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A new formula on analytic Combinatory

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Abstract

I want to prove that the ratio, Γ_n^m , of the product of any m consecutive positive integers, $n(n+1)(n+2)\dots(n+m-1)$, divided by the product of the first m positive number, $m!$, is equal to summation of $i=1\dots n$ or

$$\Gamma_n^m = \sum_{i=1}^n \Gamma_i^{m-1}.$$

I demonstrate this formula by a very simple recurrence method, therefore these properties must be known to everyone, especially to students.

1. Introduction

Number theory plays a big role in computer science and is currently part of mathematics courses [1]. Certainly I do research in theoretical physics alone because of my illnesses. And after a heart operation I wanted to change my mind and I came across google on the sum of the arithmetic series.

But I found that the methods are very long then I went to have a coffee and I had a simple idea to solve these problems simply.

When I came back home, I wrote the article [2] which is the basis of this work with article [3]. I wanted to generalize the series that I found in [3].

I found a new formula in combinatory analysis. Consequently Using the well-known formulas of combinatorial analysis

$$C_n^m = \frac{n!}{m!(n-m)!}, \quad \Gamma_n^m = C_{n+m-1}^m = \frac{(n+m-1)!}{m!(n-1)!} \quad (1.1)$$

The new combinatorial analysis formula is:

$$\left[\Gamma_n^m = \sum_{i=1}^n \Gamma_i^{m-1} \right] \quad (1.2)$$

Combinatorial probability consistently uses the formulas of combinatorial analysis first developed by Newton and later by Pascal. I want to prove formula (1.2) by recurrences in chapter two. The addition of odd numbers using the fingers is in the appendix.

2. The new formula of analytic Combinatory

We want to determine Γ_n^m :

$$\Gamma_n^m = \sum_{i=1}^n \Gamma_i^{m-1} \quad (2.1)$$

I assumed in my work [2] that there is a polynomial

$$P_n^m(n) = \Gamma_n^m \quad (2.2)$$

With
$$P_n^m(n) = a_1(n)^m + a_2(n)^{m-1} + \dots + a_{m+1} \quad (2.3)$$

To determine the coefficients a_i it suffices to take $n=1, \dots, m$ in (2.2). And so we obtain a linear system whose resolution gives a_i .

A- The numbers start from 1.

$$\Gamma_n^0 = 1 \text{ car } 0! = 1. \quad (2.4)$$

B- The summation

$$\Gamma_n^1 = \sum_{i=1}^n \Gamma_i^0 = n.$$

C- Gauss sums of the arithmetic series

$$\Gamma_n^2 = \sum_{i=1}^n \Gamma_i^1 \quad (2.5)$$

We are going to follow our method (2.2) which supposes that the polynomial:

$$P_2(n) = x_1(n)^2 + x_2(n) + x_3 = \Gamma_n^2 \quad (2.6)$$

Therefore we determine the three variables x_i by calculating Γ_n^2

For $n=1, 2, 3$ we find that: Γ_n^2 is equal to: 1, 3, 6.

And by solving the linear system which is written in matrix form:

$$\begin{pmatrix} (2)^2 & (2)^1 & 1 \\ (3)^2 & (3)^1 & 1 \\ (1)^2 & (1)^1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} \text{ la solution est } \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \quad (2.7)$$

The solution is then:

$$\frac{1}{2}n^2 + \frac{1}{2}n = \frac{n(n+1)}{2} = \frac{(n+1)!}{2(n-1)!}$$

Therefore
$$\left[P_2(n) = \frac{(n+1)!}{2(n-1)!} = \Gamma_n^2 \right] \quad (2.8)$$

D-Expression of Γ_n^3 .

The sequence of numbers is Γ_n^3 :

$$\Gamma_n^3 = \sum_{i=1}^n \Gamma_i^2 \quad (2.9)$$

We will follow my method (2.2) which assumes that:

$$P_3(n) = x_1 (n)^3 + x_2 (n)^2 + x_3 (n) + x_4 = \Gamma_n^3 \quad (2.10)$$

Therefore we determine the three variables x_i by calculating Γ_n^3

For $n=1,2,3,4$ and we find: 1,4,10,20 . And by solving the linear system which is written in matrix form:

$$\begin{pmatrix} 2^3 & 2^2 & 2^1 & 1 \\ 3^3 & 3^2 & 3^1 & 1 \\ 4^3 & 4^2 & 4^1 & 1 \\ 1^3 & 1^2 & 1^1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \\ 20 \\ 1 \end{pmatrix} \text{ The solution is: } \begin{pmatrix} 1/6 \\ 1/2 \\ 1/3 \\ 0 \end{pmatrix} \quad (2.11)$$

So
$$P_3(n) = \frac{1}{6}(n)^3 + \frac{1}{2}(n)^2 + \frac{1}{3}(n) = \frac{1}{6}n(n+1)(n+2) \quad (2.12)$$

But:
$$\frac{1}{6}n(n+1)(n+2) = \frac{(n+2)!}{3!(n-1)!}$$

Consequently
$$\left[P_3(n) = \frac{(n+2)!}{3!(n-1)!} = \Gamma_n^3 \right] \quad (2.13)$$

E- Expression of Γ_n^4 :

$$\Gamma_n^4 = \sum_{i=1}^n \Gamma_i^3 \quad (2.14)$$

We will follow my method (2.2) which assumes that

$$P_4(n) = x_1(n)^4 + x_2(n)^3 + x_3(n)^2 + x_4(n)^1 + x_5 = \Gamma_n^4 \quad (2.15)$$

Therefore we determine the three variables x_i by calculating Γ_n^4 For $n=1,2,3,4,5,6$ and we find: 1,5,15,35,70 . And by solving the linear system which is written in matrix form:

$$\begin{pmatrix} (2)^4 & (2)^3 & (2)^2 & (2)^1 & 1 \\ (3)^4 & (3)^3 & (3)^2 & (3)^1 & 1 \\ (4)^4 & (4)^3 & (4)^2 & (4)^1 & 1 \\ (5)^4 & (5)^3 & (5)^2 & (5)^1 & 1 \\ (1)^4 & (1)^3 & (1)^2 & (1)^1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 5 \\ 15 \\ 35 \\ 70 \\ 1 \end{pmatrix} \quad (2.16)$$

The solution is:

$$\begin{pmatrix} 1/24 \\ 1/4 \\ 11/24 \\ 1/4 \\ 0 \end{pmatrix}$$

And

$$P_4(n) = \frac{1}{24}(n)^4 + \frac{1}{4}(n)^3 + \frac{11}{24}(n)^2 + \frac{1}{4}(n)^1 = \frac{1}{24}n(n+1)(n+2)(n+3) \quad (2.17)$$

But $24 = 4!$, $n(n+1)(n+2)(n+3) = \frac{(n+3)!}{(n-1)!}$

Then:

$$\left[P_4(n) = \frac{(n+3)!}{4!(n-1)!} = \Gamma_n^4 \right] \quad (2.18)$$

F- Expression of Γ_n^5 :

We Write:

$$\Gamma_n^5 = \sum_{i=1}^n \Gamma_i^4 \quad (2.19)$$

The polynomial is:

$$P_5(n) = x_1(n)^5 + x_2(n)^4 + x_3(n)^3 + x_4(n)^2 + x_5(n)^1 + x_6 \quad (2.20)$$

The matrix is:

$$\begin{pmatrix} (2)^5 & (2)^4 & (2)^3 & (2)^2 & (2)^1 & 1 \\ (3)^5 & (3)^4 & (3)^3 & (3)^2 & (3)^1 & 1 \\ (4)^5 & (4)^4 & (4)^3 & (4)^2 & (4)^1 & 1 \\ (5)^5 & (5)^4 & (5)^3 & (5)^2 & (5)^1 & 1 \\ (6)^5 & (6)^4 & (6)^3 & (6)^2 & (6)^1 & 1 \\ (1)^5 & (1)^4 & (1)^3 & (1)^2 & (1)^1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 6 \\ 21 \\ 56 \\ 126 \\ 252 \\ 1 \end{pmatrix} \quad (2.21)$$

The solution is:

$$\begin{pmatrix} 1/120 \\ 1/12 \\ 7/24 \\ 5/12 \\ 1/5 \\ 0 \end{pmatrix} \quad (2.22)$$

So

$$P_5(n) = n(n+1)(n+2)(n+3)(n+4)(n+5)/120$$

Because $120 = 5!$.

We deduce that:
$$\left[P_5(n) = \frac{(n+5)!}{5!(n-1)!} = \Gamma_n^5 \right] \quad (2.23)$$

3- Appendix: The addition of odd numbers using the fingers

This work is a continuation of my article [4] on the addition of odd numbers. On fingers 1 and 2 we notice that $1+3=4=2 \times 2$, on fingers 1, 2 and 3, $1+3+5=3 \times 3$, on the 6 fingers $1+3+5+7+9+11=6 \times 6$ and thus we find the result directly.

And I found great interest by students (10 years) by this method and is the reason I mention it.

4- references

- [1] G. Andrews, Number Theory, Dover edition (1994)
- [2] M. Hage Hassan , A new method for the derivation of the expressions of Arithmetic Series , 2018 (Hal, research gate)
- [3] The Sum and the Squares of Odd and Natural Numbers
May 2022
- [4]- Derivation of the sum of odd numbers by arithmetic and geometric Methods December 2021 (Research Gate)