 \end{bmatrix} = \frac{1}{2} T(q_1) \begin{bmatrix} 0 \\ \omega_1 \end{bmatrix} = \frac{1}{2} \bar{T}(q_1) \omega_1, \quad (8)$$

where $\bar{T} = [-\bar{q}_1 \quad (\eta_1 I + S(\bar{q}_1))^\top]^\top$ because the first column of $T(q_1)$ vanishes.

Rotation matrix can also be constructed using quaternion via Rodrigues' formula. A rotation from frame \mathcal{W}^b to \mathcal{W}^a can be formulated as

$$R_1 = I + 2\eta_1 S(\bar{q}_1) + 2S^2(\bar{q}_1). \quad (9)$$

III. PROBLEM FORMULATION

Consider a robotic sensor network consisting of n quadcopters deployed in a convex space $\mathcal{Q} \subseteq \mathbb{R}^d$, and their connection topology represented by a connected undirected graph $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n)$. In this work, we are employing the Delaunay

graph and Delaunay triangulation to generate the Voronoi tessellation in (1). The corresponding Laplacian of this graph is denoted by $\mathcal{L}_n \in \mathbb{R}^{n \times n}$.

In the coverage control problem, the locational optimisation with consensus performance index can be constructed from (2) into

$$\tilde{H}(p) = H(p) + \frac{1}{2}(p - C_V)^\top \hat{\mathcal{L}}_n (p - C_V). \quad (10)$$

with $\hat{\mathcal{L}}_n = \mathcal{L}_n \otimes I_d \in \mathbb{R}^{nd \times nd}$, $p = [p_1^\top, \dots, p_n^\top]^\top \in \mathbb{R}^{nd}$ and $C_V = [C_{V_1}^\top, \dots, C_{V_n}^\top]^\top \in \mathbb{R}^{nd}$.

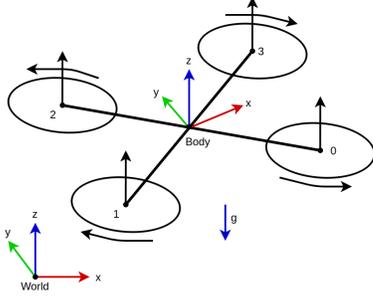


Fig. 1: Coordinate frame of a quadcopter.

Quadcopter $i \in \mathcal{V}_n$ in the network has position, velocity, attitude and angular rate denoted by $p_i \in \mathcal{Q}$, $v_i \in \mathbb{R}^3$, $q_i^c \in \mathbb{H}$ and $\omega_i^c \in \mathbb{R}^3$, respectively. The coordinate frames are illustrated in Fig. 1 where we use the ENU coordinate convention. The motions of a quadcopter can be classified into two components: translation and rotation. The translational motion is imposed by the attitude and the total thrust of the propellers. In inertial frame, the mass-normalised translational dynamics of the quadcopter is governed by

$$\ddot{p}_i = q_i^c \cdot \bar{f}_i \cdot q_i^{c*} - \bar{g} = R_i^c \bar{f}_i - \bar{g}, \quad (11)$$

where $q_i^c \in \mathbb{S}^3$ is the unit quaternion denoting the current attitude of the quadcopter, $\bar{g} = [0 \ 0 \ g]^\top$ the gravitational vector, for g being the gravitational acceleration, and $\bar{f}_i = [0 \ 0 \ f_i]^\top$ the thrust control input, for f_i being the total thrust input. Following (8), the rotational motion of the quadcopter is governed by

$$\dot{q}_i^c = \frac{1}{2} q_i^c \cdot \begin{bmatrix} 0 \\ u_i^\omega \end{bmatrix} = \frac{1}{2} \bar{T}(q_1) u_i^\omega, \quad (12)$$

where in this paper the control input for the rotational motion is the angular rate u_i^ω .

With the transformed constrained optimisation problem and the defined quadcopter dynamics in (11) and (12), the objectives of this work are to design the quaternion-based attitude and distributed coverage controllers which guarantee the convergence within a given settling time independent of the initial values.

IV. FINITE-TIME CONTROL DESIGN

A. Translational Control

In the following control design, we propose a distributed coverage controller with non-static Voronoi centroid that still guarantees the convergence to the optimal position and velocity

in finite time independent of the initial positions with information only from neighbouring agents.

Recall the performance index of the coverage problem as defined in (10). The corresponding optimal point of this optimisation given by $p^* = C_V - \frac{1}{2} M_V^{-1} \hat{\mathcal{L}}_n \tau_v$ for some vector $\tau_v \in \mathbb{R}^{nd}$. Given a connected graph, it follows that the last term vanishes due to the zero eigenvalue of the Laplacian matrix $\hat{\mathcal{L}}_n$ such that $(p_1 - C_{V_1})^* = \dots = (p_n - C_{V_n})^*$. In other words, we could say that the objective function $\tilde{H}(p)$ is optimal when the position of the robots converge to the optimal point $p^* = C_V$ and the consensus is achieved. The optimal position of the velocity is $v^* = \dot{C}_V$

For all agent $i \in \mathcal{V}_n$, consider the following errors: $\tilde{\zeta}_i = [\tilde{p}_i^\top \ \tilde{v}_i^\top]^\top$ and $\tilde{\zeta}_{ij} = [\tilde{p}_{ij}^\top \ \tilde{v}_{ij}^\top]^\top$ where $\tilde{p}_i = p_i - C_{V_i}$, $\tilde{p}_{ij} = \text{sgn}(\tilde{p}_i - \tilde{p}_j) |\tilde{p}_i - \tilde{p}_j|$, $\tilde{v}_i = v_i - \dot{C}_{V_i}$, $\tilde{v}_{ij} = \text{sgn}(\tilde{v}_i - \tilde{v}_j) |\tilde{v}_i - \tilde{v}_j|$. Since there are two terms to optimise in (10), by employing these errors, we design a controller consisting of centroid stabiliser and the consensus stabiliser. The proposed centroid stabiliser, which is responsible for driving the robots toward the centroids, is expressed as

$$u_i^g = -\kappa_g \text{sgn}(\tilde{\zeta}_i) (|\tilde{\zeta}_i|^{\frac{m_v}{n_v}} + |\tilde{\zeta}_i|^{\frac{p_v}{q_v}}), \quad (13)$$

with $\kappa_g = [k_{gp} \ k_{gv}]^\top$, for $k_{gp}, k_{gv} > 0$, and some positive odd integers m_v, n_v, p_v, q_v , for $m_v > n_v$ and $p_v < q_v$. Similarly, the consensus stabiliser, assigned to maintain the consensus, is given by

$$u_i^c = -\kappa_c \sum_{j=1}^n a_{ij} \text{sgn}(\tilde{\zeta}_{ij}) (|\tilde{\zeta}_{ij}|^{\frac{m_v}{n_v}} + |\tilde{\zeta}_{ij}|^{\frac{p_v}{q_v}}), \quad (14)$$

with $\kappa_c = [k_{cp} \ k_{cv}]^\top$ and $k_{cp}, k_{cv} > 0$. Hence, the augmented controller reads

$$u_i^f = u_i^g + u_i^c + \bar{g}. \quad (15)$$

To analyse the performance of the designed control protocol, we require the following lemmas.

Lemma 2 ([17]). *Let $x_1, x_2, \dots, x_n \geq 0$. Then $\sum_{j=1}^n x_j^a \geq (\sum_{i=1}^n x_i)^a$, for $a \in (0, 1)$*

Lemma 3 ([17]). *Let $x_1, x_2, \dots, x_n \geq 0$. Then $\sum_{j=1}^n x_j^a \geq n^{1-a} (\sum_{i=1}^n x_i)^a$, for $a > 1$*

Lemma 4 ([17]). *The equilibrium point of the scalar system*

$$\dot{x} = -\alpha x^{\frac{a}{b}} - \beta x^{\frac{c}{d}}, \quad x(0) = x_0,$$

where $\alpha, \beta > 0$, and a, b, c, d are positive odd integers satisfying $a > b$ and $c < d$, is finite-time stable with the settling time given by

$$T < T_{\max} := \frac{1}{\alpha} \frac{b}{a-b} + \frac{1}{\beta} \frac{d}{d-c}.$$

In the following theorem, we present our next result about the finite-time convergence of the proposed coverage control protocol.

Theorem 1 (Convergence of Finite-time Coverage Controller). *Consider a group of n agents connected via a connected Delaunay graph $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n)$ with agent dynamics defined in (12)*

and (11). Then, there exist some constants $\kappa_1, \kappa_2 > 0$ such that the finite-time coverage problem can be solved by employing the coverage control protocol (15) with settling time given by

$$T_f < T_{\max}^f := \frac{1}{\kappa_1} \frac{n_v}{m_v - n_v} + \frac{1}{\kappa_2} \frac{q_v}{q_v - p_v}, \quad (16)$$

where m_v, n_v, p_v, q_v are positive odd integers satisfying $m_v > n_v$ and $p_v < q_v$.

Proof. Using the translational controller (15), the translational dynamics of the quadcopter can equivalently be expressed as

$$\dot{\zeta}_i = A\tilde{\zeta}_i + Bu_i^f = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tilde{\zeta}_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i^f. \quad (17)$$

Define a Lyapunov function: $V^f(\tilde{\zeta}(t)) = \frac{1}{2} \sum_{i=1}^n \tilde{\zeta}_i^2(t)$ With the system dynamics in (17), the time derivative of the candidate function is given by

$$\dot{V}^f(\tilde{\zeta}) = \dot{V}^g(\tilde{\zeta}) + \dot{V}^c(\tilde{\zeta}). \quad (18)$$

The centroid stabiliser in the first term of (18) can be expanded into

$$\dot{V}^g(\tilde{\zeta}) \leq -\lambda_{\min}^g \sum_{i=1}^n \tilde{\zeta}_i \operatorname{sgn}(\tilde{\zeta}_i) (|\tilde{\zeta}_i|^{\frac{m_v}{n_v}} + |\tilde{\zeta}_i|^{\frac{p_v}{q_v}}), \quad (19)$$

in which we already utilise the smallest eigenvalue of $A - B\kappa_g$ denoted by λ_{\min}^g . By using the fact that $|\tilde{\zeta}_i| = \tilde{\zeta}_i \operatorname{sgn}(\tilde{\zeta}_i)$ along with Lemmas (2) and (3), the centroid stabiliser term could be written as

$$\begin{aligned} \dot{V}^g(\tilde{\zeta}) &\leq -\lambda_{\min}^g \left(n^{\frac{n_v - m_v}{2n_v}} \left(\sum_{i=1}^n \tilde{\zeta}_i^2 \right)^{\frac{m_v + n_v}{2n_v}} + n^{\frac{q_v - p_v}{2q_v}} \left(\sum_{i=1}^n \tilde{\zeta}_i^2 \right)^{\frac{p_v + q_v}{2q_v}} \right) \\ &= -\lambda_{\min}^g \left(n^{\frac{n_v - m_v}{2n_v}} (2V^f)^{\frac{m_v + n_v}{2n_v}} + n^{\frac{q_v - p_v}{2q_v}} (2V^f)^{\frac{p_v + q_v}{2q_v}} \right). \end{aligned} \quad (20)$$

Similarly, the inequality of the consensus stabiliser from the second term of (18) can be expressed as

$$\dot{V}^c(\tilde{\zeta}) \leq -\lambda_{\min}^c \sum_{i=1}^n \tilde{\zeta}_i \sum_{j=1}^n a_{ij} \operatorname{sgn}(\tilde{\zeta}_{ij}) (|\tilde{\zeta}_{ij}|^{\frac{m_v}{n_v}} + |\tilde{\zeta}_{ij}|^{\frac{p_v}{q_v}}), \quad (21)$$

where λ_{\min}^c is the smallest eigenvalue of $A - B\kappa_c$. By utilising the property of the adjacency matrix and also the fact that $|\tilde{\zeta}_{ij}| = \tilde{\zeta}_{ij} \operatorname{sgn}(\tilde{\zeta}_{ij})$, by employing Lemmas 2 and 3, the consensus stabilizer term could be written as

$$\begin{aligned} \dot{V}^c(\tilde{\zeta}) &\leq -\frac{\lambda_{\min}^c}{2} \left(n^{\frac{n_v - m_v}{2n_v}} \left(\sum_{i=1}^n \sum_{j=1}^n a_{ij} \tilde{\zeta}_{ij}^2 \right)^{\frac{m_v + n_v}{2n_v}} \right. \\ &\quad \left. + n^{\frac{q_v - p_v}{2q_v}} \left(\sum_{i=1}^n \sum_{j=1}^n a_{ij} \tilde{\zeta}_{ij}^2 \right)^{\frac{p_v + q_v}{2q_v}} \right) \end{aligned} \quad (22)$$

To analyze the graph, consider two adjacency matrices of connected undirected graphs \mathcal{G}_α and \mathcal{G}_β denoted by $A_\alpha = [a_{ij}^{2n_v/m_v+n_v}] \in \mathbb{R}^{n \times n}$ and $A_\beta = [a_{ij}^{2q_v/p_v+q_v}] \in \mathbb{R}^{n \times n}$, respectively. The corresponding Laplacians are given by \mathcal{L}_α and

\mathcal{L}_β . It follows that the inequality of the consensus stabiliser can equivalently be expressed as

$$\begin{aligned} \dot{V}^c(\tilde{\zeta}) &\leq -\frac{\lambda_{\min}^c}{2} \left(n^{\frac{n_v - m_v}{2n_v}} (2\tilde{\zeta}^\top \mathcal{L}_\alpha \tilde{\zeta})^{\frac{m_v + n_v}{2n_v}} \right. \\ &\quad \left. + n^{\frac{q_v - p_v}{2q_v}} (2\tilde{\zeta}^\top \mathcal{L}_\beta \tilde{\zeta})^{\frac{p_v + q_v}{2q_v}} \right), \end{aligned} \quad (23)$$

with $\tilde{\zeta} = [\tilde{\zeta}_1^\top, \dots, \tilde{\zeta}_n^\top]^\top \in \mathbb{R}^{nd}$. Applying the Courant-Fischer theorem of the Laplacian matrices, $\tilde{\zeta}^\top \mathcal{L}_\alpha \tilde{\zeta} \geq \lambda_2^\alpha \|\tilde{\zeta}\|^2$ and $\tilde{\zeta}^\top \mathcal{L}_\beta \tilde{\zeta} \geq \lambda_2^\beta \|\tilde{\zeta}\|^2$ for $1_{nd}^\top \tilde{\zeta} = 0_{nd}$, leads (23) to

$$\begin{aligned} \dot{V}^c(\tilde{\zeta}) &\leq -\frac{\lambda_{\min}^c}{2} \left(n^{\frac{n_v - m_v}{2n_v}} (4\lambda_2^\alpha V^f)^{\frac{m_v + n_v}{2n_v}} \right. \\ &\quad \left. + n^{\frac{q_v - p_v}{2q_v}} (4\lambda_2^\beta V^f)^{\frac{p_v + q_v}{2q_v}} \right). \end{aligned} \quad (24)$$

By adding (20) and (24) followed by some re-arrangements, the time derivative of the Lyapunov function can be written as

$$\begin{aligned} \dot{V}^f(\tilde{\zeta}) &\leq -\frac{1}{2} n^{\frac{n_v - m_v}{2n_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\alpha)^{\frac{m_v + n_v}{2n_v}}) (2V^f)^{\frac{m_v + n_v}{2n_v}} \\ &\quad - \frac{1}{2} n^{\frac{q_v - p_v}{2q_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\beta)^{\frac{p_v + q_v}{2q_v}}) (2V^f)^{\frac{p_v + q_v}{2q_v}}. \end{aligned} \quad (25)$$

By denoting $\xi = \sqrt{2V^f}$ and $\dot{\xi} = 2\dot{V}^f/\sqrt{2V^f}$ for $V^f(\tilde{\zeta}) \neq 0$, we have

$$\begin{aligned} \dot{\xi} &\leq -\frac{1}{2} n^{\frac{n_v - m_v}{2n_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\alpha)^{\frac{m_v + n_v}{2n_v}}) \xi^{\frac{m_v + n_v}{n_v}} \\ &\quad - \frac{1}{2} n^{\frac{q_v - p_v}{2q_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\beta)^{\frac{p_v + q_v}{2q_v}}) \xi^{\frac{p_v + q_v}{q_v}}. \end{aligned} \quad (26)$$

Choosing positive odd integers m_v, n_v, p_v, q_v satisfying $m_v > n_v$ and $p_v < q_v$ and employing Lemma 4 with the Comparison Principle [21] yield the boundary of the settling time expressed as

$$T_f < T_{\max}^f := \frac{1}{\kappa_1} \frac{n_v}{m_v - n_v} + \frac{1}{\kappa_2} \frac{q_v}{q_v - p_v},$$

with

$$\begin{aligned} \kappa_1 &= \frac{1}{2} n^{\frac{n_v - m_v}{2n_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\alpha)^{\frac{m_v + n_v}{2n_v}}), \text{ and} \\ \kappa_2 &= \frac{1}{2} n^{\frac{q_v - p_v}{2q_v}} (2\lambda_{\min}^g + \lambda_{\min}^c (2\lambda_2^\beta)^{\frac{p_v + q_v}{2q_v}}). \end{aligned}$$

It can be observed that the system is finite-time stable, i.e., $\lim_{t \rightarrow T_{\max}^f} V^f(\tilde{\zeta}) = 0$, implying that $\lim_{t \rightarrow T_{\max}^f} \|\tilde{\zeta}\| = 0$. \square

By utilising the translational control input in (15), we may obtain the thrust via $f_i = (u_i^f)^\top R_i^c [0 \ 0 \ 1]^\top$.

B. Rotational Control

Since the translational motion depends on the rotational motion, we need to design the attitude controller which guarantees the finite-time stability.

Given the current and desired attitudes of i -th quadcopter denoted by $q_i^c = [\eta_i^c \ \bar{q}_i^{c\top}]^\top$ and $q_i^d = [\eta_i^d \ \bar{q}_i^{d\top}]^\top$, respectively, the error quaternion can be obtained via $q_i^e = q_i^{c*} \cdot q_i^d = T(q_i^{c*})q_i^d = [\eta_i^e \ \bar{q}_i^{e\top}]^\top$. For controller analysis, an error vector is also defined as follows:

$$e_{q_i^e} = \begin{bmatrix} 1 \mp \eta_i^e \\ \bar{q}_i^e \end{bmatrix}. \quad (27)$$

Differentiating this error yields the error dynamics expressed as

$$\dot{e}_{q_i^e} = \frac{1}{2} \bar{T}(q_i^e) u_i^\omega. \quad (28)$$

In this attitude control scheme, by employing the error vector, the angular-rate control command is defined as

$$u_i^\omega = -\kappa_\omega \text{sgn}(\tilde{e}_{q_i^e}) \left(|\tilde{e}_{q_i^e}|^{\frac{m_w}{n_w}} + |\tilde{e}_{q_i^e}|^{\frac{p_w}{q_w}} \right) \quad (29)$$

with $\tilde{e}_{q_i^e} = [\bar{T}(q_i^e)]^\top e_{q_i^e}$ and $\kappa_\omega > 0$.

The following theorem states our first result on the attitude controller of a quadcopter.

Theorem 2 (Convergence of Finite-time Rotational Controller). *Let the attitude dynamics of a quadcopter be given by (12) and the error vector between the current and desired attitudes given by (27). Then, given the control protocol (29), there exist some positive constants κ_ω such that the equilibrium point of the error vector is finite-time stable with settling time given by*

$$T_a < T_{\max}^a := \frac{1}{\kappa_\omega} \left(\frac{n_w}{m_w - n_w} + \frac{q_w}{q_w - p_w} \right), \quad (30)$$

where m_w, n_w, p_w, q_w are positive odd integers satisfying $m_w > n_w$ and $p_w < q_w$.

Proof. Define a Lyapunov function:

$$V^a(e_{q_i^e}) = \frac{1}{2} e_{q_i^e}^\top e_{q_i^e}. \quad (31)$$

Taking the derivative of the Lyapunov function yields

$$\dot{V}^a(e_{q_i^e}) = -\kappa_\omega \frac{1}{2} \tilde{e}_{q_i^e}^\top \text{sgn}(\tilde{e}_{q_i^e}) \left(|\tilde{e}_{q_i^e}|^{\frac{m_w}{n_w}} + |\tilde{e}_{q_i^e}|^{\frac{p_w}{q_w}} \right), \quad (32)$$

where $\tilde{e}_{q_i^e} = [\bar{T}(q_i^e)]^\top e_{q_i^e}$ and the error dynamics with the proposed control command have been utilised. Since $|\tilde{e}_{q_i^e}| = \tilde{e}_{q_i^e}^\top \text{sgn}(\tilde{e}_{q_i^e})$, (32) can be expressed as

$$\dot{V}^a(e_{q_i^e}) = -\kappa_\omega \frac{1}{2} \left(\left(\tilde{e}_{q_i^e}^2 \right)^{\frac{m_w + n_w}{2n_w}} + \left(\tilde{e}_{q_i^e}^2 \right)^{\frac{p_w + q_w}{2q_w}} \right) \quad (33)$$

Substituting $2V^a = \tilde{e}_{q_i^e}^2$ to 33 leads to

$$\dot{V}^a(e_{q_i^e}) = -\kappa_\omega \frac{1}{2} \left((2V^a)^{\frac{m_w + n_w}{2n_w}} + (2V^a)^{\frac{p_w + q_w}{2q_w}} \right). \quad (34)$$

By taking $\varrho = \sqrt{2V^a}$ and $\dot{\varrho} = 2\dot{V}^a / \sqrt{2V^a}$, for $V^a > 0$, (33) can equivalently be rewritten as

$$\dot{\varrho} = -\kappa_\omega \varrho^{\frac{m_w}{n_w}} - \kappa_\omega \varrho^{\frac{p_w}{q_w}}. \quad (35)$$

Therefore, utilising the comparison principle [21] and Lemma 4 with some positive odd integers m_w, n_w, p_w, q_w , for $m_w > n_w$ and $p_w < q_w$, we may conclude that the settling time of the attitude system can be expressed as

$$T_a < T_{\max}^a := \frac{1}{\kappa_\omega} \left(\frac{n_w}{m_w - n_w} + \frac{q_w}{q_w - p_w} \right),$$

and the system is finite-time stable, i.e., $\lim_{t \rightarrow T_{\max}^a} V^a(e_{q_i^e}) = 0$, implying that $\lim_{t \rightarrow T_{\max}^a} \|e_{q_i^e}\| = 0$ \square

Based on Eqs. (30) and (16), the computation of the boundary of the settling time of this coverage controller is indeed dependant to some controller parameters and the algebraic graph

topology but independent to the initial values. Furthermore, the quadcopters will reach the optimal position and velocity within the settling time $T_{\text{sys}} = T^a + T^f < T_{\max}^a + T_{\max}^f$.

To obtain the desired quaternion, given translational control input (15) and desired heading ψ_i^d , let a heading vector $x_i^c = [\cos \psi_i^d \sin \psi_i^d 0]^\top$. Then, we may have a rotation matrix $R_i^d = [x_i^d \ y_i^d \ z_i^d]$ composed of $z_i^d = u_i^f / \|u_i^f\|$, $y_i^d = z_i^d \times x_i^c / \|z_i^d \times x_i^c\|$, and $x_i^d = y_i^d \times z_i^d / \|y_i^d \times z_i^d\|$.

V. SIMULATION

In this section, a numerical simulation is provided for validation of the proposed control protocols. The simulation is performed on Gazebo simulator with Mavros controller package and Robot Operating System (ROS) [23]. A computer with Linux-based operating system, 3.2-GHz processor, and 16-GB RAM is utilised to run the simulation. The screenshot of the simulator can be observed in Fig. 2.

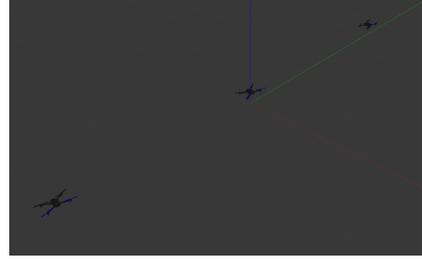


Fig. 2: Quadcopters on Gazebo Simulator.

In this simulation, we deploy 16 quadcopters randomly in a space. These quadcopters are equipped with sensors to acquire a planar space whose boundaries are $\{(0, 0), (0, 2), (2, 2), (2, 0)\}$. We also consider the altitude of the quadcopter to be constant and the quadcopters will only adjust the $x - y$ position and velocity. The planar information distribution of this scenario is uniform, that is, $\phi(x, y) = 1$. The parameters of the controller are $m_v = m_w = 5, n_v = n_w = 3, p_v = p_w = 3, q_v = q_w = 5, \kappa_\omega = 0.8, \kappa_g = 0.5, \kappa_c = 0.4$. The Delaunay graph used in this scenario has the smallest non-zero eigenvalue of $\lambda_2 = 0.4615$. Based on Theorems 2 and 1, we may obtain the estimate maximum settling time is $T_{\text{sys}} = 20.21\text{s}$.

By applying the control protocols in (29) and (15) to the quadcopter as modelled in (12) and (11), we obtain the resulting trajectory of robots and the Voronoi partition illustrated in Fig. 3. We can also observe the error convergence trajectory $\|p_i - C_{V_i}\|$ and the objective function of this finite-time case depicted in Figs. 4a and 4b, respectively.

Fig. 3 demonstrates that the controller has successfully drive the robots such that their positions align with their centroids. Since the density function is uniform within the boundary, we observe that the number of robot deployed per a unit area is constant. Fig. 4a verifies that the error between the position and the optimal position is minimised before the expected settling time T_{\max} . In Fig. 4b, we also see the convergence of the objective function to an optimal value when the centroids are reached. These simulation results verifies that the protocol (15) can successfully solve the coverage control problem and drive

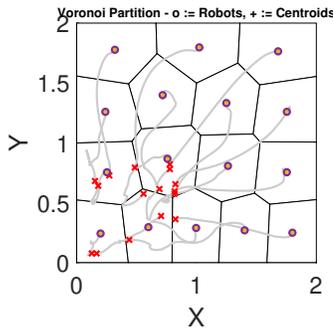
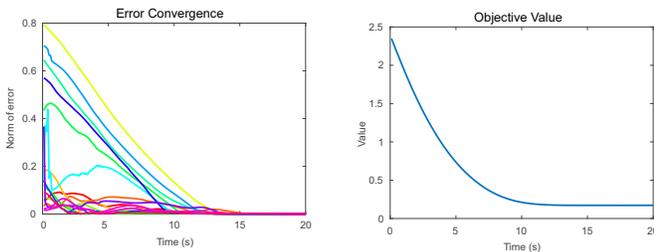


Fig. 3: Trajectories and optimal centroidal Voronoi regions with double-peak distribution.



(a) Convergence result of the centroid errors of coverage problem with double-peak distribution.

(b) Convergence result of the objective function of coverage problem with double-peak distribution.

Fig. 4: Convergence result of the objective function of coverage problem with double-peak distribution.

the quadcopters close to the optimal positions with a finite-time convergence.

VI. CONCLUSION

In this work, some control protocols to overcome the coverage control problem of quadcopter sensor networks with finite-time stability in a distributed manner have been investigated. By reformulating the locational optimisation problem, the translational control protocol was designed to drive the quadcopters to follow the position and velocity of the Voronoi centroid from the coverage control problem. The translational control command was then supplied to the rotational controller to calculate the desired attitude of the quadcopter. Since the planar translation of the quadcopter was coupled with its attitude, we also proposed a rotational control protocol for each quadcopter based on quaternion to follow the desired attitude. Those translational and rotational protocols were carefully analysed using the finite-time stability theory to ensure that the quadcopters position and velocity converge to the Voronoi centroid position and velocity within a designed settling time, independent of the initial values. Through simulation on the Gazebo simulator with ROS, we have validated the performance of the proposed control protocols.

REFERENCES

[1] A. Okabe, A. Suzuki, Locational optimization problems solved through Voronoi diagrams, *European Journal of Operational Research* 98 (3) (1997) 445–456. doi:10.1016/S0377-2217(97)80001-X.

[2] J. Cortes, S. Martinez, T. Karatas, F. Bullo, Coverage Control for Mobile Sensing Networks, *IEEE Transactions on Robotics and Automation* 20 (2) (2004) 243–255. doi:10.1109/TRA.2004.824698.

[3] S. Salhi, Facility Location: A Survey of Applications and Methods, *Journal of the Operational Research Society* 47 (11) (1996) 1421–1422. doi:10.1057/jors.1996.180.

[4] S. G. Lee, Y. Diaz-Mercado, M. Egerstedt, Multirobot Control Using Time-Varying Density Functions, *IEEE Transactions on Robotics* 31 (2) (2015) 489–493. doi:10.1109/TRO.2015.2397771.

[5] J. Cortés, S. Martínez, F. Bullo, Spatially-distributed coverage optimization and control with limited-range interactions, *ESAIM - Control, Optimisation and Calculus of Variations* 11 (4) (2005) 691–719. arXiv:0401297, doi:10.1051/cocv:2005024.

[6] L. C. Pimenta, V. Kumar, R. C. Mesquita, G. A. Pereira, Sensing and coverage for a network of heterogeneous robots, *Proceedings of the IEEE Conference on Decision and Control* (2) (2008) 3947–3952. doi:10.1109/CDC.2008.4739194.

[7] H. F. Parapari, F. Abdollahi, M. B. Menhaj, Coverage control in non-convex environment considering unknown non-convex obstacles, *2014 2nd RSI/ISM International Conference on Robotics and Mechatronics, ICRoM 2014* (2014) 119–124. doi:10.1109/ICRoM.2014.6990887.

[8] Y. Kantaros, M. Thanou, A. Tzes, Distributed coverage control for concave areas by a heterogeneous Robot-Swarm with visibility sensing constraints, *Automatica* 53 (2015) 195–207. doi:10.1016/j.automatica.2014.12.034.

[9] S. Martinez, Distributed interpolation schemes for field estimation by mobile sensor networks, *IEEE Transactions on Control Systems Technology* 18 (2) (2010) 491–500. doi:10.1109/TCST.2009.2017028.

[10] M. Schwager, M. P. Vitus, S. Powers, D. Rus, C. J. Tomlin, Robust adaptive coverage control for robotic sensor networks, *IEEE Transactions on Control of Network Systems* 4 (3) (2017) 462–476. doi:10.1109/TCNS.2015.2512326.

[11] Y. Kantaros, M. M. Zavlanos, Distributed communication-aware coverage control by mobile sensor networks, *Automatica* 63 (2016) 209–220. doi:10.1016/j.automatica.2015.10.035.

[12] Y. Mei, Y. H. Lu, Y. C. Hu, C. S. Lee, Deployment strategy for mobile robots with energy and timing constraints, *Proceedings - IEEE International Conference on Robotics and Automation 2005* (3) (2005) 2816–2821. doi:10.1109/ROBOT.2005.1570540.

[13] S. P. Bhat, D. S. Bernstein, Finite-time stability of continuous autonomous systems, *SIAM Journal on Control and Optimization* 38 (3) (2000) 751–766. doi:10.1137/S0363012997321358.

[14] S. Khoo, L. Xie, Z. Man, Robust finite-time consensus tracking algorithm for multirobot systems, *IEEE/ASME Transactions on Mechatronics* 14 (2) (2009) 219–228. doi:10.1109/TMECH.2009.2014057.

[15] H. Du, C. Yang, R. Jia, Finite-time formation control of multiple mobile robots, *6th Annual IEEE International Conference on Cyber Technology in Automation, Control and Intelligent Systems, IEEE-CYBER 2016* (2016) 416–421. doi:10.1109/CYBER.2016.7574861.

[16] J. Wang, H. Liang, Z. Sun, S. Zhang, M. Liu, Finite-time control for spacecraft formation with dual-number-based description, *Journal of Guidance, Control, and Dynamics* 35 (3) (2012) 950–962. doi:10.2514/1.54277.

[17] Z. Zuo, L. Tie, A new class of finite-time nonlinear consensus protocols for multi-agent systems, *International Journal of Control* 87 (2) (2014) 363–370. doi:10.1080/00207179.2013.834484.

[18] Z. Zuo, Nonsingular fixed-time consensus tracking for second-order multi-agent networks, *Automatica* 54 (2015) 305–309. doi:10.1016/j.automatica.2015.01.021.

[19] C. Wang, H. Tnunay, Z. Zuo, B. Lennox, Z. Ding, Fixed-Time Formation Control of Multirobot Systems: Design and Experiments, *IEEE Transactions on Industrial Electronics* 66 (8) (2019). doi:10.1109/TIE.2018.2870409.

[20] A. R. Abdulghany, Generalization of parallel axis theorem for rotational inertia, *American Journal of Physics* 85 (10) (2017) 791–795. doi:10.1119/1.4994835.

[21] H. K. Khalil, *Nonlinear systems*; 3rd ed., Prentice-Hall, Upper Saddle River, NJ, 2002.

[22] R. Brescianini, Dario; Hehn, Markus; D’Andrea, *Nonlinear Quadcopter Attitude Control Technical Report* (2020) 0–11.

[23] J. Lim, *mavros_controllers - Aggressive trajectory tracking using mavros for PX4 enabled vehicles* (Mar. 2019). doi:10.5281/zenodo.2652888.