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Potential semiotic synergy in *TouchTimes* multiplication

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Abstract: Inspired by the notion of synergy, the sharing of signs between tangible and digital artefacts, we explore signs created by students that emerge through their use of TouchTimes (TT), a multitouch iPad application. We use Peirce to study the tripartite function of the signs (symbol, icon, index) available, as a way to study potential synergies in TT, and how they compare to signs found in other visual models of multiplication.

Keywords: Multiplication, visual representations, TouchTimes, synergy, semiotics, multitouch.

Introduction

Figure 1 shows a drawing made by a grade three (9-year-old) student who had been asked to draw what it would look like to solve the following problem using the multitouch application *TouchTimes* (TT) (Jackiw & Sinclair, 2019): “A bunch of buttons fell on the floor. Nick gathered them in heaps of eight buttons. He made five heaps. How many buttons are there?” We wanted to use this drawing to motivate our interest in exploring the nature and status of visual images used in the teaching and learning of multiplication. The drawing has familiar aspects to it, such as the unit-of-units (of circles) drawn on the right, which can be found in Davydovian (1992) representations of multiplication. It also contains unusual elements, such as the fingers on the left or the two-part structure, with the vertical line down the middle. The left side illustrates the original unit of 8, which is produced in TT by pressing eight fingers on the screen (even though there are ten on the drawing!). And the right side shows the symbolic expression as well as the unit (large lasso) of 5 units of 8 circles.

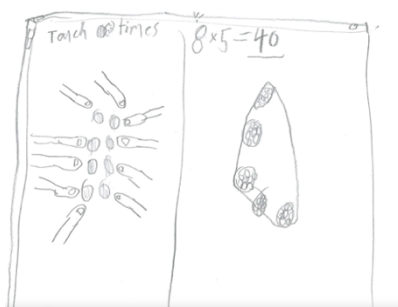


Figure 1: Student drawing of 8×5

The complexity of this drawing and its relation to the dynamic, tangible tool prompted us to wonder how it relates to commonly-used visual images of multiplication. Given the multiple ways there are of conceptualising multiplication, and the semiotic differences between them, we are interested in exploring how the use of TT might support the kind of *synergy* described by Mariotti and Montone (2020). We show that the material nature of Peirce’s indexical signs helps explain this synergy.

Visual models and representations of multiplication

Multiplication is a challenging concept for learners and researchers have advocated for the use of visual models or representations (Davydov, 1992). Kosko (2020) uses multiplicative scheme theory to argue that the development of multiplicative reasoning in children moves from counting by ones

to conceiving of conceptual multiplicative representations. He argues that visual conceptual multiplicative representations can promote or limit engagement in multiplicative reasoning. However, there are a variety of ways to conceive of multiplication and each representation will stress some elements over others.

Maffia and Mariotti (2018) distinguish two models of whole number multiplication, the repeated-sum model and the array model, which are represented, respectively, in Figure 2a and 2b. They show how different properties of multiplication can be justified using each model. They also acknowledge that while the former model is pervasive in school mathematics, it has limitations, and that many researchers have argued for increased use of the latter model—particularly to support students’ understanding of multiplication as a binary operation (in contrast to the unary operation of addition).

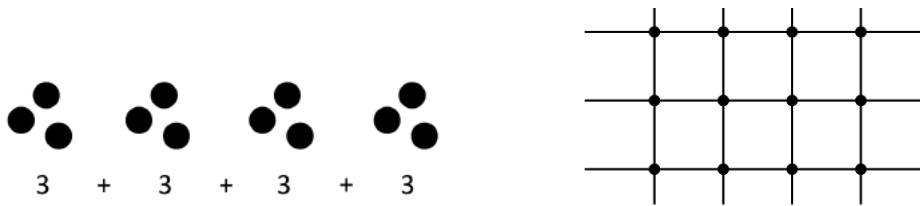


Figure 2: (a) Repeated sum model; (b) Array model (in Maffia & Mariotti (2018))

For example, Clark and Kamii (1996) use the image shown in Figure 3a to contrast additive and multiplicative thinking. While the top representation draws on the repeated sum model, the bottom one highlights the two levels of inclusion involved in multiplicative thinking—both the horizontal one and the vertical one. Looking at the vertical component of the representation, there is a *many-to-one* relation expressed by the three arrows between the bottom row and the middle one; each of the three single points is moved to the next level where it is conceived of as a single unit. Boulet (1998) reformats the Clark and Kamii representation using a Davydovian approach (Figure 3b). Davydov bases multiplication in measurement so that multiplication is related to a transfer of units. According to Davydov (1992), unitization is when “a person purposely alters a unit of count” (p. 10). The horizontal encasing that was visually expressed in Clark and Kamii is absent. This focuses attention to the many-to-one relation, or, in Davydovian terms, unitising: three becomes one.

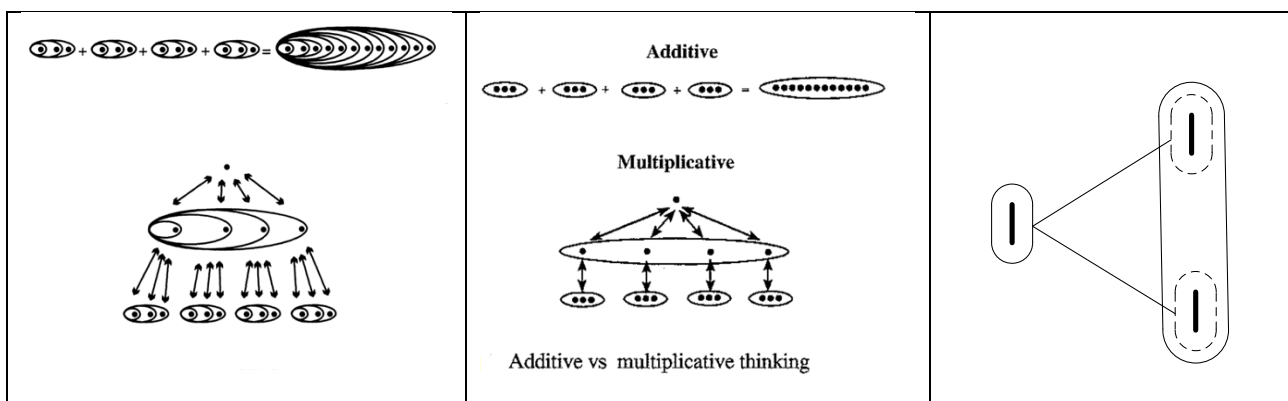


Figure 3: (a) The inclusive model; (b) The unitising model; (c) The splitting model

Confrey’s (1994) splitting model (Figure 3c) depicts an original unit, which is then “split” to create duplicate versions of itself simultaneously, which contrasts with the sequential repetition of repeated sum model. The splitting model highlights a *one-to-many* relation in addition to equal grouping.

Finally, Vergnaud (1988) proposes a t-table (Figure 4) as a way of highlighting the four quantities involved in multiplication. He distinguishes two relationships: scalar and functional. The scalar relationship (vertical) is the ratio between two values of a variable: for example, the ratio between the number of cars is 5 : 1, which is the same as the ratio between the number of tires 20 : 4. The functional relationship (horizontal) is the ratio between two values of two distinct variables. For example, the functional relationship between the number of cars and the number of tires is 4.

The number of cars	The number of tires
1	4
5	x

Figure 4: The t-table functional, quarternary model of multiplication

While each of the authors cited above argues for the advantages of their model, we are interested in the question of how these various models might be complementary—if they each emphasise different aspects of multiplications and if student flexibility is valued, then surely exposure to multiple models is desirable. However, little research has focused on how students handle these different models.

Theoretical Framing

We adopt a semiotic approach drawing on the work of Peirce, who argues that understanding is based in signs. He distinguished three types of signs (icon, index, symbol) which differ in terms of the nature of the relationship between the signified and the signifier. Icons operate according to likeness or resemblance between the signifier and the signified. If iconic signs become conventional codes within particular cultures, they may become symbols, which have an arbitrary relationship with that to which they refer, as is the case for most spoken language as well as numbers and signs such as + and -. Figure 2a combines symbols (such as ‘3’) and icons (the discs that resemble objects such as tokens). In Figure 2b, there are iconic elements in that the lines and intersections resemble either a rectangle or a cartesian coordinate system, but for some people, the image operates more like a symbol, if there is no resemblance apprehended. Indices are quite different from the other two in emphasizing the material link between signifier and signified.

Unlike icons and symbols, indexical signs are bound to the context in important ways—they “show something about things, on account of their being physically connected with them” (Peirce, 1894/1998, p. 5). As in the case of smoke billowing from a chimney indicating that the fireplace is in use, the smoke *indexes* the fire. The index is a sign that is materially linked or coupled to “its object”. According to Peirce (1932), an index

refers to its object not so much because of any similarity or analogy with it, (...) as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other. (2.305)

Verbal indexical signs perform this link to context very effectively, for instance, terms like “this” or “that” or even pronouns like “she”, are meaningful only in relation to a given context. The arrows in Figures 3a and 3b act as indices in that they materially couple the three dots to the single dot; they are not similar to the transfer, but they index it. Similarly, the dotted lines in Figure 3c index a transformation of the object from one that has been split (copied) to one that belongs to the product.

Drawing on the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), Mariotti and Montone (2020) use the notion of *synergy* to refer to the common signs that emerge from students' interactions with different artefacts, which are related to the same mathematical concept. In their study, Mariotti and Montone had students translate and reflect a concave quadrilateral, first with paper and pins which fold and anchor, followed by a similar activity in a dynamic geometry environment. Mariotti and Montone argue that in the sharing of two artefacts, the new meanings emerging from the two artefacts synergistically interweave with each other to “reach the meanings that were intended” (p. 118).

We are interested in the synergy that occurs within the use of one single artefact, TT, in which there are signs that can be associated with different conceptual models of multiplication. This was exemplified in Figure 1, which shows the double unitising of the circles *as well as* the fivefold ‘copying’ of the 8 circles on the left. We would like to understand how this kind of synergy can arise and hypothesise that this can be done by studying the types of signs (in Peirce’s typology) produced in TT. In other words, in order to identify opportunities for synergy, we analyse the TT environment in terms of its different signs, looking to see how they might relate to the different representations described in the previous section.

TouchTimes: A brief introduction

When first opened, TT is divided in half by a vertical line (Figure 5a). Whichever side of the screen is first touched by the finger(s) of the user’s hand, results in a different coloured disc (termed a ‘pip’) appearing beneath each finger and the numeral corresponding to the number of visible pips being displayed at the top of the screen (Figure 5b). The numeral adjusts instantly when fingers are added or removed, whether temporally in sequence or simultaneously, and represents the multiplicand. In order to preserve each pip (that otherwise vanish), the finger(s) must maintain continuous screen contact. When a user taps on the other side of the line with her second hand, bundles of pips (called ‘pods’) appear with each contact (Figure 5c). The number of pods is the multiplier and each pod contains a duplicate of the pip configuration, matching both the relative locations created by the pips and the colours of those pips (Figure 5d). When pods are created, a second numeral also appears, separated from the first by the multiplication sign (‘×’). If the user presses the array icon on the top right corner of the screen, the pips rearrange themselves into rows and the pods into columns.

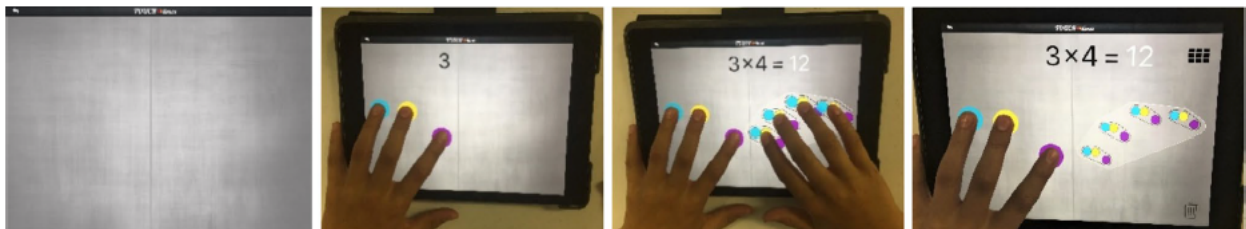


Figure 5 (a) The initial screen of TT; (b) Three fingers placed on the left; (c) Four fingers placed on the right; (d) Lifting the four fingers to see the unit of pods

Although TT provides a tangible, dynamic environment for creating multiplicative relations, and therefore differs significantly from the static representations shown above, it also aims to highlight certain aspects of multiplication and provides multiple opportunities for making different signs. The design of TT draws extensively from Davydov’s (1992) model, with the first action being the creation

of a unit, and the second action between the creation of a unit of units. Practically, this means a reversal of the repeated sum model since the unit size comes first—and therefore, we would read Figure 5d as ‘three, four times’ rather than as ‘three groups of four’, which is how the expression 3×4 is described in most North American classrooms. The array icon transforms the unit-of-units display into an array, which turns the repeated sum representation into a unit by unit array.

In addition to emphasizing *unitising*, TT also emphasizes the *splitting* idea in the sense that the ‘copies’ (literally, multi-plying) are produced simultaneously (pod-fingers touch the screen all-at-once). There is also the one-to-many relation that is emphasized when a pip finger is lifted or pressed since a change in pip on the left produces a change in each and every one of the pods on the right—we refer to this as a *spreading* idea. Finally, multiplication in TT is expressed as a *co-varying* operation between pips and pods (embodied by the use of two hands), with either quantity being dynamically changeable and visibly affecting the other.

In terms of *embedding*, the sequence needed to produce a multiplicative statement is shown in Figure 6. These four steps parallel the quarternary aspect of Vergnaud’s description of multiplication (see also Askew, 2018). That is, there are four numbers involved in multiplication. In the first step, there is the determination of the number of pips, this is a variable amount, that is, it can change (at any time). The second step is to see those pips as a unit, or a pod (“1”). This idea is expressed in TT by embedding the pips by a border. The third step determines the number of pods, this is also variable. And the final step is to see these multiple pods as a unit, the product. Again, the four numbers are the number of pips, one pod, the number of pods, and one product.

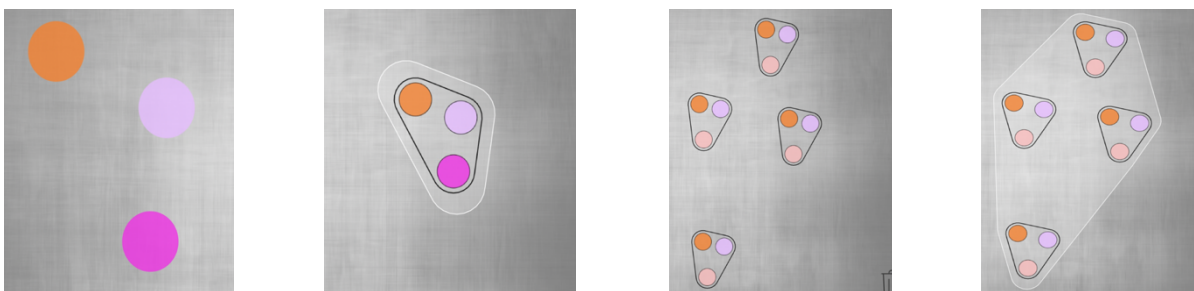


Figure 6 (a) Three pips; (b) A unit of three pips (a pod); (c) Four pods; (d) A unit of four pods (a product)

A semiotic analysis of TT

In terms of the signs at play in TT, there are numerous similarities and differences with the models we have described above, as well as significant polysemy. Some similarities include the use of circular (points, discs) signs to represent objects; the use of encircling to represent sets of objects; and, the combination of number and operation symbols with icons and indices. TT differs significantly from the other representations in three ways: (1) the use of colour; (2) the tangibility—the representations are seen *and* touched; and, (3) the temporality.

Colour is used to emphasise the idea of copying found in the splitting model, when three differently coloured pips created on one side of the screen are replicated on the other. Colour thus enriches the iconicity of ‘object to the split’. Groups or units can therefore differ internally, unlike with the circular discs, repeated sum or the dots of the unitizing models. While the number of pods indexes the

functional relationships of Vergnaud, colour indexes the scalar relationship (1 : 3). Figure 7 shows two drawings made by grade three (9-year-old) students in response to the button question (see introduction). These students had used TT during one class period and the teacher proposed the button question in order to see whether the students could connect a more traditional multiplication problem with their experiences using TT. We use these drawings simply to exemplify how TT signs might be re-used by learners. In Figure 7a, colour is used to index the split of the original unit into 5 pods.

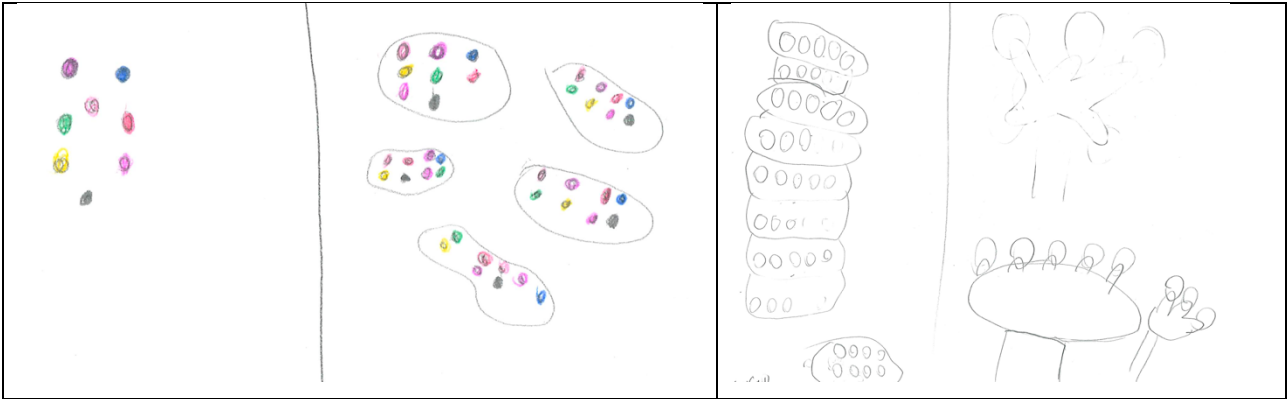


Figure 7 (a) Coloured pips; (b) 5 pods and 8 fingers

Unlike the iconic status of the circles in the repeated sum and the unitizing models, the coloured discs in TT have an indexical quality since they literally index the touch of fingers. In addition, the colour and the shape of the pods serve to index the placement of the pip-fingers. Colour and shape thus perform the same indexical function as the arrows in Figure 3a and 3b, but through different signs. The drawing of the hand in place of the pips on the right side of Figure 7b signifies the indexical relation between the use of one’s fingers and the internal make-up of the pods drawn on the left. Similarly, as mentioned above, the splitting model uses a dotted line to index the double nature of the split element as both a copy of the original and as a part of the product. The encircling of the pod on its own and then of the pods altogether seems to have a similar indexical function.

The temporal aspect of TT introduces yet other difference. For example, if one focuses only on the final product, as in the image shown in Figure 5d, the meanings of the signs are different from those produced by the sequence shown in Figures 5a, 5b, 5c. The right-hand side of Figure 5d resembles the repeated sum image shown in Figure 2a and, indeed, could be seen iconically as four groups of three. But in the temporal production of the multiplicative expression, the unit of 3 is produced first, and then copied four times, so that the sign includes both the image on the left *and* the image on the right—it functions indexically as a trace of the finger touches.

There are two other aspects of temporality that functions in tandem and both involve the potential action of fingers. First, the potential to lift or press a pip-finger functions as an index in that it literally “shows something about things”, which is that changing the number of pips will change each and every pod—which is the one-to-many idea that is represented in Figure 8a and aligns with Boulet’s model. In prior research (Bakos & Pimm, 2020), the repeated lifting and pressing of fingers has been described as “dancing” (because of the simultaneous rhythmic change of all of the pods at once), and is a common mode of interaction for learners. In this dynamic dancing gesture, attention is drawn to a fundamental element of multiplication, that of spreading. Spreading is articulated in the work of Confrey (1994) who first proposed an alternate multiplicative metaphor based on children’s facility

with the action of splitting. Dancing draws attention to the horizontal embedding of Clark and Kamii, which hints towards the general since the iterative pattern of the embedding can be easily extended. This possibility of horizontal variation emphasises both the horizontality and verticality of multiplication as a reference transformation.

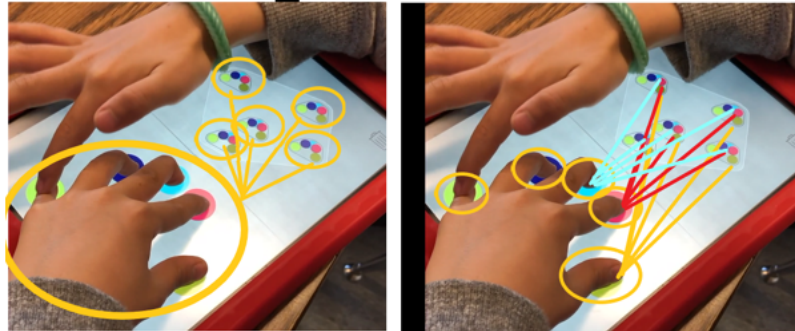


Figure 8: (a) Focus on pods; (b) Focus on pips

The second aspect of temporality is the potential to remove or add any pod (by lifting or pressing a pod-finger) which shows that changing the number of pods will change the product. This is the many-to-one idea represented in Figure 8b, which aligns with Clark and Kamii. Figure 8 highlights an additional polysemic aspect at play in TT.

Our analysis shows the different ways in which TT shares signs with other representations, even though these are quite distinct kinds of artefacts from one another. Focusing on the signs created helps explain how TT might function as an environment that promotes semiotic synergy. In addition, the temporality and tangibility of TT elicit more indexical signs, in lieu of iconic ones, which establishes a dynamic connection between the object and “the senses or memory of the person for whom it serves as a sign”, to re-quote Peirce.

Conclusion

In this paper, we have analysed different visual representations of multiplication (which draw on different mathematical models of multiplication) that have been discussed in the research literature, including the repeated-sum model, the array model, the unitising model and the splitting model. Our goal was to identify the different aspects of multiplication emphasised in these representations so that we could compare them semiotically to *TouchTimes* and also explore the potential for semiotic synergy found within TT. Our analysis revealed that the signs in TT are polysemic, indexical and multiple. While this might lead to conceptual confusion (see Izsák & Beckmann, 2019), we argued that it increased the potential for synergy. The three drawings offered in the paper suggest that students can in fact work with the multiple meanings, and can choose to use some more explicitly than others. The presence of the hands in two of the drawings strengthens our hypothesis that the greater indexicality of TT signs can better support embodied understandings of multiplication.

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