



**HAL**  
open science

# Signing about elementary algebra in Austrian Sign Language: What signs of the notion of variable can represent

Flavio Angeloni, Annika M Wille, Christian Hausch

► **To cite this version:**

Flavio Angeloni, Annika M Wille, Christian Hausch. Signing about elementary algebra in Austrian Sign Language: What signs of the notion of variable can represent. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03765017

**HAL Id: hal-03765017**

**<https://hal.science/hal-03765017>**

Submitted on 30 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Signing about elementary algebra in Austrian Sign Language: What signs of the notion of variable can represent

Flavio Angeloni<sup>1</sup>, Annika M. Wille<sup>2</sup> and Christian Hausch<sup>3</sup>

Universität Klagenfurt, Austria;

<sup>1</sup>[flavioan@edu.aau.at](mailto:flavioan@edu.aau.at); <sup>2</sup>[annika.wille@aau.at](mailto:annika.wille@aau.at); <sup>3</sup>[christian.hausch@aau.at](mailto:christian.hausch@aau.at)

*Languages have a significant impact on mathematics learning, with visual languages and spoken languages in particular differing. The study presented here investigates how one can sign about notions in elementary algebra in different ways in Austrian Sign Language. In particular, the focus is on the question of what sign language signs of the notion of variable can represent and thus how they may impact on the understanding of variables. Distinctions from spoken language are identified. The study is part of a larger investigation into communicating about elementary algebra in sign languages.*

*Keywords: Language, learning mathematics in sign language, classifiers, elementary algebra.*

## Introduction

Sign languages are full-fledged natural languages with their own grammars. They are in no way inferior to spoken languages (Beecken et al., 2014) and can express concrete as well as abstract things just as those. Therefore, communication about mathematics is just as possible as in spoken language. There is not only one sign language used around the world but many different ones (Braem, 1995). Their development does not necessarily coincide with that of the respective national spoken languages. Sign languages organize ideas and convey content, meaning and significance in a different way than spoken languages. That means a person who thinks in sign language thinks differently than one who is thinking in spoken language (Grote, 2010). Therefore, the conceptual understanding that is typically oriented toward hearing contexts – i.e., based on the characteristics and features of spoken languages – often does not match the conceptual understanding of students who use sign languages, and thus may not be appropriate for this audience (Krause, 2016, 2019; Krause & Wille, 2021; Wille & Schreiber, 2019). The study presented here is part of a longer series of investigations on the topic “sign language and mathematics education”. The specifics of sign languages will be considered with the aim of developing and evaluating approaches and concepts for teaching mathematics in sign language. Language plays an essential role in both teaching and learning mathematics (Fleming, 2007; Thürmann & Vollmer, 2017). Therefore, the research focuses first on sign language, especially on its lexemes and classifiers, two important components of sign languages. The first multiphase investigation concerns specifically talking about mathematical activities in elementary algebra. The mathematical focus is on different aspects of variables (see below): object aspect, substitution aspect and calculus aspect (Malle, 1993). The linguistic focus is on the Austrian Sign Language (ÖGS). The general interest of the study is to investigate how these aspects occur in ÖGS and how mathematical notions and their aspects are represented in this sign language. Knowledge of these two points about sign language is necessary to teach mathematics in this language. In the following, the general question is specified in terms of the object aspect of variables: Which lexemes (signs) are used for the object aspect of variables in ÖGS? In what way are classifiers used for this? What are the

differences between sign language and spoken language in this regard? High knowledge of these topics about language is on the one hand necessary for the translation between spoken language and ÖGS regarding mathematical teaching and learning. On the other hand, it provides the basis for the next investigations in mathematics education regarding sign language.

## **Aspects of variables**

The word “variable” corresponds to its etymology: It can appear in different forms, denote different things and it can have different aspects. For example, there are “word variables”. These are single words or groups of words that are representative of something else – e.g. of numbers (Akinwunmi, 2012; Küchemann, 1978; Malle, 1993). The “usual” variables used in mathematics are the “letter variables”. According to Malle (1993) at least three aspects can be identified for variables: object aspect (the variable is an unknown or unspecified object of thought), substitution aspect (the variable is a placeholder into which numbers may be inserted) and calculus aspect (the variable is a sign without meaning, but which may be operated with according to certain rules) (Malle, 1993; Schoenfeld & Arcavi, 1988; Wille, 2008). If a variable is considered under the object aspect, then the object of thought itself can be different: a figure, a number, a number as a quantity of something etc. Thus, with a single word in spoken language such as “number”, one can refer to various things. Therefore, “variable” can denote many different things. When such terms are translated into other languages, phenomena such as diversification may well occur. In translation studies the term “diversification” means the phenomenon in which for one word in the original language there are several words in the target language (Koller, 2011). Vice versa there is a “gap”, if for one word there exists no translation. Thus, the first question that arises at this point is:

- *Which sign language’s sign or signs are used for the manifold term “variable” (from the point of view of the object aspect)?*

## **Classifiers in the sign languages**

Classifiers have an important function in sign languages. Classifiers (CL) or depicting handshapes are a complex and a highly discussed topic in sign language research. Definitions often diverge. In the presented research a classifier is an element whose meaning is related to the context. It represents “entities” based on their characteristic features and is involved in a morphologically complex structure. With a classifier objects or processes are classified based on common features (e.g. the index finger represents a “person”) (Zwitserslood, 2012). Classifiers can be used to refer to the property, position and movement of the signified (Beecken et al., 2014). They follow certain rules of sign language grammar. For instance, if they refer to a noun, they are signed after the sign for that noun. If classifiers refer to a verb, they are signed before the sign for that verb. For the classification of classifiers there are several possibilities concerning semantics or linguistic context. Regarding the mathematical background of the study, first of all, the following classification is made: semantic classifiers (representing a “stylized” shape of the object: e.g., flat hand for cars or tables, claw hand shape for a clock or picture frame, etc.), instrumental classifiers (which denote objects according to how they are handled) and size and shape classifiers (representing size, extent, etc. of an object). The last group of classifiers – if it is considered to be a part of classifiers – can be further divided into static classifiers (to represent the size and shape of the object) and tracing classifiers (by moving one

or both hands they outline the shape or size of that object) (Beecken et al., 2014; Zwitserlood, 2012). The use of classifiers for real objects (including humans and animals) is quietly clear. However, when signing about mathematical notions – as about variables – the second question arises:

- *Which classifiers are used when signing about variables with regard to the object aspect?*

The formation of new groups of classifiers related to the mathematical nature of the represented objects is not excluded here. Since mathematical notions in the school context mostly originated from spoken languages the third question investigated is:

- *What differences exist between spoken language (German) and ÖGS regarding the object aspect of variables?*

## **Learning environment**

In each session of the learning environment the students get tasks<sup>1</sup> concerning two persons that have to distribute brochures. In the beginning they have one thousand brochures to distribute, but it is unknown how many the one person and how many the other person is going to distribute. Then one person gives a stack with two hundred brochures to the other person. The students are now asked to answer two questions about the amount of the brochures that each person has – before and after the stack of two hundred brochures was moved from one person to the other person.

The students have to perform those tasks in the given order according to the dialogic principle “me-you-we” (Green & Green, 2018; Ruf & Gallin, 1995, 1999). This principle provides for three phases. In the “me-phase” each student deals with the problem alone. The student does not necessarily have to come up with a solution in this phase. Notes or sketches of a solution idea are sufficient – that means: “this is how I do it”. In the “you-phase” the students exchange ideas with each other (in pairs of two or in small groups): “how do you do it?”. Results and open questions are to be recorded here. In the “we-phase” the results of the “you-phase” are presented, discussed and compared in the plenum in order to arrive at a joint solution and find a convention: “this is how we will do it”. The phases “this is how I do it”, “how do you do it?” and “this is how we will do it” can be rephrased from the point of view of sign languages as follows: “this is how I sign it”, “how do you sign it?” and “this is how we will sign it”. However, the last phase cannot always take place in this form, because there are rarely conventions for mathematical terms in ÖGS. The basic idea of the learning environment presented here is that the students become as active as possible – not only mathematically, but first of all in their language, the Austrian Sign Language. The influence of spoken language should be reduced as much as possible, on the one hand with the aim to not disturb the natural communication in ÖGS, on the other hand to not distort the observations of sign language communication. Spoken language could transport elements into ÖGS that are not compatible with the specifics of a sign language and specifically of the ÖGS. Therefore, from the beginning the material was developed in a sign language perspective and not as a translation of a material that was developed in a spoken language. The material contains no text, but only pictures (comics) and QR codes linking to videos

---

<sup>1</sup> Task card no. 1 at [https://me.aau.at/~awille/mathe\\_in\\_oegs\\_variablen\\_01.html](https://me.aau.at/~awille/mathe_in_oegs_variablen_01.html); task card no. 2 at [https://me.aau.at/~awille/mathe\\_in\\_oegs\\_variablen\\_02.html](https://me.aau.at/~awille/mathe_in_oegs_variablen_02.html); task card no. 3 at [https://me.aau.at/~awille/mathe\\_in\\_oegs\\_variablen\\_03.html](https://me.aau.at/~awille/mathe_in_oegs_variablen_03.html)

in ÖGS with explanations and tasks. The students are able to watch these videos as often as they want to on their own smart phones. All videos are signed by a deaf person who also has ÖGS as basic language. A script for the videos is not used in order to avoid any interference errors (that are typical expressions of spoken language but not of ÖGS). Mathematical terms (e.g. variable), for which no signs (in ÖGS) are known, are not mentioned in the videos and in the sessions. These terms are not even finger spelled.

## Methods

The research took place in form of two 60 minutes sessions in July and August 2021– with three people in the first and four people in the second session. The participants were deaf adults (age range 30 to 65) and their basic language was ÖGS. Basic language means the language in which a person thinks – his or her “inner” language. Adults were selected for this first part of the study because they were the most competent in ÖGS of those available. They were no more familiar with variables from their school background. Two teachers moderated the sessions. They announced which tasks had to be solved and answered technical questions. The order of the tasks is given as a structured interview. After the “you”-phase, the teachers led the “we”-discussion as an unstructured interview: getting the participants in the discussion and asking further questions (e.g. why is it like you say?). Both sessions were recorded by video and the participants were aware of that. So that is a direct observation with continuous monitoring. The videos were glossed (transcribed with glosses) according to the notation system of (Prillwitz & Wudtke, 1988), but simplified for the aim of the investigation. Glossing means the practice of writing down a sign language text sign-by-sign. Afterwards, the classifiers and signs of the notion of variable were determined. The data were then analyzed according to the principle of content analysis (Kuckartz, 2018).

## Findings

With regard to the core of the tasks – the number, i.e., the quantity of brochures that are stacked – five different classifiers occur (Fig. 1). Since the meaning of the classifiers can only be deduced from the context this was also considered. Here are transcribed examples of each that the participants used:

- P1: WIEVIEL CL<sub>1</sub>-STAPEL FLAVIO WIEVIEL CL<sub>1</sub>-STAPEL SANDRA  
*How many pieces does Flavio have? How many pieces does Sandra have?*
- P2: ZUSAMMEN EIN-TAUSEND IX CL<sub>2</sub>-STAPEL CL<sub>2</sub>-STAPEL  
*In total there are one thousand [stacked brochures].*
- P1: SANDRA DA EIN-TAUSEND CL<sub>3</sub>-STAPEL  
*Sandra has one thousand [pieces].*
- P3: ERSTENS DREI CL<sub>4</sub>-STAPEL CL<sub>4</sub>-STAPEL CL<sub>4</sub>-STAPEL  
*At the beginning there are three stacks.*
- P1: ZWEI-HUNDERT CL<sub>5</sub>-GEBEN  
*[Flavio] gives 200 [brochures] to [Sandra].*
- P6: HAUFEN GEBEN WEISS-NICHT CL<sub>1</sub>-STAPEL WIEVIEL WEISS-NICHT  
*From the pile [Flavio gives her] some. But you don't know how many there are in the pile.*

For statements about the unknown or unspecified number of brochures on a stack the first classifier was used (Fig. 1a). It is introduced by the “question sign” WIEVIEL (*how many*). In the recordings this classifier occurs when unknown quantity is concerned. The execution of the classifier, moreover, reveals what exactly it refers to: The splayed dominant hand (“sh” hand shape) moves upwards. It represents every single brochure on the stack. Since those brochures are a lot the movement of the hand is smooth. So, the sign was used to denote the unknown or unspecified number of brochures. In particular, the information that the brochures are stacked are by the sign transmitted as well.



Fig. 1a. CL<sub>1</sub>-STAPEL (*stack*)



Fig. 1b. CL<sub>2</sub>-STAPEL (*stack*)



Fig. 1c. CL<sub>3</sub>-STAPEL (*stack*)



Fig. 1d. CL<sub>4</sub>-STAPEL (*stack*)



Fig. 1e. CL<sub>4</sub>-STAPEL (*stack*)



Fig. 1f. CL<sub>5</sub>-STAPEL-GEBEN (*to give a stack*)

Figure 1. Classifiers and a “simple” sign for the representation of a known or unknown quantity of stacked brochures

However, with this classifier one additional information is given simultaneously to the information about the unknown quantity. The dominant hand moves upwards, then downwards and once again a little upwards. This can be a representation of the possible height of the stack depending on what height the signing person imagines. This means that the classifier additionally conveys how large the range of the variable is. Summing up, ÖGS can express with a single sign the following: there are many brochures, they form stacks, we don’t know their height, it can be zero to something, that however will not be “too much”. The other classifiers occur in relation to a known quantity: the classifiers CL<sub>2</sub> and CL<sub>3</sub> in Figs. 1b and 1c represent the one thousand stacked brochures. The difference between the two classifiers is the repetition. So it is with CL<sub>3</sub> clearly represented that there are multiple stacks. With the fourth and the fifth classifier the stacks have the same height as if they

would have been divided into three stacks of the same height. The sixth classifier (Fig. 1f) is an instrumental one that together with the verb “to give”. It represents how the stack (of 200 brochures) is given to the other person.

Out of the topic “classifiers”, at the lexical level the sign NOCH-NICHT (*not yet*, see Fig. 2) occurs. It refers many times to a missing (piece of) information. A few examples (from study unit 1 and 2):

P6:               NORMAL STÜCK MUSS ERST HAUFEN [pfff]. WIEVIEL STÜCK NOCH-NICHT  
                  GEBÄRDEN NOCH-NICHT NOCH-NICHT  
*Usually, it should be given at first how many pieces there are. However, this has not been said.*

P6:               WO GEBÄRDEN EIN-TAUSEND WO. NOCH-NICHT GEBÄRDEN NOCH-NICHT  
*Where has it been said that it was one thousand? Where has it been said? This has not been said?*



Figure 2. the sign NOCH-NICHT (*not yet*)

## Discussion

Regarding the first question of which sign(s) exist(s) for “variable” under the object aspect in this context, the sign for the first classifier CL<sub>1</sub>-STAPEL (*stack*) can be identified (Fig. 1a). This sign can be interpreted as a “sign variable” for the word variable “number” from the spoken language and as a variable itself in terms of the object aspect. Moreover, the sign NOCH-NICHT (*not yet*, Fig. 2) was used to express that an information is unknown.

Regarding the second question of which classifiers are used and how, it can be stated that the classifier CL<sub>1</sub>-STAPEL (*stack*) is used to express the indeterminacy of the number of stacked prospects. This classifier differs from the others (for a known quantity) especially in the movement of the dominant hand, which moves once upwards, once downwards and once again upwards, but not the same height as before. This could be related to the fact, that the height (the quantity) is unknown. If the quantity is known, other classifiers are used – and each of them expresses a different detail about the signified: size, how stacks look, if all stacks have the same size etc. However, since a classifier is very closely related to the signified, a strong diversification can be assumed for other contexts as well: Depending on what a variable refers to different signs could be used to define that variable. One reason for this phenomenon could be the nature of the sign languages themselves: they are complex languages that have many slides also on the side of the “factual information” (Schulz von Thun, 2011). Thus, the need for a single equivalent sign for the word variable “quantity” does not seem to exist. Rather, for teaching mathematics in ÖGS, it should be necessary to know how and under which aspect a variable is used in order to render this variable according to the peculiarities and characteristics of sign

language. Due to the difference in the movement of the dominant hand, the definition of a new group of classifiers could be possible and meaningful.

In this first part of the study, it was possible to state, on the one hand, possibilities with which signs and with which type of classifiers the object aspect of a variable in ÖGS can be represented. On the other hand, regarding the third question about the differences between spoken language (German) and Austrian Sign Language, it was found that in the sign language communication about the object aspect of the variable a lot of information was given simultaneously: In a single sign such as CL<sub>1</sub>-STAPEL (*stack*, Fig. 1a) it is expressed whether the quantity is large or small, what shape or arrangement is meant and in which domain the quantity is located. In contrast, the word “quantity” is not enough to convey all this information in the spoken language. In the ÖGS this is possible in a natural way and can thus describe and represent variables very accurately. This indicates that a translation of (written or spoken) texts – e.g. of teaching material or of teacher explanations – from spoken language into ÖGS should be clearly detached from the source text. It is not only about transferring the texts or the grammar of ÖGS, but also about which information is transferred and how. Typically, for example, in mathematics teaching one could speak about the “quantity of stacks” in this context. In this phrase it is explicitly given that there is a stack and we talk about its quantity. It remains implicit that we do not know the quantity at the moment or in which range this quantity is. With signs, in contrast, more of the implicit information is expressed explicitly. This does not mean that ÖGS is “underrated” compared to spoken language and therefore needs more information and explanations to express complex and abstract content in a meaningful way. Rather, that one must take into account and can use the representational power of sign language. The next step in the study will be to investigate the other aspects of variable and to vary the context.

## References

- Akinwunmi, K. (2012). Das Verallgemeinern mathematischer Muster. In K. Akinwunmi (Ed.), *Dortmunder Beiträge Zur Entwicklung und Erforschung des Mathematikunterrichts Ser: v.7. Zur Entwicklung Von Variablenkonzepten Beim Verallgemeinern Mathematischer Muster* (1st ed., pp. 93–117). Springer Vieweg. in Springer Fachmedien Wiesbaden GmbH.
- Beecken, A., Keller, J., Prillwitz, S., & Zienert, H. (2014). *Grundkurs Deutsche Gebärdensprache: Stufe I; Arbeitsbuch* (4th ed.). *Gebärdensprachlehre: Vol. 3*. Signum Verlag.
- Braem, P. B. (1995). *Einführung in die Gebärdensprache und ihre Erforschung* (3., überarb. Aufl., 4. - 5. Tsd). *Internationale Arbeiten zur Gebärdensprache und Kommunikation Gehörloser: Vol. 11*. Signum-Verlag.
- Fleming, M. (November 2007). *Languages of schooling within a European framework for languages of education: Learning, teaching, assessment. Intergovernmental Conference*. Prague.
- Green, N., & Green, K. (2018). *Kooperatives Lernen im Klassenraum und im Kollegium: Das Trainingsbuch* (8th ed.). Klett/Kallmeyer.
- Grote, K. (2010). Denken Gehörlose anders? Auswirkungen der gestisch-visuellen Gebärdensprache auf die Begriffsbildung. *Das Zeichen - Zeitschrift Für Sprache Und Kultur Gehörloser*(85), 310–319.



- Koller, W. (2011). *Einführung in die Übersetzungswissenschaft* (8th ed.). UTB GmbH.
- Krause, C. M. (2016). *DeafMath - ein Projekt zum Einfluss der Gebärdensprache auf Mathematikverständnis*. 50. Jahrestagung der Gesellschaft für didaktik der Mathematik, Heidelberg.
- Krause, C. M. (2019). What you see is what you get? – Sign language in the mathematics classroom. *Journal for Research in Mathematics Education*, 50, 84–97. <https://doi.org/10.5951/jresmetheduc.50.1.0084>
- Krause, C. M., & Wille, A. M. (2021). Sign Language in Light of Mathematics Education:: An exploration within semiotic and embodiment theories of learning mathematics. *American Annals of the Deaf*, 166, 358–383. <https://doi.org/10.1353/aad.2021.0025>
- Küchemann, D. (1978). Children's understanding of numerical variables. *Mathematics in School*, 7(4).
- Kuckartz, U. (2018). *Qualitative Inhaltsanalyse. Methoden, Praxis, Computerunterstützung* (4th ed.). Beltz.
- Malle, G. (1993). *Didaktische Probleme der elementaren Algebra*. Springer.
- Prillwitz, S., & Wudtke, H. (1988). *Gebärden in der vorschulischen Erziehung gehörloser Kinder: Zehn Fallstudien zur kommunikativ-sprachlichen Entwicklung gehörloser Kinder bis zum Einschulungsalter*. Verlag hörgeschädigte Kinder.
- Ruf, U., & Gallin, P. (1995). *Ich mache das so! Wie machst du es? Das machen wir ab. Sprache und Mathematik, 1. - 3. Schuljahr*. Lehrermittelverlag.
- Ruf, U., & Gallin, P. (1999). *Ich mache das so! Wie machst du es? Das machen wir ab. Sprache und Mathematik, 4.- 5. Schuljahr bzw. 5. - 6. Schuljahr*. Lehrermittelverlag.
- Schoenfeld, A. H., & Arcavi, A. (1988). On the Meaning of Variable. *The Mathematics Teacher*, 81(6), 420–427. <http://www.jstor.org/stable/27965869>
- Schulz von Thun, F. (2011). *Miteinander reden 1: Störungen und Klärungen*. Allgemeine Psychologie der Kommunikation (49th ed.). Rowolt Taschenbuch Verlag GmbH.
- Thürmann, E., & Vollmer, H. J. (2017). Sprachliche Dimensionen fachlichen Lernens. In M. Becker-Mrotzek & H.-J. Roth (Eds.), *Sprachliche Bildung - Grundlagen und Handlungsfehler* (pp. 299–320). Waxmann Verlag GmbH.
- Wille, A. M. (2008). *Aspects of the concept of a variable in imaginary dialogues written by students*. Cinvestav-UMSNH. 32nd Conf. of the Int. Group for the Psychology of Mathematics Education (PME), Mexico.
- Wille, A. M., & Schreiber, C. (2019). Explaining geometrical concepts in sign language and in spoken language - a comparison. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Freudenthal Group & Freudenthal Institute, Utrecht University an ERME.
- Zwitzerlood, I. (2012). Classifiers. In R. Pfau, M. Steinbach, & B. Woll (Eds.), *Handbooks of Linguistics and Communication Science. Handbücher zur Sprach- und Kommunikationswissenschaft: Vol. 37. Sign Language: An International Handbook* (pp. 158–187). Walter de Gruyter GmbH.