

**Appendix 1.** Detailed explanation of the calculation of the correlation coefficient between the species abundances and their functional dissimilarities.

Given an assemblage composed of  $N$  species with relative abundances  $p_i (i = 1, 2, \dots, N)$  such that  $0 < p_i \leq 1$  and  $\sum_{i=1}^N p_i = 1$ , let  $d_{ij}$  be the functional dissimilarity between species  $i$  and  $j$  with  $d_{ii} = 0$  and  $d_{ij} = d_{ji}$ .

The mean dissimilarity  $d_{ij}$  for all pairs of species in the assemblage  $i \neq j$  is defined as:

$$\bar{d} = \left[ 2 \sum_{i>j}^N d_{ij} \right] / [N(N-1)] \quad (\text{A1})$$

The mean pairwise product  $p_i \times p_j$  of the species relative abundances  $i \neq j$  is

$$\bar{\pi} = S / [N(N-1)] = \left[ 2 \sum_{i>j}^N p_i p_j \right] / [N(N-1)] \quad (\text{A2})$$

where  $S$  is the Simpson index of diversity  $S = 1 - \sum_{i=1}^N p_i^2 = 2 \sum_{i>j}^N p_i \times p_j$ .

Let  $d = \{d_{ij}\}_{i>j}$  and  $\pi = \{p_i p_j\}_{i>j}$ , then the variance of both variables and the covariance and correlation between the variables are defined respectively as:

$$\text{Var}(d) = \left[ 2 \sum_{i>j}^N (d_{ij} - \bar{d})^2 \right] / [N(N-1)] \quad (\text{A3})$$

$$\text{Var}(\pi) = \left[ 2 \sum_{i>j}^N (p_i p_j - \bar{\pi})^2 \right] / [N(N-1)] \quad (\text{A4})$$

$$\text{Cov}(d, \pi) = \left[ 2 \sum_{i>j}^N (d_{ij} - \bar{d})(p_i p_j - \bar{\pi}) \right] / [N(N-1)] \quad (\text{A5})$$

$$\text{Cor}(d, \pi) = \frac{\text{Cov}(d, \pi)}{\sqrt{\text{Var}(d)\text{Var}(\pi)}} \quad (\text{A6})$$

Shimatani's (2001) balance component is defined as  $B = \text{Cov}(d, \pi) \times N(N-1)$ .

It follows that  $Q = \left[ \bar{d} \times \bar{\pi} + \text{Cor}(d, \pi) \sqrt{\text{Var}(d)\text{Var}(\pi)} \right] [N(N-1)]$ .

As indicated by Shimatani, Rao's quadratic diversity  $Q$  is related to the well-known formula of the expected value of a product:

$$\begin{aligned} \frac{Q}{N(N-1)} &= E(d) \times E(\pi) + Cov(d, \pi) \\ &= E(d) \times E(\pi) + Cor(d, \pi) \times \sqrt{Var(d)} \times \sqrt{Var(\pi)} \end{aligned} \quad (A7)$$

Where  $E(d)$  and  $E(\pi)$  are the expected values of  $d$  and  $\pi$ , respectively.

Unlike Shimatani's balance component,  $Cor(d, \pi)$  represents the strength of the interaction between the species abundances and their functional dissimilarities in relative terms. Therefore, it may be easier to interpret than the balance component itself.

As any correlation coefficient,  $Cor(d, \pi)$  varies between -1 and 1. For example, if  $d_{ij} = p_i p_j$  for all  $i \neq j$ , then  $Cor(d, \pi) = 1$ , whereas if  $d_{ij} = \max(p_i, p_j) - (p_i p_j)$  for all  $i \neq j$ , then  $Cor(d, \pi) = -1$ .

## References

Shimatani, K. (2001) On the measurement of species diversity incorporating species differences. *Oikos* 93: 135–147.