Optimal Tenurial Contracts Under Both Moral Hazard and Adverse Selection

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Abstract

This paper determines the optimal tenurial contract between a monopoly landlord and a tenant protected by limited liability under both adverse selection (based on the tenant's ability) and moral hazard (based on the tenant's choice of effort). We identify different optimal contracts depending on the tenant's outside option. For intermediate values, there is a threshold of tenant ability depending on the outside option level below which the optimal contract is a separating sharecropping contract and a pooling one otherwise. We also find that an increase in the outside option does not monotonically increase the tenant's optimal effort.

Keywords: adverse selection, limited liability, moral hazard, tenurial contract.
JEL codes: D82, O12, Q15.

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Introduction

The design of optimal tenurial contracts has long concerned economists because it is a challenging question for growth, agriculture, and development. The implications for efficient production differ widely according to whether wage, fixed rent, or sharecropping contracts are implemented. As reviewed in the following section, studies of asymmetric information are now key to understanding tenancy contracts. This paper seeks to provide a better understanding of agricultural land tenure contracts in an economy characterized by asymmetric information and protection for tenants. More precisely, we make a novel contribution to the theoretical literature on tenurial contracts. We consider that, along with moral hazard, there is also adverse selection. Moreover, we study how changes in the legal protection of tenants and/or changes in the level of a country's development may affect agricultural productivity via optimal tenurial contracts.

We examine how tenurial contracting between a monopoly landlord and a tenant protected by limited liability should be optimally designed under both adverse selection and moral hazard. We assume that the output generated by the tenant is observable and verifiable but that its distribution is influenced by the tenant's effort, which is private information. Moreover, we assume that the cost of the effort borne by the tenant is influenced by his ability, which is also private information. The tenant benefits from a positive outside option that can be interpreted in different ways. It may reflect the tenant's bargaining power over the landlord ensured by legal protection (Banerjee et al. (2002)). It may also be correlated to the level of the country's development, with a higher degree of development providing the tenant with better opportunities because of a more developed labor market. This outside option has a major impact on our results.

Because of moral hazard with limited liability and adverse selection, the landlord may have to give limited liability rent and/or information rent to the tenant. The landlord must find the optimal trade-off between efficiency and the extraction of these rents, which leads to the following optimal contracts.[1]

To begin, it is worth describing three benchmarks. When information is symmetric, the optimal contract is efficient and is a pure rental contract. When information becomes asymmetric, there is a trade-off between efficiency and rent extraction. Under moral hazard alone, the conflict between efficiency and limited liability rent extraction diminishes as the level of the outside option rises (it even ceases for high levels). Sharecropping contracts appear to be optimal for low and intermediate levels. Under adverse selection alone, conflict arises between efficiency and information rent extraction. The optimal contract is a sharecropping contract except for the most-able tenant, who is offered an efficient agreement.

Let us come back to the mixed informational setting and detail the properties of the optimal contract. Firstly, when the outside option is low, we find that, under adverse

selection, moral hazard, and limited liability, a sharecropping contract with a constant share for any level of ability emerges endogeneously as optimal. In this case, the landlord leaves only a limited liability rent to all types. Because the limited liability constraint is binding regardless of the tenant's ability, the contract must be fully pooling. We obtain the same contract as under moral hazard alone with a low level of outside opportunity. The landlord's trade-off leads to no distortions with respect to moral hazard alone.

Secondly, when the outside option is high enough such that the limited liability constraint is always slack, we then find the standard optimal contract under adverse selection alone. Only the trade-off between efficiency and information rent matters for the principal. Distortions occur such that the optimal contract is a pure rental contract for the highest ability (as with moral hazard alone) and a sharecropping contract for lower abilities, i.e. shares are distorted below their efficient (or moral hazard with high outside opportunity) levels.

Thirdly, when the outside option has intermediate values, the overall trade-off comes into play. We find new results for optimal contracts: 1) There exists a threshold of tenant ability below which the optimal contract is a sharecropping contract with an ability increasing share and a constant share otherwise. 2) The contract is thus separating for ability lower than this threshold because the limited liability constraint is slack and pooling otherwise because this constraint is binding. 3) The optimal agreement is not simply a mix between the two preceding contracts. The overall trade-off implies that the landlord benefits from having as many levels of ability as possible constrained by the limited liability rent rather than the information rent. This provides an incentive to increase the share left to the tenant.

We now state the implications of the optimal contract for the level of effort exerted by the tenant. We show that effort invariably increases with ability whatever the information setting. When ability increases, the marginal cost of effort decreases. This implies that the tenant is induced to exert more effort. By contrast this is not the case with the level of the outside option. Under moral hazard alone, a greater outside option forces the landlord to offer a higher share, which induces the tenant to provide more effort. Under adverse selection alone, the outside option does not affect the share, and so does not affect effort. As a consequence of combining adverse selection and moral hazard, we find that an increase in the outside option does not monotonically increase the tenant's optimal effort. Indeed, when the outside option increases such that the limited liability constraint is slack, moral hazard is no longer an issue and it becomes optimal for the landlord to offer separating contracts that distort the sharing rule, and so the optimal effort, downward.

We summarize the main insights of the mixed informational context in the following table.

INCLUDE TABLE 1

Finally, we extend the model by assuming that high-skilled tenants also have access to better outside labor opportunities. More precisely, we assume that the tenant's outside option is linearly type-dependent. We find that a type-dependent outside option increases the pooling nature of the contract proposed by the landlord. Indeed, bunching now affects low abilities and not only high abilities. However, this new range of pooling no longer follows from the liability protection but from a better outside option for high-skilled tenants.

These results have consequences for agricultural and development economics. First, in an economy where both adverse selection and moral hazard co-exist, we should not observe a share decreasing in ability as under moral hazard alone. Moreover, we should observe heterogeneity in contracts offered to the tenant but less than under adverse selection alone. This last phenomenon should be amplified if the outside option is linearly type-dependent.

Second, consider a given probability distribution for abilities. We observe that when the outside option is low, a single contract is the best offer. When it is high, it is optimal for the landlord to offer a menu of contracts. For intermediate values, there are many contracts but not a full menu, i.e. there is less heterogeneity. We can conclude that homogeneity in contracts should be higher in less developed economies and/or in economies providing little tenant protection.

Our results also have substantial consequences for economic policy. Improving tenants' security by giving them more bargaining power and/or an increase in the level of the country's development, which increases their outside options may have ambiguous consequences for their productivity. Assume that a government implements a policy that increases the outside options (like Operation Barga in the Indian state of West Bengal studied by Banerjee et al. (2002)). Depending on the levels of tenants' abilities, the consequences may be dramatically different. An increase in the outside options could lead to only the most highly-skilled tenants increasing their effort while the less-skilled tenants reduce theirs.

The rest of the paper proceeds as follows. Section 2 presents the related literature on agricultural tenancy. Section 3 presents the model. In section 4 we develop the landlord's program. Section 5 considers the optimal tenurial contracts under both adverse selection and moral hazard; after studying the social optimum, we then derive the optimal contract under moral hazard or adverse selection alone and then we consider the mixed framework. In section 6 we consider the extension where the tenant's outside option is linearly type-dependent. Section 7 concludes. All proofs are in the Appendix.

The related literature

The literature on agricultural tenancy is abundant. Many articles seek to explain the existence of sharecropping as well as the existence of fixed rental and wage rental contracts. Following Sen (2011), the literature has mainly focused on three types of explanations for the existence of different tenurial contracts.

The first is based on the trade-off between risk sharing and incentives (Stiglitz (1974)). Since the tenant is the residual claimant of the output, fixed-rent contracts provide strong incentives, but the tenant bears the entire risk. With wage contracts, no risk is borne by the tenant, but he has incentives to shirk. Sharecropping exists as a compromise between the risk and incentive effects of a pure wage contract and a fixed-rent contract.

The second explanation centers on the adverse selection problem. According to Hallagan (1978), the tenants' abilities are private information, i.e. the landlords cannot distinguish between them. By choosing between pure wage, fixed-rent, and share contracts, the tenants are screened, generating valuable information for the landlords, who can allocate ressources efficiently. Allen (1982) shows that share contracts can also be efficient by considering two dimensions of ability that are unobservable. Introducing adverse selection explains the co-existence of multiple contracts. Chaudhuri and Maitra (2001) provide empirical corroboration for this result by using the experience of an agent as a proxy for his skill.

The third explanation focuses on moral hazard, which establishes that sharecropping can solve opportunistic behavior by the landlord and/or the tenant. Eswaran and Kotwal (1985) develop a double-sided moral hazard model in which both the landlord and the tenant can shirk. They show that sharecropping may be the optimal contract since it minimizes efficiency losses from the two types of shirking. Depending on the exogenous abilities of the landlord and the tenant, fixed wage or fixed rental contracts may also emerge. Ghatak and Karaivanov (2014) extend the analysis of Eswaran and Kotwal (1985) by introducing endogenous matching in a double-sided moral hazard model. They show that sharecropping may not emerge in equilibrium while it is optimal for an exogenously given pair of types. They conclude that sharecropping is less likely with endogenous matching compared to random matching. Ghatak and Pandey (2000) develop a multitask model in which the tenant has joint moral hazard in the choice of effort and risk. Under moral hazard in effort alone, the fixed rent contract is optimal, while under moral hazard in risk-taking alone, the fixed wage contract becomes optimal. Sharecropping contracts can emerge only when there is moral hazard in both effort and risk. Dam and Pérez (2012) show that when the agent exerts effort in multiple tasks, share contracts are optimal if the effort cost function is not separable in efforts.

Limited liability also plays a central role in explaining tenancy contracts.[2] Because of limited liability, the landlord must ensure a minimum income level to the tenant for

each realization of output. Shetty (1988) first explained the role of limited liability in the context of agrarian relations. He considers that a difference in initial wealth implies a difference in liability of the tenants. He finds that for the more wealthy tenants, fixed-rental contracts are optimal while the less wealthy tenants receive share contracts. From Shetty's model, Ray and Singh (2001) fully characterize the complete structure of tenancy contracts. Basu (1992) and Sengupta (1997) show that in the presence of moral hazard in the choice of techniques, landlords may prefer share tenancy to ensure that tenants choose less risky projects.

From an incentives viewpoint, this paper describes a mixed model where true moral hazard follows adverse selection. Thus, it differs from a mixed model with "false" moral hazard. With false (resp. true) moral hazard, the effort affects the variable observed by the principal deterministically (resp. stochastically). In the former, the agent receives an information rent only and the usual adverse selection trade-offs arise (e.g. Laffont and Tirole (1993)). In the latter, the agent obtains a limited liability and/or an information rent and the trade-offs are not simply a mix between moral hazard and adverse selection trade-offs (e.g. Ollier and Thomas (2013), At and Thomas (2017) or Gottlieb and Moreira (2015)). We also consider that the contract offered by the principal is linear and the agent is protected by limited liability. The novelty is the presence of a positive outside option. Indeed, until now, these models have only considered a null outside option. It follows that the limited liability rent is still greater than the reservation utility. In this case, it is always optimal for the principal to lower the limited liability rent instead of the information rent (if any). Thus a fully pooling contract emerges as optimal.

The Model

A landlord employs a tenant to crop his land. The landlord and the tenant are both risk neutral.[3] The tenant is protected by limited liability and has an outside option $m > 0$. The landlord has a monopoly power over the tenant. The output, y, is random and equal to

$$\begin{aligned} &y_H \text{ with probability } e, \\ &y_L \text{ with probability } 1-e. \end{aligned}$$

with $y_H > y_L > 0$. In the following we denote $\Delta y = y_H - y_L$. The output is said to be "high" ("low") if $y = y_H$ (if $y = y_L$). This is common knowledge and verifiable. The variable $e \in [0,1]$ is the effort exerted by the tenant.

Effort e costs the tenant $c(e,\theta)$. The parameter $\theta \in \Theta = \left[\underline{\theta}, \overline{\theta}\right]$ reflects the ability of the tenant. The higher the tenant's ability, the higher the value of θ. We assume that the tenant's ability affects the cost of effort he incurs in the following manner: ability

reduces both the cost of effort and the marginal cost of effort. Moreover, the cost is convex in θ. By contrast, effort is costly, even at the margin. So the cost is increasing and convex in e. For simplicity, we assume the following property.

Assumption 1. *The cost function is quadratic:* $c(e,\theta) = \frac{e^2}{2\theta}$.

Effort and ability are both private information of the tenant.[45] However, the landlord knows that θ is drawn from a probability distribution represented by a cumulative distribution function $F(\theta)$ and a density function $f(\theta) > 0$ on Θ. We assume that $f(\theta)$ satisfies the following adjusted monotone hazard rate property.

Assumption 2. *Let* $\theta_0 \in (\underline{\theta}, \overline{\theta}]$, *so* $\forall \theta < \theta_0$

$$\frac{F(\theta_0) - F(\theta) + \frac{\theta_0 f(\theta_0)}{\theta_0^2 f(\theta_0) + \int_{\theta_0}^{\overline{\theta}} \theta f(\theta) d\theta} \int_{\theta_0}^{\overline{\theta}} \theta f(\theta) d\theta}{f(\theta)} \tag{1}$$

is decreasing in θ.

This regularity assumption avoids pooling at the optimal contract due to undesirable properties of the probability distribution. Notice that if $\theta_0 = \overline{\theta}$, (1) is simply $\frac{1-F(\theta)}{f(\theta)}$.

As they are commonly observed in agriculture, we consider linear contracts. More specifically, a general contract is defined by two parameters $\langle s, r \rangle$ which are, respectively the share of the product kept by the tenant and the certain payment made by the tenant to the landlord. Note that a pure rental contract is associated with $s = 1$ and $r > 0$, a pure wage contract is associated with $s = 0$ and $r < 0$, and a sharecropping contract is associated with $s \in (0, 1)$ and $r \geq 0$.

For a given contract and a pair (e, θ), expected utilities are

$$L = (1 - s)(e\Delta y + y_L) + r, \tag{2}$$

for the landlord, and

$$T = s(e\Delta y + y_L) - r - \frac{e^2}{2\theta}, \tag{3}$$

for the tenant.

The timing is as follows (see figure 1). In stage 1, the tenant learns his ability θ. In stage 2, the landlord offers a linear contract $\langle s, r \rangle$. In stage 3, the tenant exerts an effort. In stage 4, verifiable and observable output is generated; share and payment take place.

INCLUDE FIG. 1

The landlord's program

Since Myerson (1982), we know that there is no loss of generality in restricting our attention to direct revelation mechanisms. So the landlord offers a contract $\langle s(\hat{\theta}), r(\hat{\theta})\rangle$ specifying the sharing rule and the payment according to the tenant's report $\hat{\theta}$ on his ability. She is then confronted with the incentive, participation, and limited liability constraints.

INCENTIVE CONSTRAINTS When both adverse selection and moral hazard are present, the landlord observes neither the tenant's ability nor his effort. She may induce revelation then obedience. Appealing to backward induction, let us analyze the moral hazard incentive constraints then the adverse selection incentive constraints.

Effort. If the landlord wants to induce the level of effort e, the moral-hazard incentive constraints are, given (3), $\forall e, \tilde{e} \in [0,1]$

$$e = \arg\max_{\tilde{e}\in[0,1]} s(\hat{\theta})(\tilde{e}\Delta y + y_L) - r(\hat{\theta}) - \frac{\tilde{e}^2}{2\theta}.$$

That is, the tenant is better off exerting e than any other effort $\tilde{e}$. Simple algebra gives the moral hazard effort exerted by the tenant

$$e(s(\hat{\theta}), \theta) = s(\hat{\theta})\Delta y\theta. \tag{4}$$

So the tenant's effort increases (linearly) with the sharing rule. Indeed, the higher s, the higher the power of incentives of the linear contract. When ability increases, the marginal cost of effort decreases. This implies that the tenant is induced to exert higher effort. So the effort is increasing in θ.

Revelation. Let $t(\hat{\theta}, \theta)$ be the tenant's expected utility when he reports $\hat{\theta}$ and his type is θ. Given the tenant's expected utility (3) and effort (4), we have

$$t(\hat{\theta}, \theta) = s(\hat{\theta})(e(s(\hat{\theta}), \theta)\Delta y + y_L) - r(\hat{\theta}) - \frac{e^2(s(\hat{\theta}), \theta)}{2\theta},$$

which gives after simplification

$$t(\hat{\theta}, \theta) = s(\hat{\theta})y_L - r(\hat{\theta}) + \frac{s(\hat{\theta})^2\Delta y^2\theta}{2}. \tag{5}$$

Let us denote by $T(\theta)$ the rent obtained by the tenant when his type is θ. So the adverse selection incentive constraints are $\forall \theta, \hat{\theta} \in \Theta$

$$\text{(IC)} \qquad T(\theta) = t(\theta, \theta) \geq t(\hat{\theta}, \theta). \tag{IC}$$

That is, the tenant obtains a higher expected utility when he reports the true θ than any other type $\hat{\theta}$. We obtain the following lemma.

Lemma 1. *The contract is incentive compatible if and only if,* $\forall \theta \in \Theta$

$$\text{(IC}') \qquad T'(\theta) = \frac{s(\theta)^2 \Delta y^2}{2}, \quad s'(\theta) \geq 0. \tag{IC$'$}$$

To ensure truthful revelation by the tenant, the rent must increase at the rate $\frac{s(\theta)^2 \Delta y^2}{2}$. This specifies how the rent must change with θ to compensate for the tenant's incentives to underestimate his ability. Moreover, the sharing rule must be non-decreasing. The greater the tenant's ability, the greater the share he should receive.[6]

Participation constraints The participation constraints ensure that the tenant cannot be forced to sign the contract. So his expected utility must cover at least his outside option, m. The participation constraints are written as $\forall \theta \in \Theta$

$$\text{(PC)} \qquad T(\theta) \geq m. \tag{PC}$$

But since $T(\theta)$ is strictly increasing from (IC$'$), this reduces to

$$\text{(PC}') \qquad T(\underline{\theta}) \geq m. \tag{PC$'$}$$

The participation of all types is guaranteed provided that the participation of the lowest ability is.

Limited liability constraints The limited liability constraints are such that the tenant will not be able to make the landlord a payment $r(\theta)$ in excess of what the tenant himself receives. Since $y_H > y_L$, his minimal revenue is $s(\theta)y_L$. We have $\forall \theta \in \Theta$

$$\text{(LLC)} \qquad s(\theta)y_L - r(\theta) \geq 0. \tag{LLC}$$

Using (3), the limited liability constraints become, $\forall \theta \in \Theta$

$$\text{(LLC}') \qquad T(\theta) \geq \frac{s(\theta)^2 \Delta y^2 \theta}{2} = \ell(\theta). \tag{LLC$'$}$$

Thus $\ell(\theta)$ is the minimal rent that the tenant must get when the tenant's ability is θ. This is the so-called limited liability rent obtained by the tenant.

THE PROGRAM Combining (2), (3), and (4), the landlord's objective function can be rewritten as

$$\int_{\underline{\theta}}^{\overline{\theta}} \left\{ s(\theta)\Delta y^2\theta + y_L - \frac{s(\theta)^2\Delta y^2\theta}{2} - T(\theta) \right\} f(\theta)d\theta. \tag{6}$$

The landlord's problem is to $\max_{\langle s(\theta),T(\theta)\rangle}(6)$ subject to (IC′), (PC′), and (LLC′).

The Optimal Tenurial Contracts

The Social Optimum

To improve the analysis it is useful to study certain properties of the social optimum. The social surplus is the expected output less the cost of effort incurred by the tenant

$$W = L + T = e\Delta y + y_L - \frac{e^2}{2\theta}.$$

The first best effort $e^{FB}(\theta)$ maximizes the expected surplus for $\theta \in \Theta$, we find

$$e^{FB}(\theta) = \Delta y\theta. \tag{7}$$

The first best effort is such that the social expected marginal benefit, Δy, is equal to the social marginal cost, $\frac{e}{\theta}$. Note that ability provides incentives to provide greater effort. Actually, the first best effort is increasing in ability since a higher ability reduces the social marginal cost without affecting the expected social marginal benefit. From (4), note that if the sharing rule s is equal to 1, the landlord can induce the first best effort. Thus we can define $s(\theta) = 1$ as the efficient sharing rule.

Moral Hazard Alone

In this informational context, the landlord observes the ability of the tenant but cannot observe his effort. Hence, under moral hazard alone the adverse selection incentive constraints are now irrelevant. This implies that the participation constraints can no longer be reduced to (PC′). Thus the relevant participation constraints are (PC). The landlord's problem is reduced to choosing the contract $\langle s(\theta), T(\theta)\rangle$ that maximizes the integrand of (6) subject to (PC) and (LLC′). We obtain the following proposition.

Proposition 1. *Under moral hazard and limited liability, the optimal share is*

$$s_{MH}(\theta) = \begin{cases} \frac{1}{2}, & \text{if } m \leq \frac{\Delta y^2\theta}{8}, \\ \frac{\sqrt{2m}}{\Delta y\sqrt{\theta}}, & \text{if } \frac{\Delta y^2\theta}{8} < m < \frac{\Delta y^2\theta}{2}, \\ 1, & \text{if } m \geq \frac{\Delta y^2\theta}{2}. \end{cases}$$

The moral hazard effort is increasing in ability and non-decreasing in outside option.

Three cases arise depending on which constraints, participation (PC) and/or limited liability (LLC′), are binding.

In the first case, the optimal contract is a sharecropping contract. Only (LLC′) matters because the outside opportunity m is low. Examining the integrand of (6), we notice that the landlord has an interest in decreasing the rent T. Ideally, it would be decreased until (PC) becomes binding. But when m is sufficiently low ($m \leq \frac{\Delta y^2 \theta}{8}$), this violates (LLC′) because the rent T would be set at a level lower than the limited liability rent ℓ. Since this is impossible, this implies a rent for the tenant because he gets more than his reservation payoff. This is the so-called limited liability rent. So, the landlord's choice of s is the result of a trade-off between efficiency and limited liability rent extraction. It follows that it is optimal to distort s from the first-best, i.e. $s < 1$, in order to reduce the tenant's rent. Moreover, the sharing rule is independent of ability because the primitives of the model (i.e. the probability of success and the cost function) satisfy the conditions identified by Ollier and Thomas (2013, Corollary 1, (ii)).

In the third case, the optimal contract is a pure rental contract. In this case, the reservation payoff is sufficiently high, $m \geq \frac{\Delta y^2 \theta}{2}$, to ensure that when (PC) is binding, (LLC′) is automatically satisfied. The tenant obtains no rent and the landlord can implement the efficient sharing rule $s = 1$ because the effort exerted by the tenant (4) corresponds to the optimal social effort (7). Once again, the sharing rule is ability independent.

The second case is an intermediary situation where both participation and limited liability constraints are binding. This results in the optimality of the sharecropping contract. But now, the sharing rule depends on ability.

We also find that when the ability (respectively outside option) of the tenant, θ (respectively m), increases, the contract offered by the landlord implies that the effort does not decrease. Banerjee et al. (2002) obtain similar results which they call the bargaining power effect, "an increase in the tenant's bargaining power, with everything else held constant, leads to an increase in his share and his productivity".

Adverse Selection Alone

In this informational context, the landlord observes neither the effort nor the ability of the tenant. However, the limited liability constraints are now irrelevant. This implies that the participation constraints can be reduced to (PC′). The landlord's problem is reduced to choosing the contract $\langle s(\theta), T(\theta) \rangle$ that maximizes the integrand of (6) subject to (PC′) and (IC′). We obtain the following proposition.

Proposition 2. *Under adverse selection, the optimal share is*

$$s_{AS}(\theta) = \frac{1}{1 + \frac{1-F(\theta)}{\theta f(\theta)}}.$$

The adverse selection effort is increasing in ability and constant in outside option.

The optimal contract is a pure rental contract for the highest ability, $s_{AS}(\bar{\theta}) = 1$ (since $1 - F(\bar{\theta}) = 0$), and a sharecropping contract, $s_{AS}(\theta) < 1$, for lower abilities $\theta < \bar{\theta}$. Moreover, under Assumption 2 (here $\theta_0 = \bar{\theta}$), the share is increasing in ability. These properties correspond to those of the standard adverse selection contract, implying a trade-off between efficiency and information rent extraction. So the tenant gets an information rent, except $\underline{\theta}$, to avoid his incentives to mimic lower abilities. To limit this costly rent, it is optimal for the landlord to distort the efficient sharing rule downward, i.e. $s(\theta) = 1, \forall \theta$. This cost is reflected by the term $1 - F(\theta)$ in $s_{AS}(\theta)$. This represents the proportion of ability that can mimic θ if a contract is offered for this ability. Obviously, this term is null for the highest ability because it cannot be mimicked. It is useless to distort the efficient sharing rule obtained under moral hazard alone for this ability.

We also find that the effort is increased by ability (as in moral hazard alone) but is not affected by the outside option (contrarily to the intermediate case in moral hazard alone).

Moral Hazard and Adverse Selection

The optimal contract crucially depends on the set of types for which the limited liability constraint (LLC′) is binding. So it is useful to state a number of properties of this constraint in the following lemma.

Lemma 2. *Let θ^* be the lowest type for which the limited liability constraint (LLC′) is binding. Let $\tilde{s}(\theta)$ be an incentive compatible path for the sharing rule. The limited liability constraint is such that*

i- $\tilde{\ell}'(\theta) \geq \tilde{T}'(\theta)$;

ii- *A-* *If* $\tilde{\ell}(\underline{\theta}) \geq \tilde{T}(\underline{\theta})$ *then* $\theta^* = \underline{\theta}$,
B- *If* $\tilde{\ell}(\bar{\theta}) < \tilde{T}(\bar{\theta})$ *then* $\theta^* \notin \Theta$,
C- *If* $\tilde{\ell}(\underline{\theta}) < \tilde{T}(\underline{\theta})$ *and* $\tilde{\ell}(\bar{\theta}) > \tilde{T}(\bar{\theta})$ *then* $\theta^* \in \Theta$;

iii- $\tilde{s}'(\theta) = 0$ *when (LLC′) is binding.*

Point i- states that, for any incentive-compatible sharing rule, the rent $\tilde{T}$ increases more slowly than the limited liability rent $\tilde{\ell}$. So the latter can cross the former at most once and from below. But in order to get an incentive-feasible sharing rule, $\tilde{\ell}$ cannot be strictly higher than $\tilde{T}$. Hence the three cases exhibited in point ii-. In case A, the limited liability constraint must be binding for all abilities θ if the path $\tilde{\ell}$ starts at a higher level than $\tilde{T}$. In case B, this constraint is never binding if the path $\tilde{\ell}$ ends at a lower level than $\tilde{T}$. In the last intermediate case C, it is binding for the right of the

distribution only. Point iii- states that the sharing rule is constant, so pooling inevitably occurs, when the limited liability constraint is binding, i.e. for $\theta \geq \theta^*$.

As suggested by Proposition 1, this lemma provides the intuition that different optimal contracts exist. Their existence depends on the values of the outside option m. Among these contracts, we identify two polar cases corresponding to extreme values of m. They will be referred to as the moral hazard contract and adverse selection contract. They are stated in the following proposition.

Proposition 3. *Under moral hazard, adverse selection and limited liability, the optimal share is such that,* $\forall \theta \in \Theta$

i- if $m \leq m_L^*$

$$s_L^*(\theta) = \frac{1}{2} \tag{8}$$

ii- if $m > m_H^*$

$$s_H^*(\theta) = \frac{1}{1 + \frac{1-F(\theta)}{\theta f(\theta)}} \tag{9}$$

where $m_L^* = \frac{\Delta y^2 \underline{\theta}}{8}$ *and* $m_H^* = \frac{\Delta y^2 \overline{\theta}}{2} - \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\theta f(\theta)}{\theta f(\theta) + 1 - F(\theta)}\right)^2 \frac{\Delta y^2}{2} d\theta$.

This proposition requires some comments. When m is sufficiently low, $m \leq m_L^*$, the optimal contract is a sharecropping contract with a constant share regardless of the tenant's ability. This is case i-. In such a case, the limited liability constraint is binding whatever the tenant's ability. The landlord faces the same trade-off between efficiency and limited liability rent extraction as under moral hazard alone. According to point iii- in Lemma 2, the contract must be fully pooling. But the moral hazard contract is already ability-independent. Thus the optimal contract with the addition of adverse selection is exactly the same as for moral hazard, i.e. we get the optimal share $s_L^*(\theta) = \frac{1}{2} = s_{MH}(\theta)$. We can conclude that low values of m lead to the moral hazard contract.

Case ii- arises if m is sufficiently high, $m > m_H^*$. Then the optimal contract is a pure rental contract for the highest ability, $s_H^*(\bar{\theta}) = 1$, and a sharecropping contract, $s_H^*(\theta) < 1$, for lower abilities $\theta < \bar{\theta}$. Moreover, the share is increasing in θ under Assumption 2 (here $\theta_0 = \overline{\theta}$). These properties correspond to those of the standard adverse selection contract, previously identified, implying a trade-off between efficiency and information rent extraction. Indeed, the tenant gets an information rent, except $\underline{\theta}$, since the limited liability rent is never binding because of the high level of the reservation payoff.

Finally, it follows that ability and outside option have equivalent effects on effort with respect to the preceding informational settings.

Extreme values of m lead to optimal contracts that are well known in incentives theory (e.g. Ollier and Thomas (2013) for i-, and Baron and Myerson (1982) for ii-). The optimal contracts for intermediate values of m are the real novelty from an incentives theory viewpoint. They are presented in the following proposition and in figure 2.

Proposition 4. *Under moral hazard, adverse selection, and limited liability, the optimal share is such that*

i- if $m_L^* < m \leq m_I^*$

$$s^*(\theta) = \frac{\underline{\theta} f(\underline{\theta}) + \frac{1}{\underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta}{\underline{\theta} f(\underline{\theta}) + \frac{2}{\underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta}, \qquad \forall \theta \in \Theta, \tag{10}$$

ii- if $m_I^* < m \leq m_H^*$,

$$s^*(\theta) = \begin{cases} \dfrac{1}{1 + \dfrac{F(\theta^*) - F(\theta) + \dfrac{1}{\theta^* f(\theta^*)} \int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta}{\dfrac{\theta^{*2} f(\theta^*) + \int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta}{\theta f(\theta)}}} & \text{if } \theta \leq \theta^*, \\ \dfrac{\theta^* f(\theta^*) + \frac{1}{\theta^*} \int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta}{\theta^* f(\theta^*) + \frac{2}{\theta^*} \int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta} > \frac{1}{2} & \text{if } \theta \geq \theta^*, \end{cases} \tag{11}$$

with

$$\int_{\underline{\theta}}^{\theta^*} \frac{s^*(\theta)^2 \Delta y^2}{2} d\theta + m = \frac{s^*(\theta^*)^2 \Delta y^2 \theta^*}{2}, \tag{12}$$

and

$$m_I^* = \left(\frac{\underline{\theta} f(\underline{\theta}) + \frac{1}{\underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta}{\underline{\theta} f(\underline{\theta}) + \frac{2}{\underline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta} \right)^2 \frac{\Delta y^2 \underline{\theta}}{2}. \tag{13}$$

The effort is increasing in ability.

When m has intermediate values, there are two optimal contracts depending on the level of m. Let us first examine case ii- where m has "high" intermediate values. Under Assumption 1, the optimal contract is a sharecropping contract with an ability increasing share when $\theta \leq \theta^*$ and a constant share otherwise. It is separating for abilities lower than the threshold θ^* because the limited liability constraint is slack and

pooling because otherwise this constraint is binding. According to Proposition 3, for low (resp. high) abilities, the tenant receives an information (resp. limited liability) rent. The landlord must find the optimal trade-off between efficiency and these rents. To limit these costly rents, it is again optimal to induce distortions. The term $F(\theta^*) - F(\theta) + \frac{\theta^* f(\theta^*)}{\theta^{*2} f(\theta^*) + \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta$ reflects these costs. More precisely, the first two components are related to information rent and the third to limited liability rent. Indeed, since the contract is pooling for $\theta > \theta^*$, the incentives to mimic a low ability only play a role from θ^*. So the proportion of ability that is actually more efficient than θ is henceforth $F(\theta^*) - F(\theta)$. Moreover, for ability higher than θ^*, it is shown that[7]

$$\frac{\theta^* f(\theta^*)}{\theta^* f(\theta^*) + \frac{1}{\theta^*} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta} = \frac{2s^*(\theta^*) - 1}{s^*(\theta^*)}.$$

But the optimal sharing rule is higher than $\frac{1}{2}$, its minimal constant level when the tenant gets a limited liability rent for all levels of his ability (see case i- in Proposition 3). It follows that $2s - 1$ is positive and $\frac{1}{\theta^*} \frac{2s^*(\theta^*)-1}{s^*(\theta^*)} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta$ measures the cost of such a rent over the interval $(\theta^*, \bar{\theta}]$.

Equation (12) allows us to define θ^* as an implicit function of m. Let us denote it $\theta^*(m)$. It can be shown that θ^* is increasing in m.[8] So when the outside option decreases, the interval over which the limited liability is binding increases. It follows that when m has "low" intermediate values, θ^* is equal to the lowest ability $\underline{\theta}$. This corresponds to case i-. Thus, the tenant obtains only a limited liability rent. The optimal contract is a sharecropping contract with a constant share higher than a half for any abilities.

However, the trade-offs have more subtle consequences because the optimal contracts in Proposition 4 are not simply a mix of cases i- and ii- in Proposition 3. This is stated more precisely in the following propositions.

Proposition 5. *The optimal sharing rules are such that*

$$\begin{aligned} i\text{-} \quad & s_H^*(\theta) < s^*(\theta), \quad \forall \theta \leq \theta^*, \\ ii\text{-} \quad & s_L^*(\theta) < s^*(\theta^*)|_{\theta^* = \underline{\theta}}. \end{aligned}$$

Part i- of this proposition states that when the contract for intermediate values of m is separating, the optimal share is higher than the adverse selection one. How can this be explained? The crucial point is that, according to (IC$'$), the tenant's rent $T(\theta)$ increases at a lower rate when there is pooling, i.e. $s'(\theta) = 0$. It is well known that pooling inevitably arises when the limited liability constraint is binding. Thus the landlord leaves even less rent to the right of the distribution when the junction between the rent $T(\theta)$ and the limited liability rent $\ell(\theta)$ arises early in the set Θ, especially before it would do with the sharing rule path $s_H^*(\theta)$. To do that, it is optimal to increase the sharing rule since the rate of the rent $T'(\theta)$ increases with it. However, increasing the

sharing rule for the left of the distribution is also costly in terms of information rent left to lower abilities. So as long as m is not too low, it is optimal not to decrease θ^* to $\underline{\theta}$.

However, when m has intermediate but low values, the first effect dominates the second and $\theta^* = \underline{\theta}$ becomes optimal. So it is optimal to leave a limited liability rent for all abilities. But, from (LLC′), we know that this rent increases with the share. So according to part ii- of the proposition, the landlord leaves a higher rent than in the moral hazard contract. In conclusion, in order to avoid too high a rent to the right of the distribution, the landlord prefers to leave a higher limited liability rent than in the moral hazard case.

Since the optimal share is either constant or increasing in ability, we deduce that the optimal level of effort is increasing, as until now. But what about m? Under moral hazard alone, we find the standard result, i.e. the optimal level of effort is increasing in the outside option. Under adverse selection alone, there is no effect. Combining adverse selection and moral hazard leads to less straightforward results as shown in the following proposition.

Proposition 6. *For a given level of tenant ability, $\theta_0 \in [\underline{\theta}, \overline{\theta})$, the optimal effort*

i- is increasing if $m \leq m_0$,

ii- is strictly decreasing if $m_0 < m \leq m_H^*$,

iii- is constant if $m > m_H^*$,

with m_0 is such that $\theta^*(m_0) = \theta_0$.

For $\theta_0 = \overline{\theta}$, the effort is non decreasing in m.

The interesting result is that an increase in the outside option of the tenant does not monotonically increase his optimal effort. Under moral hazard only, Banerjee et al. (2002) also find this negative effect which is the consequence of the possibility of the tenant being evicted. In our paper, this negative effect is a consequence of adding adverse selection to moral hazard. Higher θ^* increases the range of types for which (LLC′) is slack and it is optimal to increase downward distorsions to reduce the information rent. So, for a given θ, the optimal sharing rule and consequently the optimal level of effort decrease. In other words, when the outside option increases, the moral hazard problem becomes less of an issue, which makes it worth separating types and so distorting the sharing rule downward. Hence, a better legal protection of tenants and/or an increase in the level of the country's development may have ambiguous consequences for the productivity because of asymmetric information.

An Example

In this example, we consider that θ is uniformly distributed on $[1,2]$ and $\Delta y = 1/2$. The optimal share is

- $s_L^* = \frac{1}{2}$, for $m \leq 0.031$
- $s^*(\theta) = \begin{cases} \frac{5}{8}, & \text{for } 0.031 < m \leq 0.049 \\ \left(\frac{\theta^*}{8} + \frac{1}{2\theta^*}\right)\theta, & \text{for } 0.049 < m \leq 0.177 \text{ and } \theta \leq \theta^* \\ \frac{1}{2} + \frac{\theta^{*2}}{8}, & \text{for } 0.049 < m \leq 0.177 \text{ and } \theta \geq \theta^*, \end{cases}$
- $s_H^* = \frac{\theta}{2}$, for $m > 0.177$.

In figure 2, we draw the path of the optimal share for two values of m, namely $m_0 = 0.067$ and $m_1 = 0.088$, leading to the following ability thresholds $\theta_0^* = 1.3$ and $\theta_1^* = 1.5$. We draw the path of the optimal shares under both adverse selection and moral hazard. Recall that for lower values of m we find pooling contracts and for higher values of m we find the same separating contract as under adverse selection alone (dotted). We see that in economies where both adverse selection and moral hazard co-exist, less heterogeneity in contracts should be observed, and this is especially true for less developed economies and/or in economies that provide little protection for tenants. For abilities between θ_0^* and θ_1^*, the contracts are separating for tenants with high outside option ($m = 0.088$) and are pooling otherwise ($m = 0.067$).

INCLUDE FIG. 2

Now, setting $\theta_0 = 1.5$, we find $m_0 = 0.088$ as the solution to $\theta^*(m_0) = 1.5$, from (12). The optimal effort as a function of m under both moral hazard and adverse selection, is

- $e_L(m) = \frac{3}{8}$ for $m < 0.031$,
- $e^*(m) = \begin{cases} \frac{15}{32} & \text{for } m \in (0.031, 0.049], \\ \frac{3}{8} + \frac{3\theta^*(m)^2}{32} & \text{for } m \in [0.049, m_0], \\ \left(\frac{\theta^*(m)}{8} + \frac{1}{2\theta^*(m)}\right)\frac{9}{8} & \text{for } m \in [m_0, 0.177], \end{cases}$
- $e_H(m) = \frac{9}{16}$ for $m \geq 0.177$.

In figure 3, we represent the evolution of the optimal efforts under both moral hazard and adverse selection, for three different values of ability, $\theta = 1.3$, $\theta = 1.5$, and $\theta = 1.8$.

INCLUDE FIG. 3

The effect of a variation in the outside option on the optimal effort is non monotonic whatever the level of the tenant's ability but the scale differs. A policy affecting the outside option should take into account the levels of tenants' abilities because the consequences could be dramatically different. Consider for example that the outside option increases from $m_0^{1.5}$ to $m_0^{1.8}$ then only the higher skilled tenant increases his effort, the lowest ones decrease theirs.

Type-dependent outside option[9]

We consider the extension where high-skilled tenants also have access to better outside labor opportunities. More precisely, we assume that the tenant's outside option is linearly type-dependent such that $m(\theta) = \mu_0 + \mu_1\theta$, with $\mu_0 > 0$ and $\mu_1 > 0$. The participation constraints now become $\forall\theta \in \Theta$

$$(\text{PC}'') \qquad T(\theta) \geq m(\theta).$$

The main implication of (PC$''$) is the following. The information rent is still equal to the difference between the rent $T(\theta)$ and the outside option $m(\theta)$. But now, this implies that the rate of the information rent, $T'(\theta) - m'(\theta) = \frac{s(\theta)^2\Delta y^2}{2} - \mu_1$, is no longer necessarily positive as with a constant outside option. It is interesting to analyze how this affects the preceding moral hazard/adverse selection trade-offs and thus the optimal contract.

From Maggi and Rodriguez-Clare (1995), we already know that the relative values of μ_1 are crucial for the analysis. In the following we consider low, intermediate, and high values.

Proposition 7. *With a linear type-dependent outside option, the constant-outside-option optimal contract is modified if the outside option increases with ability:*

i- at a high rate;

ii- or at an intermediate rate but with a sufficiently high intercept.

If one the two conditions mentioned in the proposition is not met, the optimal contracts offered in the above analysis are essentially the same. Thus, these conditions are such that the type-dependent outside option influences the properties of the optimal contract. The main consequence is the occurence of pooling at the bottom and not only at the top. Therefore, the type-dependent outside option contributes to less heterogeneity in the menu of contracts.

Conclusion

We have developed a model of a tenurial contract when the revenue generated by the tenant is affected by both adverse selection and moral hazard. We find that the level of

the outside option (or the level of tenants' protection) plays a crucial role in the optimal contract and consequently in the tenant's choice of optimal effort.

This article is a first step in studying the optimal tenurial contract under both moral hazard and adverse selection and the repercussions of changes in tenants' legal protection. It remains to be seen how our results are affected if we embed the problem studied here in a dynamic framework, especially by assuming that the tenant may invest in land quality maintenance.

Appendix

Proof of Lemma 1

Incentive constraints require $\forall \theta \in \Theta$

$$t_1(\theta, \theta) = 0 \tag{14}$$

$$t_{11}(\theta, \theta) \leq 0. \tag{15}$$

So (14), using (5), leads to

$$s'(\theta)(y_L + s(\theta)\Delta y^2 \theta) - r'(\theta) = 0.$$

Totally differentiating (14), (15) is equivalent to $t_{12}(\theta, \theta) \geq 0$, that is

$$s'(\theta)s(\theta)\Delta y^2 \geq 0 \Leftrightarrow s'(\theta) \geq 0. \tag{16}$$

Moreover, since $T(\theta) = t(\theta, \theta)$ we get

$$T'(\theta) = t_1(\theta, \theta) + t_2(\theta, \theta) = t_2(\theta, \theta) = \frac{s(\theta)^2 \Delta y^2}{2}. \tag{17}$$

where the second equality comes from (14) and the third from (5). Equations (16) and (17) together lead to the lemma.

Proof of Proposition 1

The landlord's program is

$$\max_{\langle s(\theta), T(\theta) \rangle} s(\theta)\Delta y^2 \theta + y_L - \frac{s(\theta)^2 \Delta y^2 \theta}{2} - T(\theta)$$

s.t.

$$T(\theta) \geq m \quad (\lambda)$$

$$T(\theta) \geq \frac{s(\theta)^2 \Delta y^2 \theta}{2} \qquad (\xi).$$

The Lagrangian is

$$\mathcal{L} = s(\theta)\Delta y^2\theta + y_L - \frac{s(\theta)^2\Delta y^2\theta}{2} - T(\theta) + \lambda\left(T(\theta) - m\right) + \xi\left(T(\theta) - \frac{s(\theta)^2\Delta y^2\theta}{2}\right).$$

Necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial T(\theta)} = -1 + \lambda + \xi = 0 \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial s(\theta)} = \Delta y^2\theta\left(1 - s(\theta)(1+\xi)\right) = 0 \tag{19}$$

$$\lambda \geq 0, \lambda\left(T(\theta) - m\right) = 0 \tag{20}$$

$$\xi \geq 0, \xi\left(T(\theta) - \frac{s(\theta)^2\Delta y^2\theta}{2}\right) = 0. \tag{21}$$

From (18), we cannot simultaneously have $\xi = 0$ and $\lambda = 0$, i.e. the participation and the limited liability constraints cannot both be slack. So let us study the three remaining cases.

Case 1: Let $\xi = 0$. From (18), we have $\lambda = 1$. Plugging in (19) and (20), we get $s(\theta) = 1$ and $T(\theta) = m$. From (21), we deduce the existence condition, i.e. $T(\theta) - \frac{s(\theta)^2\Delta y^2\theta}{2} \geq 0$ or $m \geq \frac{\Delta y^2\theta}{2}$.

Case 2: Let $\lambda = 0$. From (18), we have $\xi = 1$. Plugging in (19), we get $s(\theta) = 1/2$. From (21), we get $T(\theta) = \frac{\Delta y^2\theta}{8}$. From (20), the existence condition is $T(\theta) - m \geq 0$ or $m \leq \frac{\Delta y^2\theta}{8}$.

Case 3: Let $\xi > 0$ and $\lambda > 0$. So the two constraints are binding. Equations (20) and (21) lead to the solution $s(\theta) = \frac{\sqrt{2m}}{\Delta y\sqrt{\theta}}$ and $T(\theta) = m$.

To prove that the moral effort is increasing in θ, it suffices to combine (4) and $s_{MH}(\theta)$.

To prove that the moral effort is non-decreasing in m, recall that the effort increases with the share by (4). Moreover, it is straightforward to show that s: 1) does not depend on m if $m \leq \frac{\Delta y^2\theta}{8}$ and $m \geq \frac{\Delta y^2\theta}{2}$, 2) increases with m otherwise.

Proof of Proposition 2

The proof is standard and is omitted. However, the reader can also follow the proof of Proposition 3 below while ignoring the limited liability constraint (so its associated Kuhn and Tucker multiplier) and focusing on $T(\underline{\theta}) = m$.

Using (4) and $s_{AS}(\theta)$, it is straightforward to show the effect of θ and m on the effort.

Proofs of Lemma 2

Consider the constraint (LLC′). From Lemma 1, we know that $T(\theta)$ increases at rate $\frac{s(\theta)^2\Delta y^2}{2}$. After differentiation, $\ell(\theta)$ increases at rate $s'(\theta)s(\theta)\Delta y^2\theta + \frac{s(\theta)^2\Delta y^2}{2}$.

From Lemma 1, $\tilde{s}'(\theta) \geq 0$ since this sharing rule is incentive compatible. So we necessarily have

- if $\tilde{s}'(\theta) > 0$, $\tilde{\ell}(\theta)$ increases faster than $\tilde{T}(\theta)$
- if $\tilde{s}'(\theta) = 0$, $\tilde{\ell}(\theta)$ and $\tilde{T}(\theta)$ increase at the same rate. This is i-.

So $\tilde{\ell}(\theta)$ can only cross $\tilde{T}(\theta)$ from below at most once at $\theta = \theta^*$, i.e. $\tilde{\ell}(\theta^*) = \tilde{T}(\theta^*)$. This implies that

- when $\tilde{\ell}(\underline{\theta}) = \tilde{T}(\underline{\theta})$, we have $\tilde{\ell}(\theta) \geq \tilde{T}(\theta), \forall\theta \geq \underline{\theta}$. To satisfy (LLC′), we must have $\theta^* = \underline{\theta}$,
- when $\tilde{\ell}(\overline{\theta}) < \tilde{T}(\overline{\theta})$, we have $\tilde{\ell}(\theta) < \tilde{T}(\theta), \forall\theta \in \Theta$. So (LLC′) is never violated and we have $\theta^* \notin \Theta$,
- when $\tilde{\ell}(\underline{\theta}) < \tilde{T}(\underline{\theta})$ and $\tilde{\ell}(\overline{\theta}) > \tilde{T}(\overline{\theta})$. So (LLC′) is violated for the right of the distribution only. Hence $\theta^* \in (\underline{\theta}, \overline{\theta})$. This is ii-.

Moreover, when $T(\theta) = \ell(\theta)$ on an interval, $T(\theta)$ and $\ell(\theta)$ must increase at the same rate. This implies $s'(\theta) = 0$. This is iii-.

Proof of Propositions 3 and 4

Following Seierstad and Sydsaeter (1987), the landlord problem is an optimal control problem with a mixed constraint (LLC′) in which $T(\theta)$ and $s(\theta)$ are the state variables and $x(\theta)$ such that $s'(\theta) = x(\theta) \geq 0$ is the control variable. Moreover, (PC′) is an initial condition.

We associate the adjoint variable $\lambda(\theta)$ with $T(\theta)$ and $\xi(\theta)$ with $s(\theta)$. Let $\eta(\theta)$ be the Kuhn and Tucker multiplier associated with (LLC′).

The Hamiltonian is

$$\mathcal{H} = \left\{ s(\theta)\Delta y^2\theta + y_L - \frac{s(\theta)^2\Delta y^2\theta}{2} - T(\theta) \right\} f(\theta) + \lambda(\theta)\frac{s(\theta)^2\Delta y^2}{2} + \xi(\theta)x(\theta).$$

The Lagrangian is

$$\mathcal{L} = \mathcal{H} + \eta(\theta)\left(T(\theta) - \frac{s(\theta)^2\Delta y^2\theta}{2} \right).$$

The necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial x(\theta)} = \xi(\theta) \leq 0, \quad x(\theta)\frac{\partial \mathcal{L}}{\partial x(\theta)} = x(\theta)\xi(\theta) = 0, \tag{22}$$

$$\lambda'(\theta) = -\frac{\partial \mathcal{L}}{\partial T(\theta)} = f(\theta) - \eta(\theta), \tag{23}$$

$$\xi'(\theta) = -\frac{\partial \mathcal{L}}{\partial s(\theta)} = -\left[\left(\Delta y^2\theta - s(\theta)\Delta y^2\theta\right)f(\theta) + \lambda(\theta)s(\theta)\Delta y^2 - \eta(\theta)s(\theta)\Delta y^2\theta\right]. \tag{24}$$

The transversality conditions are

$$\lambda(\underline{\theta}) \leq 0, \lambda(\underline{\theta})(T(\underline{\theta}) - m) = 0; \quad \lambda(\overline{\theta}) = 0, \tag{25}$$

$$\xi(\underline{\theta}) = \xi(\overline{\theta}) = 0. \tag{26}$$

The complementarity slackness condition is

$$\eta(\theta)\left(T(\theta) - \frac{s(\theta)^2\Delta y^2\theta}{2}\right) = 0, \quad \eta(\theta) \geq 0. \tag{27}$$

The proof proceeds in successive steps.

Step 1. We begin by formulating a conjecture. It will be checked during the proof that it is satisfied.

Conjecture 1. *We consider that* $\xi(\theta) = 0$ *when* $\theta \in [\underline{\theta}, \theta^*]$.

In other words, $\xi(\theta)$ is null when the limited liability is slack or binding at θ^*. On the one hand, this implies that the contract is separating since, from (22), we have $x(\theta) > 0$. On the other hand, this requires that $T(\theta)$ is not constrained by (LLC′). So this conjecture is equivalent to seeking a separating sharing rule as long as $T(\theta)$ is only constrained by its rate in (17). This is a usual practice in adverse selection models, given the monotone hazard rate property stated in Assumption 1.

Step 2. We prove the following lemmas.

Lemma 3. *Consider that* $\theta^* \in \Theta$. *Thus*

$$\lambda(\theta) = \begin{cases} -(F(\theta^*) - F(\theta)) - \frac{1}{\theta^*}\frac{2s-1}{s}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta, & \text{if } \theta \leq \theta^*, \\ -\frac{1}{\theta}\frac{2s-1}{s}\int_{\theta}^{\overline{\theta}} uf(u)du + \frac{\xi(\theta)}{\theta s\Delta y^2}, & \text{if } \theta > \theta^*, \end{cases} \tag{28}$$

where s *is constant.*

Proof. By definition of θ^*, (LLC′) is binding over $[\theta^*, \overline{\theta}]$. From the proof of Lemma 2, we already know that $s(\theta)$ must be constant. Let s be this constant. Integrating (24),

we obtain

$$\xi(\overline{\theta}) - \xi(\theta) = -\int_{\theta}^{\overline{\theta}} \Delta y^2 \big(uf(u) + s\lambda(u) - suf(u) - su\eta(u)\big)du$$

or, using (26)

$$\xi(\theta) = \int_{\theta}^{\overline{\theta}} \Delta y^2 \big(uf(u) + s\lambda(u) - suf(u) - su\eta(u)\big)du. \tag{29}$$

Adding then substracting $s\theta f(\theta)$, the integrand is equal to, after simple computations

$$s\Delta y^2 \left(\theta f(\theta)\frac{1-2s}{s} + \lambda(\theta) + \theta(f(\theta) - \eta(\theta))\right).$$

Using (23), then integrating (29) we get (28) when $\theta > \theta^*$.

Over $[\underline{\theta}, \theta^*]$, $\xi(\theta) = 0$ by Conjecture 1.

So using the second part of (28) we have

$$\lambda(\theta^*) = -\frac{1}{\theta^*}\frac{2s-1}{s}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta. \tag{30}$$

Moreover, let $E(\theta) = \int_{\theta}^{\overline{\theta}} \eta(u)du$. So using (23) and (25), we get

$$\lambda(\theta) = -(1 - F(\theta)) + E(\theta). \tag{31}$$

From (27), $\eta(\theta) = 0$ when $\theta < \theta^*$ and $\eta(\theta) > 0$ when $\theta \geq \theta^*$. So using (31), we have

$$\lambda(\theta) = -(1 - F(\theta)) + E(\theta^*). \tag{32}$$

Combining (30) and (32) at θ^*, we get

$$E(\theta^*) = (1 - F(\theta^*) - \frac{1}{\theta^*}\frac{2s-1}{s}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta. \tag{33}$$

Plugging $E(\theta^*)$ into (32) gives the first part of (28). ■

Lemma 4. *Let $\theta \in [\underline{\theta}, \theta^*]$, we have*

$$s(\theta) = \frac{1}{1 - \frac{\lambda(\theta)}{\theta f(\theta)}}. \tag{34}$$

Proof. Over this interval, we have $\xi(\theta) = 0$ by Conjecture 1. Then $\xi'(\theta) = 0$ on $(\underline{\theta}, \theta^*)$.

By definition of θ^*, we also have $\eta(\theta) = 0$ on $[\underline{\theta}, \theta^*)$. Using (24) we get

$$\begin{aligned}&\left(\Delta y^2\theta - s(\theta)\Delta y^2\theta\right)f(\theta) + \lambda(\theta)s(\theta)\Delta y^2 = 0\\ \Leftrightarrow &s(\theta) = \frac{\theta f(\theta)}{\theta f(\theta) - \lambda(\theta)}.\end{aligned}$$

Simplifying by $\theta f(\theta)$ implies (34). ■

Step 3: Proof of i- in Proposition 3. In this case $\theta^* = \underline{\theta}$. Thus (LLC′) is binding $\forall\theta \in \Theta$. This implies in particular that $T(\underline{\theta}) = \ell(\underline{\theta})$. Given (PC′), it follows that $\ell(\underline{\theta})$ must be higher than m. From (25), this implies $\lambda(\underline{\theta}) = 0$. Moreover, $\xi(\underline{\theta}) = 0$ using (26). Plugging these values into the second part of (28), we get (8). After computations of $\ell(\underline{\theta})$, the existence condition of this case is $m \leq \frac{\Delta y^2\underline{\theta}}{8}$. ■

Step 4: Proof of ii- in Proposition 3. This case assumes that the limited liability constraint (LLC′) is slack $\forall\theta \in \Theta$. Thus $\theta^* > \overline{\theta}$. Plugging into the first part of (28) we have $\lambda(\theta) = -(1 - F(\theta))$. From Lemma 4, we find (9). This solution leads to $s'(\theta) > 0$ since Assumption 1 holds. It follows that $x(\theta) > 0$. So using (22), we check that $\xi(\theta) = 0$, as postulated. Moreover $\lambda(\underline{\theta}) = -1 < 0$, so (25) leads to $T(\underline{\theta}) = m$. Using (17), it follows that

$$T(\theta) = \int_{\underline{\theta}}^{\theta} \frac{s(u)^2\Delta y^2}{2}du + m. \tag{35}$$

According to Lemma 2, the existence condition of this case is $\ell(\overline{\theta}) < T(\overline{\theta})$. Using (9) and $s(\overline{\theta}) = 1$, this condition boils down to $m > \frac{\Delta y^2\overline{\theta}}{2} - \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\theta f(\theta)}{\theta f(\theta)+1-F(\theta)}\right)^2 \frac{\Delta y^2}{2}d\theta$. ■

Step 5: Proof of ii- in Proposition 4. In this case, $\theta^* > \underline{\theta}$. Let $\theta \in [\underline{\theta}, \theta^*]$. By continuity at $\theta = \theta^*$, we have $s(\theta^*) = s$. Thus combining (34) and (28), we get

$$s(\theta) = \frac{1}{1 + \frac{F(\theta^*) - F(\theta) + \frac{1}{\theta^*}\frac{2s(\theta^*)-1}{s(\theta^*)}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta}{\theta f(\theta)}}. \tag{36}$$

Therefore, at $\theta = \theta^*$ we get

$$\begin{aligned}s(\theta^*) &= \frac{1}{1 + \frac{\frac{1}{\theta^*}\frac{2s(\theta^*)-1}{s(\theta^*)}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta}{\theta^* f(\theta^*)}}\\ \Leftrightarrow s(\theta^*)&\left(\theta^* f(\theta^*) + \frac{1}{\theta^*}\frac{2s(\theta^*)-1}{s(\theta^*)}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta\right) = \theta^* f(\theta^*)\\ \Leftrightarrow s(\theta^*)&\left(\theta^* f(\theta^*) + \frac{2}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta\right) = \theta^* f(\theta^*) + \frac{1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta)d\theta,\end{aligned}$$

which gives the second part of (11).

Then, straightforward algebra yields

$$\frac{2s(\theta^*)-1}{s(\theta^*)} = \frac{\theta^* f(\theta^*)}{\theta^* f(\theta^*) + \frac{1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta}. \tag{37}$$

Thus inserting (37) into (36) gives the first part of (11).

Using Assumption 1, $s'(\theta) = x(\theta) > 0$. So $\xi = 0$ from (22) as conjectured.

Moreover, using the second part of (11), it is straightforward to check that

$$s(\theta^*) > \frac{1}{2}. \tag{38}$$

So $\lambda(\theta)$ in the first part of (28) is strictly negative at $\underline{\theta}$. Thus (25) leads to $T(\underline{\theta}) = m$. Using (17), it follows that

$$T(\theta) = \int_{\underline{\theta}}^{\theta} \frac{s(u)^2 \Delta y^2}{2} du + m.$$

Inserting into (LLC′) at θ^* gives (12).

Finally, from (30), $\lambda(\theta^*) < 0$ also implies $s(\theta^*) < 1$. ■

Step 6: Proof of i- in Proposition 4. From (8) and (38), we have $s^*(\theta^*) > \frac{1}{2} = s_L(\theta^*)$. So we get $\ell_L^*(\theta^*) < \ell^*(\theta^*)$ since $\ell(\theta)$ increases with $s(\theta)$. In particular, it is true at $\theta^* = \underline{\theta}$. So there exists some m such that $\ell_L^*(\underline{\theta}) < m < \ell^*(\underline{\theta})$. Thus $T^*(\underline{\theta}) = \ell^*(\underline{\theta})$ and, according to Lemma 2, (LLC′) is binding $\forall \theta \in \Theta$ and $s^*(\theta)$ is constant. Using the second part of (11) and $\theta^* = \underline{\theta}$, we get (10). ■

Step 7: Proof of $\theta^{*\prime}(m) \geq 0$. Differentiating (12) gives after simplifications

$$\frac{dm}{d\theta^*} = -\int_{\underline{\theta}}^{\theta^*} \frac{\partial s^*(\theta)}{\partial \theta^*} s^*(\theta) \Delta y^2 d\theta + s^{*\prime}(\theta^*) s^*(\theta^*) \Delta y^2 \theta^*. \tag{39}$$

So $\theta^{*\prime}(m) \geq 0$ if the RHS of (39) is nonnegative. This is the case if

$$\begin{cases} \frac{\partial s^*(\theta)}{\partial \theta^*} \leq 0 \\ s^{*\prime}(\theta^*) \geq 0. \end{cases} \tag{40}$$

First, using the definitions of $E(\theta)$ and θ^*, and (27), we have

$$E'(\theta^*) = -\eta(\theta^*) < 0. \tag{41}$$

Second, from (32) and (34), we get $\text{sgn}\left(\frac{\partial s^*(\theta)}{\partial \theta^*}\right) = \text{sgn}(E'(\theta^*)) < 0$ as required (make use of (40) and(41)).

Third, let us compute $E'(\theta^*)$ more precisely. To do so, we use (33). We get

$$E(\theta^*) = (1 - F(\theta^*)) - \frac{1}{\theta^*}\frac{2s^*(\theta^*) - 1}{s^*(\theta^*)}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta.$$

So,

$$E'(\theta^*) = -f(\theta^*) + \frac{2s^*(\theta^*) - 1}{s^*(\theta^*)} f(\theta^*) + \frac{1}{\theta^{*2}}\frac{2s^*(\theta^*) - 1}{s^*(\theta^*)}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta - \frac{s^{*\prime}(\theta^*)}{s^{*2}(\theta^*)}\frac{1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta.$$

Factorizing $\frac{1}{\theta^* s^*(\theta^*)}$ and rearranging the first three terms gives

$$\frac{1}{\theta^* s^*(\theta^*)}\left(-\theta^* s^*(\theta) f(\theta^*) + 2\theta s^*(\theta^*) f(\theta^*) - \theta^* f(\theta^*) + \frac{2s^*(\theta^*) - 1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta\right)$$

After simplification, then factorization of $s^*(\theta^*)$, we get

$$\frac{1}{\theta^* s^*(\theta^*)}\left(s^*(\theta^*)\left(\theta^* f(\theta^*) + \frac{2}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta\right) - \theta^* f(\theta^*) - \frac{1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta\right) = 0,$$

where the equality follows from the second part of (11).

Finally,

$$E'(\theta^*) = -\frac{s^{*\prime}(\theta^*)}{s^{*2}(\theta^*)}\frac{1}{\theta^*}\int_{\theta^*}^{\overline{\theta}} \theta f(\theta) d\theta.$$

From (41), we get $s'(\theta^*) > 0$ as required in (40). ■

Step 8: Sufficient conditions. Since the Hamiltonian is linear in x and T and the mixed constraint is linear in T, necessary conditions are also sufficient if the Hamiltonian is concave in $s(\theta)$ and $\ell(\theta)$ is convex in $s(\theta)$. We get

$$\frac{\partial^2 \mathcal{H}}{\partial s(\theta)^2} = -\Delta y^2 \theta f(\theta) + \lambda(\theta)\Delta y^2 < 0$$

because λ is negative, and

$$\frac{\partial^2 \ell}{\partial s(\theta)^2} = \Delta y^2 \theta > 0.$$

■

Proof of Proposition 5

Using (34), $s(\theta)$ is increasing in $\lambda(\theta)$. So the proposition is proved if $\lambda^*(\theta) > \lambda_H^*(\theta)$ when $\theta \leq \theta^*$. According to (32), this is true if $E(\theta^*) > 0$. Yet, this is the case since

$E'(\theta^*) < 0$ from (41) and $E(\theta^*)|_{\theta^*=\bar{\theta}} = 0$ from (33). ■

Proof of Proposition 6

Proof of *i-*. If $m \leq m_L^*$, resp. $m_L^* < m \leq m_I^*$, then from (8), resp. (10), the optimal sharing rule does not depend on m and the effort is constant. However, there is an upward jump in s at $m = m_L^*$ since $s^*(\theta) = \frac{\underline{\theta} f(\underline{\theta}) + \frac{1}{\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta}{\underline{\theta} f(\underline{\theta}) + \frac{2}{\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta} > \frac{1}{2} = s_L^*(\theta)$.

If $m_I^* < m \leq m_0$, since $\theta^{*'}(m) > 0$ we deduce that $\theta^*(m) \leq \theta_0$, and so from (11), the optimal sharing rule is $s^*(\theta) = \frac{\theta^* f(\theta^*) + \frac{1}{\theta^*} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta}{\theta^* f(\theta^*) + \frac{2}{\theta^*} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta}$. We know from the proof of Proposition 2-3-step 7 that $s'(\theta^*) > 0$. Moreover, since $s'(\theta^*) > 0$, we have $\frac{\underline{\theta} f(\underline{\theta}) + \frac{1}{\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta}{\underline{\theta} f(\underline{\theta}) + \frac{2}{\underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta} \leq \frac{\theta^* f(\theta^*) + \frac{1}{\theta^*} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta}{\theta^* f(\theta^*) + \frac{2}{\theta^*} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta}$. Thus, over $[m_L^*, m_0]$, the share is either constant, or increasing in m, as is the optimal effort.

Proof of *ii-*. If $m_0 < m \leq m_H^*$, we have $\theta^*(m) \geq \theta_0$, and so from (11), the optimal sharing rule is $s^*(\theta) = \frac{1}{F(\theta^*) - F(\theta) + \frac{\theta^* f(\theta^*)}{\theta^{*2} f(\theta^*) + \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta} \int_{\theta^*}^{\bar{\theta}} \theta f(\theta) d\theta}{\theta f(\theta)}}$. We know from the proof of Proposition 2-3-step 7 that $\frac{\partial s^*(\theta)}{\partial \theta^*} < 0$. Since $\theta^*(m)$ increases with m, the effort is decreasing in m.

Proof of *iii-*. If $m > m_H^*$, then from (9) the optimal sharing rule does not depend on m and neither does the effort.

Finally, if $\theta_0 = \bar{\theta}$ then we always have $\theta^*(m) \leq \theta_0$ for $m_I^* < m \leq m_H^*$ and so, we deduce that the effort is monotonically increasing in m. ■

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Notes

[1]Notice that the optimal contracts under moral hazard and adverse selection in combination are not a simple mix of contracts under moral hazard or adverse selection taken separately.

[2]The concept of limited liability is the one used by Stiglitz and Weiss (1981) in the context of a credit market.

[3]Using detailed data on individual contracts from modern North American agriculture, Allen and Lueck (1999) find some support for models that assume risk-neutral contracting parties.

[4]According to Allen (1982), ability can be interpreted in two ways: in the first, it is the ability to make good decisions like planting, irrigation or harvesting; in the second, ability is the quality of a factor like the quality of the land.

[5]Asymmetric information is more an issue with urbanization and absentee landlords. We thank a referee for this remark.

[6]Braido (2008) empirically finds a strong inverse relation between tenancy and land quality, but he does not provide theoretical explanations for it.

[7]See the Appendix, subsection , Step 6, equation (37).

[8]See Step 7 in subsection of the Appendix.

[9]A more formal analysis is available in the on-line supplement.

Table and figures

Table 1. Main Insights of the Mixed Informational Context. This table provides a comparison of our main results between the first best (FB), adverse selection alone (AS), moral hazard alone (MH) and adverse selection with moral hazard.

	Outside option	Contract	Share with ability	Effort/Share with outside option	Mixed share with MH&AS
FB	-	Pure rental	Constant	Constant	-
MH	1. High 2. Intermediate 3. Low	1. Pure rental 2. Sharecropping 3. Sharecropping	1. Constant 2. Decreasing 3. Constant	1. Constant 2. Increasing 3. Constant	-
AS	-	Pure rental for the highest ability, sharecropping otherwise	Increasing	Constant	-
Mixed	1. High 2. Intermediate 3. Low	1. Same as AS 2. Sharecropping 3. Same as MH 3	1. Same as AS 2. Increasing then constant or Constant 3. Same as MH 3	1. Same as AS 2. Non-decreasing then decreasing then constant 3. Same as MH 3	1. Same as AS 2. Higher then lower than AS and Higher than MH 3 3. Same as MH 3

FIGURE 1

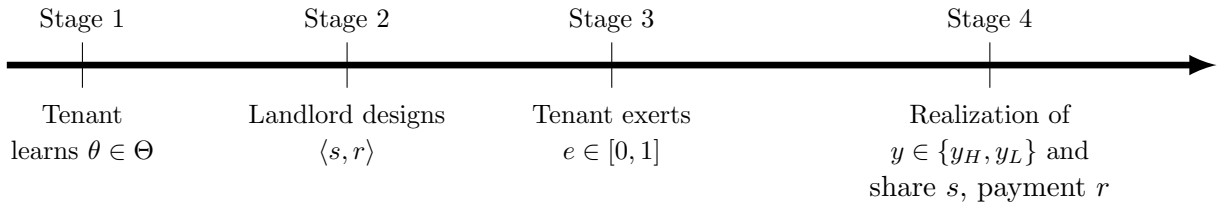

Figure 1: The sequence of events.

FIGURE 2

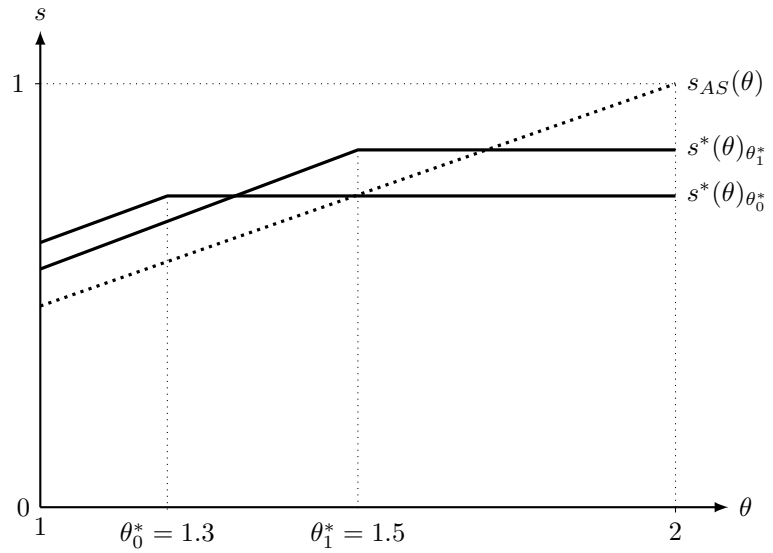

Figure 2: The optimal shares under both adverse selection and moral hazard for $m_0 = 0.067$ and $m_1 = 0.088$ leading to $\theta_0^* = 1.3$ and $\theta_1^* = 1.5$.

FIGURE 3

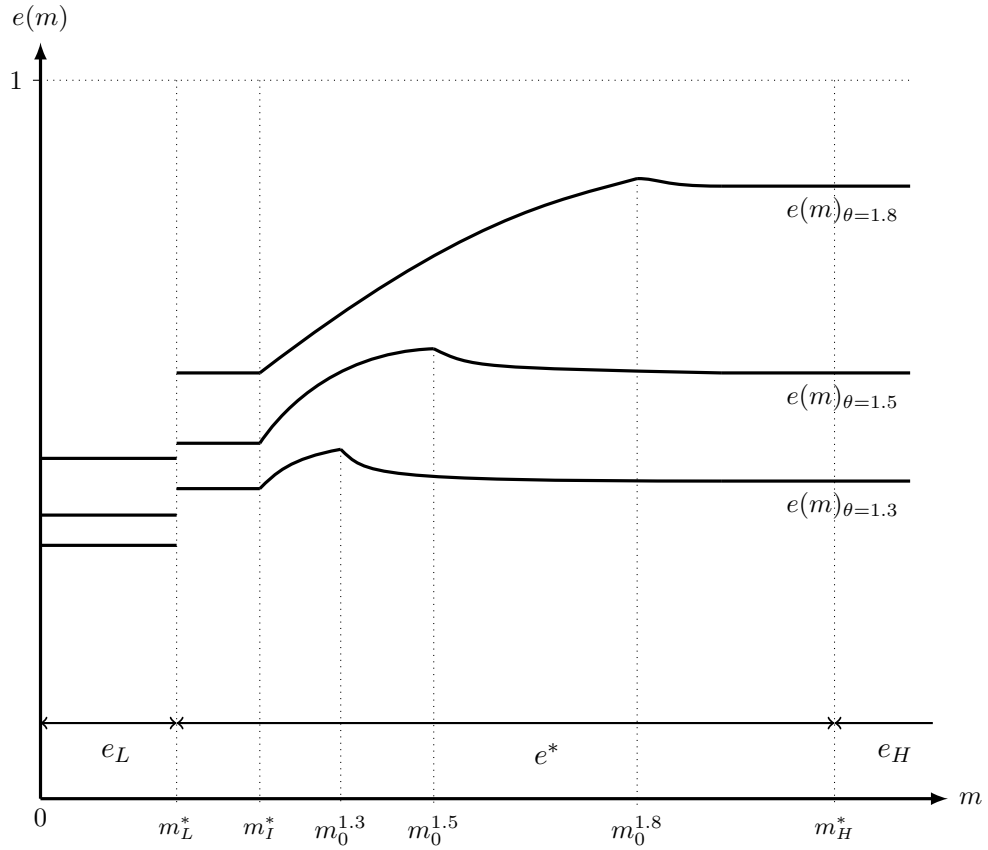

Figure 3: The optimal efforts as a function of m under both moral hazard and adverse selection, for three different values of ability, $\theta = 1.3$, $\theta = 1.5$, and $\theta = 1.8$..