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Bar-model introduction in late primary school

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The Singapore bar-model has recently arrived in many Spanish classrooms, where it has been identified as an effective modelling tool for problem-solving in primary schools. This model introduces a pictorial approach prior to the abstract symbolic resolution. Schools usually change over to this scheme by introducing it in grade 1 (6/7 years old) and progressing yearly with the children. In this study we perform a pre-post analysis of 71 fifth graders. An experimental group of 16 children were introduced to the Singapore bar-model, while the control group, the remaining 55 children, solved the exact same set of problems using their usual symbolic-only approach. Our results show statistically significant improvements in the performance of the experimental and the control groups in the cognitive domains of “knowing” and “applying” as well as in the content domain of geometry.

Keywords: Problem solving, mathematics education, Singapore bar-model, classroom research.

Introduction

Problem-solving is one of the key aspects in mathematical education and maybe one of the most difficult skills to teach. Many students and many teachers struggle with this issue, where solutions come from a complex mixture of creativity and pattern-recognition. Teaching mathematical modelling literature agrees on problem-solving being some cycle (Perrenet & Zwaneveld, 2012) where reality is transformed into a mathematical model whose mathematical solution is found and, later, transformed back into reality.

According to DeWindt-King and Goldin (2003) “a representation is any configuration (of characters, images, concrete objects, etc.) that can denote, symbolize or, otherwise “represent” something else” (p. 2). The representation of the problem data can be a way to emphasize, understand and link the most important information, so that practice can be seen as a useful tool for the students in problem solving (Heagarty & Kozhevnikov, 1999). The scientific community agrees that representing data is useful for improving performance in problem solving, although there is no single way to represent data. The works of Fagnant and Vlassis (2013) and Heagarty and Kozhevnikov, (1999) show several ways, focusing mostly on the schematic representations. Appropriate representation has positive effects on children's performance, which reinforces the importance of working on this content from an early age (Uesaka et al., 2007).

Spanish tradition tends to rely only on the symbolic representation of mathematical models, and other types of representations are hard to find in classrooms and/or textbooks. Among teachers and students, the usual scheme is known as the “*Datos-Operación-Solución*” scheme (which translates as “data-operations-solution”, *DOS*, from now on). Under this scheme, children are asked to organize problem-solving in the three steps shown in practice in Figure 1. First, they write down “Data” and all numerical data that appears in the problem statement, usually with units. Then, children have to

perform the needed calculations (*Operations*). Finally, in the *Solution* block, they write down a full sentence stating the result back in the context of the problem.

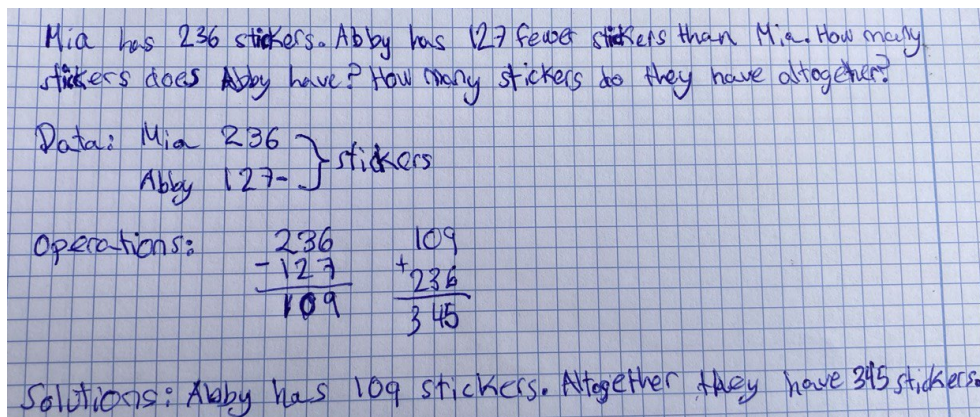


Figure 1: DOS-Scheme implemented by an 11-year-old from a bilingual school (English & Spanish)

It was in the 1980s when Singapore brought in the idea of modeling quantities and their relationships with bars, avoiding the more abstract symbolic-only approach and giving students a pictorial way to represent a problem’s mathematical model. According to Blum and Leiss (2005) modelling cycle, shown in Figure 2, BMM allows the students to obtain a model of the situation prior to the mathematizing step. This approach clearly contrasts with the DOS, which moves directly towards the mathematical work without performing any modelling.

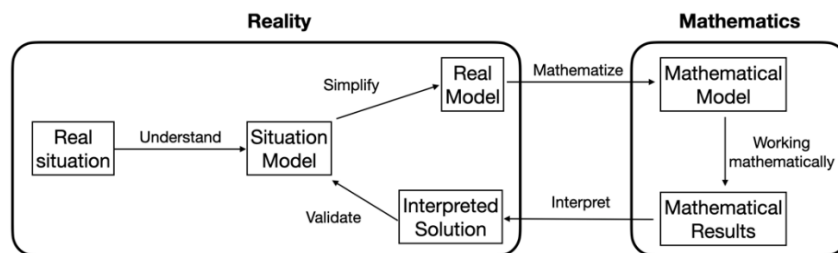


Figure 2: Blum and Leiss modelling cycle

The bar method model (BMM from now on) uses as a fundamental mainstay the concept of the "bar", which is a pictorial diagram that allows the organization of the abstract information, converting it into understandable data (Walker, 2005). Using this method, it is possible to represent data from arithmetic and algebraic problems (word problems), which not only allows us to transform abstract words and concepts into pictorial representations, but also helps to understand and analyze the problem itself (Yeap, 2011). According to Kho (1987) and Kho et al. (2009), once the students have been able to represent the data using the bars, they are much more likely to understand the situation and, therefore, to solve the problem.

Since its introduction in Singapore, the BMM has been a centerpiece in mathematics, beginning in the first grade of primary education. According to Ng and Lee (2009), in the first two years, they usually work through pictorial representations of known objects, to encourage students to have specific references to the data. Subsequently, the concept of the "bar" is introduced, with which they continue to work throughout their school years. Many Spanish schools have started to use Singapore’s textbooks. Usually, the method is introduced in grade 1 and progresses with the children as they

advance in their schooling years. In those schools, older kids are thus taught with the DOS scheme while their younger schoolmates are introduced to this pictorial representation. This situation leads to the main research question for this study: Can grade 5 students, who have only been taught to use abstract symbolic modelling, also benefit from the introduction of BMM?

Methodology

This research follows a pre-test/post-test design. This study has been carried out in a middle-sized school in Segovia, Spain. All 76 fifth grade students have participated in it, though 5 of them have not been included since they are identified as special-education students. The remaining 71 children are divided into 4 classrooms, each one lead by a different math teacher. Three of the classrooms (55 students) have been assigned as the control group, and the remaining one (16 students) has acted as our experimental group, since their teacher was looking for new approaches to problem-solving methods, given the difficulties their students had on this topic.

The intervention was carried out as follows: For over a month, daily, both experimental and control groups solved the same two problem inventories consisting of a total of 53 word-problems, all able to be solved with the help of BMM. It is important to remark that these children have never been exposed to pictorial models before our interventions. The control group worked with their usual DOS scheme. The word-problem inventory was agreed upon by the four classroom teachers and divided into three sections: additive structure with natural numbers, additive structure with decimal numbers and, finally, multiplicative structures and fractions (that were also represented with bars). With respect to TIMSS' content-domain classification, all the problems fall in the group *numbers*, while regarding cognitive domains, 49% classify as *applying*, 17% as *reasoning*, and 34% as *knowing*.

The first sessions for the experimental group were entirely dedicated to the pictorial representation of the problem statement, introducing BMM by example. In the first two sessions, all questions were removed from the problems' statements to avoid the mechanical use of DOS scheme by the students. Only when students got used to modelling the situation with bars and could pose and answer several questions from their drawn bar-model were the original questions to the problems reintroduced.

To assess the effectiveness of the intervention we used two similar sets of questions released from TIMSS (2011) as pre-test and post-test. The first set had already been configured and tested in the Spanish context in Fraile's PhD (2017) while the second was designed to match this one with a similar weight (shown in Table 1) in both content domains (numbers, data display and geometric shapes and measures) and cognitive domains (applying, reasoning, and knowing).

Table 1: Total number and percentage of the questions in each domain for the pre- and post-tests

	Date	Question content domain			Question cognitive domain		
		Numbers	Data	Geometry	Applying	Reasoning	Knowing
Pre-test	28/04/2020	7 (63%)	2 (18%)	2 (18%)	4 (36%)	4 (36%)	3 (27%)
Post-test	08/06/2020	9 (60%)	4 (27%)	2 (13%)	5 (33%)	5 (33%)	5 (33%)

Among all the questions in the pre-test, three of them threw up unexpected results and were reintroduced to better understand the children's motivations for their answers, redesigning them as diagnostic questions, where incorrect responses allow the teacher to understand where the mistakes may be (Wylie & William, 2006).

Analysis

Students' answers analysis has been carried out using R software. We start by comparing problem solving skills before our intervention between the experimental and control groups, according to their results on the pre-test. The results shown in Table 2 may suggest some bias in favor of the experimental group, but the chi-square test p-value is 0.82, showing no significant differences between the number of correct answers in both groups.

Table 2: Post-test average results in a 0-10 scale for each group in the studied domains. In brackets, pre-test average results and difference between both measurements.

	Student's content domain			Student's cognitive domain		
	Numbers	Data	Geometry	Applying	Reasoning	Knowing
Experimental	8.1 (5.7+2.4)	9.0 (6.9+2.1)	7.2 (6.6+0.7)	8.2 (7.8+0.4)	8.1 (5.6+2.5)	8.0 (4.4+3.6)
Control	5.9 (4.6+1.3)	8.0 (6.9+1.1)	2.6 (4.7-2.2)	5.6 (6.3-0.7)	6.9 (5.3+1.6)	5.4 (2.9+2.5)

The results suggest better problem-solving skills in the experimental group, as well as an improvement of this group after the intervention. Six repeated measurement Anovas were performed to determine statistically significant differences between both groups. These results are shown in Table 3.

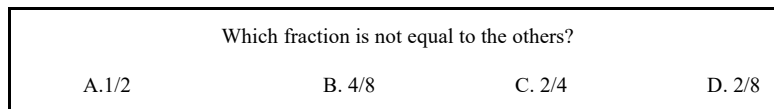
Table 3: Anovas' summary for the differences between both groups

Student's content domain			Student's cognitive domain		
Numbers	Data	Geometry	Applying	Reasoning	Knowing
F(1,69)=8.055 p<0.1 (.)	F(1,69)=0.737 p=0.39	F(1,69)=13.72 p<0.001 (***)	F(1,69)=11.2 p<0.01 (**)	F(1,69)=1.572 p=0.21	F(1,69)=10.71 p<0.01 (**)

From the data shown in Table 2, the only significant difference in content domain has been in geometry, with an average difference of 2.9 points between groups due to the worse results in the post-test for the control group. In numbers and in data the differences have been smaller, with 1.1 and 1 average points respectively, only significant to a 90% for the data domain. With respect to cognitive domains, two out of the three categories have had a significant increase for the experimental group showing p-values in the Anova tests below 0.01.

As mentioned before, three questions in the pre-test were introduced in the post-test. Let us start our analysis with the following question (TIMSS Ref.M051091) that falls into the content domain “numbers” and the cognitive domain “knowing”.

Figure 3: Question TIMSS Ref.M051091 that was included (in Spanish) in the pre-test



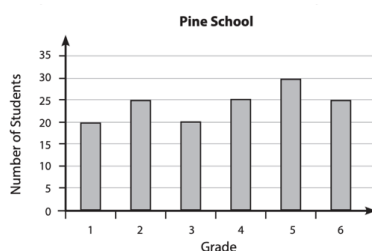
The pre- and post-test results of our fifth graders are shown in Table 4. No significant differences between experimental and control groups have been found in the answers of both groups in the pre-test (Chi-square test p-value=0.18) nor in the proportion of correct answers (Binomial test p-value=0.06). After consultation with teachers and students we understood that “not equal” might be ambiguous for the children. In the post-test we used the unambiguous term “not equivalent” obtaining a non-statistically significant 14% increase in the correct answers. Experimental and control groups differ in favor of the experimental group in the post-test (Chi-square test p-value=0.02; Binomial test p-value=0.004) where 94% of the children in the experimental group can answer correctly.

Table 4: Comparison between pre- and post-test in exp. and cont. groups in M051091 question

	Pre-test					Post-test				
	A	B	C	D	Other	A	B	C	D	Other
Cont.	21 (38%)	5 (9%)	3 (5%)	21 (38%)	5 (9%)	16 (29%)	3 (5%)	3 (5%)	27 (49%)	6 (11%)
Exp.	2 (13%)	2 (13%)	0 (0%)	11 (69%)	1 (6%)	0 (0%)	1 (6%)	0 (0%)	15 (94%)	0 (0%)

The second question that was reevaluated is shown in Figure 4. TIMSS provides the four answers A. 20, B. 25, C. 30 and D. 35. Trying to improve its usage as a diagnostic question, we changed the solutions to A. 35, B. 7 (number of empty gaps in the graph), C. 180 (6 grades times 30 students) and D. 145 (students represented in the graph). A remark on Fraile’s PhD annotations (Fraile, 2017, p.110) suggested one might further modify in the post-test the answer B. 7 to B’. 30, which was a common mistake, probably due to 30 being the only numerical data in the exercise.

Figure 4: Second question (TIMSS Ref.M051117) that was included (in Spanish) in the pre-test



In the Pine School there is room in each grade for 30 students. How many more students could be in the school?

- A. 20
- B. 25
- C. 30
- D. 35

Due to the changes in the test answers, we only analyze the proportion of correct answers. Table 5 shows a higher success rate in the post-test of the experimental group, with 75% of correct answers, with respect to the 44% of the control group but also an increase in the mistakes towards the new option B'.30. The binomial test for the two groups p-value of 0.108, with no statistical significance.

Table 5: Comparison between pre- and post-test in exp. and cont. groups in M051117 question

	Pre-test					Post-test				
	A	B	C	D	Other	A	B'	C	D	Other
Cont.	26 (47%)	0 (0%)	18 (33%)	6 (11%)	5 (9%)	24 (44%)	8 (15%)	11 (20%)	10 (18%)	5 (9%)
Exp.	8 (50%)	1 (6%)	6 (38%)	1 (6%)	0 (0%)	12 (75%)	2 (13%)	3 (19%)	1 (6%)	0 (0%)

The last question that was reevaluated (see Figure 5) deals also with basic facts about fractions and falls also in the content domain “numbers” and the cognitive domain “knowing”. It is an open question where students are asked to write their own answer.

Figure 5: Question TIMSS Ref.M041299 that was included (in Spanish) in the pre-test

Tom ate $\frac{1}{2}$ of a cake, and Jane ate $\frac{1}{4}$ of the cake. How much of the cake did they eat altogether?

TIMSS suggests $\frac{2}{6}$ as a common mistake, resulting from adding the two numerators and the two denominators. Our analysis of the pre-test showed that $\frac{1}{6}$ is also a common answer. Table 6 shows the results for $\frac{3}{4}$ (or equivalent) answers, $\frac{2}{6}$, $\frac{1}{6}$, other answers and blanks for both tests. No statistically significant differences are found between pre- and post-tests for any of the two groups, but, again, the experimental group obtains a higher rate of correct answers in the post-test compared to the control group, throwing a p-value of 0.064 in the Binomial test.

Table 6: Comparison between pre- and post-test in exp. and cont. groups in M041299 question

	Pre-test					Post-test				
	$\frac{3}{4}$	$\frac{2}{6}$	$\frac{1}{6}$	Blank	Other	$\frac{3}{4}$	$\frac{2}{6}$	$\frac{1}{6}$	Blank	Other
Cont.	15 (27%)	9 (16%)	7 (13%)	3 (5%)	21 (38%)	18 (33%)	8 (15%)	2 (4%)	5 (9%)	22 (40%)
Exp.	6 (38%)	3 (19%)	0 (0%)	1 (6%)	6 (38%)	9 (56%)	2 (13%)	1 (6%)	0 (0%)	4 (25%)

Conclusions

Not surprisingly, both experimental and control groups have benefited on average from the systematic work in problem-solving. A deeper analysis shows that there are three domains in which we have found significant differences between groups at 95% significance-level. With respect to content domain, significant differences have only been found in the geometry block, while the numbers block, throws a p-value only below 0.1. Regarding cognitive domains, “applying” and “knowing” show a statistically significant difference in favor of the experimental group, consistent with the fact that these blocks have been the focus of 83% of the intervention.

The international success average of fourth graders in question M051091 assessing the concept of equivalent fractions is 44%, while Spanish students obtain a much lower rate of 30%. One academic year later, our fifth graders are close to the international average with a slight difference (not statistically significant) in favor of the experimental group and a 45% average success rate. After the intervention, where bars were used to represent fractions, there is a non-statistically significant increase in the results of both groups which now shows a much clearer difference in the experimental group, where only one student misses this question. The differences between groups in the post-test are statistically significant and prove a better grasp of the basic facts of fractions for this group.

The international success average for fourth graders in Question M051117 where they are asked to read and interpret data from a graph is 54%, while Spanish students obtain a lower rate of 50%. In the pre-test there are nearly no differences between the two groups, with C. 180 being the most common wrong answer. The average success for this question is 48% in the pre-test. Option B was replaced by 30 respondents in the post-test, where 14% of the children chose this modified answer (only one chose number 7 in the first version), a clear sign of a better choice for a diagnostic question.

The international success average for fourth graders in Question M041299 where they are asked to add fractions $\frac{1}{2}$ and $\frac{1}{4}$ is 23% showing, again, Spanish students a much lower rate of 14%. In the pre-test, our fifth graders obtained a better result on average of 30%, with no significant differences between groups. After the intervention, the percentage of correct answers for the control group is 33% and 56% for the experimental group, without statistical significance.

According to the literature, BMM can help children’s problem-solving skills when it is taught in the early years. This study suggests that the positive effects of this approach can be attained even as late as 5th grade and that further analysis and interventions should be tested to better understand how it affects the different cognitive domains and to establish with elder children (or even adults) if the opportunity window to improve problem-solving skills with the use of BMM ever closes.

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