Appendix 3. On the relationship between the newly proposed parametric measures of community dissimilarity and a number of classical single-point measures of resemblance.

Given two plots *U* and *V*, let x_{Uj} and x_{Vj} be the abundances of species *j* ($j = 1, 2, ..., N$) in both plots. First, note that as x_{Uj} and x_{Vj} are both nonnegative, we have for $\alpha > 0$ $\sum_j \left| x_{Uj} - x_{Vj} \right|^\alpha \le \sum_j \left(\max_j (x_{Uj}, x_{Vj}) \right)^\alpha$ and thus $\sum_j \left| x_{Uj} - x_{Vj} \right|^\alpha \le \sum_j x_{Uj}^\alpha + \sum_j x_{Vj}^\alpha \le \sum_j \left(x_{Uj} + x_{Vj} \right)^\alpha$, and for $\alpha \ge 1$, $\sqrt[\alpha]{\sum_j \left(x_{Uj} + x_{Vj} \right)^\alpha} \le \sqrt[\alpha]{\sum_j x_{Uj}^\alpha} + \sqrt[\alpha]{\sum_j x_{Vj}^\alpha}$ (Minkowski's inequality).

Therefore, the following inequalities are satisfied: $D^\alpha \le \Delta^\alpha \le M^\alpha$ for $\alpha > 0$ and $L^\alpha \le D^\alpha$ for $\alpha \ge 1$ (for the definition of the parametric dissimilarities, see the main text).

For $\alpha = 2$, the dissimilarity index Δ^α is related to several resemblance measures published in the literature:

• Morisita-Horn similarity index (Morisita 1959; Horn 1966):

$$S_{MH} = \frac{\sum_j 2 x_{Uj} x_{Vj}}{\left[\left(\sum_j x_{Uj}^2 / X_U^2 \right) + \left(\sum_j x_{Vj}^2 / X_V^2 \right) \right] X_U X_V} \tag{A1}$$

where $X_U = \sum_j x_{Uj}$ and $X_V = \sum_j x_{Vj}$ are the total abundances in plots *U* and *V*, respectively. The Morisita-Horn similarity index thus corresponds to

$$S_{MH} = \frac{\sum_j 2 p_{Uj} p_{Vj}}{\sum_j p_{Uj}^2 + \sum_j p_{Vj}^2} \tag{A2}$$

where x_{Uj} and x_{Vj} are the *relative* abundances of species *j* in plots *U* and *V*, respectively.
Accordingly, a dissimilarity index can be defined as $D_{MH} = \sqrt{1 - S_{MH}}$:

$$\begin{aligned} D_{MH} &= \sqrt{1 - \frac{\sum_j 2 p_{Uj} p_{Vj}}{\sum_j p_{Uj}^2 + \sum_j p_{Vj}^2}} = \sqrt{\frac{\sum_j p_{Uj}^2 + \sum_j p_{Vj}^2 - \sum_j 2 p_{Uj} p_{Vj}}{\sum_j p_{Uj}^2 + \sum_j p_{Vj}^2}} \\ &= = \sqrt{\frac{\sum_j \left(p_{Uj} - p_{Vj} \right)^2}{\sum_j p_{Uj}^2 + \sum_j p_{Vj}^2}} \end{aligned} \tag{A3}$$

The index D_{MH} thus corresponds to index Δ^α (eq. 6b of the main text) computed from species relative abundances with $\alpha = 2$.

• Similarity ratio (e.g. Wishart 1969; Janssen 1972):

$$S_{SR} = \frac{\sum_j x_{Uj} x_{Vj}}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 - \sum_j x_{Uj} x_{Vj}} \tag{A4}$$

A dissimilarity index can be defined as $D_{SR} = \sqrt{1 - S_{SR}}$:

$$D_{SR} = \sqrt{1 - \frac{\sum_j x_{Uj} x_{Vj}}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 - \sum_j x_{Uj} x_{Vj}}} = \sqrt{\frac{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 - \sum_j x_{Uj} x_{Vj} - \sum_j x_{Uj} x_{Vj}}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 - \sum_j x_{Uj} x_{Vj}}}$$
$$= \sqrt{\frac{\sum_j \left(x_{Uj} - x_{Vj} \right)^2}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 - \sum_j x_{Uj} x_{Vj}}} \tag{A5}$$

• A similarity index derived from Sokal and Sneath (1963) as shown in Pavoine and Ricotta (2014):

$$S_{SN} = \frac{\sum_j x_{Uj} x_{Vj}}{2\sum_j x_{Uj}^2 + 2\sum_j x_{Vj}^2 - 3\sum_j x_{Uj} x_{Vj}} \tag{A6}$$

with the corresponding dissimilarity index which is defined as $D_{SN} = \sqrt{1 - S_{SN}}$:

$$D_{SN} = \sqrt{1 - \frac{\sum_j x_{Uj} x_{Vj}}{2\sum_j x_{Uj}^2 + 2\sum_j x_{Vj}^2 - 3\sum_j x_{Uj} x_{Vj}}} = \sqrt{\frac{2\sum_j x_{Uj}^2 + 2\sum_j x_{Vj}^2 - 3\sum_j x_{Uj} x_{Vj} - \sum_j x_{Uj} x_{Vj}}{2\sum_j x_{Uj}^2 + 2\sum_j x_{Vj}^2 - 3\sum_j x_{Uj} x_{Vj}}}$$
$$= \sqrt{\frac{2\sum_j \left(x_{Uj} - x_{Vj} \right)^2}{2\sum_j x_{Uj}^2 + 2\sum_j x_{Vj}^2 - 3\sum_j x_{Uj} x_{Vj}}} \tag{A7}$$

All these dissimilarity measures satisfy the complementarity requirement *sensu* Clarke et al. (2006). That is, the measure takes its maximum value (1) when the two plots *U* and *V* have no species in common.

In addition, starting from D^α (eq. 6a of the main text), for $\alpha = 2$ we can define a new dissimilarity index $D_{RP} = D^2$. The corresponding similarity index can be thus defined as:

$$S_{RP} = 1 - D_{RP} = 1 - \frac{\sum_j \left(x_{Uj} - x_{Vj} \right)^2}{\sum_j \left(x_{Uj} + x_{Vj} \right)^2}$$
$$= \frac{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 + 2\sum_j x_{Uj} x_{Vj} - \sum_j x_{Uj}^2 - \sum_j x_{Vj}^2 + 2\sum_j x_{Uj} x_{Vj}}{\sum_j \left(x_{Uj} + x_{Vj} \right)^2}$$
$$= \frac{4\sum_j x_{Uj} x_{Vj}}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 + 2\sum_j x_{Uj} x_{Vj}} \tag{A8}$$

Generalizing the previous indices we obtain:

$$S\left(x_U, x_V; m\right) = \frac{\frac{m}{2}\left(2\sum_j x_{Uj} x_{Vj}\right)}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 + \left(\frac{m}{2} - 1\right)\left(2\sum_j x_{Uj} x_{Vj}\right)} \tag{A9}$$

with

$S\left(x_U, x_V; 0.5\right) = S_{SN}$;
$S\left(x_U, x_V; 1\right) = S_{SR}$;
$S\left(x_U, x_V; 2\right) = S_{MH}$ (if relative abundances are used);
$S\left(x_U, x_V; 4\right) = S_{RP}$

and

$$D\left(x_U, x_V; m\right) = \sqrt{1 - S\left(x_U, x_V; m\right)} = \sqrt{\frac{\sum_j \left(x_{Uj} - x_{Vj}\right)^2}{\sum_j x_{Uj}^2 + \sum_j x_{Vj}^2 + \left(\frac{m}{2} - 1\right)\left(2\sum_j x_{Uj} x_{Vj}\right)}} \tag{A10}$$

where $D\left(x_U, x_V; 2\right)$ and $D\left(x_U, x_V; 4\right)$ correspond to eq. 6b and eq. 6a of the main text for $\alpha = 2$, respectively.

The parametric dissimilarities D^α, Δ^α and the other dissimilarity measures presented above can be further put into an even wider context with the general formulas:

$$S\left(x_U, x_V; m, \alpha\right) = \frac{\frac{m}{2}\left(\sum_j \left(x_{Uj} + x_{Vj}\right)^\alpha - \sum_j x_{Uj}^\alpha - \sum_j x_{Vj}^\alpha\right)}{\sum_j x_{Uj}^\alpha + \sum_j x_{Vj}^\alpha + \left(\frac{m}{2} - 1\right)\left(\sum_j \left(x_{Uj} + x_{Vj}\right)^\alpha - \sum_j x_{Uj}^\alpha - \sum_j x_{Vj}^\alpha\right)} \tag{A11}$$

$$D\left(x_U, x_V; m, \alpha\right) = \sqrt[\alpha]{\frac{\sum_j \left|x_{Uj} - x_{Vj}\right|^\alpha}{\sum_j x_{Uj}^\alpha + \sum_j x_{Vj}^\alpha + \left(\frac{m}{2} - 1\right)\left(\sum_j \left(x_{Uj} + x_{Vj}\right)^\alpha - \sum_j x_{Uj}^\alpha - \sum_j x_{Vj}^\alpha\right)}} \tag{A12}$$

$D\left(x_U, x_V; m, \alpha\right)$ satisfies the complementarity requirement as for two plots without shared species $\sum_j \left(x_{Uj} + x_{Vj}\right)^\alpha - \sum_j x_{Uj}^\alpha - \sum_j x_{Vj}^\alpha = 0$.

For $\alpha = 2$, we find again the generalizations given above ($S\left(x_U, x_V; m\right)$ and $D\left(x_U, x_V; m\right)$). Note also that $\sum_j \left(x_{Uj} + x_{Vj}\right)^\alpha - \sum_j x_{Uj}^\alpha - \sum_j x_{Vj}^\alpha = \sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}$.

In addition, $D\left(x_U, x_V; 2, \alpha\right)$ and $D\left(x_U, x_V; 4, \alpha\right)$ correspond to eq. 6b and eq. 6a of the main text, respectively.

For $m = 0.5$, we obtain a generalization of the Sokal and Sneath similarity index (presented above) and its associated generalized dissimilarity index:

$$S\left(x_U, x_V; 0.5, \alpha\right) = \frac{0.5\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}}{2\sum_j x_{Uj}^\alpha + 2\sum_j x_{Vj}^\alpha - 3\left(0.5\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}\right)} \tag{A13}$$

$$D\left(x_U, x_V; 0.5, \alpha\right) = \sqrt[\alpha]{\frac{2\sum_j \left|x_{Uj} - x_{Vj}\right|^\alpha}{2\sum_j x_{Uj}^\alpha + 2\sum_j x_{Vj}^\alpha - 3\left(0.5\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}\right)}} \tag{A14}$$

Finally, for $m = 1$, we have a generalization of the similarity ratio (presented above) and its associated dissimilarity index:

$$S\left(x_U, x_V; 1, \alpha\right) = \frac{\frac{1}{2}\left(\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}\right)}{\sum_j x_{Uj}^\alpha + \sum_j x_{Vj}^\alpha - \frac{1}{2}\left(\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}\right)} \tag{A15}$$

$$D\left(x_U, x_V; 1, \alpha\right) = \sqrt[\alpha]{\frac{\sum_j \left|x_{Uj} - x_{Vj}\right|^\alpha}{\sum_j x_{Uj}^\alpha + \sum_j x_{Vj}^\alpha - \frac{1}{2}\left(\sum_j \sum_{k=1}^{\alpha-1} C_\alpha^k x_{Uj}^k x_{Vj}^{\alpha-k}\right)}} \tag{A16}$$

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