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# Students' Perception of Change in Graphs: An Eye-tracking Study

Aylin Thomaneck<sup>1</sup>, Maike Vollstedt<sup>1</sup> and Maike Schindler<sup>2</sup>

<sup>1</sup>University of Bremen, Germany, [thomaneck@uni-bremen.de](mailto:thomaneck@uni-bremen.de), [vollstedt@math.uni-bremen.de](mailto:vollstedt@math.uni-bremen.de)

<sup>2</sup>University of Cologne, Germany, [maike.schindler@uni-koeln.de](mailto:maike.schindler@uni-koeln.de)

*Changes and dependencies of two different quantities are characteristics of functional relationships, which are often visualized in graphs. Hence, for graph interpretation it is necessary to perceive and interpret change. This paper focuses on how students perceive change in graphs. Since eye tracking is a promising research tool to approach thinking processes, we conducted an exploratory case study with two participants: We used eye tracking and stimulated recall interviews to examine this method's potential for studying individual processes in perceiving change in graphs. From the observed eye movements and given interpretations, we were able to illustrate that students' individual approaches in perceiving change in graphs can be related to different levels of covariational reasoning.*

*Keywords: Eye tracking, eye movements, functions, graphs, covariational reasoning.*

## Introduction.

Graphs, which are one of the external representations of functions, are pervasive in our lives and therefore also an important topic for mathematics education since they represent functional contexts (Friel et al., 2001). However, the meaning of a graph is not immediately apparent (e.g., Freedman & Shah, 2002). To understand data, represented in graphs, it is important to be able to interpret graphs and especially to perceive the relationship between the values of two quantities.

Some studies have already been conducted on how students reason when working with two different quantities, focusing on how they change in relation to one another. Research focuses, for instance, on different kinds of covariational reasoning (Johnson, 2015). Yet, how students proceed when interpreting empirical graphs and their change is investigated only to a limited extent. Knowledge on this can help to improve the learning and teaching of graph interpretation and in particular of covariational reasoning. To shed light on graph interpretation processes, eye tracking (ET) appears to be a promising research tool. Since ET has—to the best of our knowledge—not yet been used to study students' empirical graph interpretation, we first want to investigate the method's potential itself: whether it is possible to draw conclusions from eye movements of students interpreting graphs and to infer students' approaches of perceiving change in graphs. We first ask a methodological research question RQ1: *Is it possible to infer students' perception of change in graphs from their eye movements and given interpretations?* If it turns out that such inferences are possible, it is of interest to approach the empirical research question RQ2: *What approaches do the participants use when perceiving change in graphs and what levels of covariational reasoning does this reveal?*

## Theoretical Framework.

### Graph interpretation and covariational thinking in mathematics education.

Graphs play an important role in everyday life, as they are used, for example, in order to visualize the development of stock market prices, temperature curves, or training processes. Due to this high applicability for real-world phenomena, and because graphs represent also inner-mathematical functional relationships suitably, graphs are central contents in mathematics education. Graphs do not reveal their meaning immediately, which is a general characteristic of mathematical objects, as Duval (2006) describes:

Mathematical objects, in contrast to phenomena of astronomy, physics, chemistry, biology, etc., are never accessible by perception or by instruments (microscopes, telescopes, measurement apparatus). The only way to have access to them and deal with them is using signs and semiotic representations. (p. 107)

Data visualized in graphs must be understood by inferring their interrelationships, their background, and meaning. To meet these demands, it is crucial for pupils “to identify certain types of changes and dependencies, which are part of common events in the real world, as well as to become familiar with their representations” (Eisenmann, 2009, pp. 73–74). Numerous studies revealed that students may have difficulties dealing with representations of functions and that certain types of errors are common due to a lack of covariation understanding (e.g., iconic interpretations, interval/point confusion, slope/height confusion) (Leinhardt et al., 1990). Working with functions requires dealing with two different quantities, focusing on how these quantities change in relation to one another. Thompson and Carlson (2017) emphasize that students’ need to develop an appropriate idea of the relationship between values of two quantities in their work with functions. They argue that “variational and covariational reasoning are fundamental to students’ mathematical development” (p. 423). Since the graphs used in our study always consider changes in two quantities (time/distance covered and velocity, or time and filling level), we restrict ourselves to covariational reasoning in this paper. Thompson and Carlson (2017) distinguish six major levels of covariational reasoning, which show a continuum of students’ conceptions of covariation: *no coordination*, *precoordination of values*, *gross coordination of values*, *coordination of values*, *chunky continuous covariation*, *smooth continuous covariation* (p. 441). The first four levels can be considered as preliminary stages of conceptualizing covariation. Students whose covariational reasoning can be classified in the two highest levels perceive actual covariation, as only then the change in one quantity affects changes in the other quantity. Therefore, our study focuses on chunky and smooth continuous covariation, since we intend to describe students’ covariational reasoning when describing change in empirical graphs. In general, *continuous covariation* involves the perception of change in one variable simultaneously with changes in another variable. More specifically, in a *chunky* continuous covariation, the changes are perceived in intervals with a fixed, but not necessarily the same, size. The focus is on the values *at the end* of each interval and how they change as compared to the end of the following interval. In contrast, in a *smooth* continuous covariation the change is perceived as increasing or decreasing. The focus is on how the values change *within* an interval.

Some studies have already investigated how changes and covariation in functions are identified and understood (e.g., Johnson, 2015). Yet, it has hardly been explored how change is perceived in the interpretation of graphs and in how far covariational reasoning is involved. These processes of interpreting graphs and perceiving change within them are complex and may differ between individuals, so that ET appears to be a promising method to investigate these processes on a micro-level. We believe that with the help of eye movements, cognitive processes in the interpretation of graphs and in particular the perception of change in graphs as well as students' approaches and use of covariational reasoning may be inferred: This is why we investigate the use of ET and its potential for this purpose in this paper.

### **Eye tracking in mathematics education research.**

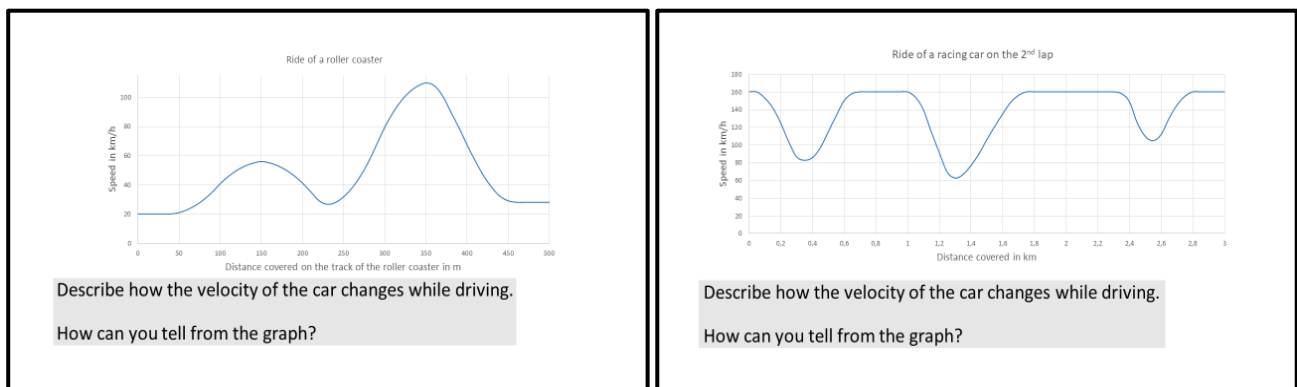
ET describes the capturing of person's eye movements, which can be visualized in a video of participants' field of view, with a wandering dot indicating the gaze. Studies on eye movements have considerably increased over the last years (König et al., 2016). In a mental-oriented view of ET, which we also adopt for our study, ET studies are used to infer cognitive processes from eye movements, i.e., "to use eye movements as a window to cognition" (König et al., 2016, p. 2). The prerequisite for this is the eye mind hypothesis (EMH). The EMH presumes a close relationship between what persons fixate on and what they process (Schindler & Lilienthal, 2019). However, it has been revealed that this assumption cannot easily be taken for granted in mathematics education. For example, Schindler and Lilienthal (2019) found that there are instances in which eye movements cannot easily be mapped to cognitive processes and that even if this is possible, the interpretation of eye movements is often ambiguous. Therefore, they call for domain-specific theories for the interpretation of eye movements. One domain in which ET has rarely been used is graph interpretation. Nevertheless, ET seems to be beneficial for studying students' graph interpretation: Strohmaier et al. (2020) emphasize that ET lends itself to the use of visualizations of mathematical objects since the work with visualizations requires persons to process visualized information with multiple gazes. To investigate the potential of ET for the analysis of graph interpretation processes, we conducted the study that is presented in this paper: It explores the potential of ET for analyzing graph interpretation and perception of change in particular. In addition, we will empirically focus on how change in graphs is perceived by students and on if and how eye movements can be used to infer students' covariational reasoning.

### **Method.**

#### **Sample, task design, and setting.**

In our exploratory study, we analyzed eye movements of two university students during graph interpretation tasks, who volunteered to be participants for our study. They were told that they will participate in a study on functions at secondary level. The participants, Gerrit (age 21; engineering and management student with a focus on production engineering; high affinity for mathematics) and Elias (age 28; a teacher student for German and history; low affinity for mathematics) were selected because they have different profiles in terms of their professional background and mathematics affinity. Further, they were not familiar with empirical graphs so that they might show interesting approaches to interpret them. In addition, being university students, they could probably express rich information about their cognitive processes in the interviews.

This paper presents an excerpt from a larger study that examined participants' eye movements while interpreting graphs in different situational contexts (see Figure 1 for two examples) as inspired by the Shell Centre for Mathematical Education (1985)). Each unit consisted of five analogous tasks with different demands. The focus in this paper is on the first task, which asked the students to describe the change in the graph (see Figure 1). The tasks were presented on a screen. There was no time restriction for working on the tasks. The study took place in a quiet room with the participant and the first author of the paper administrating the tasks. The participants sat on a firm chair in front of a table where the monitor was located. Before the questions and the graph were presented, the students were familiarized with the situational context and the graph by presenting them with a digital task sheet on the monitor showing the graph and an introductory text about the situational context.



**Figure 1: Examples of Task 1 (translated to English)**

### **Eye tracker and ET data.**

For data collection we used a wearable eye tracker: Tobii Pro Glasses 2 (50 Hz, binocular, infrared, 45 g, built-in microphone). The tasks were presented on a 24" screen (60 Hz, viewing distance: 60 cm). First, a single-point calibration procedure was performed. Under ideal conditions, gaze estimation is  $0.62^\circ$  (Tobii Pro, 2017). In our study, the accuracy was  $1.1^\circ$  on average, which corresponds to 1.15 cm in the screen. This inaccuracy was taken into account in the task design and data interpretation. Before solving the tasks, Gerrit and Elias passed an additional 9-point calibration verification so that we could later check the measurement's accuracy. Since we wanted to study students' interpretation process and especially their perception of the change in graphs, we considered all eye movements relevant and decided to analyze raw data (Holmqvist & Andersson, 2017), i.e., eye movements as displayed in gaze-overlaid videos.

### **Stimulated recall interview based on gaze-overlaid video.**

In our study, we combined ET with stimulated recall interviews (SRIs) using gaze-overlaid videos, similar to Schindler and Lilienthal (2019) since ET has not been used so far to investigate student's perception of change and therefore it is still unclear how to interpret the eye movements in this context. Stimulated recall is a technique that "gives participants a chance to view themselves in action as a means to help them recall their thoughts of events as they occurred" (Nguyen et al., 2013, p. 2). We used gaze-overlaid videos as stimulus in our study, in which the participants can watch their eye movements as a wandering dot. They were supplemented with students' utterances during task processing. Gaze-overlaid videos represent a strong stimulus because they make eye movements

visible, which are usually not conscious (Stickler & Shi, 2017). Thus, it is particularly important to keep the time span between the ET recordings and the SRI short. We therefore conducted the interviews directly, after only a short break for data transfer, so we can assume that they were still very much aware of their thoughts, which were then additionally recalled by the strong stimulus of visible eye movements as trigger. The participants had the possibility to stop the video themselves to explain their thoughts. In addition, the interviewer was able to pause the video and to invite the participants to express their thoughts. The participants wore ET glasses even during the SRI. Here we made use of the scene camera and the built-in microphone of the eye tracker to record the verbal utterances and gestures. This procedure was explained to the participants before data collection.

### **Data analysis.**

To prepare the data analysis, the students' utterances while working on and answering the tasks were transcribed together with a description of the eye movements taking place during this process. In addition, the utterances of the interviewer and the interviewees in the SRI were transcribed. These elements were arranged in one document in neighboring columns to make visible what happened simultaneously and what the SRI refers to. Data analysis followed Schindler and Lilienthal (2019) since this approach is particularly suitable for domains in which ET has rarely been used before and where it is not yet clear how eye movements can be interpreted: The steps of qualitative content analysis (Mayring, 2014) were applied and—due to the explorative and descriptive nature of the research questions—categories were developed inductively. We distinguished between gaze categories that describe gaze patterns and interpretation categories that describe the cognitive processes associated with the respective gazes. Eye movements and cognitive processes, as described by the students in the SRIs, were then mapped for all data.

## **Results and Discussion.**

### **Research Question 1: Feasibility of inferring students' perception of change in graphs.**

Our methodological research question (RQ1) asked whether it is possible to infer students' perception of change in graphs from their eye movements. Results indicate that the participants were able to explain their eye movements in the SRI. Their utterances gave information about their cognitive processes while interpreting graphs and perceiving the change within them. This is the prerequisite, since we have as a basis only utterances and interpretations of the participants' eye movements. We cannot observe the cognitive processes directly, but can only get closer with the help of the participants' eye movements and interpretations. However, like Schindler and Lilienthal (2019) we found that a certain eye movement pattern could not always be clearly assigned to one cognitive process (see Table 1).

Nevertheless, conclusions about approaches can be drawn based on the eye movements that the participants used when perceiving change and interpreted in the SRI. For instance, we were able to observe certain eye movement patterns, consisting of different gazes, that were interpreted by the participants (e.g., following the course of the graph, jumping between a point on one axis and a point on the graph, looking on several different points on the graph in succession). These eye movements were used to perceive the change in the graphs to eventually be able to describe it in order to answer

the task. The participants referred to these gaze sequences, which differed between them, in the SRI. Using this information, we were able to infer approaches when perceiving change in graphs.

Eye movement pattern (identified in the gaze-overlaid video)	Cognitive process (described by the student in the SRI)
The gaze follows the course of the graph	Grasp the graph (e.g., the course or properties) Grasp the situational context (e.g., thinking about driving curves with a racing car)
Fixations on turning points of graphs	Focus on prominent parts of the graph (e.g., turning points) Grasp the graph (e.g., the course or properties)

**Table 1: Exemplary eye movements from task 1 and students' interpretation**

### **Research Question 2: Students' perception of change and covariational reasoning**

Our empirical research question (RQ2) addressed the approaches the participants use when perceiving change in graphs and whether levels of covariational reasoning can be revealed from this. As mentioned above, we were able to observe differences regarding the occurring eye movements between the two participants. Elias followed the course of the graph or a graph section with his gaze particularly often. Afterwards, he interpreted this mostly as trying to grasp the situational context and sometimes the graph (e.g., course or properties). Moreover, he often looked at several different points on the graph in succession. He explained that this eye movement pattern served the same intention (grasping the situational context or graph). Also, Elias used gestures to support his gaze, for example by following a section of the graph with his finger. This was never the case for Gerrit, who also used other eye movements particularly frequently. His gaze often jumped between a point on one of the axes and a point on the graph. He explained that he was reading a particular point or value from the graph. Sometimes he added that he used this to grasp the graph. In addition, he often looked at turning points of the graph. He interpreted this in the SRI as focusing on prominent parts of the graphs. These results indicate that Elias perceived the change in the graphs by looking at the graph and following it with his gaze and, in some instances, his fingers, or by making sense of it by looking at several different points. He explained the change of velocity of a racing car in a car race as follows.

Elias: The car starts. Probably, most likely enters a curve, therefore drives slower here, then comes out of the curve, drives faster again, drives a straight stretch, then drives a steep curve, so narrow, must brake very hard in any case. Then drives relatively quickly out of the curve again, then again a long stretch and then again a small curve. [utterance translated from German to English by first author]

It is clear from the utterance that he focuses on the change of velocity within the intervals, i.e. increasing or decreasing. This is characteristic for smooth continuous covariational reasoning.

Gerrit's perception of change was apparently different. He also said several times that he focused on prominent parts and read values there:

Gerrit: Ehm, so first the velocity decreases to 0.35 km. Then it increases again to the original 160 km/h from 600 m to 1 km. Then it drops again to about 60 km/h. And then increases again to 160 km/h after 1.3 kilometers. From 1.8 km to 2.3 km keeps that, eh the car its velocity. And then drops again to 105 km/h approximately at 2.5 km/h and then rises again to its original 160 km/h until 2.8 km and then keeps that until 3 km. [utterance translated from German to English by first author]

In this utterance, Gerrit defined intervals and read off a value for each endpoint of an interval in order to describe the change of velocity, what is typical for chunky continuous covariational reasoning. Yet, he did not disregard the change of the graph between two of such points: He described whether the graph increased, decreased, or was constant in the respective interval. This hints—at least to a certain extent—also at smooth covariational reasoning.

## Discussion and outlook.

In this paper, we have illustrated that it is possible to interpret students' eye movements when interpreting graphs in order to obtain information about their perception of change in graphs. Our tentative results indicate that ET in combination with SRI seems to be a suitable method for studying the perception of graphs. In addition, we were able to infer different approaches regarding the perception of change and different levels of covariational reasoning for Elias and Gerrit. Our results provide evidence that certain eye movement patterns are typical of certain levels of covariational reasoning. For future research, it would be of interest, for instance, whether the relations found between Elias and Gerrit's gaze patterns and approaches can also be found for other individuals. Moreover, we only focused on smooth and chunky covariational reasoning, i.e. the two highest levels of covariational reasoning according to Thompson and Carlson (2017). Thus, it might be valuable to test and further sharpen the insights from this exploratory paper on a larger data set, and with middle school students to maybe find different approaches in interpreting the change in graphs and related levels of covariational reasoning and its preliminary stages. Besides, it should be examined what implications arise from our initial result. Elias, whose approach can be related to smooth covariational reasoning, for example, often referred to the situational contexts and related the change of velocity directly to the circumstances of the situational context (by saying that the car is driving a curve and therefore brakes, etc.). Whether there is a direct connection between the kind of covariational reasoning and the extent to which the situational context is referred to and how this affects the further interpretation of the graph remains to be studied. Even though our case study provided only a glimpse on these aspects, we think that it may be a first step towards investigating students' interpretation of graphs and in particular their perception of change in graphs using eye tracking.

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