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An Approach Inspired by Quantum Mechanics for the Modeling of Large Power Systems.

Pierrick Guichard, Nicolas Retière, Didier Mayou

Abstract—The ever increasing complexity of power systems mandates a further improvement of the numerical methods used for their simulation. We propose a new approach that is inspired by an exact analogy with quantum mechanics, but requires no specific knowledge of quantum physics. This approach allows in particular to use local methods commonly applied in quantum mechanics, such as Lanczos’ algorithm. It is applied to computing the load flow and extended to simple dynamic studies where the nodes are described by swing equations. The first set of results are for an ideal 2D-lattice grid to illustrate the method and its main features. Then a contingency analysis by the proposed approach is performed for various realistic power systems, including a model of the European network. This demonstrates the interest of the new approach and its efficiency when compared with state-of-the-art methods, especially for large power system.

Index Terms—Load flow, Local approach, Lanczos algorithm, Hamiltonian formalism, Contingency analysis.

I. INTRODUCTION

Physics-based simulation has been used for the planning and analysis of power systems for decades now. Numerous approaches have been designed and commercialized and alternatives are still under study. They enhance the abilities of simulation tools in the context of the ever-growing complexity and vulnerabilities of power systems due to the increasing share of renewable energy sources, the development of active distribution networks and DC networks, the reduction of the security margin ... For steady-state operations, traditional challenges are improving the convergence rate and decreasing the computational cost of iterative methods [2] [3], reducing the dimensionality of the problem by the use of Krylov spaces [43]–[46] or defining non-iterative methods such as Holomorphic Embedding Load-flow Methods (HELM) [20]. More recently, some data-driven methods based on Artificial Intelligence have also been designed, although it remains delicate to trust such methods entirely [17] [18] [19]. For dynamic operations described by a set of differential algebraic equations, speeding up the calculation and extending the time range of simulators are some issues arising from the increasing share of non-conventional loads and sources connected to networks through power electronics converters [4] [5].

To accelerate the computation of the electrical state of the network, hardware-based approaches are also used. They take advantage of the development of parallel computing

and GPU-based strategies. Parallelization through local partitioning techniques [8] have been developed and applied to optimal power flows [9], transient problems [11] [10], and contingency analysis [12]. Parallelization can be coupled with probabilistic methods [15] and Newton–Raphson solvers [14]. A GPU-based implementation of parallelized screening of contingencies is performed in [13]. Nonetheless, such HPC strategies require an adapted hardware which restricts their use [16].

We present here a new and original iterative approach inspired by quantum mechanics to solve power system equations in a linear regime. This approach takes advantage of a locality property. Locality means that under certain circumstances, an event has little impact on distant parts of the network. A typical example is the failure of a line of the network. In that case, the electrical current tends to redistribute to the lines which are at short distance to the faulty link. This notion of locality can be further extended to the evolution on small time intervals. After a local perturbation, the electrical network will vary continuously from its initial state. Yet after a small time, only few dynamic states in the vicinity of the initial one are influenced, as it will be detailed in section (IV). Obviously at larger times, all the system’s states might be influenced by the initial perturbation.

Already in the 1980s, methods based on local computation were proposed for contingency analysis [6] [7]. But in this paper, we propose an original approach directly inspired by complex quantum systems which however requires no specific knowledge of quantum mechanics. The load flow equations are written in a Hamiltonian formalism which is then extended to incorporate the dynamics of simple generators at low frequency. It provides an exact mapping of the electrical system onto a problem of a quantum particle on a grid described by a Schrödinger-like equation. This exact analogy allows us to use the very powerful numerical techniques developed in quantum physics to solve large complex problems where a locality principle also applies [42]. We expect from this approach to provide a new viewpoint on phenomena and properties that arise in power systems.

For a quantum particle on a large grid, well-known approaches rely on the projection onto a Krylov space and tridiagonalization techniques such as the Lanczos algorithm. These methods are designed to exploit the locality principle and are known to be very efficient for accelerating simulations. They avoid numerically demanding exact operations such as matrix diagonalization or inversion for large systems [41]. We propose to apply these efficient tools to the field of power systems. In particular, a high computational speedup

P. Guichard and D. Mayou are at Univ. Grenoble Alpes, CNRS, Institut NEEL, F-38042 Grenoble (France) (email: pierrick.guichard@neel.cnrs.fr; didier.mayou@neel.cnrs.fr)

N. Retière is at Univ. Grenoble Alpes, CNRS, Grenoble INP, G2Elab, F-38000 Grenoble, France (e-mail: nicolas.retiere@univ-grenoble-alpes.fr).

is expected for the computation of steady states, opening to an efficient method for computing LODFs (Line Outage Distribution Factors) which is tested on the Pantagrue model of the European network [48] [49]. Note that in addition to its attractive computational features, the proposed formalism also provides a new insight into the modeling of power flows, benefiting from mathematical tools of quantum mechanics and distinguishing our work from the traditional use of Krylov space methods such as the conjugate gradient descent.

This paper is divided into five sections after the present introduction. Section (II) presents the electrical model of the power system. We introduce a vector space S and its basis constituted by node and line states and at any time the electrical state of the grid is represented by a state ψ which belongs to this vector space S . We introduce also the Hamiltonian operator \mathbb{H} which acts on this space S and from which the power equations are derived. The paper focuses mainly on load flows but we will also indicate how the proposed approach can be easily extended to simplified dynamic studies. Section (III) presents the Lanczos method and how the locality principle is expressed in the Hamiltonian formulation. Section (IV) gives simple static and dynamic applications on an ideal 2D-lattice grid. Section (V) presents a series of applications to various realistic power systems including a network model of the European grid. These applications focus on the problem of line outage and on the subsequent redistribution of power transmission in the grid. The rate of convergence and speed of computation are compared with state-of-the-art methods before concluding. An appendix is provided for ease of reading.

II. DERIVATION OF THE HAMILTONIAN FORMALISM

A. The Hamiltonian of the steady load flow equations

A power grid can be modeled by a graph with N_{nodes} nodes and N_{lines} lines. Every line l has two adjacent nodes i_l and j_l such that $i_l < j_l$ and every pair of adjacent nodes i and j defines a line $l(i, j)$. Every line l is modeled by a susceptance B_l . At every node $n = 1 \dots N_{\text{nodes}}$, a power load or source P_n is connected. Under DC assumptions [26], the voltage is a complex number $V_n := e^{i\theta_n}$ (supposed to be unitary and with a small phase angle θ_n , so it can be approximated by $V_n := e^{i\theta_n} \simeq 1 + i\theta_n$). It is assumed that all the parameters of the electrical network are known. The steady state of the system is assumed to be simply described at every node by the following equations [31], [36], involving the Laplacian matrix \mathbb{L}

$$P = \mathbb{L}\theta, \quad \text{with} \begin{cases} \mathbb{L}_{ij} = -B_{l(i,j)} \text{ if } i \neq j \\ \mathbb{L}_{ii} = \sum_{j \neq i} B_{l(i,j)} \end{cases} \quad (1)$$

Rather than solving this Laplacian system for θ , we propose an alternative and original approach by introducing the vector $\psi := \begin{bmatrix} \psi^N \\ \psi^L \end{bmatrix}$ belonging to a vector space S of dimension $N_{\text{nodes}} + N_{\text{lines}}$. Its components are proportional to the differences between the phases θ and therefore directly related to the electrical currents $(I_l)_{1 \leq l \leq N_{\text{lines}}}$ through the lines of the network

$$\forall l = 1 \dots N_{\text{lines}}, \quad \psi_l^L := \sqrt{B_l}(\theta_{i_l} - \theta_{j_l}) = \frac{1}{\sqrt{B_l}} I_l \quad (2)$$

More compactly ψ is given in function of θ by $\psi = \begin{bmatrix} (0) \\ \mathbb{H}_{\text{LN}}\theta \end{bmatrix}$ where \mathbb{H}_{LN} of dimension $N_{\text{lines}} \times N_{\text{nodes}}$ is related to the Laplacian matrix from Eq. (6) by $\mathbb{L} = \mathbb{H}_{\text{LN}}^\dagger \cdot \mathbb{H}_{\text{LN}}$ and has null coefficients except for

$$\forall l = 1 \dots N_{\text{lines}}, \quad \begin{cases} [\mathbb{H}_{\text{LN}}]_{l,i_l} = \sqrt{B_l} \\ [\mathbb{H}_{\text{LN}}]_{l,j_l} = -\sqrt{B_l} \end{cases} \quad (3)$$

The new unknowns ψ verify the system of equations

$$\mathbb{H}\psi = \mathbb{P} \quad (4)$$

where \mathbb{H} is a Hamiltonian matrix of dimension $(N_{\text{nodes}} + N_{\text{lines}}) \times (N_{\text{nodes}} + N_{\text{lines}})$ and \mathbb{P} is a vector of nodal powers of dimension $(N_{\text{nodes}} + N_{\text{lines}}) \times 1$ defined by

$$\mathbb{H} := \begin{bmatrix} (0) & \mathbb{H}_{\text{LN}}^\dagger \\ \mathbb{H}_{\text{LN}} & (0) \end{bmatrix} \quad \mathbb{P} := \begin{bmatrix} P \\ (0) \end{bmatrix} \quad (5)$$

Eq. (1) was obtained from Ohm's law $I = \mathbb{Y}V$ (where I is the vector of nodal injected currents, \mathbb{Y} is the nodal admittance matrix, and V is the vector of voltages) by left-hand multiplication with V^\dagger , linearization and assumption of direct current. Consequently, the mathematical structure of Eq. (1) is exactly the same as the structure of Ohm's law because \mathbb{Y} and \mathbb{L} are both Laplacian matrices corresponding to the same graph. The Hamiltonian formalism therefore also applies to this case by replacing P by I , θ by V and \mathbb{L} by \mathbb{Y} . In this analogous system, $|\psi_l^L|^2$ corresponds to the power dissipated in line l by Joule effect (see equation (2)), so that the total square modulus of ψ^L is the total power dissipated in the circuit. In the particular case where a unit current is injected at two different nodes of the network, the square of the 2-norm of ψ is the equivalent resistance of the network between the two injection points.

B. Generalisation of the Hamiltonian formalism to a simplified dynamic model

A basic dynamic description of power systems is provided by the so-called swing equations. It is worth noting that even if very simplified, this approach is still used for fundamental works on dynamic issues at low frequency [30], [50]–[52]. The swing equations are expressed by

$$\mathbb{M}\ddot{\theta} + \mathbb{D}\dot{\theta} = P - \mathbb{L}\theta \quad (6)$$

where $(M_n)_{1 \leq n \leq N_{\text{nodes}}}$ are inertia parameters and $(D_n)_{1 \leq n \leq N_{\text{nodes}}}$ are damping values stored respectively in the matrices $\mathbb{M} := \text{Diag}(M_1, \dots, M_{N_{\text{nodes}}})$ and $\mathbb{D} := \text{Diag}(D_1, \dots, D_{N_{\text{nodes}}})$. For nodes corresponding to power loads, the related mass and damping parameters are approximately zero. Obviously, this model does not include detailed dynamics of synchronous machines nor regulators but it is a realistic starting point for a first extension of the Hamiltonian formalism to low frequency

dynamic studies. These equations are a system of second order differential equations with N_{nodes} nodal variables. This system is transformed into a system of first order differential equations using the vectors ψ of the space S that represent the instantaneous electrical states of the grid

$$\psi = \begin{bmatrix} \psi^N \\ \psi^L \end{bmatrix} = \begin{bmatrix} i\mathbb{M}^{\frac{1}{2}}\dot{\theta} \\ \mathbb{H}_{LN}\theta \end{bmatrix} \quad (7)$$

where $\mathbb{M}^{\frac{1}{2}} := \text{Diag}(\sqrt{M_1}, \dots, \sqrt{M_{N_{\text{nodes}}}})$. A state ψ contains all the information necessary to determine the evolution of the electrical grid. In this mechanical analogy the square of the 2-norm of the node state ψ^N , respectively line state ψ^L , is twice the system's kinetic energy, respectively, potential energy. The square of the 2-norm of ψ is thus twice the mechanical energy of the spring-mass system. Incorporating the dynamics in the Hamiltonian \mathbb{H} and the source term \mathbb{P}

$$\mathbb{H} := \begin{bmatrix} -i\mathbb{M}^{-1}\mathbb{D} & \mathbb{M}^{-\frac{1}{2}}\mathbb{H}_{LN}^\dagger \\ \mathbb{H}_{LN}\mathbb{M}^{-\frac{1}{2}} & (0) \end{bmatrix} \quad \mathbb{P} := \begin{bmatrix} \mathbb{M}^{-\frac{1}{2}}P \\ (0) \end{bmatrix} \quad (8)$$

leads to a system which is strictly equivalent to the swing equations

$$i\frac{\partial\psi}{\partial t} = \mathbb{H}\psi - \mathbb{P}. \quad (9)$$

Equation (9) is similar to a Schrödinger equation where the state ψ is the wave-function of the particle, \mathbb{H} is its Hamiltonian and \mathbb{P} is a source term similar to the injection of a particle in a system. This analogy gives a convenient starting point for applying concepts and efficient mathematical formulations of quantum physics. In this article we focus mainly on the use of a powerful computational tool, the Lanczos method, which is widely used in quantum physics and we show its interest in the context of power systems.

III. LANCZOS METHOD AND LOCALITY PRINCIPLE IN THE HAMILTONIAN FORMULATION

The physical behavior of the static electrical state of the network obeys Eq (4) which admits an analytical solution $\psi = -\lim_{z \rightarrow 0} [z\mathbb{I} - \mathbb{H}]^{-1}\mathbb{P}$. From a theoretical point of view, this expression is simple and solvable. Numerically its computation requires about $(N_{\text{lines}} + N_{\text{nodes}})^3$ operations, as it involves solving a linear system. Similarly, Eq (9) governing the dynamical regime admits a simple analytical solution $\psi(t) = \exp(-i\mathbb{H}t)\psi(0) + \int_0^t \exp(-i\mathbb{H}(t-t'))\mathbb{P}(t') dt'$. Numerically its computation is also cubic in the size of the system because it involves the computation of a matrix exponential. For small to medium sized networks, this computational burden remains moderate, but it quickly becomes prohibitive for larger sizes. Indeed, usual solvers for power systems do not take advantage of the underlying physical properties, especially locality. Here, we propose a new solving method that exploits this locality property as it is usually done in quantum physics.

A. The Krylov space

The locality principle, which is at the heart of the present study, translates mathematically in how the Hamiltonian \mathbb{H} acts on vectors by multiplication. The Hamiltonian is an operator that connects the line and their adjacent node states. Therefore, when applied to any initial vector q_1 localized in some part of the graph, it produces a vector q_2 whose components lie in the adjacent states of q_1 . If \mathbb{H} is applied again, it gives the adjacent states of q_2 . Thus, the surroundings of the initial state q_1 are gradually explored by applying powers of \mathbb{H} to q_1 . Eq. (4) is a linear system whose solution can be expanded on a basis involving the first powers of the Hamiltonian matrix. Therefore, a local approximation of the solutions ψ of Eq. (4) can be written under the form $\tilde{\psi} = f(\mathbb{H})q_1$, where f is a polynomial of a sufficiently high degree \bar{N} and q_1 is an initial state. Similarly, since the solution of Eq. (9) involves the exponential of the Hamiltonian matrix, it can be approximated by the first few powers of its series expansion under the same form. This approximation $\tilde{\psi} = f(\mathbb{H})q_1$ belongs to the Krylov space $K_{\bar{N}}(\mathbb{H}, q_1)$ generated by \mathbb{H} and defined as

$$K_{\bar{N}}(\mathbb{H}, q_1) := \text{Span}(q_1, \mathbb{H}q_1, \dots, \mathbb{H}^{\bar{N}-1}q_1). \quad (10)$$

This space is spanned by the images of q_1 and the first \bar{N} powers of \mathbb{H} . Its dimension \bar{N} must be chosen sufficiently large to reduce the relative error ϵ of the approximation defined as:

$$\epsilon(\bar{N}) := \frac{\|\tilde{\psi} - \psi\|^2}{\|\psi\|^2} \quad (11)$$

Its reciprocal function $\bar{N}(\epsilon)$ characterises the number of iterations necessary to achieve an approximation up to the error ϵ of the exact state ψ .

B. The Lanczos algorithm

The Lanczos algorithm can be used for constructing an orthonormal basis of the Krylov space [34], [35] which is convenient to express the solutions of Hamiltonian power systems equations (4) and (9). The algorithm proceeds by a tridiagonalization of the Hamiltonian \mathbb{H} . In this paper, we consider the case where the Hamiltonian is Hermitian and the Lanczos procedure is simplified. This is obvious if we consider the load flows described by equation (4). If we want to use the same approach for dynamic issues, the damping should be assumed to be zero. But even if there is some damping, a similar approach is possible using the so-called bi-orthonormal Lanczos method [38] [39]. To initialize the Lanczos algorithm, a unit state vector q_1 is selected. We define $\alpha_1 := q_1^\dagger \mathbb{H} q_1$. Then for the second step, we build $q_2 := \frac{\mathbb{H}q_1 - \alpha_1 q_1}{\beta_2}$ with $\beta_2 := \|\mathbb{H}q_1 - \alpha_1 q_1\|$. The next steps generalize the process by defining

$$\forall i \geq 3, \quad \begin{cases} \alpha_i = q_i^\dagger \mathbb{H} q_i \\ \beta_{i+1} = \|\mathbb{H}q_i - \alpha_i q_i - \beta_i q_{i-1}\| \\ q_{i+1} = \frac{\mathbb{H}q_i - \alpha_i q_i - \beta_i q_{i-1}}{\beta_{i+1}} \end{cases} \quad (12)$$

Theoretically, the process stops when $\beta_{\bar{N}+1} = 0$ is reached, but in practice it is advantageous to interrupt it before this. By definition, the Lanczos vectors $(q_i)_{1 \leq i \leq \bar{N}}$ are normalized and orthogonal to each other. They form a recursion chain originating from q_1 . On this chain, the Hamiltonian can be written as a tridiagonal matrix \mathbb{T} , such that

$$\mathbb{T} = \begin{bmatrix} \alpha_1 & \beta_2 & 0 & \dots & 0 \\ \beta_2 & \alpha_2 & \beta_3 & \ddots & \vdots \\ 0 & \beta_3 & \alpha_3 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \beta_{\bar{N}} \\ 0 & \dots & 0 & \beta_{\bar{N}} & \alpha_{\bar{N}} \end{bmatrix} \quad (13)$$

$\mathbb{T} = \mathbb{V}^\dagger \mathbb{H} \mathbb{V}$ where $\mathbb{V} = [q_1 | \dots | q_{\bar{N}}]$ is the matrix of Lanczos vectors. When the algorithm is initialized with a vector q_1 which is a node (respectively line) state, q_2 is a line (respectively node) state, and q_3 a node (respectively line) state, etc. It follows that $\forall i, \alpha_i = 0$, and hence the matrix \mathbb{T} has a null diagonal.

The recursion chain (or Lanczos basis) is shown in Fig. (1). It will be used to compute an approximation of the state of the grid. Its length will be chosen sufficiently large to ensure the convergence of the quantity of interest. It could be extended to get better accuracy.

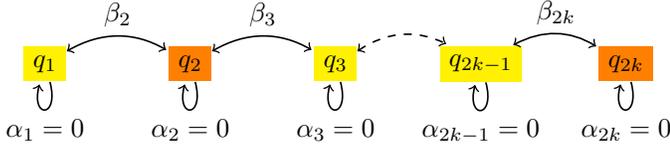


Fig. 1. Representation of the recursion chain (or Lanczos basis) with $\bar{N} = 2k$. The application of the Hamiltonian to a vector of the basis yields a linear combination of the adjacent vectors linked by the arrows. The coefficients of the combination are given by elements of the tridiagonal matrix \mathbb{T} . The node states (yellow) and the line states (orange) alternate.

C. Computation of the static solution

In order to illustrate the efficiency of the new approach we now focus on the steady state resolution. It consists in computing the solution ψ from Eq. (4) on the recursion chain of even length $q_1 \dots q_{2k}$ with $2k := \bar{N}$ originating from $q_1 = \frac{\mathbb{P}}{\|\mathbb{P}\|}$. The chain is truncated, which means that the construction of the Krylov space is not completed. Thus, the calculation yields an approximation $\tilde{\psi}$ rather than the exact solution ψ . Practically speaking, truncating the Lanczos iterations is equivalent to supposing that $\beta_{\bar{N}+1}$ is approximately zero, meaning that $\mathbb{H}q_{\bar{N}}$ decomposes only on $q_{\bar{N}}$ and $q_{\bar{N}-1}$. The solution $\tilde{\psi}$ decomposes only on the line states corresponding to the even Lanczos states

$$\tilde{\psi} = \sum_{i=1}^k \kappa_{2i} q_{2i} \quad (14)$$

with $k := \lfloor \frac{\bar{N}}{2} \rfloor$. The coefficients of the decomposition (κ_i) are obtained by inserting the expression (14) into Eq. (4). The action of \mathbb{H} on the Lanczos states (q_i) is then deduced from Eq. (12), so that each coefficient (κ_i) can be identified in the basis (q_i) . This yields the following recursive formula.

$$\begin{cases} \kappa_2 \beta_2 = \|\mathbb{P}\| \\ \kappa_{2i+2} = -\kappa_{2i} \frac{\beta_{2i+1}}{\beta_{2i+2}}, \quad \forall i = 1 \dots k-1 \end{cases} \quad (15)$$

If the recursion is performed until reaching $\beta_{\bar{N}+1} = 0$, then the exact solution ψ is obtained. Usually it is not the case, and the error ϵ from Eq (11) between the exact and approximate truncated solutions can be expressed on the Lanczos basis by

$$\epsilon(\bar{N}) = \frac{\sum_{i>k} \kappa_{2i}^2}{\sum_{i>0} \kappa_{2i}^2} \quad \text{where } 2k = \bar{N} \quad (16)$$

The numerator is the remainder of a converging series since $\|\psi\|^2 = \sum_{i>0} \kappa_{2i}^2$ and therefore it tends to zero as \bar{N} grows. From a practical point of view, if a recursion of \bar{N} iterations is performed to compute an approximate solution but the user is not satisfied with the results, the calculation can be resumed without recomputing all the coefficients $(\kappa_i)_{1 \leq i \leq \bar{N}}$ or the states $(q_i)_{1 \leq i \leq \bar{N}}$ of the previous approximation (they remain unchanged). This is an important feature of this computational approach.

IV. APPLICATION TO STATIC AND DYNAMIC STUDIES ON A 2D SQUARE LATTICE CASE

We present a first application of the Hamiltonian formalism to an ideal square lattice network with all susceptance parameters (B_i) equal to one. This illustrates the principle of locality for static and dynamic computations.

A. Static case

As we have seen in the previous section, the Lanczos vectors spread from the nodes where the source terms have non-null values. This means that the first amplitudes of the solution to be calculated with accuracy are near the source. But as the number of iterations increases the rest of the network is explored. The Lanczos algorithm can therefore be an efficient way to solve the problem locally in few iterations if the value of the solution is not needed in the whole network. This could for instance be used to accelerate the screening of contingency impacts. This is illustrated with the square lattice network shown in Fig. (2). The modulus of the power flow solution after the injection of power on one side of the network and its extraction from the other side is represented on a 3D-plot. It can be clearly seen that the exact solution is quickly approximated near the source/load nodes and propagates along the grid with the iterations. After 20 iterations, the calculation might be stopped and a good approximation of the final solution is obtained locally.

B. Dynamic case

Short time range simulations of the network may also benefit from locality. Without power injections ($\mathbb{P} = 0$) and without damping ($\mathbb{D} = 0$), the analytical expression of the solution $\psi(t)$ simplifies to $\psi(t) = \exp(-i\mathbb{H}t)\psi(0)$. It is the solution of the homogeneous linear differential equation induced in Eq. (9). Fig. (3) shows the repartition of the state

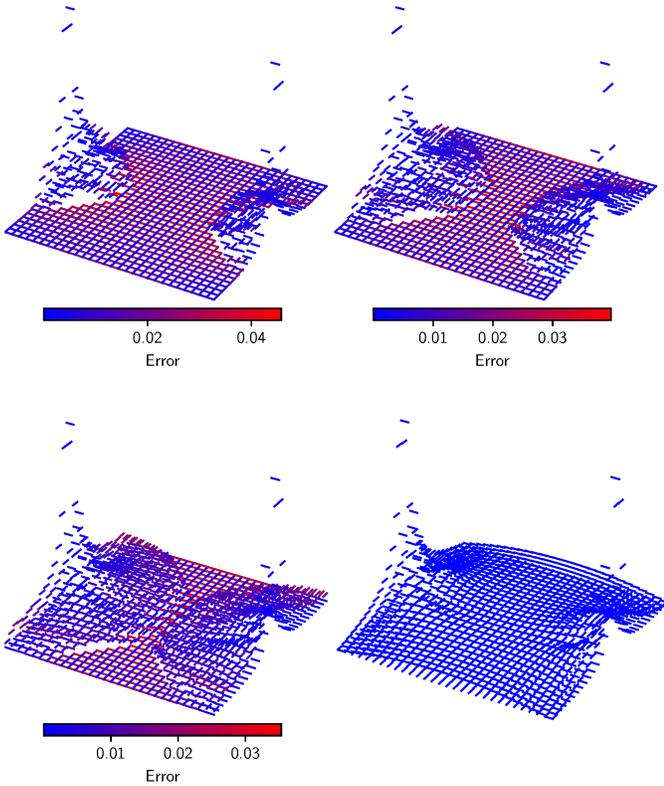


Fig. 2. Illustration of the progressive construction of the solution by considering recursion chains (Fig. (1)) of increasing length. The absolute value of current crossing a line is plotted on the vertical axis. A source and a load are placed at both sides of the network. The approximate solutions obtained from recursion chains of lengths equal to 20, 30 and 40 are shown from left to right and up to down. The exact solution is shown in the bottom right panel. The absolute value of the difference between ψ and $\tilde{\psi}$ on each line l is plotted in a color scale.

$\psi(t = 4)$ on the recursion chain, i.e., the projection of the state on the Lanczos basis. It clearly shows that beyond some range (here on the order of 10) the amplitude of the wave function decreases very sharply. This strong localization of $\psi(t = 4)$ in a finite part of the recursion chain is essential for the high precision of the Lanczos approach. It means that beyond a given length of the recursion chain, the precision of the computation of $\psi(t = 4)$ will increase exponentially or even more steeply. The approximation is computed on the Lanczos chain starting from the initial vector $q_1 = \frac{\psi(0)}{\|\psi(0)\|}$ by $\tilde{\psi}(t) = \mathbb{V} \exp(-i\mathbb{T}t)\mathbb{V}^\dagger \psi(0)$. When \mathbb{P} is not zero the additional term $\int_0^t \exp(-i\mathbb{H}(t-t'))\mathbb{P}(t') dt'$ needs to be added. It is done once again by approximating $\exp(-i\mathbb{H}(t-t'))$ by $\mathbb{V} \exp(-i\mathbb{T}(t-t'))\mathbb{V}^\dagger$. The matrix \mathbb{T} of small size \bar{N} is then diagonalised exactly to compute the exponential. Only the diagonal scalar coefficients of the exponential matrix need then to be integrated, which can be achieved at low cost by any standard integration scheme. Again the locality principle is the central reason for the good behavior of this approach. One sees clearly that in this type of situation the Lanczos-based approach is well-adapted and this is why it is commonly used for analyzing time evolution and wave-function propagation in the context of condensed matter physics or in quantum

chemistry.

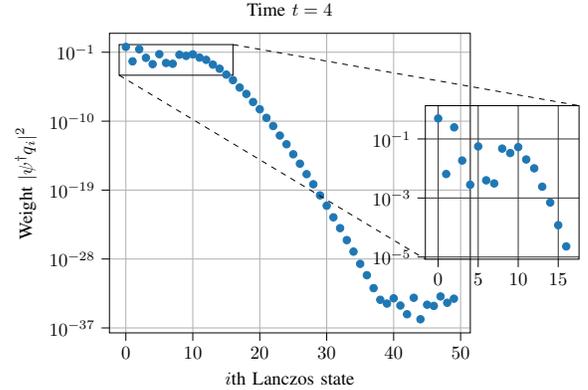


Fig. 3. This figure shows how the exact solution ψ decomposes on the Lanczos states. The weights $|\psi^\dagger q_i|^2$ of the projection of the solution ψ on the Lanczos state q_i are decreasing rapidly as the index i grows.

V. APPLICATION TO THE EFFICIENT COMPUTATION OF LODFs ON REALISTIC NETWORKS

The benefits of the proposed method are now illustrated by calculating the Line Outage Distribution Factor (LODF) [29], [33]. The LODF coefficient $\text{LODF}_l(l') := \frac{\Delta I_{l'}}{I_l}$ describes the change in current $\Delta I_{l'}$ on a line l' after the removal of a line l relatively to the previous current I_l on the line l . These numerical factors describe how a power system reacts to a line loss and are therefore employed by grid operators to assess the safety and robustness of the network in contingency analysis.

A. Computation of the approximated LODF by the Lanczos algorithm

Breaking a line l is equivalent to adding a dipole between the nodes i_l and j_l . The dipole is made by a pair of opposite power injections whose values are directly linked to the initial value of the current, chosen such that they create an opposite current in the line l . For this reason, the LODF is a static solution of Eq. (4) with a specific power injection (a dipole) and can therefore be calculated using the formula (14). For computing the LODF related to the faulty link l , the expression (14) involves an approximated solution $\tilde{\psi}$ we rather denote $\tilde{\psi}(l)$ to emphasize on its dependence on the broken line l . As the number of iterations increases, the approximation $\tilde{\psi}(l)$ tends to the exact solution $\psi(l)$ with a speed characterised by the metrics $\epsilon_l(\bar{N})$ and its reciprocal $\bar{N}_l(\epsilon)$ (defined in Eq. (11) and renamed to emphasize on the dependence on l). However, these metrics characterise the convergence of the solution related to the specific broken line l and are thus not representative of the overall convergence performance of the method on a given network. That is why their definitions are extended by averaging on the set \mathcal{L} of lines that can be broken without losing the connectivity of the network. The global convergence is assessed by computing the average error $\epsilon_{\text{avg}}(\bar{N})$ defined by

$$\epsilon_{\text{avg}}(\bar{N}) := \frac{1}{|\mathcal{L}|} \sum_{l \in \mathcal{L}} \epsilon_l(\bar{N}) \quad (17)$$

where $|\mathcal{L}|$ is the number of elements in \mathcal{L} . We also define $\bar{N}_{\text{avg}}(\epsilon)$ the reciprocal function of $\epsilon_{\text{avg}}(\bar{N})$. It characterises the number of Lanczos steps needed to achieve a given precision ϵ .

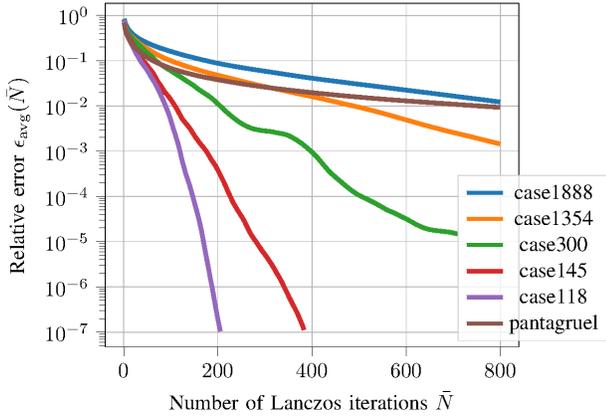


Fig. 4. Mean error $\epsilon_{\text{avg}}(\bar{N})$ for the calculation of the LODF as a function of the number of Lanczos iterations for various IEEE networks and for the European network model called Pantagruel.

Fig. (4) shows the mean error defined in Eq. (17) for various networks. The convergence appears to be at least exponential and resolution can be achieved with a good precision in less than 300 iterations, even for large networks.

B. Comparison of Lanczos method with state-of-the art results

We compare now the performance of the Lanczos algorithm for the fast computation of LODF factors with one of the most efficient methods in the state of the art: the dual approach. For one Lanczos step i such that $1 \leq i \leq \bar{N}$, the operation with the biggest cost is the dot product of the Lanczos state q_i and the Hamiltonian \mathbb{H} . The Hamiltonian is sparse and each block \mathbb{H}_{LN} and \mathbb{H}_{LN}^\dagger has only $2N_{\text{lines}}$ nonzero coefficients. As a state q_i is located on the lines or nodes only, the dot product can be performed in approximately $2N_{\text{lines}}$ elementary operations if the Hamiltonian is stored in a sparse structure. This results in the total cost being approximately $\text{Cost}(\text{Lanczos}) := 2N_{\text{lines}}\bar{N}$ elementary operations for \bar{N} Lanczos steps. The dual approach, detailed in [31] can also be conveniently derived from the Hamiltonian formalism (see the appendix). It is a method based on the flows in closed cycles of the network that uses the network's sparsity to its advantage. It has been shown to be faster than conventional nodal techniques for contingency analysis by a factor of up to 4.5. It has a cost of $\text{Cost}(\text{Dual}) = \frac{2}{3}N_{\text{loops}}^3$.

The speedup is defined as being the ratio between the cost of the dual approach and the cost of the Lanczos iterations for a number of iterations $\bar{N}_{\text{avg}}(\epsilon)$ necessary to achieve an error of magnitude ϵ

$$\text{Speedup}(\epsilon) := \frac{\text{Cost}(\text{Dual})}{\text{Cost}(\text{Lanczos})} \approx \frac{N_{\text{loops}}^3}{3N_{\text{lines}}\bar{N}_{\text{avg}}(\epsilon)}. \quad (18)$$

Table (I) presents the speedup for a threshold value of 5% for various IEEE test networks [53] [40] and the network

called Pantagruel which is an open source model of the European network [48] [49].

TABLE I
COMPARISON OF THE COSTS OF THE LANCZOS AND DUAL METHODS FOR VARIOUS NETWORKS.

Network	N_{lines}	N_{loops}	$N_{\text{avg}}(5\%)$	$\text{Speedup}(5\%)$
case30	41	12	21	0.7
case39	46	8	27	0.14
case57	80	24	41	1.4
case89	210	122	61	47
case118	186	69	55	11
case145	453	309	63	344
case300	411	112	111	10
case1354	1991	638	191	227
case1888	2531	644	343	102
case1951	2596	646	353	98
pantagruel3809	7343	3535	177	11314

As shown in Table (I), for most networks (characterised by their number of loops $N_{\text{loops}} = N_{\text{lines}} - N_{\text{nodes}} + 1$), the Lanczos method appears to run much faster than the dual approach for the calculation of the LODF. This method involves the computation of a projector (see the appendix) which requires a relatively high initial cost, but this cost is compensated for by the possibility of calculating all the LODF values of the network once the projector has been calculated. On the other hand, the Lanczos method demands a relatively small cost to perform the computation of the LODF for a specific line contingency, however the calculations must be restarted for each line of the network. If the purpose is to screen all the contingencies, then the cost is multiplied by the number of lines of the network. As such, the Lanczos method should be used in the case of an estimation of LODF required on some specific lines. It may therefore be adopted for a quick screening strategy performed in real-time by the network operator on the most vulnerable lines of the network.

C. On how the network's topology affects the number of iteration \bar{N}

For a given line l the LODF computed with the Lanczos algorithm converges with a speed that depends on the decrease of the coefficients κ_{2i} (see equation (14)). Therefore this convergence reflects the range of influence of the broken line l and depends on its relative position in the graph representing the network. This dependence is discussed here for the Pantagruel model of the European network and reveals remarkable patterns.

Fig. (5) is composed of two panels. The left panel indicates the number of contingency scenarios which could be solved for a given number of iterations (each scenario corresponding to a broken line). The number of converged scenarios with a $\epsilon = 5\%$ precision is plotted against the number of iterations \bar{N}_l . For more than half of the lines the scenarios could be solved in less than 250 iterations and many require less than 100 iterations. However, some lines need around 1500 iterations (≈ 15 times more steps) to converge. A graphical illustration of the convergence results is given in the right panel of Figure (5). The lines with a high convergence rate are located inside densely meshed areas whereas the lines

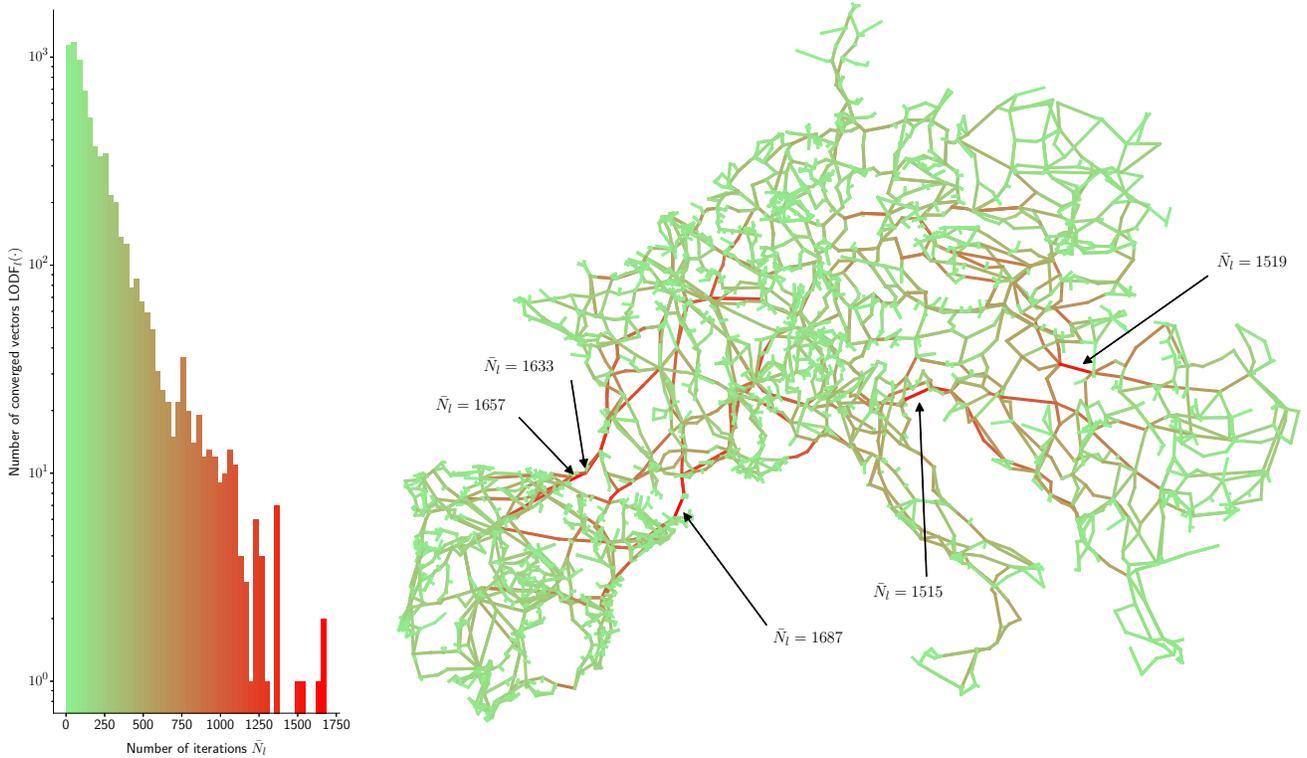


Fig. 5. Pantagruel model of the European transport network. Left panel: number of lines l needing a \bar{N}_l number of iterations to achieve a $\epsilon = 5\%$ convergence for the computation of the vector $\text{LODF}_l(\cdot)$. This histogram is plotted using 50 intervals of the same size. Right panel: Each line l is plotted with a given color depending on the number of iterations \bar{N}_l to achieve a $\epsilon = 5\%$ convergence for the computation of $\text{LODF}_l(\cdot)$. The five lines demanding the highest number of iteration $\bar{N}_l > 1500$ are indicated by arrows.

with poor convergence are situated in sparse areas. It is again an effect of locality. In dense areas, the broken lines have only a local impact, their current being redistributed in the neighbouring lines. Therefore the Lanczos approach converges quickly. In sparse area, the line loss may impact larger areas and the effect of a line outage is less local. So more iterations are required before convergence to a good approximation of the redistribution of the currents. This tendency was evaluated by carrying out the Spearman correlation test between the number of iterations \bar{N}_l and the range $R(l)$ of the redistribution of the currents defined by

$$R(l) := \left[\sum_{l'=1}^{N_{\text{lines}}} \|x(l') - x(l)\|^2 \left(\frac{\text{LODF}_l(l')}{\|\text{LODF}_l\|} \right)^2 \right]^{\frac{1}{2}} \quad (19)$$

where $x(\cdot)$ gives the geographical coordinates of the center of each line. This test yields a value of 0.85 indicating a fairly high monotonic correlation between \bar{N}_l and $R(l)$. This confirms that the lines corresponding to the slowest convergence are also those for which the outage has an impact with the largest range in the network. It is worth noting that some of these lines are known as being critical, especially at the border between France and Spain. These lines were indeed involved in recent large scale incident as mentioned in [54], [55]. In a way, the convergence rate gives an indicator of how a line is critical or not, meaning how it may impact the system's behavior locally or at system-level.

VI. CONCLUSION

This article has provided a new approach for modeling power systems which is based on a Hamiltonian formalism and an exact analogy with quantum mechanics. This analogy allows to apply mathematical methods and efficient computational techniques which are well known in quantum physics but we emphasize that it requires no a priori knowledge of quantum mechanics. The proposed approach is driven by the locality principle and the Lanczos algorithm that is used for computation is built by-design on this locality effect. It has appeared to be very efficient and able to reach speed performances beyond the state-of-the-art methods. In particular contingency analysis of various realistic power systems were performed to demonstrate some of the numerical possibilities of the method. The example of the so-called Pantagruel model of the European network shows in particular the high interest of this approach especially for large power systems. Most of the results were given for steady-state operations. Yet the Hamiltonian formalism is also suitable for dynamic simulations. We have shown how the method can be extended to dynamic modeling of power systems at low frequencies for fundamental studies of inter-area oscillations. Therefore we believe that the present analogy between quantum mechanics and power systems can be fruitful for the future modeling works performed by electrical engineering scientists and engineers.

Finally, at a more mathematical and formal level we expect that the standard tools of quantum physics such as Green's

functions (see the appendix) could be useful for power system analysis. Obviously, the dynamical model of power systems used in this paper is very basic. Yet, the proposed approach could be generalized to more advanced description of electrical networks. For example, in the case of differential evolution equations of higher order introduced by control loops or detailed description of synchronous machines, one can still get a first order differential equation analogous to Eq. (9) but with a different definition of the Hamiltonian.

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APPENDIX

A. Eigenelements and Kernel of \mathbb{H}

Here we consider first the case where there is no damping, so that the Hamiltonian \mathbb{H} defined by Eq. (8) is Hermitian.

Let us consider an eigenvector $\phi^+ = \begin{bmatrix} \phi^N \\ \phi^L \end{bmatrix}$ corresponding to a nonzero eigenvalue λ of \mathbb{H} . Then $\|\phi^N\|^2 = \|\phi^L\|^2$ and additionally $\phi^- := \begin{bmatrix} -\phi^N \\ \phi^L \end{bmatrix}$ is an eigenvector for the eigenvalue $-\lambda$. Moreover, ϕ^N is an eigenvector of $\mathbb{M}^{-\frac{1}{2}}\mathbb{L}\mathbb{M}^{-\frac{1}{2}}$ corresponding to the eigenvalue λ^2 .

We focus now more specifically on the kernel of \mathbb{H} . The kernel of \mathbb{H}_{LN} is given by

$$\ker(\mathbb{H}_{LN}) = \left\{ [\mu \dots \mu]^\dagger, \mu \in \mathbb{R} \right\} \quad (20)$$

which is a one-dimensional subspace of $\mathbb{R}^{N_{\text{nodes}}}$. It is the space orthogonal to all the nodal vectors which have a null sum (i.e., all the sources \mathbb{P} verifying Boucherot's theorem). The kernel of the Hamiltonian \mathbb{H} is deduced from the kernel of \mathbb{H}_{LN} by multiplication with the operator $\mathbb{M}^{\frac{1}{2}}$. Note that this kernel exists only if there is no damping term.

The kernel of \mathbb{H}_{LN}^\dagger is

$$\ker(\mathbb{H}_{LN}^\dagger) = \left\{ [\phi_1^L \dots \phi_{N_{\text{lines}}}^L]^\dagger \text{ such that } \forall n = 1 \dots N_{\text{nodes}}, \sum_{l=1}^{N_{\text{lines}}} s_l(n) \sqrt{B_l} \phi_l^L = 0 \right\} \quad (21)$$

where $s_l(n) \in \{-1, 1, 0\}$ gives the position of the node n relative to the orientation of the line l ,

$$s_l(n) := \begin{cases} -1 & \text{if } n = j_l \\ 1 & \text{if } n = i_l \\ 0 & \text{if } n \notin \{i_l, j_l\} \end{cases} \quad (22)$$

The kernel is constituted by the line states ϕ^L such that the currents $I_l = \sqrt{B_l} \phi_l^L$ follow Kirchoff's current law. This kernel is not modified when there is a damping term in the Hamiltonian and is also independent of the inertia terms. The dimension of this kernel is $N_{\text{loops}} = N_{\text{lines}} - N_{\text{nodes}} + 1$. Indeed, \mathbb{H}_{LN}^\dagger is a mapping from the line subspace of dimension N_{lines} to the node subspace of dimension N_{nodes} and the latter

contains a subspace of dimension one (the kernel of \mathbb{H}_{LN}) that is decoupled from the line space.

To get a basis of the line states of the kernel of \mathbb{H} , we examine all the loops $(L_k)_{1 \leq k \leq N_{\text{loops}}}$ of the network and apply a unit current to each loop. Let L_k be one such loop, and let us denote by $l_1 \dots l_q$ the lines composing the loop. Let us also define, for each line l_p joining the nodes i_{l_p} and j_{l_p} , the first node reached in the loop L_k as being $n_p \in \{i_{l_p}, j_{l_p}\}$ and $k_p := N_{\text{nodes}} + l_p$ be the index of l_p in the Hamiltonian. Then the vector family $(Y_k)_{1 \leq k \leq N_{\text{loops}}}$ defined as

$$\forall k = 1 \dots N_{\text{loops}}, \quad Y_k = \sum_{p=1}^q \frac{s_{l_p}(n_p)}{\sqrt{B_{l_p}}} e_{k_p} \quad (23)$$

is a basis of the kernel of \mathbb{H} . Note that $s_{l_p}(n_p)$ is defined by Eq. (22).

B. Calculation of the exact LODF in the Hamiltonian formalism using Green functions

For two lines l_1 and l_2 of the graph such that the loss of l_1 does not give an islanded network, the LODF is exactly given by

$$\text{LODF}_{l_1}(l_2) = -\frac{\sqrt{B_{l_2}} P_{k_2 k_1}^0}{\sqrt{B_{l_1}} P_{k_1 k_1}^0} \quad (24)$$

where $k_1 := N_{\text{nodes}} + l_1$ and $k_2 := N_{\text{nodes}} + l_2$ are the indices of the two lines in the Hamiltonian matrix and P^0 is the projector on the kernel of the Hamiltonian \mathbb{H} we will discuss later. A simple demonstration of this formula can be provided by the formalism of quantum physics, as we show now. This formalism is based on the Green's function (or resolvent operator)

$$G(z) = [z\mathbb{I} - \mathbb{H}]^{-1} \quad (25)$$

where \mathbb{I} is the identity matrix. This operator is usually used for analysis in the frequency domain with $z := \omega + i\epsilon$ where $\omega \in \mathbb{R}$ is the frequency and the infinitesimal imaginary part $i\epsilon$ ensures the convergence of Fourier integrals. It also gives the steady-state response ψ of the equation Eq. (4) to a given power injection \mathbb{P}

$$\psi = -\lim_{z \rightarrow 0} G(z) \mathbb{P} \quad (26)$$

It is now possible to give a proof of (24) using Green's functions. We define the vector e_{k_1} which is composed of zeros, except at position k_1 , where its component is 1. Let $G(l_1)(z)$ be the Green's function after the line l_1 is broken. The value of the state of the grid $\psi(l_1)$ on line k_2 after the outage of line l_1 is given by

$$\psi_{k_2}(l_1) = -\lim_{z \rightarrow 0} [G(l_1)(z) \mathbb{P}]_{k_2} \quad (27)$$

In addition, $G(l_1)$ can be expressed using the Woodbury matrix identity [37] by

$$G(l_1)(z) = G(z) + G(z)T(z)G(z) \quad (28)$$

with $T(z) := -\frac{1}{G_{k_1 k_1}(z)} e_{k_1} e_{k_1}^\dagger$. This equation is also well-known in the context of multiple scattering theory in quantum

physics and $T(z)$ is known as the T -matrix. Therefore, one has

$$\psi_{k_2}(l_1) = \psi_{k_2} - \lim_{z \rightarrow 0} \frac{G_{k_2 k_1}(z)}{G_{k_1 k_1}(z)} \psi_{k_1}. \quad (29)$$

At the same time, we have

$$G(z) = [z\mathbb{I} - \mathbb{H}]^{-1} = \sum_{k \geq 1} \frac{1}{z - \lambda_k} \phi_k \phi_k^\dagger \quad (30)$$

where (λ_k) are the eigenvalues of \mathbb{H} and (ϕ_k) is an orthonormal basis of the eigenvectors of \mathbb{H} . Thus, when z tends to zero, all terms of the sum tends to finite quantities, except for the terms with $\lambda_k = 0$ (corresponding to the kernel of \mathbb{H}). Therefore

$$\frac{G_{k_2 k_1}(z)}{G_{k_1 k_1}(z)} \xrightarrow{z \rightarrow 0} \frac{P_{k_2 k_1}^0}{P_{k_1 k_1}^0} \quad (31)$$

where P^0 is the projector on the kernel in the line subspace of S and $P_{k_2 k_1}^0$ is its matrix element. Finally, we obtain

$$\psi_{k_2}(l_1) = \psi_{k_2} - \frac{P_{k_2 k_1}^0}{P_{k_1 k_1}^0} \psi_{k_1} \quad (32)$$

Obtaining Eq. (24) is straightforward. The projector P^0 is the operator given by

$$P^0 := \sum_{k=1}^{N_{\text{loops}}} X_k X_k^\dagger \quad (33)$$

where the family (X_k) is an orthonormal basis of the kernel of \mathbb{H} . It can also be computed from any basis (Y_k) (not necessarily orthonormal) of $\ker(\mathbb{H})$ (obtained by the formula (23) for example) by the formula

$$P^0 = \sum_{k,l=1}^{N_{\text{loops}}} [\text{Gram}^{-1}(Y_1 \dots Y_{N_{\text{loops}}})]_{kl} Y_k Y_l^\dagger \quad (34)$$

where the Gram matrix of the family $Y_1, \dots, Y_{N_{\text{loops}}}$ is the matrix of the scalar products, i.e., for all indices $k, l = 1 \dots N_{\text{loops}}$,

$$[\text{Gram}(Y_1, \dots, Y_{N_{\text{loops}}})]_{kl} = Y_k^\dagger Y_l. \quad (35)$$

The computation of the projector (34) needs therefore the inversion of a linear system of dimension $N_{\text{loops}} \times N_{\text{loops}}$ where $N_{\text{loops}} = N_{\text{lines}} - N_{\text{nodes}} + 1$ and that is why its approximate computational cost is $\text{Cost}(\text{Dual}) = \frac{2}{3} N_{\text{loops}}^3$ for performing Gauss–Jordan elimination. This method can also be used for multiple line outages (as in [29] [31]).

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BIOGRAPHY

Pierrick Guichard Pierrick Guichard was born in Le Creusot, France, in 1997. In 2021, he received the Eng. degree from ENSTA Paris and a Master's degree from the Polytechnic Institute of Paris. He is a Ph.D. student at the CNRS in the Institut Néel at Grenoble. His principal research interest include the modeling of power systems and their analogy with physical models.

Nicolas Retière Nicolas Retière was born in Nantes, France, in 1969. He received the Eng. degree in 1993 and the Ph.D. degree in 1997 from the University of Grenoble Alpes, Grenoble, France. He is currently serving as a Professor of electrical engineering with the University of Grenoble Alpes where he hold several responsible positions and is currently the director of the Physics, Engineering, and Materials research department. His principal research interests include the modeling, analysis, and design of power systems.

Didier Mayou Didier Mayou was born in Cognac, France, in 1957. He received the Ph.D. degree from the University of Paris and from the Ecole Normale Supérieure, Paris, France, in 1981. He is currently Director of Research at the CNRS in the Institut Néel (CNRS and Université Grenoble Alpes). His principal research interests include the theory and modeling in condensed matter physics, nanosciences, photophysics, and, more recently, the analysis of power systems.