Advanced languages of terms for ontologies

Philippe Balbiani, Martín Diéguez, Cigdem Gencer

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Philippe Balbiani[1] Martin Diéguez[2] Çiğdem Gencer[1,3]
[1]Institut de recherche en informatique de Toulouse
CNRS-INPT-UT3, Université de Toulouse, Toulouse, France
[2]Laboratoire d'étude et de recherche en informatique d'Angers
Université d'Angers, Angers, France
[3]Faculty of Arts and Sciences
Istanbul Aydın University, Istanbul, Turkey

***Abstract*—This paper is about the integration in a unique formalism of knowledge representation languages such as those provided by description logic languages and rule-based reasoning paradigms such as those provided by logic programming languages. We aim at creating an hybrid formalism where description logics constructs are used for defining concepts that are given as arguments to the predicates of the logic programs.**

***Index Terms*—Logic programming, description logics.**

I. A short introduction

A crucial issue in the development of the semantic web is the possibility to combine rule-based systems and ontologies. There exists already several types of such combination [21], [22], [29], [32]. These approaches either build rules on top of ontologies allowing rule-based systems to use the vocabulary specified in ontologies, or build ontologies on top of rules supplementing ontological definitions by rules. None of them completely answer to the question of the combination of logic programming with description logics that we are seeking for: an hybrid formalism where description logics constructs are used for defining concepts that are given as arguments to the predicates of the logic programs. In this paper, we develop such an hybrid formalism.

This paper is organized as follows. A case study motivating the combination of logic programming with description logics that we are seeking for is presented in Section II. In Sections III and IV, we introduce the syntax and the semantics of our hybrid formalism. Decision problems are presented in Sections V, VI and VII. In Section VIII, we introduce examples. A research program is presented in Section IX.

II. A case study

Examining role-based access control and organization-based access control, we present a case study motivating the combination of logic programming with description logics that we are seeking for.

Access of subjects to objects in a computer system are permitted in accordance with a security policy embodied in an access control database. Many computer systems use the access control matrix model to represent security policies [28]. Formally, an access control matrix is a structure consisting of a set of subjects (users, processes, etc), a set of objects (files, tables, etc) and binary relations $(p_i)_{i\in I}$ between objects and subjects giving to subjects the permission to access objects. In this setting, asserting that subject a possesses permission p_i on object b comes down to asserting that p_i holds for b and a.

Access control with a lot of subjects is space-consuming. To reduce the cost of security, within the context of role-based access control (RBAC), it has been proposed that access control administrators treat sets of subjects as instances of a concept called role[1] [35]. Formally, an RBAC-structure consists of a set of subjects, a set of objects, a set of roles, a binary relation r between subjects and roles defining the roles of subjects and binary relations $(p_i)_{i\in I}$ between objects and roles giving to roles the permission to access objects. In this setting, asserting that subject a has role A comes down to asserting that r holds for a and A, whereas asserting that role A possesses permission p_i on object b comes down to asserting that p_i holds for b and A. It is possible to refine the RBAC model by including the concept of role hierarchy which allows permissions to be inherited through it. This hierarchy is specified by means of assertions of the form $A'\sqsubseteq A''$ where A' and A'' are roles. To put it simply, the idea behind RBAC is the following: in a computer system, subject a possesses a permission p on object b if and only if there are roles $A_0,\ldots,A_m$ such that r holds for a and A_0, for all positive integers $i\leq m$, $A_{i-1}\sqsubseteq A_i$ has been asserted and p holds for b and A_m.

RBAC with a lot of objects is space-consuming. To reduce the cost of security, within the context of organization-based access control (OrBAC), it has been proposed that RBAC administrators treat sets of objects as instances of a concept called view [1]. Formally, an OrBAC-structure consists of a set of subjects, a set of objects, a set of roles, a set of views, a binary relation r between subjects and roles defining the roles of subjects, a binary relation v between objects and views defining the views of objects and binary relations $(p_i)_{i\in I}$

[1]The roles in RBAC should not be mistaken for the roles in description logics. In RBAC security policies, roles correspond to sets of subjects, whereas in description logic frames, roles correspond to binary relations.

between views and roles giving to roles the permission to access views. In this setting, asserting that object b has view B comes down to asserting that v holds for b and B, whereas asserting that role A possesses permission p_i on view B comes down to asserting that p_i holds for B and A. It is possible to refine the OrBAC model by including the concept of view hierarchy which allows permissions to be inherited through it. This hierarchy is specified by means of assertions of the form $B' \sqsubseteq B''$ where B' and B'' are views. To put it simply, the idea behind OrBAC is the following: in a computer system, subject a possesses a permission p on object b if and only if there are roles $A_0, \ldots, A_m$ and there are views $B_0, \ldots, B_n$ such that r holds for a and A_0 and v holds for b and B_0, for all positive integers $i \leq m$, $A_{i-1} \sqsubseteq A_i$ has been asserted and for all positive integers $j \leq n$, $B_{j-1} \sqsubseteq B_j$ has been asserted and p holds for B_n and A_m.

It is a great pity that neither RBAC, nor OrBAC allow atomic assertions of the form $p_i(D, C)$ where C and D are, respectively, Boolean combinations of roles and Boolean combinations of views. By using assertions of that form, one may more succinctly define more precise access control policies. For instance, to say that subjects having the role A but not having the role A' possess a permission p_i on objects having the view B but not having the view B', one can simply assert that p_i holds for $B \wedge \neg B'$ and $A \wedge \neg A'$ instead of asserting that p_i holds for B'' and A'' where A'' is a new role such that for all subjects a, $r(a, A'')$ if and only if $r(a, A)$ and not $r(a, A')$ and B'' is a new view such that for all objects b, $v(b, B'')$ if and only if $v(b, B)$ and not $v(b, B')$.

Finally, it is also a great pity that neither RBAC, nor OrBAC allow conditional assertions of the form $p_i(D, C) \leftarrow p_j(D', C')$. By using conditional assertions of that form, one may more succinctly define more precise access control policies. For instance, to say that subjects having the role C possess a permission p_i on objects having the view D if subjects having the role C' possess a permission p_j on objects having the view D', one can simply say that $p_i(D, C) \leftarrow p_j(D', C')$. This is particularly interesting when p_j does not denote a permission, but an obligation corresponding to the permission denoted by p_i[2]. In that case, a conditional assertion like $p_i(D, C) \leftarrow p_j(D, C)$ expresses the deontic rule saying that subjects having the role C possess the permission p_i on objects having the view D if subjects having the role C possess the corresponding obligation p_j on objects having the view D.

Knowledge representation languages such as those provided by description logic languages [4] (allowing expressions of the form $C \sqsubseteq D$ where C and D are complex concepts) and rule-based reasoning paradigms such as those provided by logic programming languages [24], [31] (allowing expressions of the form $\alpha \leftarrow \beta_1, \ldots, \beta_n$ where α, β_1, ..., β_n are atoms) are well-known and widely used in Computer Science and Artificial Intelligence. Their integration in a unique formalism would be a natural solution for many application problems requiring the following features: allowing rule-based systems to use the vocabulary specified in ontologies and supplementing ontological definitions by rules. Hybrid knowledge bases are the main approaches proposed so far. They integrate some aspects of description logic and some aspects of logic programming [21], [22], [29], [32]. Nevertheless, they hardly address all aspects of our aim: the development of an hybrid formalism where description logics constructs are used for defining concepts that are given as arguments to the predicates of the logic programs.

III. Syntax

We introduce the syntax of our hybrid formalism.

A. Complex concepts

Let $\mathbf{VAR}$ be a countable set of *variable concepts* (with typical members denoted X, Y, etc). Let $\mathbf{CON}$ be a countable set of *constant concepts* (with typical members denoted A, B, etc) and $\mathbf{ROL}$ be a countable set of *constant roles* (with typical members denoted R, S, etc). The set of *complex concepts* (with typical members denoted C, D, etc) is defined by the rule[3]

- $C ::= X \mid A \mid \top \mid (C \sqcap D) \mid \exists R.C$,

where X ranges over $\mathbf{VAR}$, A ranges over $\mathbf{CON}$ and R ranges over $\mathbf{ROL}$. We adopt standard rules for omission of the parentheses. A complex concept C is $\mathbf{VAR}$-*free* if C contains no occurrence of a variable concept. A complex concept C is $\mathbf{ROL}$-*free* if C contains no occurrence of a constant role. For all $k \in \mathbb{N}$, the concept construct $(\exists R.)^k$ is inductively defined as follows for each $R \in \mathbf{ROL}$:

- if $k=0$ then $(\exists R.)^k C ::= C$,
- otherwise, $(\exists R.)^k C ::= \exists R.(\exists R.)^{k-1} C$.

B. Substitutions

A *substitution* is a function from $\mathbf{VAR}$ to the set of all complex concepts almost everywhere equal to the identity function [8]. To apply a substitution σ to a complex concept C amounts to replace each occurrence in C of a variable concept $X \in \mathbf{VAR}$ by the corresponding complex concept $\sigma(X)$.

C. Inclusions and equations

Concept inclusions are expressions of the form $C \sqsubseteq D$ (read "C is contained in D") for each complex concepts C, D. *Concept equations* are expressions of the form $C = D$ (read "C is equal to D") for each complex concepts C, D.

[2]We are assuming the deontic principle saying that permissions are implied by their corresponding obligations [34].

[3]The set of complex concepts we define here is the one of description logic $\mathcal{EL}$ [7]. Most of our definitions can be easily adapted to cases where other description logics are considered instead of description logic $\mathcal{EL}$ [5], [7], [14].

D. Clauses

Let $\mathbf{PRE}$ be a countable set of *predicate symbols* (with typical members denoted p, q, etc). For all $p \in \mathbf{PRE}$, let $\mathtt{ar}(p)$ be the *arity* of p. An *atom* is an expression of the form $p(C_1, \ldots, C_{\mathtt{ar}(p)})$ (read "p holds for C_1, ..., $C_{\mathtt{ar}(p)}$") where p is a predicate symbol and C_1, ..., $C_{\mathtt{ar}(p)}$ are complex concepts. *Clauses* are expressions of the form $\alpha_1, \ldots, \alpha_m \leftarrow \beta_1, \ldots, \beta_n$ (read "if β_1, ..., β_n then either α_1, ..., or α_m") where $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_n$ are atoms. *Definite clauses* are clauses of the form $\alpha \leftarrow \beta_1, \ldots, \beta_n$, *unit clauses* are clauses of the form $\alpha \leftarrow$ and *definite goals* are clauses of the form $\leftarrow \beta_1, \ldots, \beta_n$.

E. Assertions

Let $\mathbf{IND}$ be a countable set of *individual constants* (with typical members denoted a, b, etc). A *concept assertion* is an expression of the form $C{:}a$ (read "a belongs to C") where C is a $\mathbf{VAR}$-free complex concept and a is an individual constant. A *role assertion* is an expression of the form $R{:}(a, b)$ (read "a is R-related to b") where $R \in \mathbf{ROL}$ and a and b are individual constants.

F. Deductive ontologies

A *T-box* is a finite set of concept inclusions and concept equations. A *program* is a finite set of clauses. An *A-box* is a finite set of concept assertions and role assertions. A *deductive ontology* is a triple $(\mathcal{T}, \Pi, \mathcal{A})$ consisting of a T-box $\mathcal{T}$, a program Π and an A-box $\mathcal{A}$.

IV. Semantics

We introduce the semantics of our hybrid formalism[4].

A. Frames and var-interpretations

The semantics is defined in terms of *frames,* i.e. structures (W, K, Rel) where W is a nonempty set, K: $\mathbf{CON} \longrightarrow \mathcal{P}(W)$ and Rel: $\mathbf{ROL} \longrightarrow \mathcal{P}(W \times W)$. In a frame (W, K, Rel), for all $R \in \mathbf{ROL}$,

- the *R-image* of a subset S of W is the set of all $t \in W$ such that there exists $s \in S$ such that $Rel(R)(s, t)$,
- the *R-pre-image* of a subset T of W is the set of all $s \in W$ such that there exists $t \in T$ such that $Rel(R)(s, t)$,
- the *domain* of R is the set of all $s \in W$ such that there exists $t \in W$ such that $Rel(R)(s, t)$,
- the *range* of R is the set of all $t \in W$ such that there exists $s \in W$ such that $Rel(R)(s, t)$.

Obviously, in a frame (W, K, Rel), for all $R \in \mathbf{ROL}$, the domain of R is the R-pre-image of W and the range of R is the R-image of W. A *var-interpretation* on a frame (W, K, Rel) is a function V: $\mathbf{VAR} \longrightarrow \mathcal{P}(W)$. For all frames (W, K, Rel), the value of the complex concept C with respect to a var-interpretation V on (W, K, Rel) is the subset $\|C\|_V$ of W defined by

- $\|X\|_V = V(X)$,
- $\|A\|_V = K(A)$,
- $\|\top\|_V = W$,
- $\|C \sqcap D\|_V = \|C\|_V \cap \|D\|_V$,
- $\|\exists R.C\|_V = \{s \in W$: there exists $t \in W$ such that $Rel(R)(s, t)$ and $t \in \|C\|_V\}$.

Obviously, $\|C\|_V$ does not depend on V when C is $\mathbf{VAR}$-free. In that case, $\|C\|_V$ will be denoted $\|C\|$.

B. Pre-interpretations

A *pre-interpretation* on a frame (W, K, Rel) is a function I: $\mathbf{PRE} \longrightarrow \mathcal{P}(\mathcal{P}(W)^\star)$ such that for all $p \in \mathbf{PRE}$, $I(p) \subseteq \mathcal{P}(W)^{\mathtt{ar}(p)}$. For all frames (W, K, Rel) and for all pre-interpretations I on (W, K, Rel), the *value* of an atom $p(C_1, \ldots, C_{\mathtt{ar}(p)})$ with respect to a var-interpretation V on (W, K, Rel) is the element $|p(C_1, \ldots, C_{\mathtt{ar}(p)})|^I_V$ in $\{0, 1\}$ such that

- if $I(p)$ contains $(\|C_1\|_V, \ldots, \|C_{\mathtt{ar}(p)}\|_V)$ then $|p(C_1, \ldots, C_{\mathtt{ar}(p)})|^I_V = 1$,
- otherwise, $|p(C_1, \ldots, C_{\mathtt{ar}(p)})|^I_V = 0$.

C. Ind-interpretations

An *ind-interpretation* on a frame (W, K, Rel) is a function g: $\mathbf{IND} \longrightarrow W$. For all frames (W, K, Rel) and for all ind-interpretations g on (W, K, Rel), the *value* of a concept assertion $C{:}a$ is the element $|C{:}a|^g$ in $\{0, 1\}$ such that

- if $\|C\|$ contains $g(a)$ then $|C{:}a|^g = 1$,
- otherwise, $|C{:}a|^g = 0$,

and the *value* of a role assertion $R{:}(a, b)$ is the element $|R{:}(a, b)|^g$ in $\{0, 1\}$ such that

- if $Rel(R)$ contains $(g(a), g(b))$ then $|R{:}(a, b)|^g = 1$,
- otherwise, $|R{:}(a, b)|^g = 0$.

D. Models

For all T-boxes $\mathcal{T}$, a *$\mathcal{T}$-model* (or a *model* of $\mathcal{T}$) is a frame (W, K, Rel) such that for all var-interpretations V on (W, K, Rel),

- for all concept inclusions $C \sqsubseteq D$ in $\mathcal{T}$, $\|C\|_V \subseteq \|D\|_V$,
- for all concept equations $C = D$ in $\mathcal{T}$, $\|C\|_V = \|D\|_V$.

For all deductive ontologies $(\mathcal{T}, \Pi, \mathcal{A})$, a *$(\mathcal{T}, \Pi, \mathcal{A})$-model* (or a *model* of $(\mathcal{T}, \Pi, \mathcal{A})$) is a structure (W, K, Rel, I, g) consisting of a $\mathcal{T}$-model (W, K, Rel), a pre-interpretation I on (W, K, Rel) and an ind-interpretation g on (W, K, Rel) such that for all var-interpretations V on (W, K, Rel),

- for all clauses $\alpha_1, \ldots, \alpha_m \leftarrow \beta_1, \ldots, \beta_n$ in Π, if $|\beta_1|^I_V = 1$, ..., $|\beta_n|^I_V = 1$ then either $|\alpha_1|^I_V = 1$, ..., or $|\alpha_m|^I_V = 1$,
- for all concept assertions $C{:}a$ in $\mathcal{A}$, $|C{:}a|^g = 1$,
- for all role assertions $R{:}(a, b)$ in $\mathcal{A}$, $|R{:}(a, b)|^g = 1$.

Notice that in a model (W, K, Rel, I, g) of a deductive ontology $(\mathcal{T}, \Pi, \mathcal{A})$, for all var-interpretations V on (W, K, Rel),

- for all definite clauses $\alpha \leftarrow \beta_1, \ldots, \beta_n$ in Π, if $|\beta_1|^I_V = 1$, ..., $|\beta_n|^I_V = 1$ then $|\alpha|^I_V = 1$,
- for all unit clauses $\alpha \leftarrow$ in Π, $|\alpha|^I_V = 1$,
- for all definite goals $\leftarrow \beta_1, \ldots, \beta_n$ in Π, either $|\beta_1|^I_V = 0$, ..., or $|\beta_n|^I_V = 0$.

[4] In this paper, for all sets E, $\mathcal{P}(E)$ denotes the set of all subsets of E, $E^\star$ denotes the set of all tuples of elements of E and for all $k \in \mathbb{N}$, E^k denotes the set of all k-tuples of elements of E.

V. Correspondence theory

We briefly present the correspondence theory of our hybrid formalism.

Although of limited expressive power, concept constructs can be used for characterizing classes of frames. As observed by [5], [36], description logic languages are modal languages in disguise. Therefore, the following relationships that can be easily established for all frames (W, K, Rel) will not come as a surprise:

- (**1**) (W, K, Rel) is a model of $X \sqsubseteq \exists R.\top$ if and only if $Rel(R)$ is serial[5],
- (**2**) (W, K, Rel) is a model of $\exists R.\top \sqsubseteq X$ if and only if $Rel(R)$ is empty,
- (**3**) (W, K, Rel) is a model of $X \sqsubseteq \exists R.X$ if and only if $Rel(R)$ is reflexive[6],
- (**4**) (W, K, Rel) is a model of $\exists R.X \sqsubseteq X$ if and only if $Rel(R)$ is included in the identity relation on W,
- (**5**) (W, K, Rel) is a model of $\exists R.X \sqsubseteq \exists R.\exists R.X$ if and only if $Rel(R)$ is dense[7],
- (**6**) (W, K, Rel) is a model of $\exists R.\exists R.X \sqsubseteq \exists R.X$ if and only if $Rel(R)$ is transitive[8],
- (**7**) (W, K, Rel) is a model of $\exists R.\exists R.\top \sqsubseteq X$ if and only if the R-pre-image of the R-pre-image of W is empty,
- (**8**) (W, K, Rel) is a model of $\exists R.X = \exists S.X$ if and only if $Rel(R)$ is equal to $Rel(S)$,
- (**9**) (W, K, Rel) is a model of $A \sqcap B \sqsubseteq X$ if and only if $K(A)$ and $K(B)$ do not intersect,
- (**10**) (W, K, Rel) is a model of $A \sqsubseteq B$ if and only if $K(A)$ is included in $K(B)$,
- (**11**) (W, K, Rel) is a model of $\exists R.X \sqsubseteq A$ if and only if the domain of R is included in $K(A)$,
- (**12**) (W, K, Rel) is a model of $\exists R.X \sqsubseteq \exists R.(X \sqcap A)$ if and only if the range of R is included in $K(A)$.

Within our setting, elementary conditions — like "$Rel(R)$ is serial", "$Rel(R)$ is empty", etc — are first-order conditions that can be expressed as sentences in a function-free first-order language with equality based on a set of unary predicate symbols in one-to-one correspondence with **CON** and a set of binary predicate symbols in one-to-one correspondence with **ROL**. As a result, the following decision problems are of interest:

— deciding elementary definability (**DED**)
input: a T-box $\mathcal{T}$,
output:determine whether there exists an elementary condition F such that for all frames (W, K, Rel), F holds in (W, K, Rel) if and only if (W, K, Rel) is a model of $\mathcal{T}$,

— deciding concept definability (**DCD**)
input: an elementary condition F,
output:determine whether there exists a T-box $\mathcal{T}$ such that for all frames (W, K, Rel), (W, K, Rel) is a model of $\mathcal{T}$ if and only if F holds in (W, K, Rel),

— deciding elementary equivalence (**DEE**)
input: a T-box $\mathcal{T}$ and an elementary condition F,
output:determine whether for all frames (W, K, Rel), (W, K, Rel) is a model of $\mathcal{T}$ if and only if F holds in (W, K, Rel).

DED, **DCD** and **DEE** stem from the corresponding definability problems in modal logics [15]. It is not known whether **DED**, **DCD** and **DEE** are decidable[9].

VI. Deciding inclusions and equations

We present decision problems about concept inclusions and concept equations.

Let $\mathcal{T}$ be a T-box.

A concept inclusion $C \sqsubseteq D$ is a *logical consequence* of $\mathcal{T}$ (denoted $\mathcal{T} \models C \sqsubseteq D$) if for all $\mathcal{T}$-models (W, K, Rel) and for all var-interpretations V on (W, K, Rel), $\|C\|_V \subseteq \|D\|_V$. A concept equation $C = D$ is a *logical consequence* of $\mathcal{T}$ (denoted $\mathcal{T} \models C = D$) if for all $\mathcal{T}$-models (W, K, Rel) and for all var-interpretations V on (W, K, Rel), $\|C\|_V = \|D\|_V$. As a result, the following decision problems are of interest:

— deciding concept inclusions (**DCI**)
input: a concept inclusion $C \sqsubseteq D$,
output:determine whether $\mathcal{T} \models C \sqsubseteq D$,

— deciding concept equations (**DCE**)
input: a concept equation $C = D$,
output:determine whether $\mathcal{T} \models C = D$.

If $\mathcal{T}$ is **VAR**-free then **DCI** and **DCE** are in **P** [3][10]. Otherwise, it is not known whether **DCI** and **DCE** are decidable.

VII. Deciding consequences and answers

We present decision problems about logical consequences and correct answers.

Let $(\mathcal{T}, \Pi, \mathcal{A})$ be a deductive ontology.

A clause $\alpha_1, \ldots, \alpha_m \leftarrow \beta_1, \ldots, \beta_n$ is a *logical consequence* of $(\mathcal{T}, \Pi, \mathcal{A})$ (denoted $(\mathcal{T}, \Pi, \mathcal{A}) \models \alpha_1, \ldots, \alpha_m \leftarrow \beta_1, \ldots, \beta_n$) if for all $(\mathcal{T}, \Pi, \mathcal{A})$-models (W, K, Rel, I, g) and for all var-interpretations V on (W, K, Rel), if $|\beta_1|^I_V = 1$, …, $|\beta_n|^I_V = 1$ then either $|\alpha_1|^I_V = 1$, …, or $|\alpha_m|^I_V = 1$. Notice that a definite clause $\alpha \leftarrow \beta_1, \ldots, \beta_n$ is a logical consequence of $(\mathcal{T}, \Pi, \mathcal{A})$ if and only if for all $(\mathcal{T}, \Pi, \mathcal{A})$-models (W, K, Rel, I, g) and

[5]That is to say, for all $s \in W$, there exists $t \in W$ such that $Rel(R)(s, t)$.

[6]That is to say, for all $s \in W$, $Rel(R)(s, s)$.

[7]That is to say, for all $s, t \in W$, if $Rel(R)(s, t)$ then there exists $u \in W$ such that $Rel(R)(s, u)$ and $Rel(R)(u, t)$.

[8]That is to say, for all $s, t \in W$, if there exists $u \in W$ such that $Rel(R)(s, u)$ and $Rel(R)(u, t)$ then $Rel(R)(s, t)$.

[9]Description logic languages being modal languages in disguise [5], [36], the undecidability of **DED**, **DCD** and **DEE** are immediate consequences of Chagrova's Theorems [15] when description logic $\mathcal{ALC}$ is considered instead of description logic $\mathcal{EL}$.

[10]See [20] when other description logics are considered instead of description logic $\mathcal{EL}$.

for all var-interpretations V on (W, K, Rel), if $|\beta_1|^I_V{=}1$, ..., $|\beta_n|^I_V{=}1$ then $|\alpha|^I_V{=}1$, a unit clause $\alpha\leftarrow$ is a logical consequence of $(\mathcal{T}, \Pi, \mathcal{A})$ if and only if if for all $(\mathcal{T}, \Pi, \mathcal{A})$-models (W, K, Rel, I, g) and for all var-interpretations V on (W, K, Rel), $|\alpha|^I_V{=}1$ and a definite goal $\leftarrow\beta_1, \ldots, \beta_n$ is a logical consequence of $(\mathcal{T}, \Pi, \mathcal{A})$ if and only if for all $(\mathcal{T}, \Pi, \mathcal{A})$-models (W, K, Rel, I, g) and for all var-interpretations V on (W, K, Rel), either $|\beta_1|^I_V{=}0$, ..., or $|\beta_n|^I_V{=}0$. As a result, the following decision problems are of interest:

— deciding definite clauses (**DDC**)
input: a definite clause $\alpha\leftarrow\beta_1, \ldots, \beta_n$,
output:determine whether $(\mathcal{T}, \Pi, \mathcal{A})\models\alpha\leftarrow\beta_1, \ldots, \beta_n$,

— deciding unit clauses (**DUC**)
input: a unit clause $\alpha\leftarrow$,
output:determine whether $(\mathcal{T}, \Pi, \mathcal{A})\models\alpha\leftarrow$,

— deciding definite goals (**DDG**)
input: a definite goal $\leftarrow\beta_1, \ldots, \beta_n$,
output:determine whether $(\mathcal{T}, \Pi, \mathcal{A})\models\leftarrow\beta_1, \ldots, \beta_n$.

A substitution σ is a *correct answer* for the definite goal $\leftarrow\beta_1, \ldots, \beta_n$ with respect to $(\mathcal{T}, \Pi, \mathcal{A})$ if for all $(\mathcal{T}, \Pi, \mathcal{A})$-models (W, K, Rel, I, g) and for all var-interpretations V on (W, K, Rel), $|\sigma(\beta_1)|^I_V{=}1$, ..., $|\sigma(\beta_n)|^I_V{=}1$. As a result, the following decision problem is of interest:

— deciding correct answers (**DCA**)
input: a definite goal $\leftarrow\beta_1, \ldots, \beta_n$,
output:determine whether there exists a correct answer for $\leftarrow\beta_1, \ldots, \beta_n$ with respect to $(\mathcal{T}, \Pi, \mathcal{A})$.

DDC, **DUC**, **DDG** and **DCA** stem from the corresponding derivability problems in logic programming [24], [31]. It is not known whether **DDC**, **DUC**, **DDG** and **DCA** are decidable[11].

VIII. Examples

We introduce 4 examples illustrating some technical aspects of deductive ontologies: bounded recursion, unbounded recursion, non-unifiability and unifiability. We introduce as well an example about OrBAC.

A. An example about unbounded recursion

Let $(\mathcal{T}_1, \Pi_1, \mathcal{A}_1)$ be the deductive ontology where

- $\mathcal{T}_1$ is the empty T-box,
- Π_1 is the program containing the following clauses:
  - $p(\top)\leftarrow$,
  - $p(\exists R.X)\leftarrow p(X)$,
- $\mathcal{A}_1$ is the empty A-box.

Models of $\mathcal{T}_1$ are arbitrary frames. Models of $(\mathcal{T}_1, \Pi_1, \mathcal{A}_1)$ are structures (W, K, Rel, I, g) consisting of an arbitrary frame (W, K, Rel), a pre-interpretation I on (W, K, Rel) such that for all subsets U of W,

- W is in $I(p)$,
- if U is in $I(p)$ then the R-pre-image of U is in $I(p)$,

and an arbitrary ind-interpretation g on (W, K, Rel). It follows that for all complex concepts C, $(\mathcal{T}_1, \Pi_1, \mathcal{A}_1)\models p(C)\leftarrow$ if and only if there exists $k\in\mathbb{N}$ such that $\mathcal{T}_1\models C{=}(\exists R.)^k\top$.

[11]When description logic $\mathcal{ALC}$ is considered instead of description logic $\mathcal{EL}$, the undecidability of **DDC**, **DUC**, **DDG** and **DCA** can be easily proved by means of reductions from the undecidability of the reachability problem in Minsky machines.

B. An example about bounded recursion

Let $(\mathcal{T}_2, \Pi_2, \mathcal{A}_2)$ be the deductive ontology where

- $\mathcal{T}_2$ is the T-box containing the following concept inclusion:
  - $\exists R.\exists R.\top\sqsubseteq X$,
- Π_2 is the program containing the following clauses:
  - $p(\top)\leftarrow$,
  - $p(\exists R.X)\leftarrow p(X)$,
- $\mathcal{A}_2$ is the empty A-box.

Models of $\mathcal{T}_2$ are frames (W, K, Rel) such that[12]

- the R-pre-image of the R-pre-image of W is empty.

Models of $(\mathcal{T}_2, \Pi_2, \mathcal{A}_2)$ are structures (W, K, Rel, I, g) consisting of a frame (W, K, Rel) such that the R-pre-image of the R-pre-image of W is empty, a pre-interpretation I on (W, K, Rel) such that for all subsets U on W,

- W is in $I(p)$,
- if U is in $I(p)$ then the R-pre-image of U is in $I(p)$,

and an arbitrary ind-interpretation g on (W, K, Rel). It follows that for all complex concepts C, $(\mathcal{T}_2, \Pi_2, \mathcal{A}_2)\models p(C)\leftarrow$ if and only if either $\mathcal{T}_2\models C{=}\top$, or $\mathcal{T}_2\models C{=}\exists R.\top$, or $\mathcal{T}_2\models C{=}\exists R.\exists R.\top$.

C. An example about non-unifiability

Let $(\mathcal{T}_3, \Pi_3, \mathcal{A}_3)$ be the deductive ontology where

- $\mathcal{T}_3$ is the empty T-box,
- Π_3 is the program containing the following clauses:
  - $p(X)\leftarrow q(X), r(X)$,
  - $q(\exists Q.X)\leftarrow$,
  - $r(\exists R.X)\leftarrow$,
- $\mathcal{A}_3$ is the empty A-box.

Models of $\mathcal{T}_3$ are arbitrary frames. Models of $(\mathcal{T}_3, \Pi_3, \mathcal{A}_3)$ are structures (W, K, Rel, I, g) consisting of an arbitrary frame (W, K, Rel), a pre-interpretation I on (W, K, Rel) such that for all subsets U of W,

- if U is in $I(q)$ and U is in $I(r)$ then U is in $I(p)$,
- the Q-pre-image of U is in $I(q)$,
- the R-pre-image of U is in $I(r)$,

and an arbitrary ind-interpretation g on (W, K, Rel). It follows that for all complex concepts C, $(\mathcal{T}_3, \Pi_3, \mathcal{A}_3)\models q(C)\leftarrow$ if and only if there exists a complex concept D such that $\mathcal{T}_3\models C{=}\exists Q.D$ and for all complex concepts C, $(\mathcal{T}_3, \Pi_3, \mathcal{A}_3)\models r(C)\leftarrow$ if and only if there exists a complex concept D such that $\mathcal{T}_3\models C{=}\exists R.D$. Moreover, for no complex concept C, $(\mathcal{T}_3, \Pi_3, \mathcal{A}_3)\models p(C)\leftarrow$.

[12]See the 7th item in Section V.

D. An example about unifiability

Let $(\mathcal{T}_4, \Pi_4, \mathcal{A}_4)$ be the deductive ontology where

- $\mathcal{T}_4$ is the T-box containing the following concept equation:
  - $\exists Q.X{=}\exists R.X$,
- Π_4 is the program containing the following clauses:
  - $p(X){\leftarrow}q(X), r(X)$,
  - $q(\exists Q.X){\leftarrow}$,
  - $r(\exists R.X){\leftarrow}$,
- $\mathcal{A}_4$ is the empty A-box.

Models of $\mathcal{T}_4$ are frames (W, K, Rel) such that[13]

- $Rel(Q){=}Rel(R)$.

Models of $(\mathcal{T}_4, \Pi_4, \mathcal{A}_4)$ are structures (W, K, Rel, I, g) consisting of a frame (W, K, Rel) such that $Rel(Q){=}Rel(R)$, a pre-interpretation I on (W, K, Rel) such that for all subsets U of W,

- if U is in $I(q)$ and U is in $I(r)$ then U is in $I(p)$,
- the Q-pre-image of U is in $I(q)$,
- the R-pre-image of U is in $I(r)$,

and an arbitrary ind-interpretation g on (W, K, Rel). It follows that for all complex concepts C, $(\mathcal{T}_4, \Pi_4, \mathcal{A}_4){\models}q(C){\leftarrow}$ if and only if there exists a complex concept D such that $\mathcal{T}_4{\models}C{=}\exists Q.D$ and for all complex concepts C, $(\mathcal{T}_4, \Pi_4, \mathcal{A}_4){\models}r(C){\leftarrow}$ if and only if there exists a complex concept D such that $\mathcal{T}_4{\models}C{=}\exists R.D$. Moreover, for all complex concepts C, $(\mathcal{T}_4, \Pi_4, \mathcal{A}_4){\models}p(C){\leftarrow}$ if and only if there exists a complex concept D such that $\mathcal{T}_4{\models}C{=}\exists Q.D$ and $\mathcal{T}_4{\models}C{=}\exists R.D$. Of course, since for all $\mathcal{T}_4$-models (W, K, Rel), $Rel(Q){=}Rel(R)$, for all complex concepts D, $\mathcal{T}_4{\models}\exists Q.D{=}\exists R.D$.

E. An example about OrBAC

Let an OrBAC security policy be made up of

- the finite sets I, J, M and N,
- the binary relations PERM, COMP, OPTI and PROH between I and J,
- the binary relation $\mathsf{HasRole}$ between I and M,
- the binary relation $\mathsf{HasView}$ between J and N,
- the binary relation $\mathsf{HasAuthorityOn}$ on M,
- the binary relation $\mathsf{ContainsAsSubpart}$ on N.

The finite sets I, J, M and N are, respectively, the set of all roles, the set of all views, the set of all subjects and the set of all objects[14]. The binary relations PERM, COMP, OPTI and PROH between I and J correspond to the following assertions:

- "the security policy permits the i-th role to access the j-th view" for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{PERM}(\mathsf{i},\mathsf{j})$,
- "the security policy makes it compulsory for the i-th role to access the j-th view" for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{COMP}(\mathsf{i},\mathsf{j})$,
- "the security policy makes it optional for the i-th role to access the j-th view" for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{OPTI}(\mathsf{i},\mathsf{j})$,
- "the security policy prohibits the i-th role from accessing the j-th view" for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{PROH}(\mathsf{i},\mathsf{j})$.

It may happen that subjects have roles and objects have views. In this respect, the binary relation $\mathsf{HasRole}$ between I and M and the binary relation $\mathsf{HasView}$ between J and N correspond to the following assertions:

- "the m-th subject has the i-th role" for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{m}{\in}\mathsf{M}$ such that $\mathsf{HasRole}(\mathsf{i},\mathsf{m})$,
- "the n-th object has the j-th view" for each $\mathsf{j}{\in}\mathsf{J}$ and for each $\mathsf{n}{\in}\mathsf{N}$ such that $\mathsf{HasView}(\mathsf{j},\mathsf{n})$,

It may happen that subjects have authority on other subjects and objects contain other objects as subpart. In this respect, the binary relation $\mathsf{HasAuthorityOn}$ on M and the binary relation $\mathsf{ContainsAsSubpart}$ on N correspond to the following assertions:

- "the m-th subject has authority on the m′-th subject" for each $\mathsf{m},\mathsf{m}'{\in}\mathsf{M}$ such that $\mathsf{HasAuthorityOn}(\mathsf{m},\mathsf{m}')$,
- "the n-th object contains the n′-th object as subpart" for each $\mathsf{n},\mathsf{n}'{\in}\mathsf{N}$ such that $\mathsf{ContainsAsSubpart}(\mathsf{n},\mathsf{n}')$.

To express the above assertions, we will use

- the constant concepts $\mathsf{Subject}$, Object, $\mathsf{A_i}$ for each $\mathsf{i}{\in}\mathsf{I}$ and $\mathsf{B_j}$ for each $\mathsf{j}{\in}\mathsf{J}$,
- the constant roles $\mathsf{hasAuthorityOn}$ and $\mathsf{containsAsSubpart}$,
- the predicate symbols perm, comp, opti and proh of arity 2,
- the individual constants $\mathsf{a_m}$ for each $\mathsf{m}{\in}\mathsf{M}$ and $\mathsf{b_n}$ for each $\mathsf{n}{\in}\mathsf{N}$.

Let $(\mathcal{T}_5, \Pi_5, \mathcal{A}_5)$ be the deductive ontology where

- $\mathcal{T}_5$ is the T-box containing the following concept inclusions:
  - $(\mathbf{T_1})$ $\mathsf{Subject}{\sqcap}\mathsf{Object}{\sqsubseteq}X$,
  - $(\mathbf{T_2})$ $\mathsf{A_i}{\sqsubseteq}\mathsf{Subject}$ for each $\mathsf{i}{\in}\mathsf{I}$,
  - $(\mathbf{T_3})$ $\mathsf{B_j}{\sqsubseteq}\mathsf{Object}$ for each $\mathsf{j}{\in}\mathsf{J}$,
  - $(\mathbf{T_4})$ $\exists\mathsf{hasAuthorityOn}.X{\sqsubseteq}\mathsf{Subject}{\sqcap}\exists\mathsf{hasAuthorityOn}.(X{\sqcap}\mathsf{Subje}$
  - $(\mathbf{T_5})$ $\exists\mathsf{containsAsSubpart}.X{\sqsubseteq}\mathsf{Object}{\sqcap}\exists\mathsf{containsAsSubpart}.(X{\sqcap}\mathsf{C}$
- Π_5 is the program containing the following clauses:
  - $(\mathbf{DP_1})$ $\mathsf{perm}(\mathsf{A_i},\mathsf{B_j}){\leftarrow}$ for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{PERM}(\mathsf{i},\mathsf{j})$,
  - $(\mathbf{DP_2})$ $\mathsf{comp}(\mathsf{A_i},\mathsf{B_j}){\leftarrow}$ for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{COMP}(\mathsf{i},\mathsf{j})$,
  - $(\mathbf{DP_3})$ $\mathsf{opti}(\mathsf{A_i},\mathsf{B_j}){\leftarrow}$ for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{OPTI}(\mathsf{i},\mathsf{j})$,
  - $(\mathbf{DP_4})$ $\mathsf{proh}(\mathsf{A_i},\mathsf{B_j}){\leftarrow}$ for each $\mathsf{i}{\in}\mathsf{I}$ and for each $\mathsf{j}{\in}\mathsf{J}$ such that $\mathsf{PROH}(\mathsf{i},\mathsf{j})$,
  - $(\mathbf{DP_5})$ $\mathsf{perm}(X,Y){\leftarrow}\mathsf{comp}(X,Y)$,
  - $(\mathbf{DP_6})$ $\mathsf{opti}(X,Y){\leftarrow}\mathsf{proh}(X,Y)$,
- $\mathcal{A}_5$ is the A-box containing the following assertions:
  - $(\mathbf{AB_1})$ $\mathsf{Subject}{:}\mathsf{a_m}$ for each $\mathsf{m}{\in}\mathsf{M}$,
  - $(\mathbf{AB_2})$ $\mathsf{Object}{:}\mathsf{b_n}$ for each $\mathsf{n}{\in}\mathsf{N}$,

[13] See the 8th item in Section V.

[14] See Section II for a definition of the words "roles" and "views" within the context of OrBAC.

($\mathbf{AB_3}$) $\mathsf{A_i}$:$\mathsf{a_m}$ for each $\mathsf{i}\in\mathsf{I}$ and for each $\mathsf{m}\in\mathsf{M}$ such that $\mathsf{HasRole(i, m)}$,
($\mathbf{AB_4}$) $\mathsf{B_j}$:$\mathsf{b_n}$ for each $\mathsf{j}\in\mathsf{J}$ and for each $\mathsf{n}\in\mathsf{N}$ such that $\mathsf{HasView(j, n)}$,
($\mathbf{AB_5}$) $\mathsf{hasAuthorityOn}$:$(\mathsf{a_m}, \mathsf{a_{m'}})$ for each $\mathsf{m}, \mathsf{m'}\in\mathsf{M}$ such that $\mathsf{HasAuthorityOn(m, m')}$,
($\mathbf{AB_6}$) $\mathsf{containsAsSubpart}$:$(\mathsf{b_n}, \mathsf{b_{n'}})$ for each $\mathsf{n}, \mathsf{n'}\in\mathsf{M}$ such that $\mathsf{ContainsAsSubpart(n, n')}$.

($\mathbf{T_1}$) says that the set denoted by Subject and the set denoted by Object are disjoint. ($\mathbf{T_2}$) says that the set denoted by $\mathsf{A_i}$ is included in the set denoted by Subject for each $\mathsf{i}\in\mathsf{I}$. ($\mathbf{T_3}$) says that the set denoted by $\mathsf{B_j}$ is included in the set denoted by Object for each $\mathsf{j}\in\mathsf{J}$. ($\mathbf{T_4}$) says that the domain and the range of the binary relation denoted by hasAuthorityOn is included in the set denoted by Subject. ($\mathbf{T_5}$) says that the domain and the range of the binary relation denoted by containsAsSubpart is included in the set denoted by Object. Indeed, models of $\mathcal{T}_5$ are frames (W, K, Rel) such that[15]

- $K(\mathsf{Subject})$ and $K(\mathsf{Object})$ do not intersect,
- $K(\mathsf{A_i})$ is included in $K(\mathsf{Subject})$ for each $\mathsf{i}\in\mathsf{I}$,
- $K(\mathsf{B_j})$ is included in $K(\mathsf{Object})$ for each $\mathsf{j}\in\mathsf{J}$,
- the domain and range of hasAuthorityOn are included in $K(\mathsf{Subject})$,
- the domain and range of containsAsSubpart are included in $K(\mathsf{Object})$.

($\mathbf{DP_1}$), ($\mathbf{DP_2}$), ($\mathbf{DP_3}$) and ($\mathbf{DP_4}$) express the assertions corresponding to the relations PERM, COMP, OPTI and PROH. ($\mathbf{DP_5}$) is the deontic principle saying that every compulsory access is permitted. ($\mathbf{DP_6}$) is the deontic principle saying that every prohibited access is optional. ($\mathbf{AB_1}$) says that $\mathsf{a_m}$ denotes a subject for each $\mathsf{m}\in\mathsf{M}$. ($\mathbf{AB_2}$) says that $\mathsf{b_n}$ denotes an object for each $\mathsf{n}\in\mathsf{N}$. ($\mathbf{AB_3}$) and ($\mathbf{AB_4}$) are the concept assertions corresponding to the relations HasRole and HasView. ($\mathbf{AB_5}$) and ($\mathbf{AB_6}$) are the role assertions corresponding to the relations HasAuthorityOn and ContainsAsSubpart. Within the context of this example, for all $\mathsf{m}\in\mathsf{M}$ and for all $\mathsf{n}\in\mathsf{N}$, we will say that

- the security policy permits the m-th subject to access the n-th object if and only if for all models (W, K, Rel, I, g) of $(\mathcal{T}_5, \Pi_5, \mathcal{A}_5)$ and for all $\mathbf{VAR}$-interpretations V on (W, K, Rel), there exists a complex concept C and there exists a complex concept D such that $g(\mathsf{a_m})\in\|C\|_V$, $g(\mathsf{b_n})\in\|D\|_V$ and $|\mathsf{perm}(C, D)|^I_V=1$,
- the security policy makes it compulsory for the m-th subject to access the n-th object if and only if for all models (W, K, Rel, I, g) of $(\mathcal{T}_5, \Pi_5, \mathcal{A}_5)$ and for all $\mathbf{VAR}$-interpretations V on (W, K, Rel), there exists a complex concept C and there exists a complex concept D such that $g(\mathsf{a_m})\in\|C\|_V$, $g(\mathsf{b_n})\in\|D\|_V$ and $|\mathsf{comp}(C, D)|^I_V=1$,
- the security policy makes it optional for the m-th subject to access the n-th object if and only if for all models (W, K, Rel, I, g) of $(\mathcal{T}_5, \Pi_5, \mathcal{A}_5)$ and for all $\mathbf{VAR}$-interpretations V on (W, K, Rel), there exists a complex concept C and there exists a complex concept D such that $g(\mathsf{a_m})\in\|C\|_V$, $g(\mathsf{b_n})\in\|D\|_V$ and $|\mathsf{opti}(C, D)|^I_V=1$,
- the security policy prohibits the m-th subject from accessing the n-th object if and only if for all models (W, K, Rel, I, g) of $(\mathcal{T}_5, \Pi_5, \mathcal{A}_5)$ and for all $\mathbf{VAR}$-interpretations V on (W, K, Rel), there exists a complex concept C and there exists a complex concept D such that $g(\mathsf{a_m})\in\|C\|_V$, $g(\mathsf{b_n})\in\|D\|_V$ and $|\mathsf{proh}(C, D)|^I_V=1$.

To illustrate the expressive power of concept constructs, the following clauses can be added to Π_5:

($\mathbf{DP_7}$) $\mathsf{comp}(\exists\mathsf{hasAuthorityOn}.X, Y)\leftarrow\mathsf{perm}(X, \exists\mathsf{containsAsSubpart}$
($\mathbf{DP_8}$) $\mathsf{proh}(X, \exists\mathsf{containsAsSubpart}.Y)\leftarrow\mathsf{opti}(\exists\mathsf{hasAuthorityOn}.X, Y$

($\mathbf{DP_7}$) says that if the security policy permits the set denoted by X to access the set of objects containing as subpart objects of the set denoted by Y then the security policy makes it compulsory for the set of subjects having authority on subjects of the set denoted by X to access the set of objects denoted by Y. ($\mathbf{DP_8}$) says that if the security policy makes it optional for the set of subjects having authority on subjects of the set denoted by X to access the set denoted by Y then the security policy prohibits the set denoted by X from accessing the set of objects containing as subpart objects of the set denoted by Y. With our without ($\mathbf{DP_7}$) and ($\mathbf{DP_8}$), accesses of subjects to objects should be neither both permitted and prohibited, nor both compulsory and optiona: in most logical models of deontic systems, if it is prohibited to a subject from accessing some object then it is not permitted that this subject accesses that object and if it is made optional for a subject to access some object then it is not made compulsory that this subject accesses that object [34]. For this reason, it is of the utmost importance to check whether one of the following conditions holds:

- there exists a correct answer for the definite goal $\leftarrow\mathsf{perm}(X, Y), \mathsf{proh}(X, Y)$,
- there exists a correct answer for the definite goal $\leftarrow\mathsf{comp}(X, Y), \mathsf{opti}(X, Y)$.

It is also of the utmost importance to check whether there exists $\mathsf{m}\in\mathsf{M}$ and there exists $\mathsf{n}\in\mathsf{N}$ such that one of the following conditions holds:

- the security policy both permits the m-th subject to access the n-th object and prohibits the m-th subject from accessing the n-th object,
- the security policy both makes it compulsory for the m-th subject to access the n-th object and makes it optional for the m-th subject to access the n-th object.

IX. A RESEARCH PROGRAM

We present a research program. As can be seen from its presentation, this research program covers different aspects of description logics and logic programming: recursion theory with ($\mathbf{RP_1}$), computational complexity with ($\mathbf{RP_2}$), model theory and fixpoint theory with ($\mathbf{RP_3}$), automated deduction with ($\mathbf{RP_4}$), non-monotonic reasoning with ($\mathbf{RP_5}$) and ontology engineering techniques with ($\mathbf{RP_6}$) and ($\mathbf{RP_7}$). Needless to say, to carry out it, one must neither work in isolation, nor

[15] See the 9th, 10th, 11th and 12th items in Section V.

lose sight of the possible applications of the hybrid formalism developed in this paper. In other respect, with respect to expressivity, one must also compare this formalism to the main approaches proposed so far. These approaches include the above-mentioned hybrid knowledge bases [21], [22], [29], [32]. They also include approaches such as the existential rule framework [12], [33].

A. Turing-completeness

Our hybrid formalism can be seen as a programming language. It is not known whether it is Turing-complete. When description logic $\mathcal{ALC}$ is considered instead of description logic $\mathcal{EL}$, the Turing-completeness of our hybrid formalism can be easily proved by means of a reduction from the Turing-completeness of Minsky machines. Hence, the following item in our research program:

($\mathbf{RP_1}$) separate the description logics that do give rise to a Turing-complete hybrid formalism from the description logics that do not.

In particular, find simple and natural conditions on concept inclusions, concept equations and clauses such that deductive ontologies satisfying them give rise to a Turing-complete hybrid formalism.

B. Tractability

The success of the logic programming languages comes from the fact that it is relatively easy to define Turing-incomplete restrictions of clauses that can be used as a domain-specific language taking advantage of efficient algorithms developed for them [19], [27]. Thus, the following item in our research program:

($\mathbf{RP_2}$) for the description logics that do not give rise to a Turing-complete hybrid formalism, separate those that do give rise to a hybrid formalism tractable in polynomial time from those that do not.

In particular, find simple and natural conditions on concept inclusions, concept equations and clauses such that deductive ontologies satisfying them give rise to a hybrid formalism tractable in polynomial time.

C. Declarative and fixpoint semantics

In logic programming, the declarative semantics of programs is given by the usual semantics of first-order logic. It is defined in terms of Herbrand interpretations [24], [31]. In this setting, given a program, the main result is the standard characterization of its Herbrand models as the pre-fixpoints of some continuous mapping associated to it. Consequently, the following item in our research program:

($\mathbf{RP_3}$) develop the declarative and fixpoint semantics of our hybrid formalism.

In particular, given a deductive ontology, characterize its Herbrand models as the pre-fixpoints of some continuous mapping associated to it.

D. Procedural semantics

In logic programming, the refutation procedure of interest is called SLD-resolution where an inference step is based on the unifiability between the selected atom in a given definite goal and the left side of a variant of a definite clause in a given program. Hence, the following item in our research program:

($\mathbf{RP_4}$) develop the procedural semantics of our hybrid formalism.

In particular, considering the unification problem in description logics with empty T-boxes [6], adapt the related unification algorithms to the context of our hybrid formalism[16]. In this respect, the tools and techniques developed in [2], [9], [10], [25], [26], [37] might be useful.

E. Negation

By using conditional assertions of the form $\alpha_1, \ldots, \alpha_m \leftarrow \beta_1, \ldots, \beta_n, \mathtt{not}(\gamma_1), \ldots, \mathtt{not}(\gamma_o)$ where α_1, ..., α_m, β_1, ..., β_n, γ_1, ..., γ_o are atoms, one may write more expressive deductive ontologies. For instance, in our example about security policies, the deontic principle saying that every non-prohibited access is permitted and the deontic principle saying that every non-compulsory access is optional can be expressed by the following conditional assertions:

- $\mathsf{perm}(X, Y) \leftarrow \mathtt{not}(\mathsf{proh}(X, Y))$,
- $\mathsf{opti}(X, Y) \leftarrow \mathtt{not}(\mathsf{comp}(X, Y))$.

In logic programming, the declarative semantics of a program containing, possibly, negation in the right side of clauses is given by the so-called answer set semantics. It is defined in terms of stable models [23], [30]. In this setting, the question of the existence of stable models for a given program is of the utmost interest. Thus, the following item in our research program:

($\mathbf{RP_5}$) develop the answer set semantics of our hybrid formalism when programs contain, possibly, negation in the right side of their clauses.

F. Forgetting

Forgetting is an ontology engineering technique. It is achieved by eliminating from a given ontology a subset of its signature in such a way that all logical consequences up to the remaining signature are preserved. This form of knowledge compilation has important applications when an engineer who designs an ontology formulated in some language wants to import content from an existing ontology formulated in a richer language. As a result, ontology forgetting has attracted increasing attention and several algorithms have been developed to support it [16], [18]. Consequently, the following item in our research program:

($\mathbf{RP_6}$) develop the ontology engineering technique of forgetting for our hybrid formalism.

[16]The computability of the unification problem with arbitrary T-boxes is not known. In other respect, when description logic $\mathcal{ALC}$ is considered instead of description logic $\mathcal{EL}$, the computability of the unification problem either with empty T-boxes, or with arbitrary T-boxes is not known too.

G. Modularization

Modularization is an ontology engineering technique. Its importance is the result of the fact that large and monolithic ontologies are difficult to handle, whereas smaller and modular ontologies are easier to understand and use. With a view to collaboratively developing ontologies and merging independently developed ontologies into a single and reconciled one, ontology modularization has attracted increasing attention and several algorithms have been developed to support it [11], [17]. Hence, the following item in our research program:

($\mathbf{RP_7}$) develop the ontology engineering technique of modularization for our hybrid formalism.

X. Last words

Our idea of an hybrid formalism where description logics constructs are used for defining concepts that are given as arguments to the predicates of the logic programs has only one ancestor: the formalism developed in [13]. In this formalism, Boolean constructs are used for defining expressions that are given as arguments to the predicates of the logic programs, allowing clauses of the form[17]

- $\mathsf{adder}(X, Y, Z, T, U \vee V) \leftarrow \mathsf{halfAdder}(X, Y, W, U), \mathsf{halfAdder}(W, Z, T, V)$,
- $\mathsf{halfAdder}(X, Y, X \oplus Y, X \wedge Y) \leftarrow$.

where X, Y, Z, T, U, V and W denote propositional variables, $\vee$, $\oplus$ and $\wedge$ denote the Boolean constructs of, respectively, disjunction, exclusive disjunction and conjunction and adder and halfAdder are predicate symbols of, respectively, arity 5 and arity 4. Obviously, the Boolean expressions $U \vee V$, $X \oplus Y$ and $X \wedge Y$ used in these clauses can be seen as **ROL**-free complex concepts when description logic $\mathcal{ALC}$ is considered instead of description logic $\mathcal{EL}$.

Knowledge representation languages such as those provided by description logic languages and rule-based reasoning paradigms such as those provided by logic programming languages are well-known and widely used in Computer Science and Artificial Intelligence. Therefore, it is quite amazing that their integration in a unique formalism similar to the formalism proposed by [13] has not been put forward during the last 30 years. A narrow-minded explanation would consist of saying that this lack of interest is the result of the lack of importance of hybrid formalisms such as the one introduced in this paper. The case study presented in Section II and the example about OrBAC introduced in Section VIII indicate that this lack of interest might just be the result of a lack of imagination. Indeed, we believe that it is time to give space to advanced languages of terms for ontologies as introduced in Sections III and IV, to consider the decision problems presented in Sections V, VI and VII and to address the research program presented in Section IX.

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[17] See Section 4 in [13].

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