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## Modeling Topological Relations between Uncertain Spatial Regions in Geo-Spatial Databases: Uncertain Intersection and Difference Topological Model

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**Abstract**— Topological relations have played important roles in spatial query, analysis and reasoning in Geographic Information Systems (GIS) and geospatial databases. The topological relations between crisp, uncertain and fuzzy spatial regions based upon the *9-intersections model* have been identified. The research issue of topological relations, particularly, between spatial regions with uncertainties, has gained a lot of attention during the past two decades. However, the formal representation and calculation of the topological relations between uncertain regions is still an open issue and needs to be further developed. The paper provides a theoretical framework for modeling topological relations between uncertain spatial regions based upon a new uncertain topological model called the *Uncertain Intersection and Difference (UID) Model*. In order to derive all topological relations between two spatial regions with *uncertainties*, the *spatial object of type Region (A)* is decomposed in four components: the *Interior*, the *Interior's Boundary*, the *Object's Boundary*, and the *Exterior's Boundary* of *A*. By use of this definition of spatial region with uncertainties, *new 4\*4-Intersection and Uncertain Intersection and Difference (UID) models* are proposed as a qualitative model for the identification of all topological relations between two spatial regions with uncertainties. These two new models are compared with other models studied in the literature. *152 binary topological relations* can be identified by these two models. Then, the topological complexity and distance of the *152 relations* will be study in details by using the *UID model*. Based upon this study of topological complexity and distance, a conceptual neighborhood graph for the *152 relations* can be obtained. Examples are provided to illustrate the utility of these two models presented in this paper with results which can be applied for modeling GIS, geospatial databases and satellite image processing.

**Keywords**-component; *Uncertain Intersection and Difference Model*; *4\*4-Intersection Matrix*; *Uncertain Spatial Regions*; *Topological Relations*; *Geospatial Database, GIS*

### I. INTRODUCTION

Topological relations have an important significance in GIS modelling since they are the basis for spatial modelling, spatial query, analysis and reasoning. How to identify the topological relations between spatial objects is a critical point in GIS modelling.

During recent years, topological relations have been much investigated in the crisp and fuzzy topological space. The well-known *4-intersection approach* described in [1] [2], as well as the *9-intersection approach* as discussed in [3], and the *Intersection and Difference (ID) model* studied in [4][5][6], were proposed to formalize topological relations between two simple regions in the Crisp Topological Space (CTS). The *4-intersection model* is extended in [7] to deal with the topological relations between spatial objects with holes. Fig. 1 represents the *closure*, *interior* and *boundary* of a closed disk as *crisp spatial regions* as in [1][2][3][4][5] [6]. Eight topological relations of the system (*Region Connection Calculus*) *RCC8 (DC, EC, PO, TPP, NTPP, TPPI, NTPPi, and EQ)* have been identified between two simple regions by using these models in [1] [2] [3] [4] [5] [6].



Figure 1. Closure, Interior and Boundary of a Crisp Spatial Region

Geographical phenomena in GIS with *uncertain boundaries* can be modelled by *Regions with Broad Boundaries (BBRs)* as in [8]. A region with broad boundary is an extension of a region with a crisp boundary (refer to simple regions with holes as in [7]). *Objects with broad boundaries* as defined in [8] are spatial objects, whose crisp boundaries are replaced by an area expressing the *boundary's uncertainty*. The *9-intersection model* is extended in [9] to describe topological relations between *BBRs* by replacing the crisp boundary in the *9-intersection with the broad boundary*. Using this model, *44 relations* between two regions by using the *3\*3-intersection matrix* are possible. For *composite regions with broad boundaries*, there are *14 additional topological relations* [8]. Fig. 2 represents a *region with broad boundary*.



Figure 2. Region with a Broad Boundary (BBR)

Another method called *4-tuple representation* of topological relations between *BBRs* is used in [10] to infer new topological information. The *4-tuple representation* can distinguish the same topological relations as identified by the

extended 9-intersection. The 4-tuple, however, can be applied to the reasoning of topological relations between BBRs [10], because it uses the composition of topological relations between crisp regions to determine those between uncertain and vague regions.

More recently fuzzy spatial objects have been emphasized since there are spatial features which are not always crisp. Fuzzy spatial objects are those with indeterminate boundaries and uncertain spatial objects. For fuzzy boundaries, that is, boundaries that are by nature not crisp, the broad boundary represents their minimum and maximum extent. In order to derive the topological relations between fuzzy spatial objects, the 9-intersection approach was updated into the 3\*3-intersection approach in the fuzzy topological space [11] [12]. Furthermore, in these two works, a 4\*4-intersection matrix was built up by using the topological properties of fuzzy sets, and then a 5\*5-intersection matrix can be built up based on certain conditions. The 3\*3-intersection matrix in [13] [14] is derived based on the interior, boundary, and exterior of a simple fuzzy spatial region. By use of 9-intersection matrix, 44 topological relations are identified between two simple fuzzy regions (See Appendix 1. in [14]). Fig. 3 represents a region with indeterminate and uncertain boundary as simple fuzzy spatial region.



Figure 3. Two Fuzzy Spatial Regions with Uncertain Boundaries

In [14], it was shown that fuzzy spatial regions can be decomposed into four parts: the interior, the boundary of the boundary  $\partial(\partial A)$ , the interior of the boundary  $(\partial A)^\circ$  and the exterior, which are mutually disjoint as in Fig. 4.

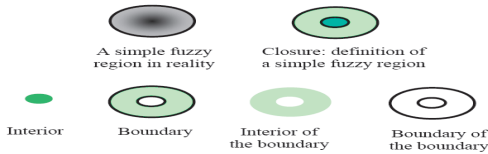


Figure 4. Interior, Boundary, Interior of the Boundary and Boundary of the Boundary of a Simple Fuzzy Region with Indeterminate and Uncertain Boundary (After X. Tang and W. Kainz in [14])

After investigation about the topological relations between two regions with uncertain boundaries compared with these models studied in the literature, we can see that some topological relations can't be identified by these models. Here are some relations presented in Fig. 5.



Figure 5. Some of Topological Relations to Identify

The question is how many topological relations there are exactly between two regions with uncertain boundaries? To answer this question, we will extend the 4-intersection and the ID models with a new definition of the uncertain boundary for regions with uncertainties.

The disadvantage of the 3\*3-intersection and 4\*4-intersection models is that the intersection operator ( $\cap$ ) is the most expensive one in terms of computation. In order to reduce the computational cost of the 3\*3-intersection and 4\*4-intersection models; and to reduce the computational complexity by avoiding spatial operations between topological components with different dimensions (**I-D** and **2-D**), we will try to reduce the number of intersections by introducing the difference operator (-).

Finally, the motivation of the paper is trying to build a complete topological model for identification of all topological relations between two spatial regions with uncertainties. This paper is also an extension of our work in [15] about the topological relations between fuzzy regions.

The structure of the paper is as follows: The definition of the uncertain spatial region is presented in Section II. The novel major contribution of our study, proposed in Section III, is a new form of 4\*4-intersection model and the Uncertain Intersection and Difference (UID) model. Section IV shows the identification of topological relations between two uncertain spatial regions by using two models (4\*4-intersections and UID) based on empty/non-empty contents. 152 topological relations can be identified by these two models. In Sections V and VI, the topological complexity and topological distance of the 152 topological relations will be studied in details by using the UID model. Based upon this study of topological complexity and topological distance, a conceptual neighborhood graph for the 152 binary topological relations can be obtained in Section VII. We present some application and the utility of this model in GIS and satellite image processing in Section VIII. We end with a discussion about results and a conclusion.

## II. DEFINITION OF UNCERTAIN SPATIAL REGION AND BOUNDARY PROPERTIES

In this section, we will develop a definition of spatial regions with uncertainties which is different from other definitions in the literature. Then, we summarize the properties of the boundary of an uncertain spatial region. This definition of spatial regions with uncertainties is taken as in our paper [15] for fuzzy spatial region.

### A. Definition of Uncertain Spatial Region

Uncertain spatial region ( $A$ ) is made up of two regions  $A_1$  and  $A_2$  with  $A_1$  and  $A_2$   $A_1 \subset A_2$  (see Fig. 6), where: (1) the interior of  $A$  is the interior of  $A_1$   $A^\circ = (A_1)^\circ$  and  $A^\circ$  is an open subset and connected; (2) the interior's boundary of  $A$  is the boundary of  $A_1$  as  $A^i = \partial(A^\circ) = \partial A_1$ , and  $A^i$  is a closed subset and connected; (3) the boundary of  $A$  is  $\partial A$  defined as the interior of the difference between  $A_1$  and  $A_2$  as  $\partial A = (A_2 - A_1)^\circ$ , and  $\partial A$  is an open subset and connected; (4) the exterior's boundary of  $A$  is the boundary of  $\partial A_2$  as  $A^e = \partial^\circ(\partial A) = \partial A_2$ , and  $A^e$  is a closed subset and connected; and (5) the intersection of all closed sets

containing  $A$  is called the *closure of  $A$* , denoted by  $A^-$ . Fig. 6 shows the four components of an *uncertain spatial region*.

This definition is considered as the decomposition of the *boundary* in [14][16][17] into *disjoint* subsets such as the *interior boundary*, the *exterior boundary* and the *interior of the boundary* with condition that is the *interior* of the *boundary* couldn't be a *non-empty set*.

We called the *boundary* ( $\partial A$ ) of  $A$  by the *uncertain boundary*. This definition of *uncertain spatial regions* is very interesting to identify all topological relations between two uncertain spatial regions that other models [8] [9] [14] can't identify. We will prove that in the next section by developing a *4\*4-intersection matrix*. In the next part, we will define the properties of the *uncertain boundary*.

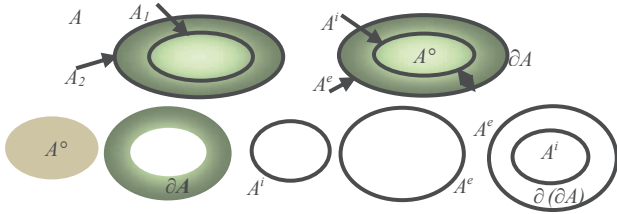


Figure 6. Interior, Boundary, Interior's Boundary, and Exterior's Boundary of an Uncertain Spatial Region

### B. Properties of Uncertain Boundary

Based on the definition of uncertain spatial regions as in [15], we find the properties of the *uncertain boundary* ( $\partial A$ ) as follows: (1)  $\partial A$  is an open subset of  $A$ ; (2)  $\partial A = \neg \emptyset$  is a *non-empty* subset; (3) the *boundary* of  $A$  ( $\partial A$ ) is the *interior* of the *difference*  $\partial A = (A^e - A^i)^\circ$  between the *exterior's boundary* ( $A^e$ ) and the *interior's boundary* ( $A^i$ ) of  $A$ ; (4) the union  $A^i \cup \partial A \cup A^e$  of the *interior's boundary* ( $A^i$ ), the *boundary* ( $\partial A$ ) and the *exterior's boundary* ( $A^e$ ) of  $A$  is a closed subset of  $A$ ; (5) the *interior boundary* of the *boundary* ( $\partial A$ ) of  $A$   $\partial^i(\partial A) = A^i = \partial(A^\circ)$  is the *interior's boundary* ( $A^i$ ); and (6) the *exterior boundary* of the *boundary* ( $\partial A$ ) of  $A$   $\partial^e(\partial A) = A^e = \partial(A^e)$  is the *exterior's boundary* ( $A^e$ ).

We also find that the intersections between  $A^\circ$ ,  $A^i$ ,  $\partial A$ ,  $A^e$  are, respectively, always empty, and the union of these parts is equal to  $A$  as follows:  $A^\circ \cap A^i = \emptyset$ ;  $A^i \cap \partial A = \emptyset$ ;  $\partial A \cap A^e = \emptyset$ ;  $A^\circ \cap A^i \cap \partial A \cap A^e = \emptyset$ ; and  $A^\circ \cup A^i \cup \partial A \cup A^e = A$ .

It also can be easily proven by the above intersections that the *interior* ( $A^\circ$ ), the *interior's boundary* ( $A^i$ ), the *boundary*  $\partial A$  and the *exterior's boundary* ( $A^e$ ) of uncertain spatial region ( $A$ ) are *mutually disjoint*.

In order to identify all possible topological relations, the condition of the mutual disjointness of these four parts of uncertain spatial regions is important to propose and construct a new method to form the intersection matrix. In the next section, a *new 4\*4-intersection matrix* and *uncertain intersection and difference model* are proposed based upon this definition of uncertain spatial region. These two models

are founded for topological relations between fuzzy regions in our work [15].

### III. CONTRIBUTIONS: 4\*4 - INTERSECTION AND UNCERTAIN INTERSECTION AND DIFFERENCE (UID) MODELS

In this section, we will develop two models to identify the topological relations between two spatial regions with uncertainties. Supposing there are two uncertain spatial regions  $A$  and  $B$ , we adopt the *interior*, *boundary*, *interior's boundary*, and *exterior's boundary* to formalize two new topological models as in the two next parts.

#### A. Contribution 1: 4\*4 - Intersection Model ( $I_{4*4}$ )

In the first contribution, the first model is a *new 4\*4 - intersection matrix* which uses the *operator* ( $\cap$ ) of *intersection*. Between these two uncertain spatial regions  $A$  and  $B$ , the *4\*4-intersection matrix* will be presented in Table I, as similar as to the building of the topological models *4-Intersection*, *9-Intersection*, and the *ID model* in [1] [2] [3] [4] [5] [6] [7] [8].

Table I. 4\*4 - INTERSECTION MATRIX

$\cap$	$B^\circ$	$B^i$	$\partial B$	$B^e$
$A^\circ$	$A^\circ \cap B^\circ$	$A^\circ \cap B^i$	$A^\circ \cap \partial B$	$A^\circ \cap B^e$
$A^i$	$A^i \cap B^\circ$	$A^i \cap B^i$	$A^i \cap \partial B$	$A^i \cap B^e$
$\partial A$	$\partial A \cap B^\circ$	$\partial A \cap B^i$	$\partial A \cap \partial B$	$\partial A \cap B^e$
$A^e$	$A^e \cap B^\circ$	$A^e \cap B^i$	$A^e \cap \partial B$	$A^e \cap B^e$

And the *4\*4-intersection model* applied to uncertain spatial regions is expressed by the following expression:

$$I_{4*4}(A, B) = \begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap B^i & A^\circ \cap \partial B & A^\circ \cap B^e \\ A^i \cap B^\circ & A^i \cap B^i & A^i \cap \partial B & A^i \cap B^e \\ \partial A \cap B^\circ & \partial A \cap B^i & \partial A \cap \partial B & \partial A \cap B^e \\ A^e \cap B^\circ & A^e \cap B^i & A^e \cap \partial B & A^e \cap B^e \end{bmatrix} \quad (1)$$

This *new 4\*4 - intersection matrix* ( $I_{4*4}$ ) given by (1) is considered as an extension of the *4-intersection model* [1] [2] [15] for uncertain spatial regions. This model is developed for fuzzy regions in our study [15].

The *intersection operator* ( $\cap$ ) is perhaps the most expensive one in terms of computation. In order to reduce the computational cost of this *4\*4-intersection model*, we will extend this model to the *uncertain intersection and difference model* in the next part.

#### B. Contribution 2: Uncertain Intersection and Difference (UID) Model

In this model, we will introduce the *difference operator* ( $-$ ). In order to avoid spatial operations between topological components with different dimensions ( $A^\circ$ ,  $B^\circ$ ,  $\partial A$ ,  $\partial B$  as **2-D**; and  $A^i$ ,  $B^i$ ,  $A^e$ ,  $B^e$  as **1-D**), we will replace the intersection between the terms  $A^\circ \cap B^i$ ,  $A^i \cap B^\circ$ ,  $A^i \cap B^e$ ,  $A^e \cap B^i$ ,  $\partial A \cap B^i$ ,  $\partial A \cap B^e$ ,  $A^e \cap B^i$  and  $A^e \cap \partial B$  in the *4\*4-intersection model* by the differences as in the matrix (see Table II).

The four intersections ( $A^\circ \cap B^\circ$ ,  $A^\circ \cap \partial B$ ,  $\partial A \cap B^\circ$ ,  $\partial A \cap \partial B$ ) with topological components with dimension **2-D** and the four intersections ( $A^i \cap B^i$ ,  $A^i \cap B^e$ ,  $A^e \cap B^i$ ,  $A^e \cap B^e$ ) with topological components with dimension **1-D** remain unchanged as similar as in [4] [5] [6] for the *ID model*.



By simplification and arrangement of this  $4*4$  - *Intersection and Difference matrix*, we obtain two *intersection matrices* and two *difference matrices* (Table III):

Table II.  $4*4$  - INTERSECTION AND DIFFERENCE MATRIX

$\cap$	$B^\circ$	$B^i$	$\partial B$	$B^e$	-
$A^\circ$	$A^\circ \cap B^\circ$	$A^i - B^i$	$A^\circ \cap \partial B$	$A^i - B^e$	$A^\circ$
$A^i$	$B^i - A^i$	$A^i \cap B^i$	$B^i - A^i$	$A^i \cap B^e$	$A^i$
$\partial A$	$\partial A \cap B^\circ$	$A^e - B^i$	$\partial A \cap \partial B$	$A^e - B^e$	$\partial A$
$A^e$	$B^i - A^e$	$A^e \cap B^i$	$B^i - A^e$	$A^e \cap B^e$	$A^e$

Table III. INTERSECTION AND DIFFERENCE MATRICES

$\cap$	$B^\circ$	$\partial B$	$\cap$	$B^i$	$B^e$
$A^\circ$	$A^\circ \cap B^\circ$	$A^\circ \cap \partial B$	$A^i$	$A^i \cap B^i$	$A^i \cap B^e$
$\partial A$	$\partial A \cap B^\circ$	$\partial A \cap \partial B$	$A^e$	$A^e \cap B^i$	$A^e \cap B^e$

-	$A^i$	$A^e$	-	$B^i$	$B^e$
$B^i$	$B^i - A^i$	$B^i - A^e$	$A^i$	$A^i - B^i$	$A^i - B^e$
$B^e$	$B^e - A^i$	$B^e - A^e$	$A^e$	$A^e - B^i$	$A^e - B^e$

At the end, the *Uncertain Intersection and Difference (UID) model* is written as follows:

$$UID_{4*4}(A, B) = \begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^i - B^i & A^i - B^e \\ \partial A \cap B^\circ & \partial A \cap \partial B & A^e - B^i & A^e - B^e \\ A^i \cap B^i & A^i \cap B^e & B^i - A^i & B^i - A^e \\ A^e \cap B^i & A^e \cap B^e & B^e - A^i & B^e - A^e \end{bmatrix} \quad (2)$$

The *UID model* given by (2) combines two different operators (*intersection and difference*). The *UID* has two advantages: first, it reduces the computational complexity by avoiding spatial operations between topological components with different dimensions, e.g.,  $A^\circ \cap B^i$ ,  $A^\circ \cap B^e$ ,  $A^i \cap B^\circ$ ,  $A^i \cap \partial B$ ,  $\partial A \cap B^i$ ,  $\partial A \cap B^e$ ,  $A^e \cap B^\circ$  and  $A^e \cap \partial B$ , with  $A^\circ$ ,  $B^\circ$ ,  $\partial A$ ,  $\partial B$  as **2-D**, and  $A^i$ ,  $B^i$ ,  $A^e$ ,  $B^e$  as **1-D**; and second, it reduces the computational cost due to only eight intersections in the matrix of *UID model*.

This *UID model* is considered as an extension of the *ID model* [4] [5] [6] [15] for uncertain spatial regions.

In general, there are  $2^{16} = 65536$  relations between two uncertain spatial regions by using the  $4*4$ -*intersection matrix* and *UID model*. For GIS applications, some conditions will limit the number of these relations as in [7] [8] [14]. However, how to find all possible topological relations between two uncertain spatial regions needs more investigation. It's done in the next section for the  $4*4$ -*intersection matrix* and *UID model*.

#### IV. IDENTIFICATION OF TOPOLOGICAL RELATIONS BASED UPON UID MODEL

For identification by *UID model*, by respecting the definition in section III, we scan all possible configurations for  $A$  and  $B$  in two different steps as following: **(1)** If the exterior's boundary of  $A$  intersects with the exterior's boundary of  $B$  ( $A^e \cap B^e = \neg\emptyset$ ), then, we search all possible topological relations between  $A$  and  $B$ , we find **105** relations; **(2)** If the exterior's boundary of  $A$  doesn't intersect with the

exterior's boundary of  $B$  ( $A^e \cap B^e = \emptyset$ ), then, we search all topological relations between  $A$  and  $B$ , we find **47** relations. The total number of topological relations identified by *UID model* between  $A$  and  $B$  is **152** relations (See Appendix). Some of these topological relations are not identified and determined in [8] [9] [14]. We give some examples in Fig. 7 and Fig. 8.

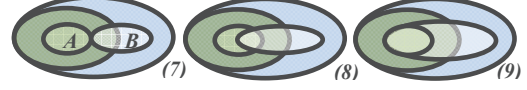


Figure 7. Examples for  $A^e \cap B^e = \neg\emptyset$

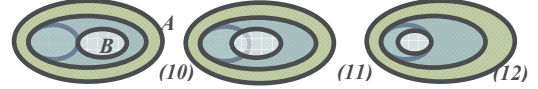


Figure 8. Examples for  $A^e \cap B^e = \emptyset$

For each relation identified in the *Uncertain Intersection and Difference (UID) Model* takes value of either empty ( $\emptyset$ ) or non-empty ( $\neg\emptyset$ ). For example, The *UID matrices* correspondent to the relations (7), (8) and (9) in Fig. 7, and to the relations (10), (11) and (12) in Fig. 8 are given, respectively, by:

$$(7) \begin{bmatrix} \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \end{bmatrix} \quad (8) \begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \end{bmatrix} \quad (9) \begin{bmatrix} \neg\phi & \phi & \phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \\ \neg\phi & \phi & \neg\phi & \neg\phi \end{bmatrix}$$

$$(10) \begin{bmatrix} \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix} \quad (11) \begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix} \quad (12) \begin{bmatrix} \neg\phi & \neg\phi & \neg\phi & \phi \\ \phi & \neg\phi & \neg\phi & \phi \\ \neg\phi & \neg\phi & \phi & \phi \\ \phi & \phi & \neg\phi & \phi \end{bmatrix}$$

To find and extract these topological relations by the  $4*4$ -*intersection* and *UID models*, we have developed and implemented these two steps on *MATLAB*. In the next section, we will study the topological complexity for *UID model*.

Comparing between the relations identified in [8] [9] [14] and our models ( $4*4$ -*intersection and UID*), we find that there are **100 new relations** which can't be discriminated and identified by other models. These **152 relations** identified by *UID model* are presented in the appendix of this paper.

#### V. TOPOLOGICAL COMPLEXITY

Intuitively, the complexity of a kind of topological relation corresponds to the number of non-empty value in its *uncertain intersection and difference (UID) model*, because the non-empty value may be further differentiated by other topological invariants (e.g., *dimension* and *separation number*) [4] [5] [6]. It is hereof natural to define the **topological complexity** denoted by  $C_T(UID)$  of the topological relations obtained by the *UID model* as follows:

$$C_T(UID) = \sum_{i=1}^4 \sum_{j=1}^4 \delta(e_{ij}) \quad (3)$$

Where  $e_{ij}$  ( $1 \leq i, j \leq 4$ ) is the value of elements in the  $i^{th}$  row and  $j^{th}$  column of the *uncertain intersection and difference matrix* defined in (3). The symbol  $\delta(\cdot)$  is a function of mapping the values of empty ( $\emptyset$ ) and non-empty

$(-\emptyset)$  onto the integers  $\mathbf{0}$  and  $\mathbf{1}$ , which can be represented by the next expression:  $\delta(\cdot) = \begin{cases} 1, & \text{When } e_{ij} = -\emptyset \\ 0, & \text{When } e_{ij} = \emptyset \end{cases}$  (4)

The **topological complexity**  $C_T(\mathbf{UID})$  is the sum of all *non-empty*  $(-\emptyset)$  values in the **UID** matrix of the relation.

For example, the topological complexity of the topological relation ( $\mathbf{UID}^{21}$ ) given by the matrix:

$$\mathbf{UID}^{21} = \begin{bmatrix} \phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ \phi & -\phi & -\phi & \phi \\ \phi & -\phi & -\phi & -\phi \end{bmatrix} \text{ is computed by using (3) :}$$

$C_T(\mathbf{UID}^{21}) = \sum_{i=1}^4 \sum_{j=1}^4 \delta(e_{ij}) = 12$  ; Another example, the topological complexity of topological relation ( $\mathbf{UID}^{17}$ ) given

$$\text{by the matrix: } \mathbf{UID}^{17} = \begin{bmatrix} -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \end{bmatrix} \text{ is also computed by}$$

$$\text{using (3) : } C_T(\mathbf{UID}^{17}) = \sum_{i=1}^4 \sum_{j=1}^4 \delta(e_{ij}) = 16$$

The **topological complexity** ( $C_T(\mathbf{UID})$ ) of the (152) kinds of topological relations obtained by the **UID** model between uncertain spatial regions can be computed by using (3), where  $C_T(\mathbf{UID})$  has eleven (11) different values: 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 and 16, listed as follows (Presented in Fig. 10 and implemented on **MATLAB**):

1.  $C_T(\mathbf{UID}) = 6$  : Group 1 of the Topological Relations: 85, 137, 139, 140, 151, 152
2.  $C_T(\mathbf{UID}) = 7$  : Group 2 of the Topological Relations: 84, 95, 133, 134, 136, 138, 142, 143, 144, 145
3.  $C_T(\mathbf{UID}) = 8$  : Group 3 of the Topological Relations: 1, 74, 75, 77, 87, 91, 94, 96, 97, 114, 116, 117, 128, 129, 132, 135, 141, 146, 147, 148
4.  $C_T(\mathbf{UID}) = 9$  : Group 4 of the Topological Relations: 2, 54, 55, 58, 65, 70, 73, 76, 78, 83, 86, 90, 92, 98, 109, 110, 113, 115, 118, 123, 124, 127, 130, 149
5.  $C_T(\mathbf{UID}) = 10$  : Group 5 of the Topological Relations: 3, 7, 37, 39, 50, 52, 56, 57, 59, 64, 66, 69, 71, 79, 82, 88, 93, 99, 104, 108, 111, 119, 122, 125, 126, 131, 150
6.  $C_T(\mathbf{UID}) = 11$  : Group 6 of the Topological Relations: 4, 6, 8, 11, 25, 26, 33, 38, 40, 45, 46, 49, 51, 60, 63, 67, 72, 80, 89, 100, 103, 105, 106, 112, 120
7.  $C_T(\mathbf{UID}) = 12$  : Group 7 of the Topological Relations: 5, 9, 12, 13, 18, 21, 24, 27, 31, 34, 41, 44, 47, 53, 61, 68, 81, 101, 107, 121
8.  $C_T(\mathbf{UID}) = 13$  : Group 8 of the Topological Relations: 10, 14, 20, 28, 32, 35, 42, 48, 62, 102
9.  $C_T(\mathbf{UID}) = 14$  : Group 9 of the Topological Relations: 15, 19, 22, 29, 36, 43
10.  $C_T(\mathbf{UID}) = 15$  : Group 10 of the Topological Relations: 16, 23, 30
11.  $C_T(\mathbf{UID}) = 16$  : Group 11 of the Topological Relations: 17

One can easily see from above list that the six types of topological relations with  $C_T(\mathbf{UID}) = 6$ , i.e., Group 1 of the

*Topological Relations*: 85, 137, 139, 140, 151, 152, are of the *lowest complexity*. The relation with  $C_T(\mathbf{UID}) = 16$ , i.e., *Group 11 of the Topological Relations*: 17 (**Completely Overlap**), is of the *highest complexity*. The topological relations with  $C_T(\mathbf{UID}) > 6$  are of higher complexity. They are completely differentiated by the proposed *uncertain intersection and difference (UID) model*. In other words, some relations of the 152 relations with are higher complexity of  $C_T(\mathbf{UID}) < 16$  cannot be classified into subclasses in terms of topology.

The above investigation has shown that on the one hand the topological complexity defined in  $C_T(\mathbf{UID})$  given by (3) can be used as a measure for the in-depth of further classification, and that on the other hand the *Uncertain Intersection and Difference (UID) model* can be used to represent effectively the potential complexity of various kinds of topological relations between two spatial regions with uncertainties.

## VI. TOPOLOGICAL DISTANCE

Since the topological relation converts from one case to another with the changes in geometry of one or two of the involved objects, such as location, orientation, shape, and size, a topological change occurs if the change in geometry of an object affects its relation with respect to another object [4] [18]. Taking Fig. 9 as example, the relation between  $A$  and  $B$  in (a) is **Disjoint**, with the location of object  $B$  moving downwards, the relation is in order changed into **Completely Meet** in (b) and **Completely Overlap** in (c). It indicates that there is different degree of similarities and the order of transformation among topological relations. Egenhofer and Al-Taha [18] utilized the *9-intersection* as a basic model to describe such order over topological relations, and provided a measure to assess how far two relations are apart from each other. In following discussion, a *similar definition of topological distance* to Egenhofer and Al-Taha [18] is taken on the basis of the newly presented uncertain intersection and difference model as similar as in [4].

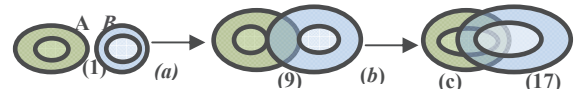


Figure 9. Illustration of the process of topological changes using the **UID** model: from (a) **Disjoint** (Relation (1)) to (b) **Completely Meet** (Relation (9)) and then to (c) **Completely Overlap** (Relation (17)) (See Appendix).

For two topological relations of the 152 relations obtained by the **UID** model between two uncertain spatial regions, the *uncertain intersection and difference matrix* of these relations is represented as  $\mathbf{UID}^k$  and  $\mathbf{UID}^m$ , respectively with  $1 \leq k, m \leq 152$ . Further, the **topological distance** denoted by  $\mathbf{TD}_T(\mathbf{UID}^k, \mathbf{UID}^m)$  between these two relations can be defined as in [13] by the following expression:

$$\mathbf{TD}_T(\mathbf{UID}^k, \mathbf{UID}^m) = \sum_{i=1}^4 \sum_{j=1}^4 \left| \delta(e_{ij}^k) - \delta(e_{ij}^m) \right| \quad (5)$$

Where the definitions of  $e_{ij}$  and  $\delta(\cdot)$  are the same as those in (4). By using (5), the **topological distance** ( $\mathbf{TD}_T$ ) between any two topological relations can be obtained. For

example, by taking the two topological relations ( $k = 5$  and  $m = 127$ ) given by the matrix of *UID* model as following:

$$UID^k = UID^5 = \begin{bmatrix} \phi & \phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ \phi & \phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \end{bmatrix} \text{ and } UID^m = UID^{127} = \begin{bmatrix} -\phi & -\phi & -\phi & \phi \\ \phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & \phi & \phi \\ \phi & \phi & -\phi & \phi \end{bmatrix}$$

**Then** the topological distance between these two relations ( $UID^5$  and  $UID^{127}$ ) can be computed by (5) as follows:

$$\begin{aligned} TD_T(UID^k, UID^m) &= TD_T(UID^5, UID^{127}) = \sum_{i=1}^4 \sum_{j=1}^4 |\delta(e_{ij}^5) - \delta(e_{ij}^{127})| \\ TD_T(UID^5, UID^{127}) &= |0-1|+|0-1|+|1-1|+|1-0|+|1-0|+|1-1|+|1-1|+|1-1| \\ &\quad +|0-1|+|0-1|+|1-0|+|1-0|+|1-0|+|1-0|+|1-1|+|1-0| \\ TD_T(UID^5, UID^{127}) &= 1+1+0+1+1+0+0+0+1+1+1+1+1+1+0+1=11 \end{aligned}$$

Similarly, topological distances can be obtained for other relations. We have developed and implemented the calculus of topological distances on *MATLAB*. From this study about the topological distance, it can be found that the topological distance between two topological relations based upon the *UID* model satisfies the following properties as in [4]:

**(1) Symmetric Property:**  $TD_T(UID^k, UID^m) = TD_T(UID^m, UID^k)$

**(2) Minimum Value Non-Minus Property:** **If**

$UID^k = UID^m$  **then**  $TD_T(UID^k, UID^m) = 0$ , in other words, the *minimum topological distance* is equal to zero when the topological relations are the same. This property is called *non-minus property* where the topological distance is already more or equal to zero  $0 \leq TD_T(UID^k, UID^m)$ . For example, by taking

$UID^k = UID^m = UID^1$ , **then**, the topological distance:  $TD_T(UID^1, UID^1) = \sum_{i=1}^4 \sum_{j=1}^4 |\delta(e_{ij}^1) - \delta(e_{ij}^1)| = 0$ .

**(3) Minimum Value Non-Zero Property:** **If**

$TD_T(UID^k, UID^m) = 1$  **then** the topological distance between these two relations is a *minimum value non-zero* and equal to one. And this property is called by a *minimum value non-zero property*. For example, the topological relation  $UID^{17}$  has three *minimum values non-zero* with three different relations which are the relations  $UID^{16}$ ,  $UID^{23}$ , and  $UID^{30}$ . We take the two relations ( $k = 17$  and  $m = 23$ ) given by the matrix of *UID* model as following:

$$UID^k = UID^{17} = \begin{bmatrix} -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \end{bmatrix} \text{ and } UID^m = UID^{23} = \begin{bmatrix} -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \end{bmatrix}$$

**Then** the topological distance is given by:

$$\begin{aligned} TD_T(UID^k, UID^m) &= TD_T(UID^{17}, UID^{23}) = \sum_{i=1}^4 \sum_{j=1}^4 |\delta(e_{ij}^{17}) - \delta(e_{ij}^{23})| \\ TD_T(UID^{17}, UID^{23}) &= |1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1| \\ &\quad +|1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1|+|1-1| \\ TD_T(UID^{17}, UID^{23}) &= 0+0+0+0+0+0+0+0+0+0+0+0+0+0+0+0=0 \end{aligned}$$

**(4) Maximum Value:** The *maximum topological distance* is equal to **14** only in two cases. The **first Case (1)** is: For

the topological relations ( $k = 63$  and  $m = 136$ ) given by the matrix of *UID* model:

$$UID^k = UID^{63} = \begin{bmatrix} -\phi & \phi & \phi & \phi \\ -\phi & -\phi & -\phi & \phi \\ -\phi & \phi & -\phi & -\phi \\ -\phi & -\phi & -\phi & -\phi \end{bmatrix} \text{ and } UID^m = UID^{136} = \begin{bmatrix} -\phi & -\phi & -\phi & -\phi \\ \phi & \phi & -\phi & -\phi \\ \phi & -\phi & \phi & \phi \\ \phi & \phi & \phi & \phi \end{bmatrix}$$

**Then** the topological distance between these two relations ( $UID^{63}$  and  $UID^{136}$ ) can be computed by (5) as follows:

$$\begin{aligned} TD_T(UID^k, UID^m) &= TD_T(UID^{63}, UID^{136}) = \sum_{i=1}^4 \sum_{j=1}^4 |\delta(e_{ij}^{63}) - \delta(e_{ij}^{136})| \\ TD_T(UID^{63}, UID^{136}) &= |1-1|+|0-1|+|0-1|+|0-1|+|1-0|+|1-0|+|1-1|+|0-1| \\ &\quad +|1-0|+|0-1|+|1-0|+|1-0|+|1-0|+|1-0|+|1-0|+|1-0| \\ TD_T(UID^{63}, UID^{136}) &= 0+1+1+1+1+0+1+1+1+1+1+1+1+1+1=14 \end{aligned}$$

And the **second Case (2)** is: the two topological relations ( $k = 84$  and  $m = 103$ ) given by the matrix of *UID* model:

$$UID^k = UID^{84} = \begin{bmatrix} -\phi & \phi & \phi & \phi \\ -\phi & \phi & \phi & \phi \\ \phi & \phi & -\phi & -\phi \\ -\phi & \phi & -\phi & -\phi \end{bmatrix} \text{ and } UID^m = UID^{103} = \begin{bmatrix} -\phi & -\phi & -\phi & -\phi \\ \phi & -\phi & -\phi & -\phi \\ -\phi & -\phi & \phi & \phi \\ \phi & -\phi & -\phi & \phi \end{bmatrix}$$

**Then** the topological distance between these two relations ( $UID^{84}$  and  $UID^{103}$ ) can be computed by (5) as follows:

$$\begin{aligned} TD_T(UID^k, UID^m) &= TD_T(UID^{84}, UID^{103}) = \sum_{i=1}^4 \sum_{j=1}^4 |\delta(e_{ij}^{84}) - \delta(e_{ij}^{103})| \\ TD_T(UID^{84}, UID^{103}) &= |1-1|+|0-1|+|0-1|+|0-1|+|1-0|+|0-1|+|0-1|+|0-1| \\ &\quad +|0-1|+|0-1|+|1-0|+|1-0|+|1-0|+|0-1|+|1-1|+|1-0| \\ TD_T(UID^{84}, UID^{103}) &= 0+1+1+1+1+1+1+1+1+1+1+1+1+0+1=14 \end{aligned}$$

**Rang Values:** Base upon the minimum and maximum values of the topological distance, we find that the topological distance values have the rang:  $0 \leq TD_T(UID^k, UID^m) \leq 14$

**(5) Triangular**

**Inequality:**

$$TD_T(UID^k, UID^m) + TD_T(UID^m, UID^n) \geq TD_T(UID^k, UID^n)$$

In the properties from (1) to (6) of the topological distance, the notations  $UID^k$ ,  $UID^m$  and  $UID^n$  represent three topological relations obtained by the *UID* model, respectively. Compared to the definition of distance in metric space, it is easily observed that the topological distance is very similar to *Manhattan distance*. Therefore, the topological distance can be understood as another measure of topological relations in addition to the topological complexity defined in the previous section.

## VII. NEIGHBORHOOD RELATIONS AND CONCEPTUAL NEIGHBORHOOD GRAPH

From the property of *minimum value non-zero*, it has shown that the *minimum distance between different topological relations is equal to one* [4]:  $TD_T(UID^k, UID^m) \geq 1$  **For**  $UID^k \neq UID^m$

Where:  $UID^k$  and  $UID^m$  represent the *uncertain intersection and difference matrix* of two different relations. Therefore we define **neighborhood relation** to be two relations satisfying the same condition of the *minimum value non-zero property*:  $TD_T(UID^k, UID^m) = 1$ . In other words:







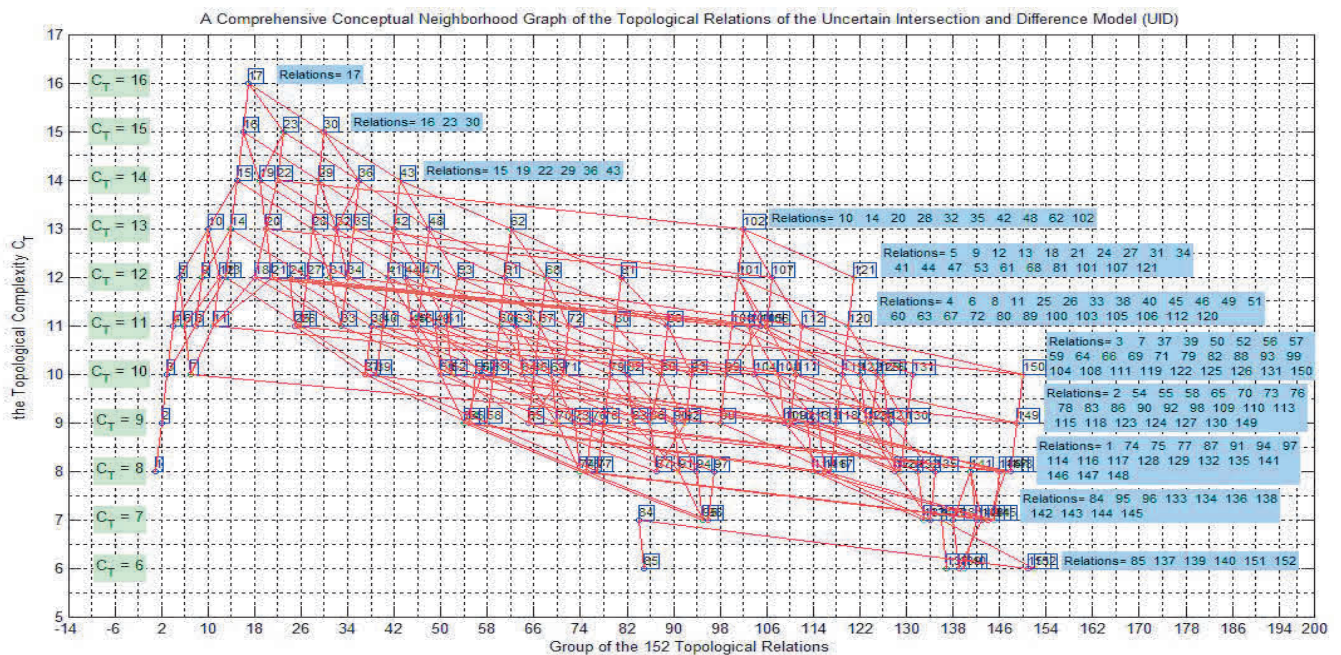


Figure 11. A Comprehensive Conceptual Neighborhood Graph of the 152 Relations under the Uncertain Intersection and Difference (UID) Model

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APPENDIX. (CONTRIBUTION *UID MODEL*): ONE HUNDRED AND FIFTY-TWO (152) TOPOLOGICAL RELATIONS BETWEEN TWO UNCERTAIN SPATIAL REGIONS BY USING THE UNCERTAIN INTERSECTION AND DIFFERENCE (*UID*) MODEL

