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Jean Prestet's *Éléments des mathématiques*: A Cartesian textbook by a Cartesian author?

Catherine Goldstein*†

Abstract

Although the idea of 'Elements of mathematics' is usually closely linked to that of geometry, some early-modern authors also proposed alternatives. Jean Prestet, a protégé of the philosopher Nicolas Malebranche, developed an analytic basis for mathematics, which he presented as largely Cartesian. Prestet's emphasis on arithmetic and algebra pushed him to subject to proof what had been hitherto seen as obvious facts, to treat symbolic expressions as integers, and to renew and extend Diophantine analysis as well as combinatorial questions. These features, in return, challenged both Prestet's publisher and Descartes's viewpoint. These interconnected aspects of Prestet's treatise, several editions of which are kept in the Russell Library, are discussed here.

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1 A book

'What is a book?', asks Roger Chartier in one of his courses at the Collège de France, emphasizing the *longue durée* of the question and the danger of an anachronistic paradigm of writing and authorship to discuss it. While a dual description was favoured during the seventeenth century—that of an object and of a work, matching for some authors the human duality of the body and the soul—, it was later enriched:

To the classical dichotomy between the two natures, corporeal and spiritual, of the book, [Fichte] adds a second one, which distinguishes in every work between the ideas it expresses and the 'form' given to them by the writing. Ideas are universal by their nature, by their destination and by their utility; they cannot therefore justify any personal appropriation.¹

Scientific works, of course, particularly challenged the relevancy of these descriptions, with their repeated claims on universal and eternal truths.² The title-page of the book that will be discussed here, however, displays the classical dichotomy in an almost emblematic way (Fig. 1). In the first edition, that of 1675, the spiritual nature seems confined to the top half-page. It unrolls a title that is both descriptive and programmatic:

Elements of mathematics, or general principles of all sciences the object of which is magnitude. Containing a short and easy method to compare these magnitudes and to discover their ratios by means of the characters of numbers and the letters of the alphabet. In which things are proved according to geometrical order and analysis is made much easier and more thoroughly treated than has been done until now.³

Then, half the remaining page is filled with an engraving, preceding the usual information about the

²The issue of authorship in early-modern science is well-known, see for a few milestones David A. Kronick, 'Authorship and Authority in the Scientific Periodicals of the Seventeenth and Eighteenth Centuries', in: *The Library Quarterly* 48-3 (1978), pp. 255-275; Kevin Dunn, *Pretexts of Authority: The Rhetoric of Authorship in the Renaissance Preface*, Stanford: Stanford University Press 1994; Domenico Bertoloni Meli, 'Authorship and Teamwork around the Cimento Academy: Mathematics, Anatomy, Experimental Philosophy', in: *Early Science and Medicine* 6 (2001), pp. 65–95; *Scientific Authorship: Credit and Intellectual Property in Science*, ed. Mario Biagioli and Peter Galison, New York & London: Routledge 2003.

³Élémens des mathématiques, ou Principes généraux de toutes les sciences qui ont les grandeurs pour objet. Contenant une méthode courte et facile pour comparer ces grandeurs & pour découvrir leurs rapports par le moyen des caractères des nombres, & des lettres de l'alphabeth. Dans laquelle les choses sont démontrées selon l'ordre géométrique, & l'Analyse renduë beaucoup plus facile, & traittée plus à fond que l'on n'a fait jusqu'ici, Paris: Pralard 1675.

¹Roger Chartier, 'Qu'est-ce qu'un livre? Métaphores anciennes, concepts des Lumières et réalités numériques', in: *Le Français aujourd'hui* 178 (2012), pp. 11–26 (quotation pp. 12–13): 'À la dichotomie classique entre les deux natures, corporelle et spirituelle, du livre, qui sépare le texte de l'objet, [Fichte] en ajoute une seconde qui distingue dans toute oeuvre les idées qu'elle exprime et la "forme" qui leur est donnée par l'écriture. Les idées sont universelles par leur nature, leur destination et leur utilité; elles ne peuvent donc justifier aucune appropriation personnelle'.



Figure 1 – Title page of the first edition of Jean Prestet's *Élémens des mathématiques*. Maynooth University, Russell Library, Sc 17 13.

publisher, here the bookseller André Pralard, in Paris. Two lines at the bottom, provide the date of publication and the important mention 'avec privilège du roy', the then-compulsory authorization to publish granted in the name of the King.

Here, the mathematical and the human elements seem to neatly match well-distinguished spiritual and corporeal natures. One might even detect a Fichtean distinction between the ideas, the content, here the results on magnitudes and their comparisons—a banal topic in a book intended for beginners, with its standard applications to the rule of three or the calculation of interest on a loan for instance—and the form of their presentation, the 'method', the 'geometrical order', and the thorough treatment promised in the title.

As explained, the form would become later the feature specifically associated with authorship. In the first edition of the *Él'emens*, however, no printed mention of an author appears.⁴ Things are different in the subsequent, heavily revised, editions, 'more ample and better digested', published in two volumes under the title of *Nouveaux Elemens des mathematiques* from 1689 onwards.⁵ In these editions, the author's name and all his titles are printed exactly in the middle of the title-page, immediately after the promise of a proper and better treatment of the material: 'Jean Prestet, prêtre, ci-devant professeur de mathématiques dans les universités d'Angers et de Nantes' (Jean Prestet, priest, previously professor of mathematics at the Universities of Angers and Nantes). But the issue of form goes beyond the personal identification of the writer, the Oratorian Jean Prestet: the mention of Angers and Nantes Universities, the use of 'analysis' and the 'letters of the alphabet' in the title, in association with the 'method', give to this issue a Cartesian resonance.

The distinction between object, form, and idea could thus suggest a frame for studying these *Élémens* in three ready-made historical fields—those of publishing, of Cartesianism, and of mathematics —each of them inheriting a range of well marked-out issues. However, what will hold our attention here is their mutual interference, and the way this interference blurred the frontiers between ideas, forms and material environment. How does Cartesianism or its derivatives intervene in the formatting of a mathematical text, or in the ideas it promotes? Did the conditions of printing direct the expression or the presentation of mathematics, and did the genre of the textbook help to spread or constrain specific convictions? And last, but not least, how can innovation, if any, manifest itself at such a cross-road? The hints in the title

⁴A handwritten identification of this author, however, has been added in most exemplars of the first edition I have seen, such as 'Jean Prestet [Or.]' in the Russell Library copy, see Fig. 1.

⁵For instance: Jean Prestet, *Nouveaux Elemens des mathematiques, ou Principes généraux de toutes les sciences qui ont les grandeurs pour objet. Seconde edition, plus ample et mieux digeree,* 2 vols., Paris: Pralard 1689, or: Jean Prestet, *Nouveaux Elemens des mathematiques, ou Principes généraux de toutes les sciences qui ont les grandeurs pour objet. Troisieme edition, plus ample et mieux digeree,* 2 vols., Paris: Pralard 1694, a copy of which is in the Russell Library. Throughout this chapter, I use an abridged form of the titles: *Élémens des mathématiques* or simply *Élémens,* either in general or to refer specifically to the first edition, and *Nouveaux Elemens des mathematiques* or *Nouveaux Elemens* for the subsequent editions. Except otherwise mentioned, all references to the *Nouveaux Elemens* are to the 1689 edition.

page will be our first guide.

2 One publisher—or several

Just one name is printed on the title page of the 1675 book: the name of the publisher, André Pralard. Born in 1635 and dead by the beginning of the eighteenth century, Pralard was not specially known for his scientific books. Most of the 400-odd titles attributed to his publishing efforts in the database of the French National Library concern religious topics: Pralard's catalogue is a material embodiment of the spiritual nebula that in early-modern France links Cartesian and Jansenist trends, in particular in connection with the Oratorian Order.⁶ In 1668, he had been jailed in the Bastille for having circulated forbidden Jansenist publications. Once freed, he seemed nonetheless to have benefitted from powerful protections and, while being received as an independent master in the printing business, he was also hired by royal authority to trace and repress libels and counterfeits.⁷

Pralard's continuing Jansenist sympathies are clearly on display in the list of books he printed: he published such luminaries as Antoine Arnauld, including his new translation of Augustine, and Louis Ellies du Pin, whose *Bibliothèque des auteurs ecclésiastiques* includes for instance the *Traitez de controverse de Cornelius Jansenius*—and also a variety of writers close to Port-Royal, such as Nicolas Fontaine, Isaac Louis Le Maistre de Sacy, Pierre Danet, Jacques de l'Oeuvre, Antoine Singlin and Alexandre Varet. To their books should be added the numerous editions of the Jansenist Oratorian Pasquier Quesnel's *Abrégé de la morale de l'Evangile*, translations of Pierre de Bérulle himself, the Oratorian Bernard Lamy's *Nouvelles réflexions sur l'art poétique* or *Démonstration de la vérité et de la sainteté de la morale chrétienne*, as well as *Vitas* of Oratorian priests, such that of François de Saintpé by Charles Cloysault, and, directly relevant for us, as we shall see, Nicolas Malebranche's *Recherche de la vérité*, the first edition of which appeared just one year before Prestet's *Élémens*. Among the mere handful of scientific books published by Pralard in the last third of the seventeenth-century, most of them are authored by Oratorians, such as Lamy's *Traité de méchanique* in 1679, his *Traité de la grandeur* in 1680 or his *Eléments de géométrie ou de la mesure du corps* in 1685 or Michel Mauduit's discourse on the gout in 1687.

Pralard obtained the authorization to publish the *Élémens des mathématiques*— the so-called King's privilege — as early as 1672.⁸ The title page displays his specific printer's mark, which reflects the

⁶Fred Ablondi, 'Bernard Lamy, Empiricisms and Cartesianisms', *History of European Ideas* 44-2 (2018), pp. 149-158; Tad Schmaltz, 'What has Cartesianisme to Do with Jansenism?', *Journal of the History of Ideas* 60-1 (1999), pp. 37-56.

⁷Léon Thévenin, 'Un libraire de Port-Royal: André Pralard', *Bulletin du bibliophile* (1961), pp. 18-38, repr. *Un libraire de Port-Royal: André Pralard*, Paris: Giraud-Badin 1962; Henri-Jean Martin, *Livre, pouvoirs et société à Paris au XVII^e siècle*, 2 vols., Genève: Droz 1984, II, p. 714 and p. 753.

⁸As far as the catalogues show, this is the earliest privilege for a scientific treatise published by Pralard. The privilege for

name of his Parisian shop, 'The Occasion' ('L'Occasion'): a central figure of a woman sailing over a (very small) winged globe, spreading out a banner with the inscription: 'Inimicos virtute superabis' (by your strength will you overcome your enemies), with a seascape in the background. The woman is the fortune, the *occasio*. The same device is used for instance in Malebranche's *Recherche de la vérité* and in most of Pralard's books—with some variants, for instance, the banner may appear as a sail, the motto may be independently printed above the picture or integrated into the banner.⁹ Such a device had been compulsory in France since François I (1539), in particular to prevent counterfeits.¹⁰

For a publisher who was not an expert in mathematics, printing this book was certainly a feat of craftsmanship at several levels: besides the symbols themselves, to which we shall come back, adjusting computations and equations to the size of the page was not an easy task (see for instance Fig. 2). Moreover, the text involves references to tables which had to be separately printed, then inserted at a given place, in order to be unfolded and used while reading the main text (see Fig. 3).¹¹

Pralard also published a second, regular, edition of the book, this time in two volumes; a new privilege had been granted on 21 February 1686, to the author himself, Jean Prestet, who transferred it to André Pralard. The privilege ran for six years from the date of the first impression, indicated as 20 September 1689 and Pralard's regular device again appears on the title page. However, the situation changes for the subsequent editions, in particular the 1694 edition in the Russell Library (see Fig. 4). Though published before the expiration date of the privilege, and using almost exactly the same layout as in 1689, in particular the alternation of red and black,¹² the title page of this exemplar is ornamented with another device, slightly askew: an armillary sphere between two angels holding a banner with the motto: 'universitas rerum ut pulvis in manu Jehovah' (the totality of things [is] like dust in the hand of

a booklet on the use of a new mathematical instrument, a pantometer, first printed by Pralard on October 31, 1675, that is, one month before the *Élémens*, is granted directly to the author of the book, the architect Pierre Bullet, and in September 1675 only. ⁹Aurélie Vertu, *Les marques typographiques d'imprimeurs et de libraires (XV^e-XIX^e siècles*, Lyon: ENSSIB 2004.

¹⁰The use of a globe was not unique to Pralard, though: Sébastien Gryphe, for instance, also used a globe of fortune. On the role of such engravings in early-modern publications, see Volker Remmert, *Widmung, Welterklärung und Wissenschaftslegitimierung: Titelbilder und ihre Funktionen in der wissenschaftlichen Revolution*, Wiesbaden: Harrassowitz 2005, transl.: *Picturing the Scientific Revolution: Title Engravings in Early Modern Scientific Publications*, Philadelphia: Saint Josephs University Press 2011; *Gateways to the Book: Frontispieces and Title Pages in Early Modern Europe*, ed. Gitta Bertram, Nils Büttner and Claus Zittel, Leiden: Brill 2021.

¹¹It should be noted, however, that the price indicated in the *Catalogue des livres imprimez chez André Pralard, libraire à Paris, ruë S. Jacques, à l'Occasion* in 1677 (available on line https://gallica.bnf.fr/ark:/12148/bpt6k131900z) is 5 pounds, which, although not cheap (roughly a week's salary for a unqualified labourer in Paris at the time) is more or less aligned with the other in-quarto books proposed in the catalogue. I do not know the terms of this publishing contract, but on this issue, see Martin, *Livre, pouvoirs et société*, II, pp. 914–921.

¹²One difference is the indication of the publisher: the name of Pralard is in black instead of red and the name of his bookshop, L'Occasion, which, as explained above, is directly linked to his official device, is missing in 1694.

ELEMENS V 20 V20 Vis. -VS +Vs +3Vs $= V_{11\frac{1}{4}} \int_{a}^{b} moitie' \frac{1}{2}V_{5} = V_{\frac{1}{4}} = V_{\frac{1}{4}}$ par V 20 rand quarré +20 petit quarre _15 sa moitié 3V5=V44 VVI- racine cherchee. VVI reste + 5 fa racine VS On trouvera pareillement que la racine de $V_5 - V_3$ eff $V \cdot V \overline{1_4} - V \cdot V \overline{1_4} - V \cdot V \overline{1_4} - V \cdot \overline{1_4} - V \cdot$ par Vs orand quarre +5 difference V 5– Sa moitié V 14 fomme $V_5 \rightarrow V_2$ moitié $V_1 \frac{1}{4} \rightarrow V_2$ -11 petit quarre _ V.V.I' -V's racine cherchee. reste +2 V.VI++Vfa racine V 2 Troisième Exemple. Pour tirer la racine de a-b/ab-+2ab. 1°. Je retranche 4aabb quarte de la partie 2ab, de a'b+2aabb+ab' quarré de la partie a+bV ab. Le refte eff $a^{i}b_{-2aabb+ab}$, dont je tire la racine $a_{-b}Vab$. 2°. J'ajoûte la racine $a_{-b}Vab$ à la partie $a_{-+b}Vab$, la fomme effi 2aVab, & je retranche la même racine de cette partie, la différence effi 2.6 V ab. 3°. Je tire V aV ab la racine de aV ab moitié de la fomme 2aV ab, & V bV ab la racine de bV ab moitié de la difference 2bV ab, & V aV ab=+V bV ab, ou V V ab=+V V ab' est la racine que je cherche: a+Vab,+2ab. par a+bVab a+bV ab a-+-bVab +a-by ab -a-+bVab grand quarre aa-+ 2abb-+ bbab fomme 2aV ab difference 2bV ab ou a'b+2aabb+ab3 fa moitié aV ab petit quarré —4aabb reste a'b—2aabb++ab³ fa moitié by ab Vavab +VbVab {racine cherchée, ou VVab +VVab3 fa racine a-bV ab Il en est ainsi des autres ; mais on connoît souvent par un fimple regard, ou en faivant à peu prés les methodes expliquées III. 39. & 40. quelles font les racines des binomes exprimez par lettres. Par exemple pour trou-ver celle du binome aa+bc+2ab/bc, je vois d'abord que 2ab/b est double du plan de *a* racine du quarré *aa* par bc est cancine de *bc* confideré comme quarré : Et ainsi je connois que a+bc est la racine du binome aa+bc+2ab/bc. Pareillement

Figure 2 – A typical page of the *Élémens des mathématiques*, 1675, p. 136. Note the repeated use of the sign for the square root, the hesitant horizontal bars to delineate their extent, and the overflow into the margins.



Figure 3 – A tipped-in plate in the *Élémens des mathématiques*, 1675, 'to be inserted p. 369'.

Jehovah). The motto and the image do not belong to Pralard, but to Jean Antoine Huguetan and, later, to his sons, a family of printers and booksellers originally from Lyon, who moved to Amsterdam at the end of the century.¹³ And although a privilege is claimed on this title page, none is reproduced in the book, in contrast to the preceding editions. All this suggests that this so-called third edition is in fact a counterfeit edition, issued with or without Pralard's knowledge. In any case, this 1694 version (like others, later) is a cheaper one: the tables, for instance, are no longer displayed separately and beautifully folded to allow a comfortable use while pondering the text; they are simply printed within the text, placed wherever it is possible. Some other versions of the book are to be found. Note that the second edition, that of 1689, was still proposed in Pralard's 1702 catalogue at the 'fair price' of 16 pounds for the two volumes. However, if, as some sources have suggested,¹⁴ the author had had hopes of becoming rich through the sales of the new edition(s) of his book, it came too late: he died about six months after the first impression of the second volume.

3 One author—or several

Jean Prestet was born in 1648 in Chalon-sur-Saône, a town in Burgundy about 120 km to the north of Lyon. His father, François Prestet, was a subaltern employee in the local county court. When he was about 20, Jean Prestet went to Paris as a servant to the Oratorian priest and philosopher Nicolas Malebranche, who also taught him mathematics and encouraged him, or perhaps helped him, to write a new mathematical textbook. These *Elements* were to serve as a companion for the first edition of Malebranche's celebrated treatise on *Search For Truth* (La Recherche de la Vérité). This circumstance

¹³Their bookshop in Lyon was called 'The Sphere'. Pralard had been trained in Lyon at Huguetan's.

¹⁴On this issue, see Katia Asselah, Arithmétique et algèbre dans la deuxième moitié du 17^e siècle français: les Élémens et Nouveaux Elemens des mathematiques de Jean Prestet, unpublished thesis, Université Paris 7 (2005), p. 3.



Figure 4 – Title page of the first volume of Jean Prestet's *Nouveaux Elemens des mathematiques*, 1694. Maynooth University, Russell Library, Sc 17 11 a.

explains why the first, anonymous edition of Prestet's book has been often attributed to Malebranche himself, for instance by John Wallis in his 1685 *Treatise of algebra both historical and practical*.¹⁵ Moreover, Prestet participated at this time in the activities of Malebranche's circle.¹⁶

On 31 December 1675, the very year when the Élémens were published, Prestet officially entered the Oratory of Jesus and spent a few years at the famous College of Juilly (Notre Dame des Vertus), before becoming a priest five years later, in 1680. He was then given several successive assignments: he was in particular sent to Nantes, in western France, to become the new professor of mathematics at the university. But he was confronted there with the hostility of the Jesuits who were already solidly implanted in the local college and controlled the training of the local elites. As a result, less than one year later, in 1681, Prestet was moved to the University of Angers (c. 80 km to the east of Nantes). Angers had also been for several years a place of violent controversies between Cartesian-Jansenist members of the Oratoire, such as Bernard Lamy, and their opponents.¹⁷ Prestet eventually returned to Paris in 1685 with the objective of working on a more developed edition of his treatise and he then participated in a variety of debates on mathematico-philosophical topics, such as negative numbers or infinity. Nonetheless, he did not renounce his religious tasks; on 22 April 1686, for instance, Jacques Bossuet thanked Louis Abel de Sainte-Marthe, the Superior General of the Oratorian order, for letting Prestet go to the Claye mission, near Paris, in order to preach to new converts, in the wake of the revocation of the Edict of Nantes.¹⁸ However, 'not considering himself sufficiently esteemed either because of his low origins or because of his former employment by Malebranche',¹⁹ Prestet left the Oratory in 1689.

The 1689 new edition of his treatise, the *Nouveaux Elemens*, bears the mark of this trajectory; it incorporates Prestet's long inaugural discourse at Angers University and some of his own interventions in the controversies of his time.²⁰ And while the first edition was dedicated to his superior, Sainte-Marthe, the dedication of the 1689 edition, reflecting Prestet's estrangement from the order, was directly to God, 'the Sovereign Lord of sciences, source, father and principle of enlightenment and truth', who 'loves simplicity' and not 'the vain self-importance of arrogant minds'. Suffering from health problems, Prestet returned to the Oratory almost immediately and was sent to an Oratorian convent in Marines, to the north of Paris, to be cared for. However, as mentioned, he died on 8 June 1690.²¹

¹⁹This quote comes from Joseph Bicaïs, reproduced in Asselah, Arithmétique et algèbre, pp. 232-233.

²⁰See for instance, Prestet, *Nouveaux Elemens* 1689, I, pp. 561–588 or II, pp. 362–372. On the controversy on negative numbers, see Paul Schrecker, 'Arnauld, Malebranche, Prestet et la théorie des nombres négatifs (d'après une correspondance retrouvée)', in: *Thalès* 2 (1935), pp. 82–90.

²¹For more details on Prestet's biography, see Louis Batterel, Mémoires domestiques pour servir à l'histoire de l'Oratoire,

¹⁵Still, Leibniz identifies Prestet as the real author of the book in a letter to Henry Oldenburg in 1675, Nicolas Malebranche, *Œuvres complètes*, ed. André Robinet, Paris: Vrin & CNRS Editions 1955-1965, XVIII, p. 109; see also Asselah, *Arithmétique et algèbre*, p. 8.

¹⁶André Robinet, 'Jean Prestet ou la bonne foi cartésienne (1648–1691)', *Revue d'histoire des sciences* 13 (1960), pp. 95–104.
¹⁷Malebranche, *Œuvres*, XVIII, pp. 111–120.

¹⁸Jacques Bénigne Bossuet, Correspondance, 15 vols., Paris: Hachette 1909-1925. 3, p. 222.

Although less well-known and versatile than other Oratorian scientists, like Bernard Lamy, or even Charles-René Reynaud, whom he had trained and who replaced him in Angers, Prestet had earned a certain reputation in his time. Leibniz, who met him and wrote extensive notes on the *Élémens* during his Parisian sojourn,²² expressed disappointment for instance upon learning that Prestet had turned to other topics once he had become a priest: 'I would like your author of *Élémens* who is in the Oratory not to give up algebra entirely, for which he has a particular talent', he wrote to Malebranche on 22 June 1679.23 In his biography of Descartes, Adrien Baillet qualifies Prestet as 'one of the most skilled mathematicians of our time'.²⁴ However, these expressions of praise turned out to be ambiguous, because they situated Prestet in the wake of Descartes, at precisely the moment when the calculus began to gain the attention of mathematicians. Leibniz, for instance, expresses his disappointment in 1693: 'As [Prestet] applied himself mostly to analysis, he could have advanced considerably this science if he had not been too tied to the ideas of Descartes' s analysis alone, which limited his views'.²⁵ The issue is delicate; it is well-known that Malebranche and his circle promoted their own, specific, version of Descartes' scientific and philosophical heritage, then partially distanced themselves from mathematical Cartesianism, more or less at the same time as Prestet distanced himself from Malebranche and the Oratory.²⁶ Claire Schwartz has recently advocated for a stronger continuity of Malebranche's thought on mathematics, against the supposed change of gear brought about by calculus.²⁷ Moreover, from the point of view of the history of mathematics, and following Leibniz's quotation, such late Cartesianism is often considered a hindrance with respect to mathematical innovation.²⁸ But how Cartesian then was Prestet really and to what extent did he remain loyal to Cartesian viewpoints? And what were the consequences for his mathematical achievements?

Bicais's necrology of Prestet described him as a 'zealous supporter of the person and philosophy

Paris: Picard 1905, IV, pp. 432-437; Robinet, Jean Prestet; Asselah, Arithmétique et algèbre.

²²On these notes, see André Robinet, *Malebranche et Leibniz: Relations personnelles*, Paris: Vrin 1955, pp. 52-60; Gottfried Wilhelm Leibniz, *Sämtliche Schriften und Briefe*, VII: *Mathematische Schriften*, 2, Berlin: Akademie Verlag 1996; Arilès Remaki, *L'art combinatoire en tant qu'art d'inventer chez Leibniz, 1672-1680*, unpublished thesis, Université de Paris (2021).

²³Malebranche, *Œuvres*, XVIII, p. 160: 'Je voudrais que votre auteur des *Élémens* qui est dans l'Oratoire, n'abandonnât pas tout à fait l'Algèbre, pour laquelle il a un talent particulier'.

²⁴Adrien Baillet, *La vie de Monsieur Descartes*, 2 vols., Paris: Horthemels 1691, I, p. 350.
²⁵Malebranche, *Œuvres*, XIX, p. 601.

²⁶This is for instance André Robinet's point of view, see Robinet, *Jean Prestet*; André Robinet, 'La philosophie malebranchiste des mathématiques', in: *Revue d'histoire des sciences* 14 (1961), pp. 205–254.

²⁷Claire Schwartz, *Malebranche: mathématiques et philosophie*, Paris: Sorbonne Université Presses 2019.

²⁸See Robinet, Jean Prestet. Katiah Asselah vindicated Prestet's work in Asselah, Arithmétique et algèbre; Katia Asselah, 'Jean Prestet: Algèbre et combinatoire dans la résolution des équations', in: Bollettino di storia delle scienze matematiche 31 (2011), pp. 9–34; see also Catherine Goldstein, 'On a Seventeenth-Century Version of the Fundamental Theorem of Arithmetic', in: Historia Mathematica 19 (1992), pp. 177–187.

of Descartes'.²⁹ Prestet himself calls Descartes an excellent master,³⁰ and praises Descartes's approach repeatedly in the *Éléments*. In the preface of the first edition, for instance, he announces that he will particularly explain Descartes's method as it is 'the most general, the most fruitful and the easiest of all'.³¹ His network of references is also quite revealing; the authors who are the most quoted are Franz van Schooten³² and Johann Hudde,³³ not only directly for their Cartesian works, but also as Prestet's sources for other earlier authors, such as Stifel or Simon Jacob from Coburg.³⁴ Except for some other Oratorians, such as Bernard Lamy, anonymously referred to as 'a person the merit of whom I extremely honor'³⁵ or writers close to them such as Pascal or Arnauld, the few other authors he mentions are criticized or presented as needing corrections or complements, such as for instance Simon Stevin on incommensurables and the extraction of roots of binomial expressions or Bachet and Viète on Diophantine problems. This position does not change substantially in the subsequent editions. In 1689, for instance, Prestet states:

But one can safely say that the method of Mr. Descartes is as much above that of Mr. Viete as this is above the others. And I do not believe that one can ever discover one that is superior to it, nor one that has at the same time as much scope & as much fruitfulness, as much facility & as much light.³⁶

Prestet also takes Descartes's side in various polemics, either real or virtual. For instance, he protests against Jacques de Billy's crediting to Fermat for a rule for the transformation of equalities, as Prestet says it comes from Descartes.³⁷ Likewise, he defends Descartes against Wallis's attacks.³⁸ A longer controversy saw him oppose members of the French Academy over Descartes's 'rule of signs', specifically Michel Rolle, whose 1684 article in the *Journal des savants* had criticized the rule for its lack of generality. Prestet argued that:

Mr. Descartes has thus perfectly distinguished in his rule all the various cases that it can contain, and has never claimed to extend it beyond its proper limits, by passing off as true or false roots that are merely imaginary. [... This] rule also retains all the convenience that

³⁸Ibid., II, preface.

²⁹Joseph Bicaïs, *Notices de l'Oratoire de France ou recherches sur les membres de cette congrégation*, Bibliothèque Méjane, Aix-en-Provence, ms 331.

³⁰Prestet, Nouveaux Elemens, I, preface.

³¹Prestet, *Élémens*, preface.

³²Ibid., p. 142, p. 146, p. 205, pp. 318-319, etc.

³³For instance: Ibid., p. 374, p. 381, pp. 412–414.

³⁴Ibid., p. 318.

³⁵Ibid., p. 351.

³⁶Prestet, *Nouveaux Elemens*, II, preface: 'Mais on peut bien avancer sans crainte que la méthode de Monsieur Descartes est autant au-dessus de celle de Monsieur Viete que celle-ci l'est au-dessus autres. Et je ne crois pas que l'on en puisse jamais découvrir qui l'emporte sur elle, ni qui ait tout ensemble autant d'étendue & de fécondité, autant de facilité, & autant de lumière'. ³⁷Ibid., II, p. 175.

he judged could be derived from it, in order to discover all the commensurable roots of a compound equality.³⁹

Putting himself forward as Descartes's principal defender, Prestet even attacked someone else's criticism of the Academy on this question, namely that of Jacques Ozanam, for using 'bad reasons to defend' a 'good cause'.⁴⁰

And while the second and other editions refer to many more names than the first, Prestet still claims that his techniques inspired by Descartes are more efficient or more complete than theirs. In all editions, for instance, he contests Clavius' assertion than geometry is necessary to solve a certain second-degree problem by showing how to solve it without any figural representation, by what he describes as a Cartesian method.⁴¹

This last assertion shows how such a Cartesian position may interfere with the way mathematics is done. More generally, what sort of textbook did such a self-proclaimed Cartesian author write? Did he solve only some pedagogical issues raised by Descartes' mathematics? Or did he integrate new mathematical developments? We now turn to these questions, using the top of the title page as our guideline.

4 Éléments

'On nous donne sans cesse des Élémens de mathématiques', complains a reviewer of the *Journal littéraire* in 1774. Already in Prestet's day, several Elements were available to the French readers. Some explicitly positing themselves under the umbrella of Euclid's *Elements*, such as *Les quinze livres des éléments géométriques d'Euclide*, translated with commentary by Denis Henrion and edited several times from 1615 onwards. Others claiming some originality, from Jean Marrois's *Elemens de mathématiques traitant des quantitez en général* in 1644 to Antoine Arnauld's *Nouveaux élémens de géométrie* in 1667 or Bernard Lamy's *Elémens de mathématiques* in 1680.

Prestet does not follow a Euclidean structure. In the preface of the 1675 edition, he announces a division in two main parts, the first one in five chapters, the second in four.⁴² The first part is devoted to the

³⁹Ibid., II, p. 364: 'Monsieur Descartes a donc parfaitement distingué dans sa règle tous les divers cas qu'elle peut renfermer, & n'a jamais prétendu l'étendre au delà de ses justes limites, en faisant passer pour vraies ou fausses des racines qui ne sont qu'imaginaires. [...Cette] règle conserve aussi toute la commodité qu'il a jugé qu'on pouvait en tirer, pour découvrir toutes les racines commensurables d'une égalité composée'.

⁴⁰Ibid., II, p. 365.

⁴¹Prestet, *Élémens*, pp. 306-307 or Prestet, *Nouveaux Elemens*, II, p. 16.

⁴²The table of contents does not reproduce these divisions: it displays 54 sections, the titles of which are not exactly those indicated in the volume itself. On the history of tables of contents, see *La Table des matières: Son histoire, ses règles, ses fonctions, son esthétique*, ed. Georges Mathieu and Jean-Claude Arnould, Paris: Classiques Garnier 2017; Christophe Schuwey,

basic arithmetical operations (including the extraction of roots), irrational magnitudes and proportions, including usual practical rules, such as the rule of three or of alloys, and an introduction to logarithms; the second part explains how to solve problems, in particular Diophantine equations and equations with one variable up to degree 4. Both parts were extended in the subsequent editions, so as to form one volume each.

A remarkable feature of Prestet's *Éléments*, particularly for a Cartesian aficionado, is of course his rebuttal of geometry.⁴³ However, his explanation for this rebuttal was based on Descartes's overall scientific framework: true science should be universal, founded, not on the senses, but instead on simple ideas, and thus favour the economy of the mental processes.

4.1 Algebra versus Geometry: Universality and Economy

In the dedication to Louis Abel de Sainte-Marthe, Prestet claims: 'There is nothing in this book that flatters the senses or the imagination. Everything in it only aims at enlightening the mind, and giving it enough strength and scope to penetrate and understand the most hidden things in science'.⁴⁴ In Arnauld's Cartesian *Nouveaux élémens*, magnitudes can be represented by lines, although it is suggested to mark them by letters in order to 'accustom the mind to conceive things in a spiritual manner without the help of any sensible image'.⁴⁵ Prestet goes further, as he completely dismisses the representation by geometrical objects:

As all sciences must tend to enlighten the mind, & to reveal to it the correct magnitude, or the closest, of unknown things, one must not value much the use of these expressions by lines, which speak only to the eyes and to the imagination.⁴⁶

^{&#}x27;La table des matières au XVIIe siècle, entre promotion commerciale et action politique', *Fabula*: « S'asseoir à la table »: La table des matières, du Moyen Âge à nos jours, http://www.fabula.org/colloques/document7267.php, 2021, consulted on 14 April 2022. In the second edition, only the second volume has a table of contents, with the titles of the chapters but no page numbers provided.

⁴³The first chapter of the second edition includes a few definitions related to geometry, but they are not exploited in the remainder of the textbook and the applications to geometry were never published, see Malebranche, *Œuvres*, XIX, pp. 619–620.

⁴⁴Prestet, *Élémens*, dedication: 'Il n'y a rien dans ce Livre qui flatte les sens ou l'imagination. Tout ce qu'il contient ne tend qu'à éclairer l'esprit, et à lui donner assez de force et d'étenduë pour pénétrer et pour comprendre ce qu'il y a de plus caché dans les sciences'.

⁴⁵Antoine Arnauld, *Nouveaux élémens de géométrie*, Paris: Charles Savreux 1667, p. 4. On the differences between Arnauld and Malebranche, see Denis Moreau, *Deux Cartésiens: la polémique Arnauld-Malebranche*, Paris: Vrin 1999 and Schrecker, 'Arnauld, Malebranche, Prestet'.

⁴⁶Prestet, *Élémens*, preface: 'Comme toutes les sciences doivent tendre à éclairer l'esprit, & à lui découvrir la juste grandeur, ou la grandeur la plus approchante des choses inconnues, on ne doit pas beaucoup estimer l'usage de ces expressions par lignes, qui ne parlent qu'aux yeux et à l'imagination'.

The fundamental science is thus one directly dealing with magnitudes: that is, arithmetic. This point of view was also the point of view of Malebranche, for whom only mathematics provided necessary truths and for whom to know truth was to know relations between magnitudes. For Malebranche, 'the notion of accuracy and the necessity of a numerical determination are much more essential than the inspection by the mind of our ideas'.⁴⁷ Prestet followed him on this point, stating for instance in the preface of the first edition that 'It seems to me obvious that this number $\sqrt{20}$ is more exactly known than the hypotenuse of a right-angle the sides of which are 2 and 4'.⁴⁸ As pointed out by Claire Schwartz, 'The first versions of the Search for thruth, as well as Prestet's Éléments were part of the project to establish a general theory of magnitude based on the number conceived then as the operator of determined measurement'.⁴⁹ But it goes beyond numbers, to the letters of algebra. Descartes's Géométrie itself is mentioned as an illustration of the fruitfulness and the ease of this extension.⁵⁰ Again, Prestet here meets Malebranche, who explains in the Search for truth: 'By algebra, I mean mostly the algebra that M. Descartes and a few others have used. [...] This is a universal science and like a key for all the other sciences'.⁵¹ The issue of generality is of course essential for Descartes. The first rule of the Regulae states that one should not direct his efforts toward particular truths or properties and the opposition between particular and general plays a key role in several mathematical controversies between Descartes and his contemporaries.⁵²

Having first insisted on the importance of numbers and the science of arithmetic, Prestet goes on:

Though arithmetic is a science on which all others depend, yet we shall explain another, more universal one, by using the letters of the alphabet. This science, which is called Algebra, is used to clarify, extend and improve, as much as one can, arithmetic, and more generally all the sciences which are related to mathematics. $[\ldots]$ It is so general that it considers all magnitudes $[\ldots]$ As one cannot give to a mind a greater reach or a greater ability than it

⁴⁷Schwartz, *Malebranche*, p. 21: 'la notion d'exactitude et la nécessité d'une détermination numérique sont bien plus essentielles que l'inspection par l'esprit de nos idées'.

⁴⁸Prestet, *Élémens*, preface: 'Il est ce me semble evident que ce nombre $\sqrt{20}$ est beaucoup plus connu que la soûtendante d'un angle droit dont les côtez sont 2 & 4'.

⁴⁹Schwartz, *Malebranche*, p. 187: 'Les premières versions de la *Recherche [de la vérité]* ainsi que les *Éléments* de Prestet s'inscrivaient dans le projet d'établir une théorie générale de la grandeur fondée sur le nombre conçu alors comme l'opérateur de mesure déterminée'.

⁵⁰Prestet, *Élémens*, p. 277.

⁵¹ 'Par l'algèbre, j'entends principalement celle dont M. Descartes et quelques autres se sont servis [...] C'est une science universelle et comme la clé de toutes les autres sciences', quoted in Robinet, 'La philosophie malebranchiste', p. 208.

⁵²See for instance, Catherine Goldstein, 'L'expérience des nombres de Bernard Frenicle de Bessy', in: *Revue de synthèse*, 4e s., 2-3-4 (2001), pp. 425–454; Catherine Goldstein, 'Routine Controversies: Mathematical Challenges in Mersenne's Correspondence', in: *Revue d'histoire des sciences* 66-2 (2013), pp. 249–273. On the issue of generality, see *The Oxford Handbook of Generality in Mathematics and the Sciences*, ed. Karine Chemla, Renaud Chorlay and David Rabouin, Oxford: Oxford University Press 2016. Although such requirement may seem obvious in the case of mathematics, it is not: as seen in some of the controversies with Descartes, it may mean a lack of interest in effectivity or a disdain for problems on integers, for instance. possesses, this science teaches us only how to spare the mind, presenting it with a whole set of ideas in the form of very short expressions.⁵³

Prestet insists several times on this point:

The method that expresses each unknown by an unknown letter is the most general of all $[\ldots]$. And a second reason to prefer it to other methods is that it assumes less knowledge in the mind of whom uses it.⁵⁴

Besides its exactness and universality, symbolic algebra has thus, for Malebranche as well as for Prestet, the capacity to spare mental effort, while supporting mental representations by concrete but general traces.⁵⁵ Here, intuition is not associated with geometrical figures, but with discrete signs, such as digits or letters. As Descartes himself wrote in his *Discours de la méthode*, the analysis of the Ancients 'is always so tied to the consideration of figures that it cannot exercise the understanding without tiring the imagination a lot.⁵⁶ Prestet goes even further from the second edition on, claiming for instance that:

Is it not here [in Geometry] that we find what we might justly call head-breakers? We have a very visible example of this in the person of Mr. Paschal. For it is believed that he died so young only because of not having sufficiently spared his strength and the scope of his imagination. The great efforts he had to make in order to imagine and unravel the prodigious number of confused lines and figures and to embrace them at a single glance provoked in him that general exhaustion of the brain which took him off in the flower of his age. One need not fear the same danger from the sciences that are set out here. Because although they lead further, & uncover many more truths, & in less time than Geometry, they nevertheless tire the mind all the less the more one delves deeply into them and one understands them better.⁵⁷

⁵³Prestet, *Élémens*, preface: 'Mais quoy que l'Arithmétique soit une science dont toutes les autres dépendent, cependant nous en expliquons une autre plus universelle, en nous servant des lettres de l'alphabeth. Cette science qu'on appelle Algebre sert à éclaircir, à étendre, et à perfectionner autant qu'on le peut faire l'Arithmétique, et généralement toutes les sciences qui se rapportent aux mathématiques. Elle est si générale qu'elle considere toutes les grandeurs... Mais ce qu'il y a de plus considérable dans cette science n'est pas son étendue et son universalité, [...] c'est la facilité qu'elle à l'esprit pour decouvrir les vérites les plus cachées et dont il seroit absolument impossible de s'éclaircir par l'arithmétique & par la géométrie ordinaire, ny par le secours d'aucune science. Comme on ne peut donner à l'esprit plus d'etendue, et plus de capacité qu'il n'en a, cette science apprend seulement à le ménager. Elle lui represente sous des expressions tres courtes un assemblage de plusieurs idées'.

⁵⁴Ibid., p. 166: 'La méthode d'exprimer chaque inconnue par une lettre inconnue est la plus generale de toutes [...] Et une seconde raison pour laquelle on doit encore la preferer aux autres methodes, c'est qu'elle suppose moins de connoissance dans l'esprit de celui qui s'en sert'.

⁵⁵Schwartz, Malebranche, p. 37.

⁵⁶René Descartes, *Œuvres*, Paris: Vrin 1996, VI, pp. 17–18: [L'analyse des Anciens] est toujours si astreinte à la considération des figures qu'elle ne peut exercer l'entendement sans fatiguer beaucoup l'imagination.

⁵⁷Prestet, Nouveaux Elemens, II, preface: 'N'est-ce pas là ce qu'on pourrait justement appeler des rompements de teste ?

EXEMPLES	DE LA MULTIPLI	CATION	
DES G	RANDEURS LITERALI	18.	
a===b a==1	b $a-b$	a + b	
a===b a==1	b $c-d$	c + d	
aa-+ab aa-+ab	bbadbd acbc	ad_bd ac+bc	
aa *bb. aa-+ 2ab	+bb. ac-bc-ad+bd.	$ac \rightarrow bc \rightarrow ad \rightarrow bd$.	
$aa \rightarrow 2ab \rightarrow bb$	2a-2ab-+bb	aa_ab-+bb	
$a \rightarrow b$	ab	a-+b	
$a_{ab} \rightarrow 2abb \rightarrow b^{3}$	—aab +2abb—bs	aab_abb+b ³	
$a^{3} \rightarrow 2a_{ab} \rightarrow abb$	a ² —2aab+abb	a ³ _aab+abb	
$a^3 \rightarrow 3aab \rightarrow jabb \rightarrow b^3$.	a'-zaab+zabb-b3.	a3 * *-1: b3.	XCVI.

Figure 5 – Examples of multiplication for literal magnitudes, Jean Prestet's Élémens des mathématiques, 1675, p. 23.

4.2 Numbers and letters

A particular feature of the treatise is the systematic treatment of numbers and letters—numbers and 'literal magnitudes', as Prestet called them suggestively—on the same footing and in parallel to each other. The extension from numbers to the letters of algebra constitutes in fact an important twist, often underestimated by the commentators.⁵⁸ In Prestet's *Élémens*, all operations, including the extraction of roots, are thus handled and explained both on usual numbers and on algebraic symbols. For instance, the multiplication is first defined for all magnitudes, as well as the rule of signs. Then details are given for the various possibilities, in particular, 'when each magnitude is expressed by letters, one immediately joins these letters and one writes before the sign + or — [according to the sign at each magnitude]'. He then continues: 'If some numbers precede the letters, one multiplies the numbers by the numbers, the letters by the letters, and one writes the product of the numbers before the product of the letters'.⁵⁹ In the same vein, besides the multiplication (and division) table for the ten first integers, Prestet offers a table for the composition of powers, which provides the development of the powers of a + b, until the power 10, see Fig. 6.

This parallel treatment is applied to fractions or incommensurable magnitudes as well. However, its

Nous en avons un exemple tout visible dans la personne de Monsieur Paschal. Car on croit qu'il n'est mort si jeune, que pour n'avoir pas assez ménagé les forces & l'étenduë de son imagination. Les grands efforts qu'il étoit nécessairement obligé de faire pour imaginer & pour démêler ce nombre prodigieux de lignes & de figures, & pour les embrasser d'une seule vuë luy attirèrent cet épuisement général du cerveau, qui l'enleva dans la fleur de son âge. On ne doit pas craindre le même danger des sciences qu'on explique ici. Car quoi qu'elles conduisent plus loin, & qu'elles découvrent beaucoup plus de véritez, & en moins de temps que la Géométrie, cependant elles fatiguent d'autant moins l'esprit qu'on les pénètre davantage, & qu'on les possède mieux'. On the issue of Pascal and imagination, see Véronique Wiel, 'Malebranche en dialogue avec Pascal: la question des contrariétés', in: *Dix-septième siècle* 224-3 (2004), pp. 461-476; Alberto Frigo, 'Pascal dans la Recherche de la vérité de Malebranche: l'imagination', in: *Les Études philosophiques* 95-4 (2010), pp. 517-534.

⁵⁸For instance, it modifies the hope for a theory of exponents, criticized by Leibniz as far as it is restricted to numbers, but nonetheless quite decisive for him. See Remaki, *L'art combinatoire*.

⁵⁹Prestet, *Élémens*, p. 21. See examples in Fig. 5.



Figure 6 – Table for the composition of powers, that is, binomial formulas up to the power 10, Jean Prestet's *Élémens de mathématiques*, 1675, inserted between p. 100 and p. 101. The *n*-th line displays the various monomials which compose the *n*-th power of a + b.

{ 1 ^{er} 20 efcus { 2° 40 efcus f pour 4 mois. { pour 5 mois	{ 3° 60 escus { pour 6 mois.	your 7 mois.			
Produits 80 -+ 200	-+ 300 -+				
fomme des produits. gain total : :	produits . So .	gains particuliers. 14 gain du 1 ^{er}			
1200 . 240 :::	360	48 gain du 2° 72 gain du 3°	abg cdg	efg	& _ghl
	(560 .	112 gain du 4	ab+cd+ef+bl ab+cd+ef	+bl ab+cd+ef+l	hl ab+cd-+ef-+hl

Figure 7 – A classical arithmetical problem, extracted from Jean Prestet's *Élémens des mathématiques*, 1675, pp. 237–239: Four merchants have made a deal where the first contributes 20 crowns during 4 months, the second 40 during 5 months, the third 60 crowns during 6 months and the fourth 80 crowns during 7 months; they have earned 240 crowns. What is the gain of each merchant (if it is proportional to the money he provided and the time during which he provided it)? On the left, the numerical solution; on the right, what Prestet calls the 'proof' of the problem, using letters to represent the various data.

role also evolves over the course of the resolution of problems: symbolic expressions are thus used to embody the generality of a problem on numbers. For instance, when Prestet explains the procedure to find the greatest common divisor of two magnitudes, he performs it first on examples such as 32 and 64, or 98 and 47. He then proves his procedure on magnitudes designated by *a* and *b*. The same occurs for classical arithmetical problems, such as 'three persons have a number of coins, the first and the second having a given number of coins more than the third, the first and the third a given number more than the second, etc; how many of coins has everyone'. While Diophantus and most of his successors state the problem in general terms, before making a concrete choice for the given quantities in order to explain the solution, Prestet does the opposite: he first presents the statement with concrete numbers. Then 'to make the question more general and to ensure that its resolution serves as a general model for any similar one', he designates these numbers by letters and proceeds with them until the end. He explains that 'By naming the known magnitudes with the first letters of the alphabet, computations are made more easily than with the characters of the numbers which are the values of these magnitudes'.⁶⁰ Both formulas and numerical answers are then provided (see Fig.7).

⁶⁰Ibid., pp. 163–164.

		Г	Table 4 ^e	pour la	refoluti	on des	puiffanc	es.			
Ta	16	1									
aa	22	16	1	2° puiffa	ince.						
a3	3aa	3ab	1 66	i	3° puiffai	nce.					E and
a4	4 23	Gaab	1 4.abb	1 1/3	T	4° puiffa	ince.				
as	5 544	10236	10aabb	sab3	6+	1	° puiffand	ce.			
ac	6a5	15146	202366	Isaabi	6264	65	1	6° puissa	nce.		
a7	1 700	21a5b	350+66	352363	21aab*	1 7 abs	1 60	1	7° puif	lance,	
aª	1 807	28266	1 56a5bb	702+63	1 (Ga3b4	28aabi	1 Sabs	1 67		8°	puissance.
a?	908	36a'b	184a°66	126asb3	1264464	84.0365	1 26aab6	0 ab7 1	68	-	19° puist
ato	1000	45236	1202766	210266	2522564	2108+65	1200360	4 saabi	10.068	1 69	10e puiff
				aland Long Long	· · · · · · · · · · · · · · · · · · ·				1	1	17 4

Figure 8 – Table for the resolution of powers, Jean Prestet's *Élémens des mathématiques*, 1675, inserted between p. 100 and p. 101. The first column contains the successive powers of *a*, and is the same as the first column of the table for the composition of powers, Fig. 6. The other cells of the *n*-th line contain the monomials which compose $(a + b)^n - a^n$ divided by *b*. They are directly derived from the cells in Fig. 6, up to a division by *b*.

Tables given for the composition of symbols or 'resolution of powers' (that is, root extractions) are also used to compute powers or roots of numbers. In such cases, Prestet designates by letters the digits of the number to be sought and explains first the procedure on these letters (this is what he calls the 'general problem'), and then on numerical examples.

To exemplify more clearly one of the numerous issues this common treatment raises, let us have a closer look at one case, namely, the extraction of the square root of 294849.⁶¹ Prestet first considers the first two characters on the left, 29, and designates them by a single letter *A*. He then extracts the largest possible square (25) from 29, and puts a = 5 as the first character of the root. He then joins the reminder 4 = 29 - 25 to the left of the two subsequent characters B = 48, thus obtaining 448. As next stage, Prestet refers to the 'Table for the resolution of powers', a symbolic table which displays and reorganizes the development of binomial powers (see Fig. 8).

In this table, one is tempted to read 2a + b as a simple addition of algebraic expressions, such that $(a + b)^2 = a^2 + b(2a + b)$: that is, 2a + b would simply be the factor multiplying *b* in the expansion of $(a + b)^2 - a^2$. However, this is not the case. Prestet uses it here to find the first digit (4) of 448. The character *b*, multiplied by 2a, should give the beginning of 448. As 2a = 10, *b* should be 4. But then Prestet states (without any comment) that 2a + b = 104: the sign + does not mean an addition any more, but the juxtaposition of the characters. Alternatively, one could say that the position of the characters is decisive and that the sign or character *a* is used sometimes as a number $a0 \cdots 0$, if needed.⁶² The

⁶¹Ibid., pp. 78-79.

⁶²However, it is striking that Prestet, or his publisher, uses a dot, and not a 0, to mark in such a case the possible 'missing' characters.

displayed operations show this ambiguity clearly and the text itself underlines the importance of the position of 2a = 10 ('I write it separately, and I write +*b* in the rank at the right', says Prestet⁶³). But the ambiguity in respect of *a*, seen in the same problem as a number or as a character, is never addressed explicitly. Prestet writes at the same time, a = 5, b = 4, and 2a + b = 104.

Prestet obtains then 2ab + bb = 416, which, once subtracted from 448, leaves 32. Once juxtaposed to C = 49 (the last two characters of the number), it leaves 3249 to consider. Reiterating the procedure provides Prestet with 3 as the third character of the root. Indeed, $543^2 = 294849$.

The same procedure is applied by Prestet to other examples of roots (up to the seventh root of numbers) and also to literal expressions, for instance to the cubic root of $a^3 + 3aab + 3abb + b^3 + 3aac + 6abc + 3bbc + 3acc + 3bcc + c^3$.⁶⁴ The procedure is completed in the next section by the approximation of true roots to any order of magnitude, if the extraction of the root does not provide an integer.

Such parallel treatment of algebraic expressions and numbers was clearly a challenge for the printer: he had, as mentioned earlier, to position long symbolic expressions on the page, and to print fractions of such literal expressions-in numbers, the numerator and denominator of a fraction are usually printed in smaller characters, up to the limit of readability. To give one other instance of the difficulties he was confronted with, let us consider another example given by Prestet, namely the division of $y^6 - 8y^4 - 124yy - 64$ by yy - 16. Prestet displays it in the same way as a division in numbers: the divisor is written under the initial dividend, the digits of the quotient are gradually written on the same line as the dividend, but separated by a small sign (a semi-circle). The first step is to compute the so-called exponent (that is, the result of the division) of y^6 by yy, which is y^4 . This term is written on the other side of the 'semi-circle', as it is done in the case of numbers. As y^4 multiplied by yy gives y^6 , and $y^6 - y^6 = 0$, Prestet 'erases' y^6 and yy, before going on with the division by the second term of the divisor -16. Usually, in numbers, one displays the erasure by a strike across the sign; as this operation is commonplace, the printer has such 'strikethrough' numbers at his or her disposal. But this is not the case for letters: thus, a number 1 is juxtaposed with the letters and this number 1 is 'strikethrough', that is, the printer puts Ay^6 instead of y^6 . And Prestet comments in the margin: 'One joins here strikethrough numbers to the letters that one says should be deleted because the printer did not have proper characters to mark them otherwise'.65

⁶³Prestet, *Élémens*, p. 78: 'je l'écris séparément, & après lui dans un rang plus à droite, j'écris +*b*'.

⁶⁴As in many textbooks, one suspects that the examples have been chosen backwards. The result here for instance is obviously a + b + c, but it is derived at length by Prestet using his general procedure and the tables.

⁶⁵Prestet, *Élémens*, p. 36: 'On a joint ici des chiffres tranchez aux lettres qu'on dit devoir estre effacées, parce que l'imprimeur n'a pas eü des caractères propres pour les marquer autrement'.

4.3 Simplicity

Scientific practice for both Descartes and Malebranche should first identify simple elements. Descartes 'supports the theory, in the *Regulae*, that science must be based on simple natures which are known by intuition'.⁶⁶ He applies the word 'simple' only to those things which are so clearly and distinctly known 'that the mind cannot further distinguish them into more distinctly known parts.⁶⁷ Simple things for Descartes should be intuitively conceived, and not defined, as definitions use more complex terms than the simple one in question. However, the issue of what is simple in mathematics is in some respect left open and Prestet addresses this issue in a variety of contexts. First of all, the epitome of simplicity, concerning magnitudes and their relations, is unity:

If we reflect on all our knowledge we will easily see that there is none which is clearer and more distinct to us than that of unity. For its nature and its properties are to be very simple, indivisible, & without composition of any parts. Not only does it measure itself, but it is also the immutable & natural rule by which we measure all the numbers that follow it up to infinity, which are nothing else than this unity repeated several times.⁶⁸

For Malebranche too, '[t]he bearer of truth, the object of knowledge by ideas, would thus be the measurable relation of equality or inequality' and unity being essentially the scale of comparison would be a decisive feature in the search for truth: integers are then seen as ratios, the first term of which is explicit, while the second is tacitly unity.⁶⁹ This also applies to magnitudes expressed by letters, for instance *a* is the same as $\frac{a}{1}$, says Prestet.

From the second edition onwards, 'simple' can be found to qualify integers in a new meaning: simple or prime numbers are those which cannot be divided by any other integer except themselves or the unity. The other integers are said to be composed. And these terms naturally extend to literal magnitude: a simple magnitude is, Prestet says, 'a linear one', where each letter appears in the first degree, such as *a* or a + b, for instance; in contrast, *ab*, but also ab + cd are said to be composed.⁷⁰

Simplicity also concerns operations and problems as well as objects. For instance, Prestet enigmati-

⁶⁶Desmond Clarke, *Descartes' Philosophy of Science*, Manchester: Manchester University Press 1982, p. 48. ⁶⁷Descartes, *Œuvres*, X, p. 418.

⁶⁸Prestet, *Élémens*, p. 5: 'Si nous réflechissons sur toutes nos connaissances nous verrons facilement qu'il n'y en aucune qui nous soit plus claire et plus distincte que celle de l'unité. Car sa nature et ses propriétés sont d'estre très simple, indivisible, & sans composition d'aucunes parties. Non seulement elle se mesure elle-même, mais elle est aussi la regle immuable & naturelle par laquelle on mesure tous les nombres qui la suivent jusques à l'infini, lesquels ne sont que cette unité même repetée plusieurs fois'.

⁶⁹Schwartz, *Malebranche*, p. 184: 'Le porteur de vérité, l'objet de connaissance par idées, ce serait donc le rapport mesurable d'égalité ou d'inégalité'. The role of the unity (or unit) for numbers, in Malebranche's *Search*, is discussed thoroughly in Ibid., pp. 166–173.

⁷⁰Prestet, *Nouveaux Elemens*, I, pp. 141–142.

cally explains: 'One conceives addition and subtraction as simple, one also conceives them as composed. The simple ones are the fundamental operations upon which all the others depend'.⁷¹ In fact, his 'composed addition' is multiplication and his composed subtraction is division. Simple problems are those 'where one assumes that some unknown magnitude considered as linear is equal to other entirely known magnitudes', that is those where the unknowns are single letters at the first power.⁷² These problems are considered to be a basis for the solution of more complicated problems (termed 'composed problems', exactly like the operations). But to solve such problems requires the identification of certain magnitudes as unknowns, a delicate issue which is never explicitly articulated by Descartes, and to which Prestet devotes detailed discussions: he explains in particular how to deal with each problem, according to the different ways of 'naming' (his term) the unknown quantities with letters and to the different rules used to transform the equations thus obtained.

4.4 Applying a Cartesian method

This global approach which is proximate to Malebranche's Cartesian program for knowledge acquisition is put into action in problem solving. Here, too, Prestet regularly alludes to Descartes's rules and preferences.

4.4.1 Never to accept anything as being true too quickly.

A remarkable feature of the *Élémens* is the number of proofs, even for what may appear at the time as trivial. The most striking example, from the second edition onwards, is Prestet's detailed proof that an integer can be decomposed into simple (that is, prime) divisors and that all possible divisors of this integer are obtained by the various multiplications of these simple ones.⁷³ According to Descartes, proofs should enlighten the mind, a requirement which is frequently mentioned by Prestet. For instance, when explaining how to check a subtraction (by adding the result to the term subtracted in order to obtain the first term), he adds:

One should not imagine, as is usually the case, that this way of examining whether or not one has not failed is a true demonstration. For besides the fact that it does not enlighten the mind, it could happen that one would be as much mistaken in the subtraction as one would be in the addition.⁷⁴

⁷¹Prestet, *Élémens*, p. 10.

⁷²Ibid., p. 149.

⁷³That is, essentially, the existence and unicity of the decomposition into prime factors, see Goldstein, 'On a Seventeenth-Century Version'.

⁷⁴Prestet, *Élémens*, p. 19: 'Il ne faut pas s'imaginer comme on le fait ordinairement que cette manière d'examiner si on n'a point failli soit une démonstration véritable. Car outre qu'elle n'éclaire pas l'esprit, il pourrait arriver que l'on se tromperait

4.4.2 Divide the difficulties and the problems so as to resolve them into simpler parts.

This Cartesian demand is mostly achieved through (algebraic) analysis. This question has been discussed above.

4.4.3 Proceed in an adequate, methodic order.

The appeal to a natural and simple order is recurrent:

One passes from the known to the knowledge of the unknown by going through all the known relations from one to the other in an orderly manner by natural, correct and very clear reasoning. The order in which one passes from the known to the unknown is what is called method or rules. And the observation of this method or of these rules, is what we call operations on the magnitudes.⁷⁵

Order may be linked to the judicious choice of the unknown which allows a more direct path, and indeed operates at several levels, including minute ones. For instance, when explaining how to get the square of an expression or a number, Prestet multiplies the digits (or letters) in a specific order, then comments that these expressions, 'disposed according to a certain order and reduced to a sum, provide the square'.⁷⁶

Order may also complete the preceding requirement. When beginning his study of the resolution of equations with one unknown, Prestet first composed multiplicatively simple expressions (that is linear ones, such as z - 2 = 0) in order to show how the coefficients—which readily provide the solutions in the first-degree case (or instance 2 is the solution of z - 2 = 0)—are combined in the case of higher degrees. Thus, he proves that the product of z - 1, z - 2, z - 3 and z + 4 is $z^4 - 4z^3 - 19zz + 160z - 120$, and examines the relations between the coefficients (1, -4, -19, 160, -120) of the equation $z^4 - 4z^3 - 19zz + 160z - 120$ and the solutions of $z^4 - 4z^3 - 19zz + 160z - 120 = 0$, here obviously (1, 2, 3, -4). On this, he comments:

It is against order to seek to know a simple thing by rendering it composite. On the contrary, what is composed must be resolved into all its different parts & having examined them well separately, one will judge with more order & light the relationship they have between each other, and the nature of the whole they compose. However, one must not imagine that I am acting against this rule, if I take simple magnitudes that are known to form wholes by their more composite products that are unknown. It is not my intention to learn to look for in this

autant dans la soustraction qu'on se seroit trompé dans l'addition'.

⁷⁵Ibid., pp. 9–10: 'L'on passe du connu à la connaissance de l'inconnu en parcourant avec ordre tous les rapports connus de l'un à l'autre par des raisonnements naturels, justes et très clairs. L'ordre que l'on donne pour passer ainsi du connu à l'inconnu, est ce qu'on appelle methode ou regles. Et l'observation de cette méthode ou de ces règles, est ce qu'on appelle opérations sur les grandeurs'. See also, among others, Ibid., p. 9, p. 168, p. 277, p. 356.

⁷⁶Ibid., p. 72.

composition what I already knew in the simple parts. But it is so that having examined well how these products are formed, I can learn methodically to solve similar ones.⁷⁷

Prestet is then able to link the combinations of the roots with the coefficients of the equation.

The issue of order also conveys echoes of past and on-going debates about analysis and the contrast between possessing a method and solving a problem by guessing:

Calculations of analysis are not made by blindly feeling one's way, as some say. On the contrary, they form the most ingenious expressions that can be invented of the most exact and the most methodical reasonings that one makes on the magnitudes.⁷⁸

4.4.4 Make enumerations and reviews.

We have already met several examples of this method of verification. In the first edition, moreover, a whole section is devoted specifically to what Prestet describes as the synthetic method,⁷⁹ which, once the simple parts of the question have been considered, systematically enumerates all the relations and combinations between them to solve a given problem. The synthesis, here understood in a way close to that of Descartes, designates the possibility of finding a result by composition of elementary or simple elements. This approach does not exclude algebra, as the different parts are usually designated by letters and the systematic study of their combinations is helped by hints from algebra. The first example is dedicated to the determination of the number of conjunctions of the '7 planets' (including the Sun and

⁷⁸Ibid., p. 277: 'les calculs de l'analyse ne se font point à tastons, comme disent quelques-uns; au contraire, elles font des expressions les plus ingénieuses que l'on puisse inventer des raisonnements les plus exacts et les plus methodiques que l'on fait sur les grandeurs'. This issue is central in Descartes's debates with Mersenne's circle, see on this issue Goldstein, 'L'expérience des nombres'; Catherine Goldstein, '1 803 601 800: de l'art des nombres à l'analyse, une autre voie ?', in *Aventures de l'analyse:*. *Mélanges en l'honneur de Christian Gilain*, ed. 'Suzanne Féry', Nancy: Presses universitaires de Nancy 2012, pp. 41–57.

⁷⁹For the delicate issue of analysis and synthesis in the seventeenth century, in particular in Descartes's thought, see Howard Duncan, 'Descartes and the Method of Analysis and Synthesis', in: *An Intimate Relation: Studies in the History and Philosophy of Science*, ed. James Brown & Jürgen Mittelstrass, Dordrecht: Springer 1989, pp. 65–80; Athanassios Raftopoulos, 'Cartesian analysis and synthesis', in: *Studies in History and Philosophy of Science* 34 (2003), pp. 265–308; Philip Beeley, 'Nova methodus investigandi: On the Concept of Analysis in John Wallis's Mathematical Writings', in: *Studia Leibnitiana* 45, H. 1 (2013), pp. 42–58; David Cunning, 'Analysis versus Synthesis', in: *The Cambridge Descartes Lexicon*, ed. Lawrence Nolan, Cambridge: Cambridge University Press 2015, pp. 7–12.

⁷⁷Ibid., p. 356: 'Il est contre l'ordre pour connaître une chose simple de la rendre composée. Il faut au contraire resoudre ce qui est composé en toutes ses différentes parties & les ayant bien examinées separément, l'on jugera avec plus d'ordre & de lumière du rapport qu'elles ont entr'elles, & de la nature du tout qu'elles composent. Il ne faut pourtant pas s'imaginer que j'agis la contre cette règle, si je prens des grandeurs simples & connues pour former des touts par leurs produits plus composez & comme inconnus. Ce n'est pas mon dessein d'apprendre à rechercher dans cette composition , ce qui m'estoit déjà connu dans [es parties simples. Mais c'est afin qu'ayant bien examiné comment ces produits se forment, je puisse apprendre méthodiquement à en resoudre de semblables'.

the Moon. . .); the problem is well-established, but Prestet assimilates these conjunctions to 'products' of the associated letters, with ab representing for instance the conjunction of the planet a and the planet b. In the case of the conjunctions, ab and ba represent the same thing, but Prestet then studies an analogous problem for words, in which the order of the letters is taken into account, as well as other situations, where the same letter can be used several times. He also revisits the famous problem of the permutations of the words in the verse 'Tot tibi sunt dotes Virgo quot sydera caelo', while respecting the rules of metrics.⁸⁰ The use of letters allows Prestet to highlight the relationship between combinations and development of algebraic formulas (like the binomial). The same reasoning is used to determine the number of divisors of monomial expressions such as a^3bbccd , and then of an integer, whose prime factors are assimilated to distinct and therefore independent letters.⁸¹

These different components are articulated by Prestet in his pedagogical practice of writing, as can been seen in many pages of the treatise, for instance:

I considered it necessary [to explain easy things] to allow those who are just starting out in the main foundations of all Analysis, so that having formed very clear and very distinct ideas of it, they could more easily recognize its uses later on, and apply it more methodically. And moreover, I believe I have explained nothing that does not need to be known, that does not deserve attention, if one wants to see what is in general the most natural order, according to which one must conduct oneself to arrive at the solution of problems.⁸²

5 In furtherance of the foregoing

Prestet's *Élémens* are thus anchored in a Cartesian framework. As we have seen, to revisit within this framework the standard questions of arithmetic and algebra for a readership of beginners means to solve numerous issues. The complex treatment of the literal magnitudes, in and of themselves or as substitute

⁸⁰See Prestet, *Élémens*, p. 342. On this verse and its manipulations during the seventeenth-century, see Ernest Coumet, 'Des permutations aux XVI^e et XVII^e siècles', in: *Actes du Colloque sur les permutations, Paris, Université René Descartes, 10-13 juillet 1972*, Paris: Gauthier-Villars 1974, pp. 277-289, repr. in *Œuvres*, t. 1, ed. Thierry Martin and Sophie Roux, Besançon: PUFC, 2016, pp. 270-290; Fernand Hallyn, 'Un artifice de peu de poids: Poésie expérimentale au XVIIe siècle', in: *Théorie Littérature Epistémologie* 10, special issue *Epistémocritique et cognition* 1 (1992), pp.19-37; Fernand Hallyn, 'L'anagramme et ses styles au XVII^e siècle', in: *Littératures classiques* 28 (1996), pp. 239-254. Prestet indeed quotes here other authors, in particular Tacquet.

⁸¹Prestet, *Élémens*, pp. 353–355.

⁸²Ibid., p. 168: 'J'ai cru [devoir expliquer des choses faciles] pour arrester un peu ceux qui commencent sur les fondements principaux de toute l'Analyse, afin que s'en estant formé des idées très claires & tres distinctes, ils pussent plus facilement dans la suite bien reconnaître ses usages, & l'appliquer plus methodiquement. Et de plus, je croy n'avoir rien expliqué qu'on ne doive sçavoir, & qui ne merite qu'on y fasse attention, si l'on veut bien appercevoir quel est en general l'ordre le plus naturel, selon lesquel on doit se conduire pour arriver à la resolution des problemes'. for numerical magnitudes, triggers semiotic ambiguities, but it also suggests fruitful crossovers. This framework was seen as too narrow to the mathematicians of Prestet's time who were involved in handling geometrical problems through calculus. However, besides the reframing of standard procedures and problems, Prestet brought various innovations, in particular from the second edition onwards.

5.1 Diophantine Analysis

The most obvious innovations concern Diophantine analysis. Most of these problems, of which only solutions in integers or fractions are required, are indeterminate, in the sense that they involve several unknowns and may have infinitely many solutions. While Diophantus usually provided one solution (fixing relations between the unknowns so as to handle one unknown only), seventeenth-century authors read and developed Diophantus in a variety of ways.⁸³

Prestet's first edition illustrates his approach through a few problems drawn from Diophantus, Bachet, and Vieta. In the first book of the second part, for instance, he proposes the problem: 'To find two magnitudes the sum of which being subtracted from their product, the reminder is a given magnitude'.⁸⁴ Prestet names *a* the given magnitude, *y* and *z* the two unknown magnitudes, and thence easily finds $z = \frac{a+y}{y-1}$, which provides infinitely many solutions. However, he spends many more lines exploring another possible choice of unknowns, 'to express the solution more or less as Diophantus does', he says, and proceeds to give several pairs of solutions for *a* = 8. We find here again the main characteristics of his treatment of the problems, intended to emphasize the effectiveness of the Cartesian method.

In the second edition, however, Prestet decided to solve 'all the questions of Diophantus because of the esteem in which it is generally held & of the care that illustrious authors have taken to comment on them'. He added 'everywhere the literal formulas of general resolutions and of infinite resolutions, such that one has in this volume as an already solved table of Diophantus's questions', hoping that 'this will be of a great help to all those who often attempt analytical researches'.⁸⁵ To see what Prestet's method brought to these problems, let us look at a typical example, that of the first case of the Question XV, in

⁸³Jean Cassinet, 'Problèmes Diophantiens en France à la fin du XVIIe siècle: J. de Billy, J. Ozanam, J. Prestet (1670–1689)', in: *Cahiers du Séminaire d'Histoire des Mathématiques de Toulouse* (1987), pp. 13–41; Goldstein, 'L'expérience des nombres'; Catherine Goldstein, 'Diophantus redivivus: is Diophantus an early-modern classic?', in: *Oberwolfach Reports* 26 (2021), pp. 38–40.

⁸⁴Prestet, *Élémens*, pp. 311–312. This is in fact a lemma for the problem XXXV in Diophantus's Book IV, with 8 the given number.

⁸⁵Prestet, *Nouveaux Elemens*, II, preface: 'J'ai résolu toutes les questions de Diophante, à cause de l'estime qu'on en fait généralement, et du soin que des Auteurs illustres ont pris de les commenter. [...] J'ai ajouté partout les formules littérales des résolutions générales et des résolutions infinies, afin que l'on eût dans ce volume comme une table déjà résolue des questions de Diophante'.

the fifth book:⁸⁶ To find three magnitudes [z, y, x], of which the two-by-two products [zy, zx, yx] having received a known magnitude [a], the sums are perfect squares. Naming *v*, *t* and *q* the sides of the three perfect squares,⁸⁷ and deriving *y* and *x* from the first equalities to replace them in the last one, Prestet obtains the algebraic expression of the problem in the form

$$zy + a = v^2$$
, $zx + a = t^2$, $yx + a = q^2 = \frac{v^2t^2 - av^2 - at^2 + a^2 + az^2}{z^2}$

He then remarks that in order to make the last term a perfect square, it is sufficient to take z = v + tor z = v - t, which transforms the numerator of this term into $v^2t^2 \pm 2avt + a^2$ (which equals the square $(vt + a)^2$ or $(vt - a)^2$). It provides indeed numerous examples with t and v arbitrary. The solution is then summarized in three parts: 'suppositions', that is, 'assumptions', infinite resolution, and examples. And in this case, Prestet proposes another even less restrictive resolution with a milder assumption on the unknowns. It has to be pointed out that no proof is given that the solutions are all different, which would indeed provide infinitely many solutions.

More spectacular cases are treated in *Nouveaux Elemens*. For instance,⁸⁸ the question XXXVII of Book V asks to cut a magnitude [4*a*] into four magnitudes [*z*, *y*, *x*, *v*], such that the six differences [z - y, z - x, y - x, ...] are perfect squares. Putting $z - y = (2t)^2$, $z - x = (2s)^2$ $z - v = (2r)^2$, Prestet finds $4a = 4v + 12r^2 - 4t^2 - 4s^2$ and thus

$$v = a - 3r^{2} + t^{2} + s^{2}$$

$$z = a + r^{2} + t^{2} + s^{2}$$

$$y = a + r^{2} - 3t^{2} + s^{2}$$

$$x = a + r^{2} + t^{2} - 3s^{2}$$

It is also necessary that y - x, y - v and x - v are perfect squares. Thus to find three squares, the differences of which are squares, Prestet puts

$$r = q + t$$
, $s = p + t$, $q = (n^2 + m^2)l$, $p = (n^2 - m^2)l$.

In this way, he is led to a triple equality, of a type he has already solved in a preceding problem, and draws *t*, under the condition that $a > 3r^2 - t^2 - (n^2 + m^2)^2$. Prestet's resolution here depends on three arbitrary parameters. For $a = \frac{5}{4}$ and the parameters 2, 1, $\frac{1}{10}$, he finds for instance the solution (Fig. 9):

⁸⁶Ibid., pp. 205-206.

⁸⁷To a modern reader, the seemingly random names of the variables is remarkable. ⁸⁸Prestet, *Nouveaux Elemens*, II, pp. 233-237.



Figure 9 – A typical presentation of the solution of a Diophantine problem (here Question XXXVII of Book V), Jean Prestet's *Nouveaux Elemens des mathematiques*, II, p. 235.

$v = \frac{21621832786373}{42878854598410}$	$\frac{9}{0}$
$z = \frac{70752550156623}{42878854598410}$	$\frac{9}{0}$
$y = \frac{70539903766863}{42878854598410}$	$\frac{9}{0}$
$x = \frac{51479986282188}{42878854598410}$	$\frac{3}{0}$

Although these solutions are new, it should be noted that their properties are those emphasized by the preceding generation of mathematicians: the size and apparent numerical complexity of the solutions warrant that they were not obtained by chance, or by blindly feeling one's way, but by a systematic method.⁸⁹ Fermat also claimed to have infinitely many solutions to some Diophantine problems, but no proof was given, either by him or any other seventeenth-century author. Prestet's algebraic formulas may

⁸⁹See Goldstein, 'Routine controversies', pp. 258-265.

seem to do the trick, but as we know now, this is not always truly the case.⁹⁰ Nonetheless, most of his solutions were new and displayed his ingenuity in the application of Cartesian algebra to Diophantine procedures. In particular, he systematized the resolution of the problem where one looks for two cubes, the sum or difference of which is equal to the sum or difference of two known cubes—a problem made famous through Fermat's 1657 challenge to the mathematical world, in particular Wallis and Frenicle.⁹¹ He shows in particular how to go from one problem (to find two cubes the difference of which is the sum of two given cubes) to its variants (here: to find two cubes the sum of which is the difference of two given cubes) in order to construct recursively new solutions, generalizing here the procedure suggested by Fermat. In the case where the sum of the given cubes is 28, besides algebraic formulas, he delivers for instance the solution recursively:

1892071224702010971769032350335	15011042268205492036870569329391
4947561551827392932621677753432'	4947561551827392932621677753432

5.2 Solving Equations

In Fermat's work, what was at stake was to find true solutions (that is, positive rational numbers) in cases where the usual Diophantine method would provide negative numbers. His procedures to find new solutions allowed him in some cases to obtain such positive solutions from negative ones. Prestet's aim was different, for the problem of negative numbers recedes under the shadow of solutions under the form of algebraic formulas. The same change occurs in the sections devoted to the solution of algebraic equations with one unknown.

If one considers only the algebraic formulas for the first degrees (up to 4), there is nothing new in Prestet's resolution. However, his presentation reveals a deep engagement on his part with the foundations of these formulas. In the case of quadratic equations, Prestet states what he calls 'the Axiom which is the principle of equalities of second-degree equations':

Two magnitudes being given, the square of any of them minus the same square minus the product of the two magnitudes plus the same product is zero.[...] If the two magnitudes are a and b, instead of saying the square of a or the square of b, we shall write zz, the square of an unknown z which fits as well a as b.⁹²

⁹⁰The issue of generality is particularly acute in Diophantine questions: in current terms, most of these equations provide algebraic curves of genus 1, which may or not have infinitely many rational solutions.

⁹¹Prestet, Nouveaux Elemens, II, pp. 256–262.

⁹²Prestet, *Élémens*, p. 358: 'Deux grandeurs estant données le quarré de celle des deux que l'on l'on voudra moins ce même quarré moins encore le plan des deux grandeurs, plus le même plan, est égale à zero. [...]. Si les deux grandeurs sont a et b au lieu de dire la quarré de a ou le quarré de b, nous écrirons seulement zz, le quarré d'une inconnuë comme z, qui convient autant à la grandeur a qu'a la grandeur b.

Prestet's point here is that -az - bz then represents minus the square of one magnitude minus the product of both (independently of the choice of z as a or b). Correspondingly, the axiom translates as $z^2 - az - bz + ab = 0$.

One therefore gets: zz- az - bz+ab = 0. Prestet insists on having this written on two lines, in order to display even more clearly the ambiguity in the interpretation of this axiom.

$$zz - az + ab = 0$$
$$- bz$$

It is the same for the axioms of the higher degree equations, for instance for the fourth degree:93

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2

$$z^{4} - az^{3} + abz^{2} - abcz + abcd = 0$$
$$- az^{3} + acz^{2} - abdz$$
$$- az^{3} + adz^{2} - acdz$$
$$- az^{3} + bcz^{2} - bcdz$$
$$bdz^{2}$$
$$cdz^{2}$$

This leads him to a tabular representation of the 'combination of equations', displaying how the coefficients of an equation are constructed from the roots. And reciprocally in the following section, to the resolution of the equations, including special procedures to find multiple roots (as in Johann Hudde's work,⁹⁴ to which Prestet refers) or integral roots. In the case of cubes for instance, he provides explicit transformations from the form of the equation provided by the axiom to the reduced form $y^3 - py - q = 0$ through which the final solution is expressed. In the second edition, all the cases of the third degree equation are discussed, and the imaginary roots are also displayed in the examples (see Fig. 10).

In her detailed presentation of the work on Prestet on equations, Asselah emphasizes the mixture of algebra and what we would call combinatorics in Prestet's approach.⁹⁵ In the first edition, at least, it corresponds to his idiosyncratic distinction between analysis and synthesis: the former operated with algebraic identities and is illustrated by the linear factorization of equations, while the latter explores the relation between coefficients and roots, as seen before.

⁹³Ibid., p. 361.

⁹⁴Karl-Heinz Haas, 'Die mathematischen Arbeiten von Johann Hudde', in: *Centaurus* 4 (1956), pp. 235-284.
⁹⁵Asselah, 'Jean Prestet'.

Réfolution générale. 366 20 3.00 $q \gg 0.5 \sqrt{C.^{1}q} + \sqrt{-qq} - \frac{1}{-p^{3}} \gg \sqrt{C.36 + \sqrt{1296-512}} \gg 4.3bb \infty$

Figure 10 – Solutions of a cubic equation in the case where $\frac{p^3}{27}$ is less that $\frac{q^2}{4}$ in Jean Prestet's *Nouveaux Elemens des mathematiques*, II, p. 393. One notes the difficulties in printing the solutions in the absence of specific signs for cubic roots and the way in which imaginary roots are presented.

5.3 Combinations

Combinations began to acquire the status of a specific topic during the seventeenth century, in particular in the work of Mersenne, Frenicle, Pascal, and later Leibniz.⁹⁶ However, Frenicle's *Abrégé des combinaisons*, although composed in the 1640s, was only published posthumously and not before 1693. Meanwhile, many partial results focused on specific problems, such as anagrams or the variations of the verse 'Tot tibi sunt dotes Virgo quot sydera caelo', we have already met. One also finds scattered identities or tables. It should be emphasized that combinatorial questions have been in some cases the nemesis of Cartesian algebra⁹⁷ and Leibniz, of course, repeatedly promoted them as a key issue for the future development of mathematics, including during his stay in Paris where he participated in Malebranche's circle.⁹⁸ Nonetheless, the role of combinations in Prestet's *Élémens* is particularly intriguing, as it changes radically between the first edition and later editions.

In the first edition, as explained above, combinatorial problems occur in the second book of the second part, which is devoted to the resolution of questions through composition rather than through algebraic resolutions, and they thus illustrate the synthetic path as opposed to the analytic path. They are also linked to the issue of enumerations, required to complete the search for truth in the Cartesian (and Malebranchist) way. Prestet here provides only examples of problems, from which a few rules are derived, based on combinations with or without repetition and order.

From the second edition onwards, however, a whole book of the first volume is devoted to the general composition of magnitudes. Prestet reorganizes the results of the first edition into three theorems (with proofs) and seventeen problems. Here, a new object is put centre-stage: the so-called alternative products,

⁹⁶Ernest Coumet, *Mersenne, Frenicle et l'élaboration de l'analyse combinatoire dans la première moitié du XVII^e siècle,* thesis, Université de la Sorbonne 1968, repr. in: *Œuvres*, t. 2, ed. Catherine Goldstein, Besançon: PUFC 2019; Remaki, *L'art combinatoire*.

⁹⁷Goldstein, 'L'expérience des nombres'; Goldstein, '1 803 601 800'.

⁹⁸ Robinet, 'La philosophie malebranchiste'.

			No	mbres des	chofes		prifes		
I. <u>I</u> .	3.4.	5-	6.	7.	8.	9.	10.	une à une	
2-	6.1 z.	20.	30.	42.	56.	72.	90.	deux à deux	
	6.24.	60.	120.	210.	336-	504-	720.	trois à trois	
	24.	I 20.	360.	840.	1680.	3024.	5040.	quatre à quatre	
	•	120.	720.	2520.	6720.	15120.	30240.	cing a cing	
			720.	5040.	20160.	60480.	151200.	fix à fix	
			-	5040.	40320.1	181440.	604800.	(ept à sept	
				•	40320.	362880.	1814400.	hnit à huit	
						62880.	628800.	neuf à neuf	
						-	3628800.	dix à dix	
1.4.1	5.64.	325.	1956.1	3699.1	109600.9	86409.9	864100.	Commes entiéres.	

Figure 11 – Table counting permutations, Jean Prestet's Nouveaux Elemens des mathematiques, I, p. 123.

which correspond to combinations of k elements among n.⁹⁹ The first theorem counts alternative planes (that is, alternative products of 2 elements): 1 for n = 2 (the product ab), 3 for n = 3 (the three products ab, ac, bc), ..., ¹⁰⁰ Prestet gives these numbers up to n = 5, concluding 'and so on to infinity'. Interestingly enough, his 'proof' (up to n = 5) relies on a relation between the result for n - 1 and n (a type of reasoning which would be formalized later as 'complete induction'). The other theorems are devoted to alternative products of higher order, which are gathered in a table.

The various problems are then all expressed in terms of abstract things (represented by letters). For instance, problem XIV searches 'all combinations of various things, when each of them can come back or be taken as many times as one wishes', problem XV requiring the numbers of such combinations (solved with the help of the table). Along the way, Prestet fixes the vocabulary of combinations and permutations (which his predecessors did not do). Concrete examples are sometimes given, but in contrast to the first edition, they only illustrate a result established directly on a collection of letters. In his own terms, 'the examples may fix and enlighten these rules'.¹⁰¹ As noted by Geoffrey Bringer, this new approach, with no reference to synthesis, is closer to that of Pascal. In particular, it uses Pascal's terminology of 'order' to designate the data of a line in the table.¹⁰² But Prestet introduces numerous new tables, with the formulas which are at the basis of their construction (Fig. 11). Moreover, the emphasis on alternative products goes together with procedures based on the reduction of each problem to a study of these products: that is, Prestet at this point adopts an original *analytical* approach to the problems of combinations.

⁹⁹We find again in the use of the word 'product' the ambiguity between juxtaposition and operation that we have noted earlier for addition.

¹⁰⁰Prestet, Nouveaux Elemens, I, p. 114.

¹⁰¹Prestet, *Nouveaux Elemens*, I, p. 126.

¹⁰²Geoffroy Bringer, Jean Prestet et la combinatoire dans les Eléments des mathématiques de Prestet, Mémoire Sorbonne Université 2018.

6 Back to a book

Malebranche recommended Prestet's Élémens in several editions of his Search for Truth until he turned with enthusiasm to works integrating the new calculus. According to the Catalogue collectif de France, one still finds copies of Prestet's Élémens in the libraries of about thirty French towns, attesting a wide use in early-modern universities and learned circles. The copy of the first edition at the Russell Library seems to have belonged to Jean Galipaud, a Father of the Oratory, born in Nantes in 1660 and known in particular for his lectures on the topic of grace at the Riom college around 1690, lectures which were accused of being Jansenistic. But bookplates and other marks in existent copies show that Prestet's book circulated beyond Oratorian circles, for instance in the Jesuit centre of La Flèche, or in the hands of Jean Joseph de la Montre, for a short time professor at the Collège royal or in the hands of Charles Penot de Tournière, associate at the Academy of Sciences in the mid-18th century. One other copy, in the Angers town library, bears the name of the Cartesian Pierre-Sylvain Régis and the book also appears in the catalogue of Montesquieu's library. It was also read outside France, in particular in England and the Netherlands. The Swiss philosopher and mathematician Jean-Pierre de Crousaz testifies to this larger diffusion of the book, explaining: 'I was in my 15th year when a French gentleman brought in the first edition of Father Prestet's Élémens des mathématiques [...I]n my eagerness to read it, I promised to explain it to him'.¹⁰³ And more or less pirated editions from 1694 onwards testify to a certain survival of the publication. Prestet's textbook is regularly mentioned in eighteenth-century treatises, whether as a general source or in view of criticising and improving it.¹⁰⁴ At a micro-level, the name of Prestet, with references to specific results, in particular on combinations or on Diophantine equations, is mentioned again in the last decades of the nineteenth century, when these topics began to successfully take off.¹⁰⁵

Prestet's book nevertheless appears as a challenge to the question raised by Roger Chartier and stated at the beginning of this chapter. On the one hand, as we have seen, Prestet constantly places himself on several levels as a simple epigone of Descartes, effacing himself as author behind the Cartesian method,

¹⁰³Malebranche, *Œuvres*, XVIII, p. 264, n.1: 'J'étois dans ma quinzième année quand un gentilhomme français fit venir la première édition des *Élémens des mathématiques* du Père Prestet. [...D]ans l'ardeur où j'étois de la lire, je lui promis que je le lui expliquerais'.

¹⁰⁴For examples of such mentions, see Jean-Pierre de Crousaz, *Réflexions sur l'utilité des mathématiques et sur la manière de les étudier, avec un nouvel essai d'arithmétique démontrée*, Amsterdam: L'Honoré and Châtelain 1715, p. 46; Charles-Etienne-Louis Camus, *Cours de mathématique*, Première partie: *Élémens d'arithmétique*, Paris: Ballard 1753, p. 452; Alexandre Savérien, *Histoire des progrès de l'esprit humain dans les sciences exactes, et dans les arts qui en dépendent ... avec un abrégé de la vie des auteurs les plus célèbres dans ces sciences*, Paris: Lacombe 1766, p. 13.

¹⁰⁵For instance: Adolphe Desboves, 'Mémoire sur la résolution en nombres entiers de l'équation $aX^m + bY^m = cZ^n$ ', in: *Nouvelles annales de mathématiques* 2s. 18 (1879), pp. 398–410, esp. p. 406; Edouard Lucas, *Théorie des nombres*, Paris: Gauthier-Villars 1891, p. 38; Paul Mansion, 'Sur un problème de Diophante d'après M. Heath d'après Jean Prestet', in: *Mathesis* 34-4 (1914), pp. 145–149, pp. 209–214.

defending Descartes from contemporary attacks, and revisiting, in the light of his method, multiple, but often classical problems, whether those of practical and commercial arithmetic or of low-degree and Diophantine equations.

On the other hand, what might appear to be a modest pedagogical exercise on Prestet's part to clarify the most delicate points of Cartesian practice, such as the choice of unknowns, leads in fact to various tensions, with their own dynamics. The constraints of the printer clash with the efforts of the teacher, whether in the arrangement of the algebraic formulas, the new signs, or the vast and abundant tables and the very large numbers. Moreover, in Prestet's wish to deliver a large-scale systematized use of literal representation, algebra passes from the status of a method with its own objects and problems and its own rules of operation, to the status of a generalized mode of writing, the mastery of which is not self-evident, as is shown by the blurring between operations and juxtaposition. The symbolic representation not only serves, at the beginning of a problem, to allow the implementation of the rules of algebra, but also becomes at the end the expression of the solutions, now as formulas. The pedagogical author seems here to be captured by his new objects themselves.

To all this is added the diversion of Cartesian disciplinary priorities by those of Malebranche's circle. By placing problems about numbers at the centre of mathematics, contrary to Descartes who did not find them universal enough, Prestet creates a new ground for exercise. One of the most striking examples is of course the transformation of the section on combinations between the first edition and those that came after it. These combinations, algebraized (or perhaps better expressed, literalized), classified, and tabulated, became a powerful tool for understanding equations and even elaborating proofs. The potential fruitfulness of these developments should not be ignored: there was no unique royal road to the future of mathematics, the one leading to calculus, but several, and probably many, some of which emerge only decades later.

Even if we discard both the Cartesian leitmotiv and a too narrow view of the future of mathematics, it is nonetheless quite difficult to evaluate the combined roles of Prestet and Pralard. Some aspects (such as the flood of algebraic formulas) are part of a global movement of which Prestet is less the author than one of the actors. Others, such as the surprising proof of the uniqueness of the prime decomposition of a number, or, even more hidden and fleeting, his reflections on the ambiguity of symbols only become visible to us because of the later importance of this point in 19th-century equation theory.¹⁰⁶

It has become commonplace to see in a text or a work, and in its important characteristics, the effect of the multiple readers, individual or collective, who have read or used it over time. But its effects operate

¹⁰⁶The complex issue of early modernity and its assessments, depending on later priorities, is also illustrated in Henk Bos, 'Tradition and modernity in early modern mathematics: Viète, Descartes and Fermat', in: *Mathematical Europe: history, myth, identity*, ed. Catherine Goldstein, Jeremy Gray and Jim Ritter, Paris: Editions de la Maison des Sciences de l'homme 1996, pp. 183-204. On the issue of ambiguity, see Jenny Boucard, 'Louis Poinsot et la théorie de l'ordre: un chaînon manquant entre Gauss et Galois ?', in: *Revue d'histoire des mathématiques* 17 (2011), pp. 41–138.

on multiple scales—from the writing of a symbol to a global vision of disciplines or to institutions of teaching—and, in mathematics, where the long term remains particularly relevant, these effects combine and interfere with temporal shifts.¹⁰⁷ The question of authorship thus arises here in a more complex way than the legally oriented tripartition suggested by Fichte. Words, often redefined, are not always a common and banal possession, any more than are the signs of the printing press; but in reverse, distinctive traces of Prestet, Malebranche, Pralard, Descartes, and others—mathematicians of the past, immediate contemporaries playing various roles, and even future readers—'dismembered and dispersed', are often intermingled in the meaning given to these words and symbols.

And I am willing that some believe there is little of my invention, not only in my first work, but even in this one; provided that it is easy to see here clearly and in good order, and to learn in depth, what one perhaps sees with less breadth and cohesion—dismembered and dispersed—in many others.¹⁰⁸

¹⁰⁷See for instance *Scientific Authorship*. The issue is illustrated in detail in Catherine Goldstein, *Un théorème de Fermat et ses lecteurs*, Saint-Denis: PUV 1995.

¹⁰⁸Prestet, *Nouveaux Elémens*, I, preface: 'Et je veux bien qu'on croye qu'il y a peu de mon invention, non seulement dans mon premier ouvrage, mais même en celuy-ci; pourvu qu'il soit facile d'y voir clairement & dans un bon ordre, & d'y apprendre à fond ce qu'on verroit peut être avec moins d'étenduê & d'enchainement comme démembré & dispersé dans un grand nombre d'autres'.