

A branch-price-and-cut algorithm for the Multi-Commodity two-echelon Distribution Problem

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1 Introduction and problem definition

The *Multi-Commodity two-echelon Distribution Problem* (MC2DP) considers a two-echelon distribution system composed of a set of suppliers \mathcal{S} , a set of distribution centres \mathcal{D} and a set of customers \mathcal{C} , where collection and delivery operations are performed. Specifically, $|\mathcal{K}|$ commodities are collected from the suppliers, sent to the distribution centres for consolidation purposes and delivered to the customers to fulfill their requests. For each commodity $k \in \mathcal{K}$, each supplier $i \in \mathcal{S}$ provides an amount $P_{ik} \geq 0$ of k , and each customer $j \in \mathcal{C}$ has a request $R_{jk} \geq 0$ for k . The collection operations are performed by an unlimited fleet of homogeneous vehicles of capacity Q^1 with direct trips from the suppliers to the distribution centres. Conversely, each distribution centre owns an unlimited fleet of homogeneous vehicles of capacity Q^2 performing routes to deliver the commodities to the customers. All vehicles can transport any set of commodities as long as their capacity is not exceeded. In addition, as in the *Commodity constrained Split Delivery Vehicle Routing Problem* (C-SDVRP) (see [1]), customers can be visited multiple times. However the request for a given commodity has to be delivered in a single visit. The aim of the MC2DP is to fulfill the customer requests not exceeding the vehicle capacities, and the available commodity amounts at the suppliers and such that the overall transportation cost is minimized.

The MC2DP can be framed in the class of the two-echelon routing problems ([2]), and

shares similarities with the *2-Echelon Capacitated Vehicle Routing Problem* (2E-CVRP). The 2E-CVRP considers a *one-to-many* setting, where a single commodity has to be delivered from a depot to a set of customers through distribution centres by means of two level of routing decisions. Conversely, the MC2DP involves multiple commodities which have to be collected from multiple suppliers and delivered to the customers (*many-to-many* setting). In addition, in the 2E-CVRP, the link between the collection and delivery echelons is commonly done by the *load synchronization* strategy ([3]): the amount of commodity sent to each distribution centre must be sufficient to serve the customers assigned to that distribution centre. In the MC2DP, the same synchronization strategy is applied, but it is associated with each commodity.

The MC2DP was presented in [4], where the authors proposed a heuristic sequential approach (collection first, delivery second). Differently from [4], we devise an exact algorithm based on a *branch-price-and-cut* (BPC).

2 Problem formulation

We define the MC2DP on a directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V} = \mathcal{S} \cup \mathcal{D} \cup \mathcal{C}$ and $\mathcal{A} = (\mathcal{S} \times \mathcal{D}) \cup (\mathcal{D} \times \mathcal{S}) \cup (\mathcal{D} \times \mathcal{C}) \cup (\mathcal{C} \times \mathcal{D})$. Each arc $(i, j) \in \mathcal{A}$ is associated with cost $C_{ij} \geq 0$. For each distribution centre $o \in \mathcal{D}$, set \mathcal{R}_o contains the feasible routes that a vehicle owned by o can perform, i.e., the non-empty cycles in the delivery echelon starting and ending at o such that the total amount of delivered commodities does not exceed vehicle capacity Q^2 . Finally, C_r is the cost associated with route r and a_{jk}^r is a binary parameter taking value 1 if r delivers commodity k to customer j and 0 otherwise.

We introduce the following variables. For all distribution centres $o \in \mathcal{D}$ and suppliers $i \in \mathcal{S}$, integer variable x_{oi} represents the number of vehicles traversing arc (o, i) . For all $o \in \mathcal{D}$, $i \in \mathcal{S}$ and $k \in \mathcal{K}$, non-negative continuous variable q_{oi}^k stores the amount of commodity k loaded at supplier i and sent to distribution centre o . Finally, for all $o \in \mathcal{D}$ and $r \in \mathcal{R}_o$, binary variable λ_r takes value 1 if route r is selected in the solution and 0 otherwise. Our extended formulation, referred to as Master Problem (MP), reads

$$\min \sum_{(o,i) \in \mathcal{A}^1} (C_{oi} + C_{io})x_{oi} + \sum_{o \in \mathcal{D}} \sum_{r \in \mathcal{R}_o} C_r \lambda_r \quad (1)$$

$$\sum_{o \in \mathcal{D}} q_{oi}^k \leq P_{ik} \quad \forall i \in \mathcal{S}, \forall k \in \mathcal{K} \quad (2)$$

$$\sum_{k \in \mathcal{K}} q_{oi}^k \leq Q^1 x_{oi} \quad \forall o \in \mathcal{D}, \forall i \in \mathcal{S} \quad (3)$$

$$\sum_{o \in \mathcal{D}} \sum_{r \in \mathcal{R}_o} a_{jk}^r \lambda_r \geq 1 \quad \forall j \in \mathcal{C}, \forall k \in \mathcal{K} \text{ s.t. } R_{jk} > 0 \quad (4)$$

$$\sum_{i \in \mathcal{S}} q_{io}^k \geq \sum_{r \in \mathcal{R}_o} \sum_{j \in \mathcal{C}} R_{jk} a_{jk}^r \lambda_r \quad \forall o \in \mathcal{D}, \forall k \in \mathcal{K}, \quad (5)$$

$$x_{oi} \in \mathbb{Z}_{\geq 0}, \forall o \in \mathcal{D}, \forall i \in \mathcal{S} \quad q_{oi}^k \in \mathbb{R}_{\geq 0}, \forall o \in \mathcal{D}, \forall i \in \mathcal{S}, \forall k \in \mathcal{K} \quad \lambda_r \in \{0, 1\}, \forall r \in \mathcal{R}_o, \forall o \in \mathcal{D}. \quad (6)$$

Objective function (1) minimizes the overall transportation cost. Constraints (2) guarantee that the amount of each commodity available at each supplier is respected. Constraints (3) ensure that a sufficient number of vehicles performs the collection operations. Constraints (4) are the *covering constraints* ensuring that customer requests are fulfilled. Constraints (5) impose the load synchronization strategy linking the collection and delivery echelons at the distribution centres. Constraints (6) define the variables.

3 A branch-price-and-cut algorithm

To solve the MP introduced in Section (2), we design a BPC algorithm, where, at each column generation iteration, we price λ_r variables. Specifically, we solve the pricing problem $\min\{\bar{C}_r \mid r \in \mathcal{R}_o, o \in \mathcal{D}\}$, where \bar{C}_r is the reduced cost of λ_r , by decomposing it per distribution centre. Solving $\min\{\bar{C}_r \mid r \in \mathcal{R}_o\}$ reduces to solve an *Elementary Shortest Path Problems with Resource Constraints* (ESPPRC) (see [5]). We tackle the ESPPRC through a label setting dynamic programming algorithm which incorporates the *ng-path* relaxation and an implicit version of the bidirectional labeling search. Our BPC implements several classical and advanced techniques ([6]) to accelerate the column generation procedure: the automatic dual pricing smoothing stabilization, a multi-phase strong branching procedure and three heuristic approaches to solve the ESPPRC. Two of these are similar to the graph reduction heuristics proposed in [5]. The third one is a novel two-phase heuristic. The first phase computes a lower bound on the value of $\min\{\bar{C}_r \mid r \in \mathcal{R}_o\}$, $o \in \mathcal{D}$ and a set of promising customer sequences: it does so by solving the ESPPRC on a modified graph, where customers are delivered with their least requested commodity. The second phase retrieves the route to be inserted in the MP by solving the ESPPRC again on several acyclic graphs, one for each customer sequence computed in the previous phase. Finally, before starting the branching phase, we look for violated valid inequalities. We consider the capacity constraints and two new families of valid inequalities based respectively on the set covering polytope and on the number partitioning problem polytope.

4 Preliminary computational results

Table 1 shows preliminary results obtained by running our BPC algorithm on six small instances considered in [4]. We compare our results with those obtained by solving the compact formulation of the MC2DP presented in [4] with the commercial solver CPLEX 12.8. For both approaches, we report the upper and lower bounds returned after a time limit of one hour and either the computational time, if the instance is solved to optimality (“-” in column LB), or the optimality gap is computed as $\frac{UB-LB}{LB} \cdot 100$. The BPC algorithm outperforms CPLEX in terms of time and solution quality: it optimally solves all instances in short computation times.

Instance				CPLEX			BPC		
$ S $	$ D $	$ C $	$ K $	UB	LB	t(s)/gap(%)	UB	LB	t(s)/gap(%)
4	2	10	2	579.52	549.03	5.55%	579.52	-	0.75s
4	2	10	2	562.34	-	1441s	562.34	-	0.47s
4	2	10	2	663.52	-	2261s	663.52	-	0.32s
4	2	15	2	771.34	654.28	17.89%	742.71	-	1.39s
4	2	15	2	790.28	682.49	15.79%	784.05	-	296.14s
4	2	15	2	896.12	742.96	20.62%	892.96	-	1.11s

Table 1: Preliminary computational results.

As future research direction, we plan to generalize the MC2DP by considering route decisions also in the collection echelon and, hence, to adapt our BPC algorithm accordingly.

References

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