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Towards a Daytime and Low-altitude Stellar Positioning System : challenges and first results

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Abstract.

Since the emergence of electronic image sensors, stellar pictures can be easily processed by computers to identify stars and measure angles : distance between light spots on the same scene or angle above the visible horizon, like the sextant does for celestial navigation.

Adding an inclinometer on the camera axis, the computer is now able to calculate the attitude angles (course, trim and list) in a local coordinate system and the position (latitude, longitude and even altitude). The last coordinate, time, may be measured by observing angular distances on the celestial sphere between stars located near the orbit of a satellite (Moon or LEO). If the sensor is on a vehicle, gyroscopes are necessary to smooth the rotations or vibrations.

Going from this appealing theory to the reality of a daytime stellar positioning system hides many challenges such as detecting stars by day, at low altitude or in bad weather. If the level of accuracy increases, many sources of error emerge, e.g. sky background, diffraction limit, vertical deflection.

Many prototypes or manufactured products have shown the potential of a stellar positioning system thanks to the maturity of technologies like inertial sensors and silicon CMOS detectors. Most of these applications are designed for altitude operations such as :

- yield the attitude angles of satellites, stratospheric balloons, high altitude aircraft or missiles ;
- adjust the inertial system of civil aircraft ;
- measure satellite parameters from immobile ground telescopes in altitude ;
- but rarely for positioning.

The last challenge lies in tracking stars from a moving vehicle during daytime at low altitude : sea or land. The lower layers of atmosphere add numerous perturbations, forcing to work by day in infrared band, where silicon sensitivity ends and InGaAs sensitivity starts (or other technologies).

This paper aims at looking for a compromise between detector technologies, stars detectability, duration of

availability and accuracy of results for a stellar positioning system on a moving vehicle on the ground or at sea.

A first parameter is the transmission of the atmosphere : when wavelength increases, scattering decreases but thermal emissions in infrared submerge starlight. Some bands obscure it but transmit photons from molecular resonance fluorescence of sunlight. A rigorous simulation of radiative transfer with local weather parameters is necessary to figure out the best band of transmission.

Secondly, the number of observable stars in each band, with a given magnitude or flux, increases with the wavelength... except in R (0.6-0.7 μ m) and I (0.7-1.0 μ m) bands where we have few data in star catalogs : the magnitudes of many stars have been calculated by comparing their known data and their spectral type, confirming the tendency.

Lastly, silicon technology for light detection is well developed thanks to public cameras and smartphones : the size and number of pixels on a sensor allow a high accuracy on angular distance measurements. In the opposite, Short Wave InfraRed detectors (SWIR: 1.0–2.5 μ m) suffer bigger pitch and poor definition. But the potential of SWIR detectors remains high, and their performances are improving quickly. Betting on this ascending technology readiness, daytime star tracking tests have been realized : simulations are confirmed, and new challenges occur.

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Introduction.

Celestial navigation consists in fixing one's position by observing stars, planets or the Moon. This study is part of the MARIS STELLA project, aiming at providing a stellar receiver for maritime navigation and, further, to compute position, attitude and time for any vehicle on land.

The system will face many challenges to work by night and by day, to detect the light of stars among the noise of the sky background or the mask of bad weather, to choose a reference of horizon in spite of the motion of the ship and, obviously, to remain quick and accurate. This article will focus on the stars we can observe from the Earth in order to look for an optimum among the parameters of accuracy, magnitude and spectral bands.

1 – Observation tool

Celestial navigation on a ship is considered as a backup system if satellite positioning is not available for a long time when crossing an ocean. For this purpose, a poor accuracy is tolerated around 2 nautical miles, justified by the hardy tools : stars observed with human eye through a sextant above the visible horizon line.

Beside the backup aspect, celestial observation may be realized by a camera, which optical sensor is mainly build with silicon technology. The spectral response of such sensor is wider than the human eye, which « technology » is based on a combination of cones and rods. The figure 1 compares these sensors, showing that a camera is able to detect more light wave-lengths and has a higher sensitivity than human eye.

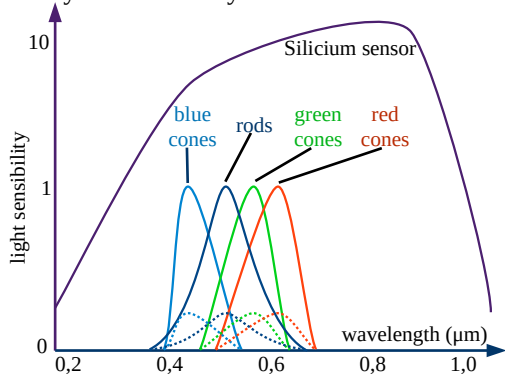


Fig 1 – Comparison of the spectral sensitivity of human eye and Silicon sensor

Furthermore, the numerous technologies of light detectors allow to deal with other bands of wavelength but we will shrink the choice to components off the shelf :

- silicon, very common on personal equipment like smartphone ;
- InGaAs for science and industry use ;
- HgCdTe soon better mastered.

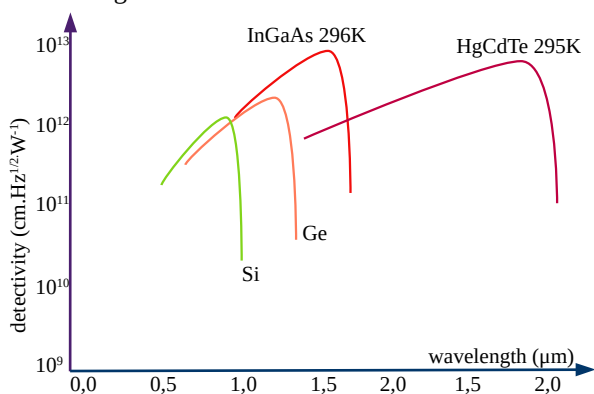


Fig 2 - Comparison of the detectivity of available detectors [1]

Silicon sensors are wide spread and well known but InGaAs and even more HgCdTe technologies are quickly improving their performances (actual mid-range segment) :

- the **size of a pixel** is small (pitch around 3 μm vs 15 μm for InGaAs or HgCdTe) ; these features allow a good **angular resolution** (for a 300 mm focal length : 2 arcsec per pixel vs 10 arcsec for others) ;
- the **definition** is high (9602x6498 vs 640x512 for InGaAs or 1280x1024 for HgCdTe) ; this is an asset to yield a wide **field of view** with many stars and hence to allow an **identification** of stars ;
- the price is around 5 000 € for silicon 9602x6498 and InGaAs 640x512 but 50 000 € for HgCdTe 640x512, impeding tests without science funding or military grant ;
- the operating temperature from -20 to +70°C doesn't need cooling for silicon sensors but InGaAs needs one or two PELTIER thermo-electric cooler to reach -20° to -40°C and nitrogen or helium cryostat for HgCdTe to work at -100° to -200°C with a good sensitivity (low dark current) ;
- the number of frames per second is almost the same for the 3 technologies ;
- dynamic range (full well capacity) is roughly equal ;
- the read-out noise is 3 e^- vs 30 e^- for infrared sensors but these are reducing this drawback ;
- quantum efficiency is nearly equivalent.

Infrared technologies are quickly refining their definition and pitch but the size limit is set by wavelength.

An angular resolution of 1 arcsec for the measure of the direction of a star means an error of 31 m on the final position. Additional errors come from horizontal reference, almanac and global calculation.

2 – Observation through the atmosphere

Star light is observed after the propagation through different layers of the atmosphere, inducing many perturbations : scattering, absorption, diffraction, blinding by Sun or Moon light.

Scattering mainly depends on the ratio of air molecule size to wavelength : the Mie parameter $x = \frac{2\pi r}{\lambda}$.

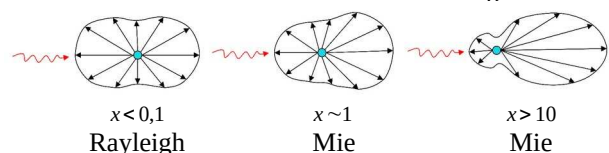


Fig 3 – Rayleigh or Mie scattering according to Mie parameter [2]

The star light is spread out in all directions, weakening the intensity observed by a detector.

Many molecules in the atmosphere may change their energy level by absorbing photons around a resonance wavelength, depending on the size and geometrical structure of the molecule. The vibration of atoms inside the molecule (bent, stretch or spin motions) and the

collisions between molecules contributes to this energy shift under pressure and temperature. Energy is released by fluorescence, emitting a photon with equal or longer wavelength.

When particles of dust and aerosols are much wider than the wavelength, the light is absorbed and changed into heat. The energy is released by emitting light in other directions with various wavelengths.

These phenomena create a background noise able to decrease the contrast with the star light.

Some layers of the atmosphere are moving with wind and turbulence, tilting the light beam and splitting the Airy spot of a star in a wider puzzle. The energy is spread wide around and the signal to noise ratio decreases. Even on a short time observation, the barycentre of the puzzle pieces may be different of the true position of the star !

Sun light is also scattered by the atmosphere. The intensity of this stray light is higher by day than most of the stars in visible band and blue wave-length 0.3 μm above all : stars are drowned in sky radiance.

Atmosphere, earth, lens and many parts of the camera are emitting radiations whose wavelength depends on their temperature like a black body. This emission of thermal infrared above 1.7 μm adds noise and decreases the contrast with star light.

The spectrum of transmission of the whole atmosphere shows absorption and scattering lines or bands where a star light faints or is obscured. Between these limits, remaining windows are described by photometric systems of bands B, V, R, I, J, H and K between 0.3 and 2.2 μm .

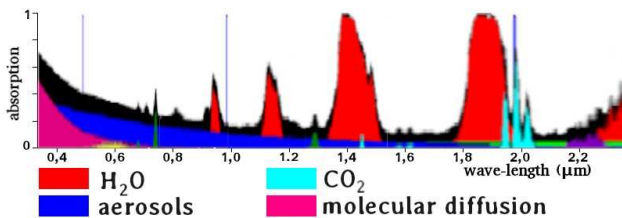


Fig 4 – Absorption of light by the atmosphere between 0.3 and 2.3 μm [4]

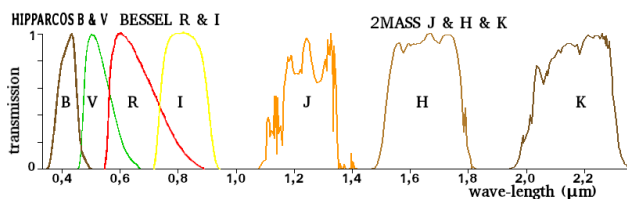


Fig 5 – Optical bands between 0.3 and 2.3 μm [3]

By day, in visible light (0.4 to 0.7 μm), the sky background is more radiant than stars. To see a bright star by daylight at low altitude, you need to reduce the surface of blinding background thus the field of view (increasing the focal length) and enhance the flux of photons (widen the diameter of the primary lens). Vega (magnitude 0) is detectable by day with 1 870 mm focal length and 400 mm of lens diameter (and a good baffle !).

From a higher altitude (greater than 3 000 m) you can detect it with smaller features but thanks to a harsh post-

processing to increase the signal to noise ratio : histogram reduction around the star spot intensity, stack of numerous images to dig the contrast... what gets shaky if the detector is rolling or pitching with its carrier (man, vehicle).

At 30-45 km high (stratospheric balloon) or in space (satellite, probe), the atmosphere density is low : stars are visible as by night if the camera is pointing far from the Earth, Moon or Sun.

NIR (0.75-1 μm) and SWIR (1-2.5 μm) bands are barely scattered compared to visible wavelength, offering a dark background even by day. But specific artifacts happen :

- pollen grains or aerosols may create halos that don't hide a star but decreases the contrast ;
- insects appear like false shining stars ;
- high altitude cirrus clouds may be invisible for the eye but bright for infrared detectors ;
- gaps between J-H and H-K bands correspond to water molecule absorption and re-emission that create a curtain of light and decreases the contrast.

Pollen grains and insects are never stationary : comparing different frames of a burst allows a separation between immobile stars and moving artifacts.

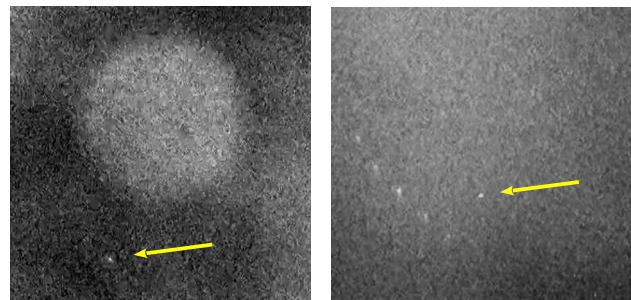


Fig 6 – left : disk of scattered sunlight by grain of pollen or aerosol ; right : trajectory of an insect beyond the arrow : Capella (α Aurigae, magJ = -1.3) shot by daylight and clear sky in Marseille on February 17th 2021 around 14:00 with ZEISS 300 mm focal length, 0.9-1.3 μm filter and C-RED2 camera [5] 0.9-1.7 μm InGaAs sensor cooled to -40°C

Cirrus clouds, visible on the images of Fig 6 like varying shades of grey, may be partly cleaned by a high pass filter. Luminous curtain of water molecule scattering bands (1.35-1.5 and 1.8-2.0 μm) may be cut by optical filters. The challenge consists in choosing the compromise between star flux and water noise because sensing too large is veiled and sensing too narrow shuts starlight :

- each filter, added to stop a scattering band of water, absorbs a few percents of the light ; combining many band pass filters increases the price and decreases the contrast of star with respect to the light curtain ;
- the wider band of wavelength you receive, the higher flux you detect from a star : sensing J, H and K band without water scattering bands would be perfect !

3 – Detectable stars from ground

Star catalogs give the position of each luminous source and a magnitude calculated for each band of wavelength :

$$mag_V = -2,5 \cdot \log_{10} \left(\frac{E_V}{E_{V_0}} \right)$$

Fig 7 – formula of apparent magnitude in V band
 E_V spectral radiant flux of the star in V band
 E_{V_0} spectral radiant flux of Vega in V band
 Lower magnitude means brighter star

For historical reasons, Vega (α Lyrae, $mag_V = 0.03$, $mag_J = -0.18$) has been chosen as the reference to measure the flux of a star, thus $mag = 0$ for the whole spectrum. But after centuries, the spectroscopy has shown many differences between stars : their magnitudes depend on the wavelength. Star catalogs now give the spectral type of a star, allowing to get a rough spectrum for all the stars in the same type : only the dilatation along vertical axis of flux depends on the magnitude.

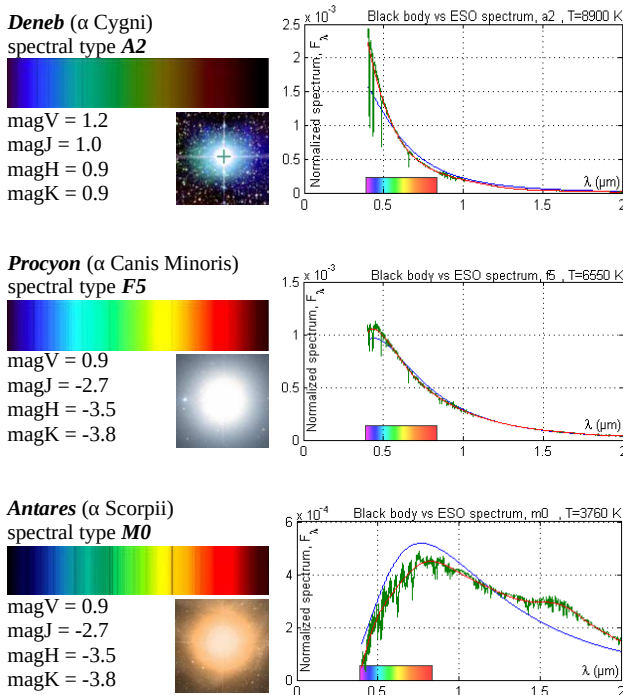


Fig 8 – spectral types [7], visible spectrum, magnitudes and images [6] of 3 stars

First result of this comparison : many stars are more radiant in infrared than in visible wavelength. Processing star catalogs shows that, for a given magnitude limit, the number of stars having a magnitude equal or lower than this limit is growing with the wavelength.

This statement requires a specific process because star catalogs are fed with data from different surveys made at different dates, from earth or from space, with various sensors :

- Harold L. JOHNSON & William W. MORGAN (1953), A. W. J. COUSINS (1976), M. BESSEL (1990) : changing definition of “Vega

normalized” system in order to adapt to sensor evolution, photomultiplier then CCD ;

- CFHT : Canada-France-Hawaii Telescope, 1979-current, visible + infrared bands B, V, R, I and Z ;
- HIPPARCOS, satellite on geostationary transfer orbit, 1989-1993 (ESA), visible band V ;
- 2MASS : Two Micron All-Sky Survey, Arizona US and Chile, 1997-2001 (NASA), infrared bands J, H and K ;
- SDSS : Sloan Digital Sky Survey, New Mexico US, 2000-current, visible + UV + infrared bands u, g, r, i, z ;
- GAIA, space probe on LAGRANGE point L₂, 2013-2024 (ESA), visible + NIR band G.

Each survey proposes a specific photometric system of bands chosen for the purpose (astrometry, spectrometry, star, galaxies, exoplanets, and so on) and the sensor (silicon, InGaAs, HgCdTe or other) of its device. The resulting catalog of light sources proposes formulas to calculate a magnitude for other photometric systems but it remains a rough estimate. Global catalogs aim at merging the data from all surveys and articles [6] but magnitudes are rarely given for R and I bands (0.7 to 1.0 μm).

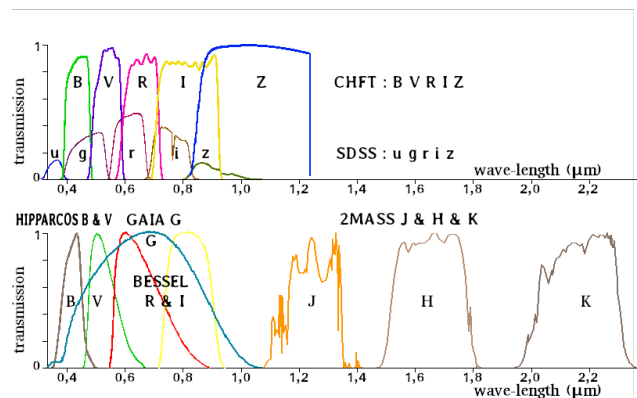


Fig 9 – filters of some astronomical survey [3]

A better estimate is the spectral type, that refers to the spectrum of starlight. The reconstruction of missing magnitudes is done with two methods :

- the normalized spectrum from ESO [7] has been used to process the spectral type of a star, vertically amplified to match with known magnitudes : the flux of the band for missing magnitude then allows a calculation with the formula of Fig 7 ;
- a normalized spectrum from ESO [7] is chosen to match optimally with known magnitudes when the spectrum of the spectral type of the star doesn't fit properly with its known magnitudes.

63 433 brighter stars are processed to provide a complete catalog and sort them by growing magnitudes and spectral bands.

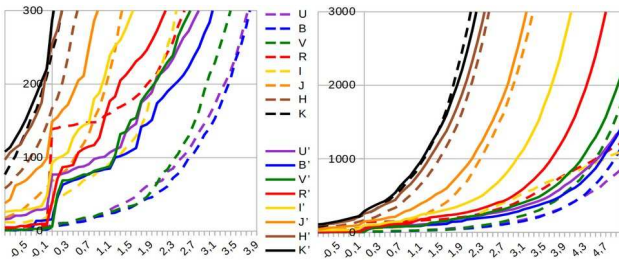


Fig 10 - number of stars given in Simbad database for U, B, V, R, I, J, H, K magnitudes in Vega system [8] (dashed lines) and after reconstruction of missing magnitudes (continuous lines) left scale till 300 stars, right scale till 3000 stars

The result in Fig 10 shows a growing number of detectable stars with wavelength, deeply benefiting infrared sensors.

Assuming that stars are homogeneously spread on the celestial sphere, Fig 11 gives an estimation for the number of detectable stars depending on magnitude, band and field of view.

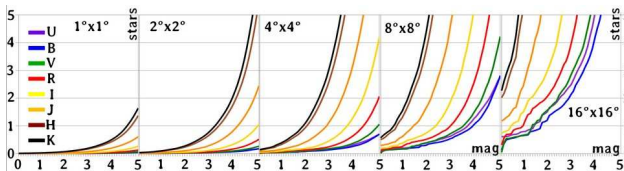


Fig 11 - number of detectable stars depending on a maximum magnitude in different bands and for various fields of view

4 – Single star or multi stars observation

Celestial navigation with a sextant consists in observing a single star because the field of view of its telescope (6 to 10°) and the performances of the human eye (scotopic vision under low-light levels with rod cells photo-receptors whose maximum sensitivity is 0.498 μm) confine the detection to stars with visible magnitude under 3.0 : Nautical Almanac contains the position of 173 stars (81 stars till magnitude 2.8 in French Éphémérides Nautiques).

The identification of the star is made through an estimated position (dead reckoning from last observed position with elapsed time, ship course and speed) and a map of the local sky (or HO249 tables) : the sailor chooses a star on the star-finder, reads its altitude Hc and azimuth Zn, adjusts the index bar and drum of the sextant on Hc and drives it to Zn. The lone bright and stationary point in the telescope is the chosen star.

The actual performances of infrared sensors lead to a compromise between accuracy of the measure (thus angular resolution, depending on pixel size) and aperture, given by f-number N (as low as possible to maximize the incoming flux from the star).

$$FoV = 2 \cdot \arctan\left(\frac{Nbr \cdot pitch}{2 \cdot f}\right) = 2 \cdot \arctan\left(\frac{Nbr \cdot pitch}{2 \cdot N \cdot D}\right)$$

Fig 12 - formulas to calculate the field of view FoV
Nbr : vertical or horizontal number of pixel
pitch : size of a pixel (or distance between pixels)
f : focal distance , N : f-number

D : diameter of primary lens

Assuming an infrared technology with Nbr = 1024, pitch = 12 μm, N = 3, D = 100 mm yields a FoV = 2.3°. If the camera is able to detect stars with magnitude lower than 4, sensor may have :

- two stars in the field of view for H or K band ;
- one star in the field of view for J band.

Assuming a silicon technology with Nbr = 6498, pitch = 3 μm, N = 3, D = 100 mm yields a FoV = 3.7°. The camera may not detect any star in the field of view for silicon spectrum (B, V, R, I bands) and magnitude lower than 4.

Seeking an accuracy around 1 arcsec, current infrared technologies (InGaAs and HgCdTe) are limited to single star detection because of :

- large pixel size (thus gross angular resolution) ;
- low definition (hence narrow field of view) ;
- sensitivity (too small magnitude limit for a reduced camera embedded on a vehicle).

A single star is not enough to fix the position of the observer because the line of positions, where

- the same star can be observed,
- at the same time,
- with the same zenithal distance (or altitude above the local horizon),

is a circle on the Earth. Sailors navigating with sextant used to shoot three celestial bodies to fix their position on the sole point of intersection of three circles.

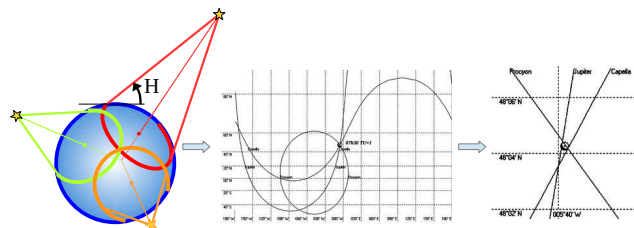


Fig 13 – three stars are necessary to fix the astronomical position thanks to sextant observations

Three single star observations are considered with three cameras separated by 120° of azimuth, for instance at 45° above the horizon : e.g. Daytime Stellar Imager (Fig 14). This device may be fixed on a vehicle, undergoing its roll and pitch, or driven by two-axis precision rotary stage to point a telescope at a bright star or at clear area of the sky. Steerable telescope increases the number of detectable stars by choosing a direction far from sunlight and moonlight, or towards a window in the cloud layers. In this case, an additional camera is used for all sky image process. However, rotary stage with 1 arcsec of accuracy is harsh to build and every moving part is a source of weakness and error.

maneuverability	two-axis
3 fields of view	0.5°×0.4°
definition	320x256 pixels
angular resolution	5.6x5.6 arcsec
thermo-electric cooler	PELTIER
spectral band	H
max magnitude H	6.4
aperture	50 mm
external dimensions	20×15×12 cm
accuracy on attitude angles	1 arcsec
accuracy on final position	30 m

Fig 14 – features of a Daytime Stellar Imager used in addition of a G.P.S. receiver but without inertial sensor nor magnetometer, for attitude determination or inertial calibration [10]

The number of available stars estimated in Fig 11 for various bands and magnitudes is based on :

- an equal density of stars on the whole celestial sphere ;
- a field of view measured horizontally with degrees of azimuth and vertically with degrees of altitude, leading to tiny fields of view near the zenith !

A more realistic method consists in :

- using a star catalog with astronomical coordinates : right ascension RA (or α) and declination Dec (or δ) of stars whose magnitude is lower than a given limit for a chosen band ;
- calculating with RA and Dec the outline of a field of view, knowing its boresight astronomical coordinates in the image center.

The result is displayed in Fig 16 with RA on vertical axis from 000° downwards to 360° and Dec on horizontal axis from 90°S to 90°N to the right. If a field of view of 1°x1° is driven to a point of the celestial sphere at RA and Dec coordinates, the number of stars under a magnitude limit is given by colors. The mean number is given in the table of Fig 15. The standard deviation for magH and mag K \leq 6 is 2.02, evidence that global approach like Fig 11 is not relevant enough.

I band represents the possible detection with silicon but infrared bands are detailed. This presentation overrates stars around the poles but shows that the number of available stars in a field of view :

- is lower than in the rough estimate of Fig 11 ;
- fluctuates with time and hemisphere ;
- is not proportional to the number of telescopes.

band	mag \leq 3	mag \leq 6
I	0.02 (0.17)	0.55 (0.81)
J	0.06 (0.26)	1.22 (1.51)
H	0.14 (0.40)	1.5 (2.02)
K	0.16 (0.43)	1.5 (2.02)

Fig 15 – mean number of stars in a field of view of 1°x1° (with standard deviation) in I, J, H and K bands

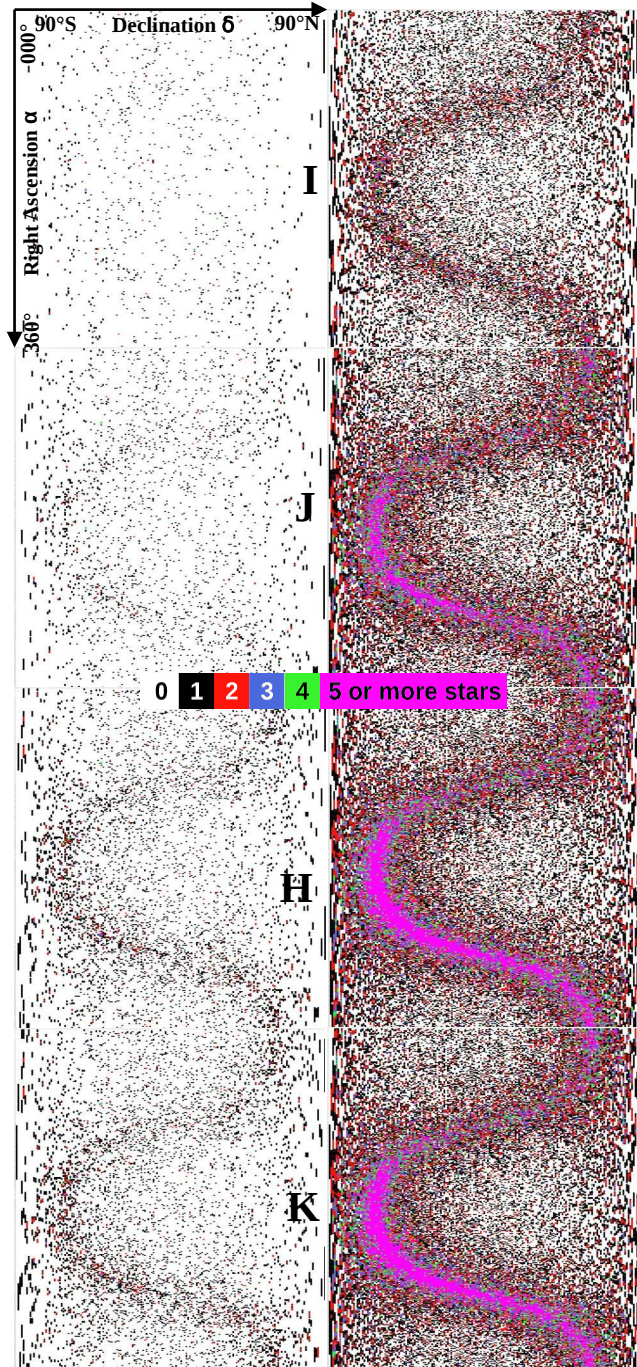


Fig 16 – “sky maps” for I, J, H, K bands displaying the number of stars in a field of view of 1°x1° with max magnitude 3 (left) or 6 (right)

Visible sensors based on silicon fail at detecting a star with magUBVI < 3 in a narrow field of view of 4°x4° due to the lack of stars (and lower magnitude limit because of sky background radiance by day), but remains efficient thanks to their performances :

- small pixel size (thus little angular resolution) ;
- high definition (hence wide field of view).

Assuming Nbr = 8 896, pitch = 3.76 μ m, N = 3, D = 42 mm , f = 126 mm yields a FoV = 15.1° and angular resolution of 6.1 arcsec. The camera may detect :

- 4 stars with magnitude lower than 3 ;
- 5 stars, with magnitude lower than 4.

Multi stars observation is interesting for many reasons :

- automatic identification of stars ;
- measure of attitude angles ;
- enhancement of accuracy ;
- measure of time.

Many algorithms are able **to identify stars** on a sky image with a field of view around 15°x15° but they need a minimum number of light sources to be robust and accurate.

algorithm name	minimum number of stars
Angle	2
Combine triangle	3
Pyramid	4
Quadrilateral	4
Pentagon	5
Quadrilateral pyramid	5
Triangular dipyramid	5

Fig 17 - minimum number of stars in the field of view for various star identification algorithms [9]

Star identification is useful :

- for a method of position fixing thanks to an horizontal (or vertical) reference but without position estimate ;
- to measure local attitude angles (course, trim and list) thanks to an horizontal (or vertical) reference.

Once the stars are identified, the astronomical coordinates of the boresight can be calculated and the angle of “horizontal” edge of the camera over the equator plane. Knowing the universal time, this leads to absolute attitude angles of the telescope in a geocentric reference frame.

An horizontal or vertical reference is necessary to project the boresight direction in a local reference system and get the **local attitude angles**.

Sextant measures on board a ship are based on the horizon line. This suffers a significant atmospheric refraction that sailors correct roughly in accordance with the altitude (Ho for observed altitude). Additional correction may be used for temperature and pressure measured on board, given with an accuracy of 0.1' (minute of arc). It doesn't take into account the changing characteristics of the atmosphere along the light-beam from horizon to sextant. Air navigation uses a bubble sextant and, in addition to the refraction, shall take into account the Coriolis correction depending on latitude and ground speed (accuracy of 1').

Seeking for 1 arcsec of precision needs an instrument able to reach the same accuracy or better. Inclinometer may be accurate on a stationary camera but, on a moving vehicle, gyroscopes are necessary to mitigate inertial errors. This compels to choose an accurate thus expensive inertial navigation system to measure the vertical gravity without accelerations of the vehicle motion. Finally, a gravity model is necessary to correct the deflection of the vertical (DoV) : extreme values are recorded 10 km South of Annapurna II (Nepal) to 109 arcsec but, far from high

mountains and some volcanic rocks or specific sediments, the deflection remains between ± 10 arcsec on continents and ± 6 arcsec at sea. The Earth Gravitational Model 2008 (EGM2008) has a spatial resolution of 2 to 9 km and the accuracy is around 1.3 arcsec [11]. A ultra-high resolution model proposes a spatial resolution of 200 m with better accuracy between latitudes ± 60° [12].

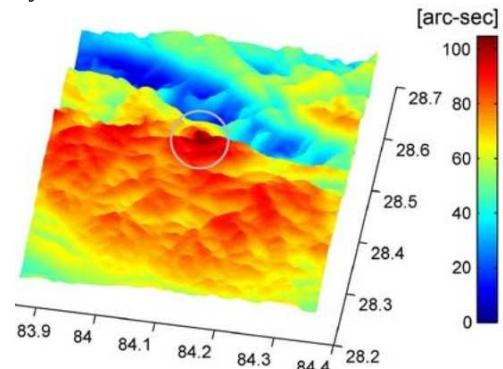


Fig 18 - extreme values of gravity deviation recorder in Nepal around Annapurna, axis graduations with latitude and longitude [12]

The accuracy in position of a single star shot by a camera is approximately equal to angular resolution of a pixel. But the boresight benefits the **enhancement of accuracy** provided by observing n star and the error is divided by \sqrt{n} . This property is used on star-trackers like ST5000 whose angular resolution is 35 arcsec but the pointing accuracy is given for 0.5 arcsec with 24 stars identified in the field of view [13].

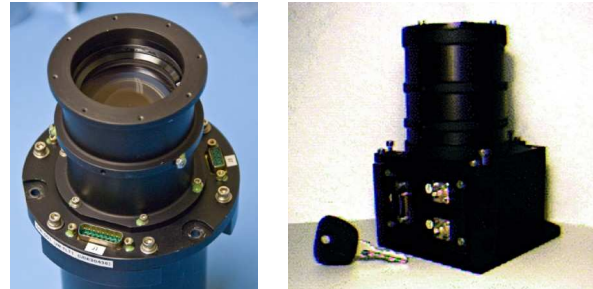


Fig 19 - ST5000 star-tracker, 7°x5° and 9.2x6.7° FOV versions, used at 40 km of altitude to measure attitudes of the gondola under a stratospheric balloon [13]

Observing many celestial bodies, natural or artificial, allows **a measure of time**. Bodies orbiting around the Earth turn like the hands of a clock whose hours are marked by stars on the celestial background. This method is used for centuries to measure universal time by observing Jupiter's satellite eclipses with a telescope and CASSINI tables (1668) or “lunar distances” between the Moon and particular bright stars on its trajectory.

The quickest satellite gets most accurate time ! Given the trajectory parameters of a detectable LEO satellite, an image showing it among identified stars, with an angular resolution of 1 arcsec per pixel, leads to universal time with 4.2 ms of accuracy (for a circular orbit in 90 minutes). Knowing that the Earth spins at 15° per hour, this yields an error of 1.9 m on a celestial fix calculated with this adjusted clock.

5 – Tests with InGaAs camera

Following features are chosen to test the detectability from ground (Marseille, altitude 50 m) by daytime :

- C-RED2 InGaAs camera 0.9-1.7 μm [5] ;
- 640x512 pixels of 15 μm ;
- read-out noise $< 30 e^-$;
- working temperature $+20^\circ$ to -40°C with 2 PELTIER thermo-electric coolers : ultra low dark current ;
- high pass filter $\lambda > 1.5 \mu\text{m}$ or low filter pass $\lambda < 1.3 \mu\text{m}$;
- lens ZEISS, focal length 300 mm, aperture $N=4$.

This yields :

- field of view $1.8^\circ \times 1.5^\circ$;
- angular resolution 10.3 arcsec per pixel.

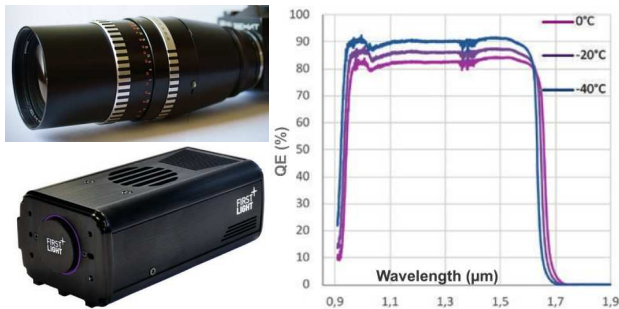


Fig 20 – ZEISS lens, C-RED2 camera and its quantum efficiency spectrum [5]

Vega ($\text{mag}_J = -0.18$ and $\text{mag}_H = -0.03$) and Capella ($\text{mag}_J = -1.29$ and $\text{mag}_H = -1.72$) are shot by clear sky around 10:00 AM in February with 1.6 ms of exposure at 600 frames per second :

- “J” band (0.9 – 1.3 μm) ;
- “H” band (1.5-1.7 μm) ;
- “J+H” (without filter thus 0.9 – 1.7 μm).

Background noise is more important in “J+H” because of water scattering band (1.3-1.5 μm). Contrast is better in “J” band (16%) and signal to noise ratio as well (SNR = 101).

Sky is clear in visible light but infrared pictures show clouds moving on the background and flattening the contrast. Information is spread on 1.5% of the dynamic range. High altitude aircrafts and insects appear like false stars. Aerosols and pollen grains scatter sunlight, creating moving halos and decreasing the contrast (Fig 6). Light reflections display artifacts because ZEISS lens is not designed for infrared wavelength.

The main lesson is the importance of :

- checking radiative transfer with all sky image, local weather measures and global atmosphere data to choose optimal directions in the sky ;
- adapting the time of exposure to noise and sky background level ;
- post-processing the images to clean false stars and increase the signal to noise ratio.

This first trial has motivated further tests with more compliant optics and efficient filters to increase the magnitude limit and expand the limits of observation in spite of bad weather conditions.

6 – Accurate visible horizon ?

The horizon line used with a sextant is too vague, because of refraction, for an accurate measure of the sextant altitude H_s to 1 arcsec, but can be considered as an horizontal plan.

For a ship or an aircraft navigating off the coast, observing this line of horizon in various azimuths, with detectors of n columns of pixels for each, is able to provide an horizontal plan with an optical error divided by \sqrt{n} . This plan may be different of “the” horizon but is parallel to it, thus perpendicular to the vertical direction of gravity. Furthermore, the local variations of the deflection of vertical may be smoothed as much as the vantage point is high above the sea.

For instance a silicon sensor with 9602x6498 pixels (horizontal x vertical) and 60° of vertical field of view, the vertical angular definition is 33.2 arcsec per pixel but the 9602 columns enhance the accuracy to 0.33 arcsec on the horizon line measure.

Curved detectors have been recently mastered to hugely improve wide field pictures with zero vignetting and scarce optical aberrations. An all sky image of 200° (thus 10° under all the horizon line) can be shot with a fish eye lens and a curved detector.

The curvature can be concave or convex, spherical for a circular field of view or cylindrical for a wide one [14]. This method could only work off the coast when the horizon line is sharp.

Such sensor could also :

- detect clouds and help to choose a window in the sky where light transmission is optimal to observe stars with a steerable telescope ;
- watch the sea surface to detect lights of other ships, identify lighthouses, measure relative bearing on landmarks and perhaps search man over board.

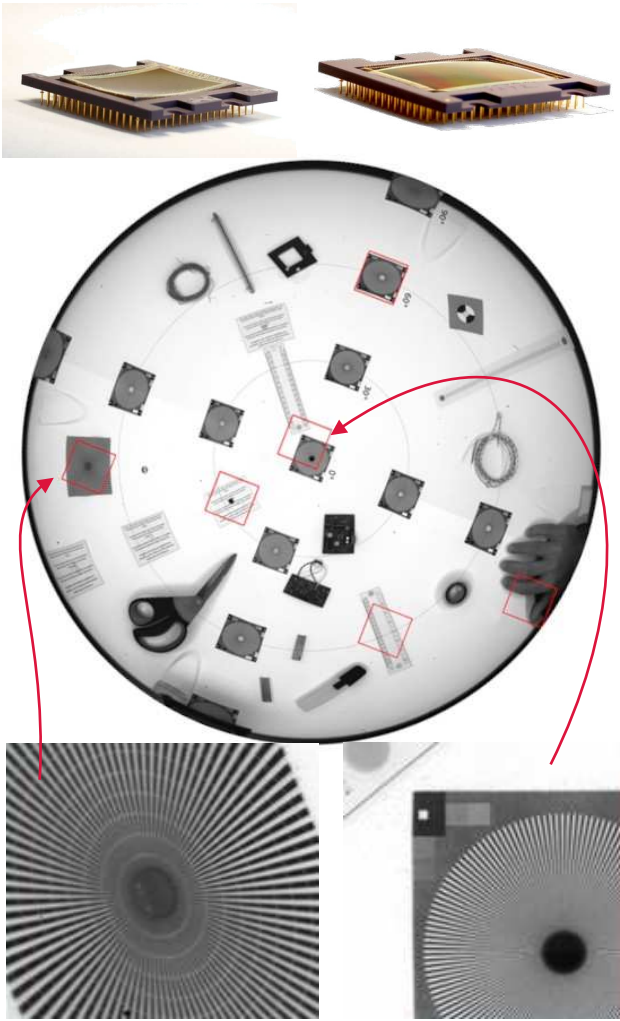


Fig 21 – concave and convex curved sensors, measure of vignetting and residual aberrations with 180° fish eye lens [14]

As a conclusion, this review of actual technologies, performances and components off the shelf for light detectors convinces that the challenge to use starlight for accurate navigation by night and day is about to be managed, waiting a few more months for the improvements promised by Moore's law. Even if precipitations still weaken or blind stars, the compromise between accuracy and availability seems to exist somewhere between I and K bands to provide a reliable stellar positioning system.

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