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# Rippling 3-Riemannian structure describing gravity: an alternative to dark matter validated by 130 galaxy rotation curves

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I present a novel theory of gravity that is in correspondence with general relativity (GR) results and resolves the missing mass problem ('dark matter'). It is a theory of emergence of local length and time *scales* not measurement. While locally length or time *scales* are constant, measurements can be arbitrary. I use light wave as measuring instrument i.e wavelength and period define one unit of length and time *scales* respectively. By energy-momentum relation in special relativity, variation in proper time and proper length is symmetric i.e. there is coequality in variation of length and time scales. This is the founding principle of the proposed theory and is evident in metric solutions to GR EFEs.

Building on the hypothesis that matter and energy are separate physical entities, I propose energy is a property of a three-dimensional probabilistic structure spanning space. Mass interacts by binding energy density which causes variations in length and time scales, mathematically equivalent to spacetime curvature in GR. The theory introduces Einstein's Reduced Equation that enables evaluating variation in local length scales from the wave vector of flow of energy and following the principle, variation in time scales can be known. Thus, gravity is defined by flow and distribution of energy. This leads to modification to EFEs and two sets of EFEs are proposed, for large and largest scales, which smoothly deform into each other. On large scale the flow of energy bounded by source mass is inwards i.e. gravity is attractive and on largest scales, the local energy density is monotonically decreasing causing repulsion. The proposed theory extends GR and key predictions include flat galaxy rotation curves and dependence of cosmological constant on space and time coordinates equivalent to a negative source mass.

The theory is validated by comparing the predicted and the observed rotation curves based on SPARC data set of late-type galaxies. The rotation curve analysis is based on fluid dynamics and defines regimes of validity of Newtonian dynamics and MOND for galactic dynamics. MOND acceleration scale  $a_0$  is given a physical meaning and is equivalent to baryonic mass  $\ll$  mass equivalence of cumulative energy density at transition radius.

## I. INTRODUCTION

The missing mass problem was first noted in 1930s by Zwicky [1, 2] and Smith [3] and later confirmed by works of Rubin and Ford [4, 5] which led to the understanding that the most mass of galaxy or galaxy clusters is non-luminous 'dark matter'. There is undeniable evidence for effects observed under the head 'dark matter' especially on largest scales. There have been two approaches to solve this problem, non-baryonic matter and modified laws of gravity.

The hypothesis of existence of cold, weakly interacting non-baryonic particle [6–9] is subjected to the correctness of general relativity (GR). However, GR is well-established experimentally within the weak conditions of solar system [10] and predictions of existence of black holes [11] and gravitational waves [12] have also been confirmed. So far, it cannot be tested beyond the gravitational field strength larger or smaller than scales within solar system and has remained incompatible with quantum theory. Therefore, completeness of GR can be contested in conditions on largest scales or on smallest scales and because most mass-energy of the universe consists of

unknown matter or energy initially unaccounted for in the theory.

According to GR, spacetime is featureless and has no intrinsic properties. It can curve and bend like a differentiable manifold or surface and is a dynamical stage on which physical phenomenon takes place. The theory has provided incredible development in our understanding of space and time where both are assumed to be dimensions with no physical rooting but this has served the theory well. Yet, there remains an enigma around understanding time and emergence of arrow of time. Results in general relativity are limited to geometry of spacetime and its effects on matter. Consequently, any physical phenomenon has to be captured in terms of metric of spacetime. This is simple and elegant but limiting as well.

MODified Newtonian Dynamics (MOND) [13–15] proposes to replace 'dark matter' by explaining Mass Discrepancy Acceleration Relation (MDAR) [16–18]. Recently, super-fluid dark matter (SFDM) theory [19–23] was presented that describes a scalar particle that condenses into a superfluid. However, both these models have limitations on largest scales.

The mystery of accelerated expansion of universe [24, 25] is mathematically resolved by non-zero cosmological constant but what is sourcing it and why it has unusually small value are still open questions. Negative mass spanning the interstellar space is discussed [26, 27]

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as a source of ‘dark energy’ and alternatively, some models unify dark energy and dark matter. However, both models are found to have limitations. Other models propose dark matter-dark energy interactions [28, 29] as possible source.

### A. This paper

I present a novel theory of gravity that extends general relativity to describe the large scale phenomenon. The theory is validated by analyzing galaxy rotation curves of 130 galaxies from SPARC data set and suggests modification to Einstein field equations (EFEs). The theory concurs with Newtonian dynamics in the inner regions of a galaxy. The distance independent dynamics predicted by the theory in the outer regions of a galaxy possibly give physical meaning to acceleration scale  $a_0$  in Milgrom’s MOND. The character of the structure proposed in the theory also preserves some work done on cold dark matter (CDM). Thus it is a hybrid theory that brings together features of MOND and CDM while preserving mathematics of general relativity.

The central idea of the paper is that variations in length and time *scales* are coequal i.e. symmetric and orthogonal. If we use light wavelength and period as instruments for defining these scales, coequality is immediate. Further, similar to quantum theory, I assume energy is a property of a structure independent of matter however matter and energy interact to produce physical phenomenon. Leveraging these principles and the ideas from literature, I propose a three-dimensional probabilistic structure of energy, *Kaal* (‘dark’) spanning space. The flow and distribution of energy near a source mass defines variations in local length and time *scales* and thus preserves the results of general relativity. On large scale, *Kaal* structure is equivalent to a superfluid and an approximate fluid dynamics model is employed to predict galaxy rotation curves.

### B. A simple visualisation

Matter and energy are separate entities. Matter resides in a sea of energy and mass interacts by binding energy density thus the sea is denser nearer the source mass. Energy density defines local length and time *scales* and is equivalent to spacetime curvature. So the flow and distribution of energy is equivalent to gravity. The bounded energy is equivalent to a superfluid which is dragged by rotating mass or vice-versa and is used to determine rotation curves.

## II. PRINCIPLE, HYPOTHESIS AND POSTULATES OF THEORY

### A. Background

I leverage upon the distinction between *scales* and measurements. While locally *scales* are constant, measurements can be arbitrary. This paper is about emergence of local *scales* that enable measurements by an observer. Lorentz transformations and general relativity capture the relation between measurements. Measurement of length can vary relative to a condition if either the underlying space curves while the length scale remains constant or if the length scale varies while the space remains flat. For this paper, I use light wave as measuring instrument i.e. wavelength and period define one unit of length and time measurement respectively. I also assume space is a flat background in correspondence with Einstein-de Sitter model. Then what we may consider as curvature of space would be the curvature in *measurement* of space as by gravitational redshift light wavelength is not constant near a source mass i.e. length *scale* varies. This preserves the mathematics of general relativity and only its interpretation is effected. Curvature in spacetime is curvature in *measurement* of length and time.

### B. Principle: Coequality in variation in length and time scales

From the results of special relativity, it is known proper time and proper length vary by the same factor. This symmetry in variation stems from energy-momentum relation  $E^2 = (mc^2)^2 + (pc)^2$  and thus similar result is obtained from metric solutions to Einstein field equations. It is immediately evident in Schwarzschild and Kerr solutions. One exception would be FLRW metric as it is based on certain assumptions and is discussed later.

Coequality in variation in length and time *scales* is the founding principle of the theory. As speed of light is constant, it is immediately evident if I use light wavelength and period for defining length and time *scales* respectively. Evaluating variations in spatial components (wave vector) can give complete information about the variations in time scale (period). However inverse argument is not valid as time cannot be resolved into three dimensions and though magnitude of variation can be known directions would remain unresolved.

### C. Hypothesis

In classical theories, energy is an abstract scalar property associated with matter and in quantum theory energy is described by wave function of the system. I take an alternative description which is similar to quantum theory: *Energy and Matter are separate physical entities.*

*Their interaction causes physical phenomenon.* I extend this hypothesis into a three-dimensional structure as the proposal.

#### D. Proposal

Though the act of measurement of time is based on the instrument, time itself at any point must exist independent of it. Thus, a three dimensional structure spanning space termed as *Kaal* ('dark') is proposed. Each point of Kaal structure (notation:  $\mathcal{K}$ ) is characterized by (1) Kaal energy density  $\kappa$  (2) time density (3) metric of 3-Riemannian manifold of space measure  $g_{mn}$ . All three entities co-exist and neither one is independent of another.

Extending the hypothesis, energy is now a property of Kaal structure and not of matter; matter interacts with Kaal structure to produce physical phenomenon. A trivial way to visualize this is mentioned in Sec. ??.

#### E. Postulates

Building on the hypothesis and proposal, following are the properties of Kaal structure:

1. Kaal energy density (KED): What we conventionally consider as the energy of matter can now be thought of as energy of a structure spanning space. KED  $\kappa$  is amount of energy at a point and is a positive definite quantity.
2. Kaal ripples is the flow of energy through Kaal structure and determines the amount of KED at any given point. Variations in Kaal ripples travel at speed of light (this would be equivalent to gravitational waves). In absence of influence of matter, Kaal ripples flow from high to low KED regions.
3. KED and time density is defined by frequency of Kaal ripples and is equivalent to rate of passage of time. Higher the KED, higher is the rate of passage of time i.e. light wave period based clock will register more ticks for an event. This is equivalent to clocks running slow near a gravitating source incorporating the type of clock.
4. Metric  $g_{mn}$ : The variation in *scales* for length measurements is captured by a 3-dimensional Riemannian manifold (notation:  $\mathcal{RIE}^3$ ). Space is flat, the curvature is not in space but in *measurement* of space. The manifold is locally Euclidean and diffeomorphic to  $\mathbb{R}^3$  i.e. length *scale* is locally constant. As KED  $\kappa \in (0, \infty)$ , the length *scale* also ranges from  $(0, \infty)$  i.e. there exist infinite diffeomorphisms [30] of any patch  $U \subset \mathcal{RIE}^3$ .
5. No rest, all motion: Energy is always flowing i.e. Kaal ripples exist at all times at all points. It will

be discussed later, how these ripples correspond to motion of mass but this is a fair assumption to make in view of constant motion of the galaxies. Kaal is thus a dynamic structure.

6. Minimizing (repulsive) nature: In regions free from influence of matter, KED tends to minimize itself locally causing expansion of Kaal structure. This expansion is continuous and monotonically decreasing i.e. a system tends to reduce its local energy density or time density. This is a naturally irreversible phenomenon unless intervened upon externally. In sloppy terms, I say a system wishes to be free of time and tends to be time-less. This property has its roots in entropy and arrow of time.

#### F. Dynamics: Interaction of mass

1. Curvature in  $\mathcal{RIE}^3$ : Mass binds (collects, pulls, increases) KED in its neighborhood which causes wavelength of Kaal ripples to vary and thus arises curvature in  $\mathcal{RIE}^3$  of length measure. Kaal ripples flow along the direction of variation in KED. If effects of KED can be ignored, this is equivalent to spacetime curvature in general relativity.
2. Motion of mass drags Kaal ripples along or vice-versa. On large scale, this is equivalent to motion of source mass dragging the Kaal structure or vice-versa. The analysis is similar to fluid dynamics where mass density of fluid is equivalent to KED. Small scale effects are not discussed in this paper.

The Kaal structure  $\mathcal{K}$  is employed to understand the local and large scale phenomenon i.e. to establish correspondence with results of special and general relativity and 'dark matter'. Local analysis is abstract and is discussed in Appendix A. It is useful to understand the probability amplitude of Kaal ripples however this is insignificant on large scale hence it can be skipped to understand large scale phenomenon.

### III. LARGE SCALE ANALYSIS

#### A. Discussion

To completely determine the three entities of Kaal structure - Kaal energy density (KED), metric  $g_{mn}$  and time density - determining metric of 3-Riemannian manifold  $\mathcal{RIE}^3$  of space measure is necessary and sufficient. Locally, KED is constant, thus length and time *scales* are constant. However, in the generalized description, mass binds KED causing deformation in Kaal structure which is equivalent to variation in KED, length and time *scales*. If KED is ignored and only the variations in length and time scales are considered, general relativity is complete. Consideration of KED leads to modification of Einstein

field equations (EFEs) and large scale dynamics as discussed later.

### 1. Bounded Kaal Energy

*Bounded Kaal Energy (BKE)* is defined as the KED enclosed between the source mass and the point of interest. Consider a spherical source mass, say a star. The KED enclosed in the spherical shell volume extending from surface of the star to radial distance  $r$  is BKE.

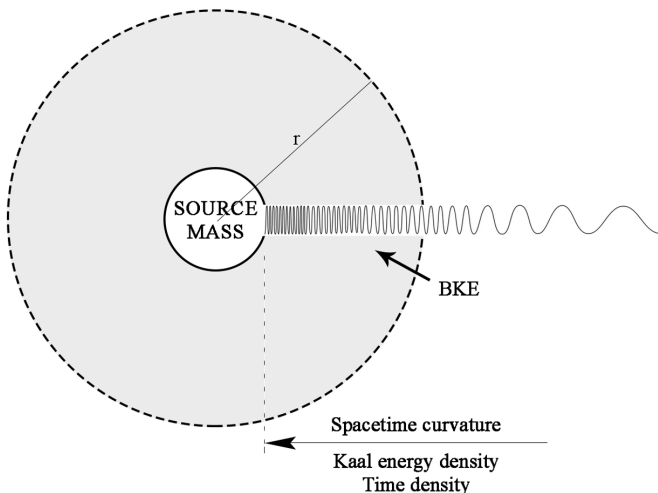


FIG. 1. An illustration representing bounded Kaal energy (BKE) near a source mass. Energy density is higher closer to the source which is equivalent to spacetime curvature in GR.

BKE is the distinction between general relativity and Kaal theory on large scale. From Sec. IIE, higher the KED, higher is the time density. If we compare this with results of general relativity, it is clear that KED increases as one moves towards the source mass i.e. Kaal structure is denser closer to the source mass as postulated.

General relativity can be understood as an approximate mathematical wrapper over Kaal theory. KED defines local length and time *scales*. Variation in KED is equivalent to spacetime curvature.

*Kaal Fluid:* By mass-energy equivalence, BKE can be treated as a fluid for analysis. At any given point, mass density of the fluid is equivalent to KED. Kaal fluid is dragged by motion of mass or inversely the flow of Kaal fluid governs motion of mass on large scale (interactions with only mass are discussed in this paper).

## B. Einstein's Reduced Equation (ERE)

### 1. Generalization

Locally, KED is constant as length and time *scales* are constant. For Kaal ripples this means  $|\vec{k}| = \omega =$

*constant*. On large scale due to deformation in Kaal structure,  $|\vec{k}| = \omega \neq \text{constant}$  i.e. the character of Kaal ripples vary and consequently length and time *scales* vary (in correspondence with general relativity). Because  $\vec{k}$  is not a constant on large scale, the corresponding length *scales* vary and correspond to the line element of 3-Riemannian manifold of length measure  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  equivalent to proper length in GR. Reiterating once again, space is flat and infinite, Kaal ripples expand or contract causing length scales to vary.

Clearly, there is strong correlation between  $\vec{k}$  of Kaal ripples and the metric  $g_{mn}$  of  $\mathcal{R}\mathcal{I}\mathcal{E}^3$ . Thus, the problem hereon is to determine the metric  $g_{mn}$  of  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  and consequently arrive at KED and time density.

**Ricci Curvature Tensor.** For 3-Riemannian manifolds, *Ricci Curvature Tensor*,  $R_{ij}$  contains necessary and sufficient information about the curvature at any point and is defined as

$$R_{ij} = R_{ijk}^k = \frac{\partial \Gamma_{ik}^k}{\partial x^j} - \frac{\partial \Gamma_{ij}^k}{\partial x^k} + \Gamma_{ik}^r \Gamma_{rj}^k - \Gamma_{ij}^r \Gamma_{rk}^k \quad (1)$$

$$R_j^i = g^{ik} R_{kj} \quad (2)$$

Contracting the Ricci Curvature Tensor, we obtain the *Ricci Scalar Curvature*,  $R$  as

$$R = R_i^i = g^{ij} R_{ij} \quad (3)$$

**Source of Curvature.** Kaal theory separates energy from matter and considers energy as property of Kaal structure  $\mathcal{K}$ . Mass can also be considered as “bounded energy” by the mass-energy equivalence. Thus, in the present framework source of curvature in  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  i.e. any variations in KED or time density must be source energy itself. Unlike the source in general relativity which is a rank-2 energy-momentum tensor  $T_{\mu\nu}$ , the source in Kaal theory is a scalar quantity, energy.

Coordinate-independent formulation is still part of this framework as the source of curvature is now a rank-0 tensor i.e. a scalar which is independent of coordinate system adopted.

**Equations.** From the postulates, mass binds energy i.e. KED increases in the neighborhood of a source mass. This causes curvature in  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  and therefore the curvature in  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  must be related with source mass-energy. However, the source of curvature is a scalar (energy), thus the relation must involve Ricci Scalar Curvature  $R$  and be of the form

$$R = a\rho_s \quad (4)$$

where  $\rho_s$  is the source energy and  $a$  is some constant.

In weak conditions, where the contribution of source energy to the source term can be ignored (Schwarzschild conditions), proposed results must correspond with the static spacetime results in general relativity when spatial

hyperspace is orthogonal to time vector in (1+3) spacetime. In such conditions, the equation (4) relating scalar curvature and source energy must take the form as (also satisfies dimensional analysis)

$$R = \frac{4\pi G}{c^4} \rho_s \quad (5)$$

where  $c$  is speed of light. In units with  $c = 1$ , this is

$$R = 4\pi G \rho_s \quad (6)$$

The numerical part in this equation is inconsequential as it can be shifted to left hand side and made part of the definition of the metric. This result can also be arrived at by contracting Einstein field equations  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$  by a timelike vector i.e projecting spacetime on three-dimensional spatial hyperspace [31]. Eq. (6) is termed as *Einstein's Reduced Equation (ERE)* for Kaal theory.

### C. From Metric to KED and time density

Once metric  $g_{mn}$  is known from solution to ERE, it must lead to determining KED and time density at a point in Kaal structure. Consider a metric solution to ERE Eq.(6) given by the following line element in some locally orthogonal coordinates  $(x^1, x^2, x^3)$  on  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  near a source mass,

$$ds^2 = p(x^1, x^2)(dx^1)^2 + q(x^1, x^2)(dx^2)^2 + r(x^1, x^2)(dx^3)^2 \quad (7)$$

where  $p, q, r$  are functions of  $(x^1, x^2)$ . It can be thought of having symmetry in  $x^3$ -plane. The metric  $g_{mn}$  then looks like

$$g_{mn} \equiv \begin{pmatrix} p(x^1, x^2) & 0 & 0 \\ 0 & q(x^1, x^2) & 0 \\ 0 & 0 & r(x^1, x^2) \end{pmatrix} \quad (8)$$

At a distance far from the source mass, say  $x^1 \rightarrow \infty$  let us assume the metric corresponds to flat manifold metric.

Since metric is a function of only  $(x^1, x^2)$ , so when an observer moves through  $\mathcal{K}$  any variation is expected only in  $(x^1, x^2)$  plane locally with respect to flat conditions. Because variations can only be carried by variations in Kaal ripples i.e. flow of energy, it can be assumed Kaal ripples flow only in  $(x^1, x^2)$  plane locally. This further implies that KED  $\kappa$  and time density are functions of  $(x^1, x^2)$  only and independent of  $x^3$ -coordinate. Thus, the variations in measurement of length and time locally must be a function of only  $(x^1, x^2)$ . This discussion implies

$$dt \rightarrow z(x^1, x^2)dt \quad (9)$$

where  $z$  is some function of  $(x^1, x^2)$ .

Now, it is typical physics of waves. If the direction of propagation of a wave is along some line in  $(x^1, x^2)$

plane, the wave vector  $\vec{k}$  can be resolved along  $(x^1, x^2)$  coordinates say as  $k_1, k_2$  such that  $|\vec{k}| = k = \sqrt{k_1^2 + k_2^2}$ . The wave frequency  $\omega$  is related with wave number  $k$  by wave velocity. Since, Kaal ripples are propagating along a line in  $(x^1, x^2)$  plane locally, the wave vector  $\vec{k}$  of Kaal ripples must only depend on changes in metric along  $(x^1, x^2)$  directions. Thus,  $\vec{k}$  depends on  $g_{11} = p(x^1, x^2)$  and  $g_{22} = q(x^1, x^2)$ . Consequently so does the frequency of Kaal ripples as  $\omega = |\vec{k}|$  in  $c = 1$  units. This is all *relative* to flat conditions.

Because of inverse relation between energy and wavelength, we have KED relative to flat conditions as

$$\kappa \equiv \left( \frac{1}{p(x^1, x^2)} + \frac{1}{q(x^1, x^2)} \right)^{-\frac{1}{2}} \quad (10)$$

where conveniently it can be considered  $|\vec{k}|_{flat} = \omega_{flat} \equiv 1$  in  $(x^1, x^2)$  plane. Consequently

$$z(x^1, x^2) \equiv \left( \frac{1}{p(x^1, x^2)} + \frac{1}{q(x^1, x^2)} \right)^{\frac{1}{2}} \quad (11)$$

i.e. time scale changes by the factor  $z(x^1, x^2)$ .

Clearly this is a general discussion using a particular coordinate system. The exact nature of metric, KED and time density depend on the solution to ERE Eq.(6) in a suitable coordinate system. A solution to EFEs in general relativity will comply with this analysis due to the underlying principle.

### D. Dynamics

Kaal theory describes the relationship between energy and length, time scales. Therefore, any description of motion of a particle or source mass is essentially a description of distribution and flow of Kaal energy density (KED). There are two conditions in which motion should be analyzed-

1. **Rest Energy Dominates:** In such conditions, length and time *scales* vary, KED is not constant i.e. variations in Kaal structure contribute significantly in mass dynamics. If effects of BKE is ignored and only variations in length and time *scales* are considered, equivalent model is (1+3) spacetime of general relativity.
2. **Kinetic Energy Dominates:** In these conditions (generally small scale), the structure of Kaal ripples determines particle dynamics. These conditions are not discussed in this paper.

Condition 1 is partially considered in general relativity where effects of BKE on dynamics are ignored and Newtonian dynamics is an approximation. If BKE is considered, dynamics will differ from present formulation especially on largest scales and experimental results obtained

with Newtonian gravity as an approximation of general relativity will lead to existence of either additional form of matter or energy.

#### IV. SOLUTIONS IN LARGE SCALE ANALYSIS

##### A. Conditions

On large scale, the rest energy of the source mass dominates. Because of this simplicity, two sets of conditions are discussed basis the contribution of source energy to bounded Kaal energy (BKE).

1. Weak source-energy conditions such as solar system: Rest energy of source mass dominates and primarily defines the BKE in its neighborhood. Any contribution of energy from motion to BKE can be ignored for analysis. This will give results in correspondence with static spacetime solutions such as Schwarzschild solution in general relativity.
2. Strong source-energy conditions: Contribution of energy from source motion to BKE cannot be ignored. This would give results equivalent to non-static spacetime in general relativity and is only discussed in this paper.

##### B. Case of Schwarzschild's Solution

Consider a slowly rotating equivalently static isotropic source mass such as our Sun. The objective is to determine how local *scales* of length and time are affected near such source from their measurements in flat conditions. The approach is similar to how one solves for Schwarzschild's solution in general relativity. Two points are to be considered -

1.  $\rho_s = 0$ , this means any contribution of energy from motion to BKE is ignored.
2. Kaal ripples exist due to slow rotation however the effects of rotation are ignored (will be considered in next section). Thus, this case implies evaluating KED bounded by the source mass.

Borrowing assumptions and conditions from Schwarzschild's solution in general relativity, consider the following as the line element of  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  in the spherical coordinates  $(r, \theta, \phi)$  near the source mass,

$$ds^2 = e^{2X(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (12)$$

where function  $X(r)$  needs to be determined. The metric in matrix form is

$$g_{mn} \equiv \begin{pmatrix} e^{2X(r)} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (13)$$

Solving for

$$R = 0 \quad (14)$$

or

$$g^{mn} R_{mn} = 0 \quad (15)$$

where  $R_{mn}$  is Ricci Curvature Tensor.

Using the metric in Eq. (13), Christoffel symbols  $\Gamma_{st}^r$  can be evaluated which are then used to determine the Ricci Curvature Tensor  $R_{mn}$ . If

$$e^{2X(r)} = \frac{1}{1 + b/r} \quad (16)$$

then

$$R_{mn} = \begin{pmatrix} \frac{b}{r^2(b+r)} & 0 & 0 \\ 0 & \frac{-b}{2r} & 0 \\ 0 & 0 & \frac{-b \sin^2 \theta}{2r} \end{pmatrix} \quad (17)$$

Consequently

$$g^{mn} R_{mn} = \left(1 + \frac{b}{r}\right) \left(\frac{b}{r^2(b+r)}\right) + \left(\frac{1}{r^2}\right) \left(\frac{-b}{2r}\right) + \left(\frac{1}{r^2 \sin^2 \theta}\right) \left(\frac{-b \sin^2 \theta}{2r}\right) = 0 \quad (18)$$

i.e.

$$R = 0 \quad (19)$$

The manifold describing the variation in length scales in the neighborhood of a source mass is scalar flat but not Ricci flat. The line element is then

$$ds^2 = \frac{dr^2}{(1 + b/r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (20)$$

This line element describes exactly the same information as the Schwarzschild's solution to EFEs in the same conditions. This is not surprising as Schwarzschild metric is a static spacetime.

Considering the flow of Kaal ripples to be entirely radial (this is an approximation), only the radial component contributes to the wave vector  $\vec{k}$  and to frequency  $\omega$ . Therefore, relative to Kaal ripples in flat conditions ( $\omega_{flat} = 1$ ),

$$\omega = k = \left(1 + \frac{b}{r}\right)^{-\frac{1}{2}} \quad (21)$$

and consequently

$$dt \rightarrow \left(1 + \frac{b}{r}\right)^{\frac{1}{2}} dt \quad (22)$$

This implies KED at a radial distance  $r$  from the source mass is  $(1 + b/r)^{-1/2}$  times the KED in flat conditions.

Without any loss of generality, complete correspondence with general relativity results can be established if the constant  $b = (-2GM/c^2)$ , leading to the line element of the form

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (23)$$

and explicitly the metric  $g_{mn}$  of  $\mathcal{RLE}^3$  is

$$g_{mn} \equiv \begin{pmatrix} \left(1 - \frac{2GM}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \quad (24)$$

As discussed earlier, KED is higher nearer to the source mass i.e. it binds (collects, increases, pulls) KED in its neighborhood. In practical conditions, source energy also contributes to bounded Kaal energy (BKE) contributing to RHS of ERE Eq.(6).

### 1. Newtonian Limit

Kaal theory gives further meaning to gravitational potential and potential energy in terms of KED. In Newtonian conditions, Eq.(21) with  $b = (-2GM/c^2)$  can be used to write KED  $\kappa_r$  at radial distance  $r$  as

$$\frac{\kappa_r}{\kappa_{flat}} = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} \approx \left(1 + \frac{GM}{c^2 r}\right) \quad (25)$$

or with  $\phi = -GM/r$ ,

$$\frac{\kappa_r}{\kappa_{flat}} = \left(1 - \frac{\phi}{c^2}\right) \quad (26)$$

$$\frac{\kappa_r - \kappa_{flat}}{\kappa_{flat}} = -\frac{\phi}{c^2} \quad (27)$$

i.e. gravitational potential at radial distance  $r$  is the (minus the) gain in KED coming from infinity.

### 2. Schwarzschild Black Hole

From the metric in Eq.(24) it is evident that at Schwarzschild's radius  $r_s = 2GM/c^2$ , Kaal ripples tend to become infinitely dense. Consequently, the length measurement tends to infinity, time measurements tend to zero and the KED approaches infinity.

$$r \rightarrow r_s \implies \kappa_r \rightarrow \infty \quad (28)$$

The results agree with the prediction of black hole in general relativity and there is additional information relating the structure of black holes and energy density. Further analysis on Schwarzschild's black hole with respect to the present theory is deferred for now.

## C. Discussion: Moving Source

For assessing cases that involve moving sources following two points need to be considered:

1. From the postulates, it is evident motion is fundamental to Kaal theory. Kaal ripples continuously carry energy through Kaal structure and therefore there are no static conditions in Kaal theory.
2. Because energy is property of Kaal structure, what is conventionally understood as kinetic energy (other energy forms are not considered in this paper) needs to be evaluated in terms of KED. This is not detailed in this paper however it is broadly discussed in Appendix B.

### 1. Kaal Fluid

KED in the neighborhood of a source mass in Schwarzschild conditions is found to be

$$\kappa_r = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} \kappa_{flat} \quad (29)$$

i.e. KED is higher near the source mass compared to KED in flat conditions at infinity. This distribution of energy surrounding the source mass can be equivalently considered as *fluid* and the source mass is analogous to a submerged sphere in a fluid. The mass density of this *Kaal Fluid* is equivalent to the KED by mass-energy equivalence. Now, this is a typical fluid dynamics problem that considers Euclidean background space. Once again, it is reiterated, space is flat but variation in scales/measurements is not.

### 2. Rotating Spherical Source

The most relevant case in large scale analysis is of rotating source mass. Generally, the source is considered to be spheroidal however for simplicity let us consider a spherically isotropic source mass. The result can then be extended to spheroidal rotating mass through appropriate spheroidal coordinates as in Kerr's solution to EFEs. Correspondence is also established with cross-terms in metric solutions to EFEs signifying rotations.

Say the radius of source is  $R_M$  and its constant angular velocity  $\omega_M$ . Surface of the rotating source mass presents one boundary condition i.e. KED in infinitesimal volume element at the surface is dragged by linear velocity  $\omega_M R_M \sin \theta$  in the direction of rotation. Consequently, KED  $(\kappa_r)_{rot}$  on the surface and thus in the neighborhood of rotating source must increase by the factor  $\gamma$

as

$$(\kappa_r)_{rot} = \gamma(\kappa_r)_{Schwarzschild} \quad (30)$$

$$\gamma = \left(1 - \frac{(\omega_M R_M \sin\theta)^2}{c^2}\right)^{-\frac{1}{2}} \quad (31)$$

in correspondence with results of special relativity and local analysis (Appendix A) and so,

$$(\kappa_r)_{rot} = \left(1 - \frac{(\omega_M R_M \sin\theta)^2}{c^2}\right)^{-\frac{1}{2}} \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} \kappa_{flat} \quad (32)$$

which implies KED is a function of  $(r, \theta)$ . Consequently, wave-vector  $\vec{k}$  and frequency  $\omega$  of Kaal ripples, metric  $g_{mn}$  of  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  are also function of  $(r, \theta)$ . This implies locally Kaal ripples flow in  $(r, \theta)$  plane. This follows from Sec. III C. There continues to be symmetry in azimuthal plane and thus  $\vec{k}, \omega, g_{mn}$  are all independent of azimuthal angle  $\phi$ . However, by virtue of rotation of the source (or inversely) and from Eq. (32), Kaal fluid is dragged along the direction of rotation i.e. in azimuthal plane and this is equivalent to frame-dragging effect in solutions to EFEs if Kaal fluid is considered to have negligible viscosity  $\eta$ . This is currently being formulated as part of continuing research however the validity of this equivalence is later in the paper tested by analyzing galaxy rotation curves in the present theory and the results therein support the proposed dynamics of Kaal structure.

The KED of an infinitesimal volume element at radial distance  $r$ , due to rotation of source mass is equivalent to the rotation of that element by angular velocity  $\omega_M R_M \sin\theta / r$ . Clearly, as  $r \rightarrow \infty$ , rotation of Kaal fluid slows down while linear velocity remains almost constant.

### 3. Flow of KED: Equator to Poles

E. (32) indicates that at a certain fixed radial distance  $r$ , KED has a maxima at the equator ( $\theta = \pi/2$ ) and minima at the poles ( $\theta = 0, \pi$ ) along the axis of rotation. Kaal ripples flow from high KED to low KED region as postulated. Therefore, Kaal ripples must flow from equatorial plane to the poles and away from the rotating source along the axis of rotation.

This flow of energy from the equatorial plane to poles results in flow of Kaal ripples radially pushing inwards towards the source analogous to how any fluid would flow in the similar situation. Thus in Schwarzschild conditions, Kaal ripples exist for slowly rotating source however these are ignored for simplicity in the problem.

### 4. Slow Rotations in Newtonian Conditions

Expanding Eq. (32) for weak conditions,

$$(\kappa_r)_{rot} \approx \left(1 + \frac{(\omega_M R_M \sin\theta)^2}{2c^2}\right) \left(1 + \frac{GM}{c^2 r}\right) \kappa_{flat} \quad (33)$$

$$\approx \left(1 + \frac{(\omega_M R_M \sin\theta)^2}{2c^2} + \frac{GM}{c^2 r}\right) \kappa_{flat} \quad (34)$$

ignoring the cross-term with  $c^4$  in the denominator. Using  $\omega_M, R_M$  for Earth, the contribution from second term is negligible. So at any point, the pressure from flow of KED towards the poles along rotational axis can be ignored and the ‘gravitational’ effects can be conveniently considered completely radial in such weak conditions.

## V. VALIDATING THE THEORY: GALAXY ROTATION CURVES

While the elements of the theory are in complete correspondence with successes of general relativity - space-time curvature, gravitational redshift, black holes, gravitational waves and others - however very large scale phenomenon remains to be resolved. Application of Newtonian dynamics to baryonic mass distribution on galactic scales fails to explain the observed rotational motion. Evidence for this discrepancy [32–34] is ambiguous and leads to either predicting existence of non-luminous matter or failure of existing gravitational theories [35, 36]. Clearly, Kaal theory modifies dynamics and in this section, I present the validation of such modification by application to rotation dynamics of galaxies. However, it is to be noted that MOND [37–41] hypothesizes modification to effective force law at low accelerations,  $a \ll a_0$ . The acceleration scale  $a_0$  is considered axiomatic and is empirically determined to be  $a_0 = 1.2 \times 10^{-10} \text{ms}^{-2}$  [42] which is interestingly stable for observations [43]. While predictions of MOND do emerge in deep MOND regime ( $a \ll a_0$ ), it is only an effective theory without any physical meaning to it [44, 45]. Newtonian and deep MOND regimes are connected with interpolation function of MOND theory. Therefore, any attempt to explain galaxy rotation curves by Kaal theory must satisfy two conditions (1) results of both Newtonian dynamics and MOND must be supported (2) there must be some correlation with MOND acceleration scale  $a_0$ .

### A. Features in galactic dynamics

1. *Baryonic Tully Fisher Relation (BTFR)*: From the data [46–48], flat rotation velocity  $V_f$  and total baryonic mass  $M$  of any galaxy are related by

$$M = AV_f^x \quad (35)$$

Further, as a consequence of MOND, BTFR is equivalent to mass-asymptotic speed relation

(MASR) [41] if  $V_f$  is an adequate proxy for  $V_\infty$ , the slope  $x = 4$  and normalization  $A = \zeta/(a_0 G)$ . Thus, galaxies with same baryonic mass have the same  $V_f$  irrespective of the distribution of mass [35, 49, 50]. So, galactic dynamics behave in accordance with Newtonian laws ( $V \propto \sqrt{M_*/R}$ ) at small radii but at larger radii depend only on the total baryonic mass ( $V_f \propto M_b^{1/4}$ ) and dependence on  $R$  is lost. Thus the dynamics “knows” about the distribution of baryonic mass [51].

2. *Acceleration discrepancy*: Assuming spherical mass distribution, the dynamical mass enclosed within radius  $r$  is simply  $M_{dyn} = rV^2/G$ . Comparing this with luminosity or baryonic mass enclosed by the same radius results in mass discrepancy [16, 52, 53] also referred to as acceleration discrepancy:  $D = a/g_N \approx M_{dyn}/M_b$  [44]. Evidently, discrepancy assumes validity of Newtonian dynamics on entirety of galactic scale. Thus modification to force law by MOND eliminates the discrepancy atleast on galactic scale. In MOND, there is no discrepancy when accelerations are below the critical value  $a_0$ . A transition should occur at  $r_M \approx V^2/a_0$  [44], after which the discrepancy should increase with increasing radius as the acceleration declines ( $a \sim r^{-1}$ ). Thus, the transition radius  $r_M$  varies from galaxy to galaxy as  $a_0$  is constant.
3. *Coupling between baryonic mass and ‘dark matter’*: Following acceleration discrepancy, there exists empirical radial acceleration relation [18] between the observed radial acceleration  $g_{obs}$  and that due to the baryonic mass,  $g_{bar}$ .

$$g_{obs} = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_\dagger}}} \quad (36)$$

Thus, distribution of ‘dark matter’ follows directly from the relation leading to the conclusion that ‘dark matter’ (DM) and baryonic mass are strongly coupled [16, 17].

4. *Surface brightness*: High mass, high surface brightness (HSB) galaxies are baryon dominated in the inner regions and become DM dominated in the outer regions. Their curves are steeply rising and can be described “maximum disk” in inner regions. While, low mass, low-surface brightness (LSB) galaxies have slowly rising rotation curves and have significant DM content at small radii which systematically increases with radius. Also, rotating gas in inner regions of low-mass galaxies seems to relate to the outer regions of high-mass galaxies [17]. Further, because HSB galaxies are baryon dominated in inner regions, these are self-gravitating i.e scaling relation between central dynamical density and central stellar density of HSB galaxies is unity.

While for LSB galaxies DM contribution progressively increases but remains coupled to baryonic mass.[54]

5. *Correspondence of features*: The connection between light and mass is summarized in Renzo’s rule: “for any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa.” This follows from the assumption that stars dominate the potential and applies to both HSB and LSB galaxies. Galaxies with strong central concentrations of stars (bulge) have steeply rising rotation curves. Thus, in HSB galaxies rotation curve rises steeply while in LSB galaxies it rises more gradually [51]. Further, bumps and wiggles in the rotation curve are also known to correspond with massive spiral arms of a rotationally supported galaxy [55].
6. *Flat rotation curves*: Rotation curves of spiral galaxies are observed to remain flat indefinitely far out [56–58]. Flat rotation curves were primary motivation for ‘dark matter’ while MOND takes flat rotation curves to be axiomatic.

To build a model from the Kaal theory that defines rotation curves, following conclusions are made from these known features in galactic dynamics:

1. Coupling between baryonic mass and ‘dark matter’ when seen along with MASR leads to flat rotation velocity  $V_f$  being dependent either on baryonic mass or on ‘dark matter’.
2. Assuming exponential disk model [45] for galaxies it is sufficient to explain the rotation curves of HSB galaxy as rotation curves of LSB galaxy mimics the outer part of HSB galaxy.
3. Rotation curves as a function of mass distribution that is based only on stellar distribution can produce discrepancy in case of LSB galaxies as some of these are known to have dominant gas mass such as DDO154 [51]
4. Existence of acceleration discrepancy assumes validity of Newtonian gravity on galactic scale. However, in deep MOND regime ( $a \ll a_0$ ) due to change in force law such discrepancy vanishes. Hence, any new dynamical theory must support both ND and MOND in their respective regimes.

## B. Fluid dynamics: How the present model predicts rotation curves

### 1. Condition for Newtonian dynamics

Bounded Kaal energy (BKE) (Sec. III A 1) is the distinguishing feature between general relativity and Kaal

theory. Therefore, conventional Newtonian gravitational law is not an universal description and becomes invalid in the present theory as the shell theorem for spherical mass distribution ignores effects of BKE. However, in weak conditions where BKE is less than source mass-energy, Newtonian law is a known valid approximation.

## 2. Model: point mass in rotating fluid

In the framework of Kaal theory, galactic dynamics is modeled analogous to motion of point mass in rotating fluid. As discussed in Sec. III A 1, Kaal fluid is mass equivalence of BKE near a source mass with negligible viscosity ( $\eta \approx 0$ ). Following are features of the first approximate model for galaxy rotation curves:

- *Fluid density:* From Eq. (29) the KED near a source mass approximately varies by  $\kappa_r = (1 - 2GM/c^2r)^{-1/2}\kappa_{flat}$ . The most massive galaxies have mass  $\sim 10^{12}M_\odot$  which gives Schwarzschild radius  $\sim 10^{15}m \approx 0.1pc$ . Thus for radial distances in order of kilo-parsecs,  $\kappa_r \approx \kappa_{flat}$  and thus  $\kappa_r$  can be considered constant indefinitely far out. Consequently, Kaal fluid can be considered as a fluid of constant mass density  $\rho_{KF} = \kappa_{flat}/c^2$ .
- *Rotation of fluid:* Due to rotation of galactic core, the fluid is dragged (or vice-versa) where each layer of the fluid rotates approximately by the same linear velocity  $V_{core}$  of the core as discussed in Sec. IV C 2. Consequently, the angular velocity falls moving away from the core while linear velocity remains constant.
- *Circular disk geometry:* Ignoring the width of galactic disk, a rotationally supported galaxy is assumed to be a flat circular disk. Thus, at radius  $r$ , a ring of BKE has mass-equivalence  $M_{KF} = 2\pi r\rho_{KF} = Nr$  where  $N = 2\pi\rho_{KF} = constant$  for a galaxy.  $N$  is termed as ‘far-out energy parameter’.
- *Effect of baryonic mass:* Assuming baryons collectively as point mass  $M_{BM}$  at radius  $r$ , the conservation of momentum gives rotation velocity of fluid ring and the baryons,

$$V_r = \frac{V_{core}}{1 + M_{BM}/M_{KF}} = \frac{V_{core}}{1 + M_{BM}/NR} \quad (37)$$

Clearly any effect of mass-energy of baryons on neighborhood Kaal fluid is ignored in this approximate formulation of rotation velocity. The presence of baryonic mass at any radius will cause the velocity to reduce from  $V_{core}$ , thus  $V_{core} = V_{max}$ .

## C. Exponential disk model

The radial light distribution of spiral galaxies is generally approximated as an exponential disk [45]

$$\Sigma(R) = \Sigma_d e^{-R/R_d} \quad (38)$$

where  $\Sigma_d$  is central surface brightness [ $\Sigma(R=0)$ ] and  $R_d$  is disk scale length.  $\Sigma_d$  characterizes if a disk is high surface brightness (HSB) or low surface brightness (LSB). Though it is a tolerable approximation however in some galaxies mass of gas can exceed that in stars as neutral gas is generally more extended [51]. Assuming an appropriate mass-to-light ratio  $\gamma^*$ , mass can be determined. For Milky Way, estimated bulge stellar mass is  $\sim 2 \times 10^{10}M_\odot$  [59], assume scale length  $R_d = 2.5kpc$  [45] and  $V_{flat} = V_{max} = 230km s^{-1}$  [55]. Thus Eq. (37)

$$V_r = \frac{V_{flat}}{1 + (\gamma^* \Sigma(R))/NR} \quad (39)$$

predicts a rotation curve for some  $N$ , correlated with the observed curve [55]. Clearly, this is a general description based on widely accepted smooth exponential disk model for galaxies. It is only presented to show that the shape of rotation curves predicted by the theory concurs with observations.

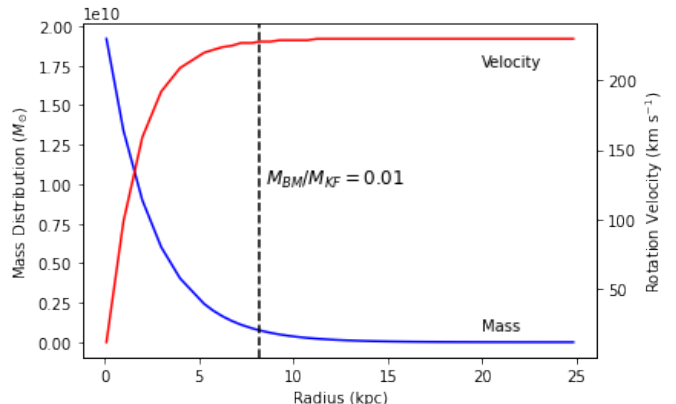


FIG. 2. The rotation curve (red solid line) for Milky Way exponential disk model as predicted by the theory is shown against exponential mass distribution (blue solid line). The rotation velocity flattens when baryonic mass is much less than mass-equivalence of ring of bounded Kaal energy i.e.  $M_{BM}/M_{KF} \ll 1$ . The shape of predicted rotation curve concurs and correlates with the observation.

## D. SPARC database: Validating against actual observed rotation curves

### 1. Data

SPARC is a representative sample of disk galaxies in the nearby universe spanning the widest possible range of

properties of galaxies (HSB and LSB) with extended rotation curves [60]. SPARC data has been used in several publications [18, 60–63] and is publicly available online at [astroweb.cwru.edu/SPARC](http://astroweb.cwru.edu/SPARC). From the preceding sections, the proposed galactic fluid model considers only smooth stellar mass distribution and ignores other elements for simplicity. Therefore, for the purposes of this study, galaxies which exhibit following properties are considered from the SPARC database:

- rotation curve rises and flattens smoothly i.e.  $V_{flat} \approx V_{max}$
- minimal or no bumps and wiggles which are due to uneven mass-energy distribution
- stars dominate the potential i.e. deviations can be attributed mostly to gas mass-energy

Thus, although all 175 galaxies were analyzed, the relevant sample size comprises of 130 galaxies ( $\approx 74\%$ ) with  $20 \text{ km s}^{-1} \leq V_{flat} \leq 321 \text{ km s}^{-1}$ . Effects of uneven mass-energy distributions are subject of subsequent research with more complex fluid rotation models.

### 2. Mass-to-light ratio $\Upsilon_*$

Luminosity is converted to stellar mass by adopting  $\Upsilon_{disk} = 0.5 M_{\odot}/L_{\odot}$  and  $\Upsilon_{bul} = 1.4 \Upsilon_{disk} = 0.7 M_{\odot}/L_{\odot}$  [60]. Thus baryonic mass at radius  $R$  is  $M_{BM}(R) = 0.5 \Sigma_{disk}(R) + 0.7 \Sigma_{bul}(R)$ . The current model only considers stellar masses for simplicity and excludes contribution of other baryonic mass: gas, dust and remnants.

### 3. Methodology

I set the likelihood function as  $L = e^{-\frac{1}{2}\chi^2}$  [64] with  $\chi^2$  given by

$$\chi^2 = \sum_R \left[ \frac{V_{obs}(R) - V_{pred}(R)}{V_{err}(R)} \right]^2 \quad (40)$$

where  $V_{pred}(R)$  is the expected rotation velocity from Eq. (37) and  $V_{obs}, V_{err}$  are observed rotation velocity and its error from the data. While the error bars differ, SPARC data is a good approximation of galaxy rotation velocities as described in several published papers.

Standard affine-invariant MCMC ensemble sampler in the Python package *emcee* [65] was employed to map posterior distributions with flat prior for  $\log N > 10$ . The Markov chains are initialized with 100 random walkers and iterated for 2000 steps. Best fit parameter value of  $N$  that maximize the probability function for each galaxy was recorded. Similar results were obtained using gaussian priors on a sample.

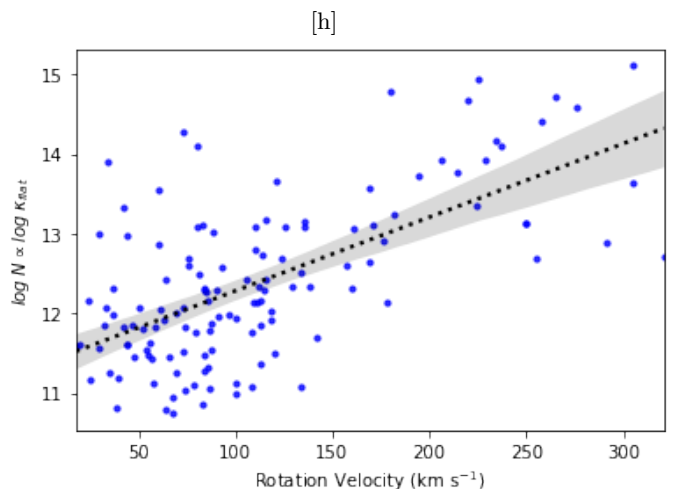


FIG. 3. Result of model fitting using *emcee*: The far-out energy parameter  $N$  for 130 galaxies against their rotation velocity  $V_{flat}$  is shown. Each dot is a galaxy. The general trend in the plot shows higher the rotation velocity, larger is the bounded energy which supports coupling between baryonic mass and dynamics. Further, any two galaxies with same  $V_{flat}$  but different  $N$ , indicates difference in slope in the inner regions of rotation curve. Higher the central core mass-energy density, steeper is the rotation curve in the inner regions. This shows agreement between Newtonian dynamics and the proposed model.

### 4. Result

Using Eq.(37) for each galaxy in the study, the predicted rotation curve fits with the observed curve for some value of  $N = 2\pi\rho_{KF} = 2\pi\kappa_{flat}/c^2$ . The curve fits validate the galactic dynamics based on proposed fluid model from the Kaal theory. In Figure VD4 the parameter  $\log N$  for each galaxy is shown with respect to its rotation velocity  $V_{flat}$ . Trend-line supports coupling between baryonic mass and dynamics as it has been observed [18, 51]. In Appendix C the predicted and observed rotation curves are shown for the sample of 130 galaxies..

Any two galaxies with same  $V_{flat}$  but different  $N$  indicate there is difference in bounded Kaal energy density which is reflected in the steepness of rotation curve. This implies there is difference in mass-energy density of the galactic cores in correspondence with Renzo’s rule. Further, if Newtonian force law is applied in inner regions of rotation curve, steeper curve indicates higher mass density. Therefore, both the theories give the same result and it can be concluded Newtonian gravity is an approximation of Kaal theory in inner regions of galaxy.

As radial distance increases, the ratio of baryonic mass to mass-equivalence of bounded Kaal energy becomes much less than unity, resulting in flat rotation curve in the outer regions. Once  $M_{BM}/M_{KF} \ll 1$  is achieved, rotation velocity is distance independent similar to MOND.

## VI. IMPLICATIONS FOR GENERAL RELATIVITY, MOND AND DARK MATTER

### A. General relativity

General relativity (GR) is a complete theory if bounded Kaal energy (BKE) is ignored and only the variation in length and time *scales* is considered. Thus, GR is a mathematical construct over Kaal theory in weak conditions. However, on the largest scales BKE cannot be ignored as this is evident from galaxy rotation curves and hence GR by itself gives incomplete description of physical phenomenon. This leads to unexplained observations being attributed to ‘dark matter’, ‘dark energy’ or modified dynamics.

Proposed Kaal theory postulates length and time scales vary due to energy bounded by mass and expansion of universe occurs due to minimizing nature of Kaal structure. Kaal structure deforms from being ‘attractive’ on large scale to being ‘repulsive’ on largest scales. Therefore, any mathematical description of flow and distribution of Kaal energy must step from being binding on large scales to being repulsive in vast, empty space on largest scales. It has to be a continuous, smooth deformation but for simplicity it can be said there must be two sets of Einstein field equations (EFEs) to completely describe the dynamics and thus the large scale effects. This leads to modifications to EFEs as now both attractive and repulsive ‘gravity’ are part of one structure.

Historically, observations have led to modifications to EFEs. Einstein introduced cosmological constant term  $\Lambda$  to his field equations to formulate for a static universe. Eventually, it was dropped when universe was found to be expanding however, it was once again introduced later to compensate for accelerating universe. Thus modifications to EFEs is not new.

The accelerated expansion of universe is currently described by introduction of cosmological constant term  $\Lambda$  in EFEs. However, in Kaal theory continuous minimization of local energy density causes accelerated expansion and the acceleration must be monotonically decreasing. Therefore any term attempting to explain repulsive effects in Kaal theory cannot be a constant and must be dependent on time and space coordinates:

$$\text{Cosmological function : } \Lambda(t, x^1, x^2, x^3) \quad (41)$$

It is cautioned that cosmological function is equivalent to a matter field and hence would form part of source lagrangian in the action thereby preserving general covariance. Now, to arrive at a simpler model which approximates to existing formulation, we can make the following assumptions:

1. expansion is isotropic
2. at any given point in time, expansion rate is inde-

pendent of spatial coordinates i.e.

$$\Lambda(t, x^1 + \delta x^1, x^2 + \delta x^2, x^3 + \delta x^3) = \Lambda(t, x^1, x^2, x^3) \quad (42)$$

$$\Lambda(t = T) = \text{constant} \quad (43)$$

$$\Lambda(t = T + \delta T) < \Lambda(t = T) \quad (44)$$

Thus, we can approximate it to be a function of only time as  $\Lambda(t)$ .

In Kaal theory, matter and energy are separate entities thus at any given point either matter or energy exists. This implies the right hand side of EFEs i.e. energy-momentum tensor  $T_{\mu\nu}$  must correspond to only KED and its flow. Classically, energy of an object is considered to have no presence beyond its physical boundaries i.e. in empty space. Though this is an incoherent description of energy of source mass in Kaal theory, the mathematical description differs only by the density and flow of BKE near a source mass and can be captured by  $T_{\mu\nu}$  itself. This is definitely valid till the limit  $M_{BM}/M_{KF} \ll 1$  on galactic scale i.e as far as Newtonian dynamics is valid, however beyond this distance the implications on EFEs if any are still being explored.

On largest scales, space is free from influence of matter, the RHS of EFEs must correspond mostly with monotonically decreasing local energy density i.e. energy-momentum tensor is defined by cosmological function which is equivalent to time varying negative mass. The effect of minimization of local KED (repulsion/expansion) is the dominant source (this can be assumed considering dark energy constitutes 70% mass-energy of universe). Cosmological implications of the proposed Kaal structure are not explored in this paper.

Now, modification to EFEs is introduced as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \begin{cases} kT_{\mu\nu} & \text{large scale} \\ kT_{\mu\nu} - \Lambda(t)g_{\mu\nu} & \text{largest scale} \end{cases} \quad (45)$$

Thus the EFEs now take two forms, first to describe spacetime when mass interacts with Kaal structure by binding it and second for largest scale, when length and time scales increase due to reduction in local energy density. The transformation between these two equations is continuous and smooth which is ignored for simplicity here. Assuming the large age of universe, it may be difficult to observe the reduction in accelerated expansion of universe over a short period of time such as a few decades.

#### 1. Considering spatial dependence

If the second assumption made in Eq. (42) is ignored, which ideally should be ignored when comparing observations from near and far universe, then we have  $\Lambda(t, x^1, x^2, x^3)$ . We continue to assume isotropy and adopting spherical coordinates we have  $\Lambda(t, r)$  defining

decreasing local KED causing expansion. Therefore, the second modified EFE takes the form

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = k \left( T_{\mu\nu} - \frac{\Lambda(t, r)}{k} g_{\mu\nu} \right) \quad (46)$$

This could mean the expansion rate measured using a galaxy in far universe may be lower compared to expansion rate measured using a galaxy in near universe.

## 2. Remarks

It should be noted that none of the modified EFEs above is an universal equation and has its own domain of application. However, it is known that dynamics become distance independent on galactic scales and cannot be explained by Newtonian dynamics. Therefore, Kaal theory and general relativity should be seen as complementing theories which agree within Newtonian regime where effects of BKE are approximated by Newtonian dynamics where  $M_{BM} \gg M_{KF}$ . On largest scales, an alternative approximation of Kaal theory is needed to explain observations.

## B. MOND

From the rotation curves results of previous section, it can be seen in both observed and predicted curves that flatness begins much before the transition radius  $r_M = V_{flat}^2/a_0$  in MOND [44]. Both MOND and Kaal theory suggest outer regions of rotation curves are distance independent. Possibly the MOND acceleration scale  $a_0$  can be said to be equivalent to the condition  $M_{BM}/M_{KF} \ll 1$ . Further, the same condition encapsulates the scale invariance property [66]. Can MOND be an approximation for galactic dynamics in Kaal theory? This result may give physical meaning [51] to acceleration scale  $a_0$  in MOND i.e. beyond transition radius the baryonic mass is negligible compared to the mass-equivalence of ring of bounded Kaal energy (BKE). However, the theory certainly lends support to MOND as an approximation for galactic dynamics.

## C. Dark matter

Kaal theory postulates existence of energy spanning space which is equivalent to superfluid on largest scales. It can be profiled similar to dark matter halos and be employed to account for dynamics in clusters and other observations. Kaal energy interacts through ‘gravity’, is collision less, non-baryonic and is dragged by (or drags) mass. Thus mass equivalence of Kaal energy density at a point can be considered as a virtual particle and does not rule out the work done on simulations based on CDM model.

Thus, proposed Kaal theory is a hybrid structure and broadly brings together work done on both MOND and CDM models while preserving mathematics of general relativity. Detailed integration of models is subject of subsequent research.

## VII. DISCUSSION: TIME

There are three objects that define the notion of time - (a) rate of passage of time (time density) (b) measure of time (c) time - within the framework of observed, observation and observer. *Rate of passage of time* is very well understood from general relativity and has been discussed previously in this paper. In Kaal theory, every particle is in some state of motion, so is a clock and its constituents always in motion; thus each tick of the clock measures the rate of passage of time. The number of ticks registered by the clock is the *measure of time*. If the time density is constant over a certain region of Kaal structure, notion of time gives the impression of being a parameter otherwise when this varies over a certain region, it is equivalent to curvature in  $\mathcal{RTE}^3$  of Kaal structure or in (1+3) spacetime of general relativity. *Time* is the total number of ticks registered by a clock for a phenomenon under observation. Thus, if phenomenon ends, so does time for it. It is the concept of ‘rest’ which muddles the understanding of time vis a vis space (‘rest’ does not exist in present framework, Kaal structure is dynamical). There is no concept of time reversal in the present framework by the very way it considers measure of time. Past and future can be determined but cannot be accessed as ticking of clock is not independent of flow of KED.

Clocks enable measure of time using an object in motion such as light or a particle. Fundamentally, clocks compare one motion with another and thus motion is fundamental for the existence of measurement of time. However, at any point time density must not be dependent on an object, it must exist independent of light or particle.

## VIII. CONCLUSIONS

The paper begins by distinguishing between *scales* and measurements; while *scales* are locally constant, measurements can be arbitrary. This is a theory of emergence of local length and time *scales*. I use light wave as an instrument of defining local length and time *scales* via its wavelength and period respectively. Using this, results from special and general relativity, the founding principle for the theory is defined - variation in length and time *scales* is coequal i.e. symmetric. Following which the theory proposes

1. matter and energy are separate physical entities and both interact to give the observed phenomenon

2. a three-dimensional probabilistic structure *Kaal* ('dark') spans space and defines energy density, time density and 3-Riemannian metric for length *scale* at each point.
3. mass binds energy density causing variation in length and time *scales* which is equivalent to space-time curvature in GR

Large scale analysis was performed for Schwarzschild conditions which defines energy density near a static source mass. Case of rotating sources is discussed and therein the dynamics was validated using galaxy rotation curves. Thus, flow and distribution of the energy causes 'gravitational' effects which is also equivalent to spacetime curvature in weak conditions. Consequently there is mathematical correspondence with results of GR while interpretation may need a revision.

A postulated property of Kaal structure is - it minimizes local energy density in regions free from influence of matter. On largest scales, this is equivalent to expansion of universe as Kaal structure expands. This minimization is monotonically decreasing and captured by *cosmological function*  $\Lambda(t, x^1, x^2, x^3)$ .  $\Lambda(t, x^1, x^2, x^3)$  is equivalent to negative mass source and thus preserves general covariance. It proposes that universe as a system tends towards a lower energy state. Distinction between matter and energy along with introduction of cosmological function leads to modification of Einstein field equations (EFEs). Though there is continuous deformation in the Kaal structure, two sets of EFEs are suggested one each for large and largest scales.

On large scale, the energy bounded by mass is equivalent to a surrounding super-fluid and thus a simple fluid

dynamics model is used as an approximation to predict the galaxy rotation curves. Model fitting was done for 130 galaxies from SPARC data set using an MCMC technique. The results (Appendix C) from the model agreed with the observed rotation curves thus validating the theory. Further, the results indicate that the model agrees with Newtonian dynamics in the inner regions of the curve and becomes distance independent, scale invariant similar to MOND in the outer regions. An universal condition  $M_{BM}/M_{KF} \ll 1$  i.e. baryonic mass being much less than mass equivalence of energy density sets the flatness of rotation curves similar to acceleration scale  $a_0$  in MOND. Energy density can also be considered as a virtual particle of the superfluid and simulated on largest scales similar to CDM to account for other 'dark matter' observations. Thus, the theory is established as a hybrid model which preserves GR.

The theory describes gravitational effects in sync with general relativity and additionally resolves missing mass problem on galactic scale. While qualitatively the theory explains other 'dark matter' observations, much work needs to be done to describe these large scale phenomenon and also arrive at cosmological implications of the theory. The novelty of the proposed theory possibly allows integration of various existing 'dark matter' models and explain the large scale phenomenon.

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## Appendix A: Local Analysis

Local analysis is discussed to establish correspondence with special relativity. It is abstract and is *not* a substitute to special relativity rather *complementary* to its results. This is not a proposal to replace any existing mathematics.

### 1. Local Kaal structure

As postulated, the Riemannian 3-Manifold  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  of length measure is locally Euclidean and diffeomorphic to  $\mathbb{R}^3$ . Here, it is then convenient to consider Kaal energy density (KED) as a *scalar field* over Euclidean flat three-space. As  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  is locally flat, this implies KED is a constant scalar field (neglecting the effect of presence of matter and effects of KED) i.e. KED is not a function of any coordinates. This implies Kaal ripples are consistent and time density is constant i.e. length and time *scales*

are constant.

#### a. Inertial and non-inertial frames

Consider local patches  $U, V \in \mathcal{K}$  such that KED in  $U$  and  $V$  is some constant function  $\kappa_U$  and  $\kappa_V$  respectively with respect to some arbitrary observer in  $U$ . Such patches in  $\mathcal{K}$  with constant KED are equivalent to inertial frames of reference. Kaal ripples in such regions flow with consistent structure i.e. with constant wavelengths and period. Thus, time and length *scales* remain constant i.e. time density and metric of  $\mathcal{R}\mathcal{I}\mathcal{E}^3$  are constant.

Then, inertial frames of references are defined as *diffeomorphisms of a local patch of Kaal structure  $\mathcal{K}$  that preserve the local structure*.

Similarly, if in a certain patch of  $\mathcal{K}$ , KED is some non-constant function of space and time parameters then these correspond to non-inertial frames of reference. The structure of Kaal ripples is also a function of these parameters and so are the local length and time *scales*.

## 2. Equations and Solutions

In this section, the effects on the Kaal structure due to mass of test particle/observer are neglected. Kaal ripples thus flow with equal speed with respect to space and time parameters (with  $c = 1$ ). Locally, the description of Kaal ripples is consistent with the equations,

$$|\nabla\Omega| = \left| \frac{\partial\Omega}{\partial t} \right| \quad (\text{A1})$$

$$\nabla^2\Omega = \frac{\partial^2\Omega}{\partial t^2} \quad (\text{A2})$$

where  $\nabla$  is the gradient operator in the adopted spatial coordinates. The relationship described by (A1) is the fundamental local *invariant* identity in Kaal theory. Wave-like motion of Kaal ripples is assumed in setting up the above equations as such a motion (flow) of energy can accommodate  $(N + 1)$  parameters in  $N$  dimensional space. Also, waves are ubiquitous in physics.

General solution of (A2) is known to be a superposition of left and right traveling waves parameterized by  $(t, x^1, x^2, x^3)$

$$\Omega = f(\vec{k}\cdot\vec{r} \pm \omega t) \quad (\text{A3})$$

where  $(\vec{k}, \omega)$  indicate the wave-vector ( $\equiv$  metric) and frequency ( $\equiv$  KED and time density) of Kaal ripples at the point of interest. These are also equivalent to momentum and energy of Kaal ripples respectively.

$\Omega$  is the *probability amplitude* of locating KED described in (A3) at the point of observation. For inertial frames of reference i.e. regions in  $\mathcal{K}$  with constant KED, the solution  $\Omega$  is a plane-wave spanning such region. Thus, the probability of locating KED at any point

in the region is unity as expected. On large scale, these ripples in Kaal structure generally do not affect the physical phenomenon but KED does; however on small scale, Kaal ripples plays a critical role not being discussed in this paper.

Based on above, (A1) is equivalent to

$$|\vec{k}| = |\omega| \quad (\text{A4})$$

which is true for Kaal ripples and light (classically) at all points in regions free from effects of matter. Kaal energy density (KED) is formulated as

$$\kappa = H\omega \quad (\text{A5})$$

where  $H$  is some constant.

This leads to considering a wave-function like entity beyond quantum scale. It is like a corrugated sheet which on a large scale can be assumed smooth, can curve like a sheet of paper does but on smaller scales corrugations effect the motion of particles.  $\Omega$  is not identical to quantum wave-function  $\Psi$  though they can be considered similar. Description of quantum systems is not discussed in this paper since interaction of matter properties other than mass is not considered here.

Consider the following form for (A3),

$$\Omega = \exp(i\vec{k}\cdot\vec{r} + i\omega t) \quad (\text{A6})$$

to work with. This is a plane wave solution with  $|\Omega| = 1$  at all points as expected. On large scale, this solution suffices as only wave-vector  $\vec{k}$  and frequency  $\omega$  are enough to describe large scale phenomenon and Kaal ripples are not significant.

### 3. Diffeomorphic Transformations

Consider two local patches  $U, V \in \mathcal{K}$  with flow of Kaal ripples defined as  $\Omega_U, \Omega_V$  respectively.  $U, V$  can be thought of as local patches on a manifold as in theory of differentiable manifolds. Clearly,  $\Omega$  is parameterized by three spatial coordinates  $(x^1, x^2, x^3) \in \mathbb{R}^3$  and one time-coordinate  $t \in \mathbb{R}$ .

Let us consider two inertial frames of reference  $\Sigma, \Sigma'$  such that  $\Sigma'$  is moving with relative constant velocity  $v$  with respect to  $\Sigma$  along one coordinate axis. Let the regions  $U, V$  correspond to these inertial frames  $\Sigma, \Sigma'$  respectively. Kaal ripples in  $U, V$  can then be described by

$$(U, \Sigma) : \exp(i\vec{k}_U \cdot \vec{r} + i\omega_U t) \quad (\text{A7})$$

$$(V, \Sigma') : \exp(i\vec{k}_V \cdot \vec{r} + i\omega_V t) \quad (\text{A8})$$

with  $|\vec{k}_U| = |\omega_U|$  and  $|\vec{k}_V| = |\omega_V|$ . From special relativity, clocks in  $\Sigma'$  run slower relative to clocks in  $\Sigma$  i.e. time density is higher in  $(V, \Sigma')$  relative to time density

in  $(U, \Sigma)$  i.e. KED is higher in  $V$  with respect to KED in  $U$  i.e.

$$\kappa_V > \kappa_U$$

Without loss of generality, using the time dilation (or length contraction) results from special relativity, it is evident time (or length) scales must be governed by

$$\kappa_V = \gamma \kappa_U \quad (\text{A9})$$

where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ . This implies

$$\omega_V = \gamma \omega_U \quad (\text{A10})$$

For the ease of understanding, let us assume the Kaal ripples are propagating in the same direction  $r$  in both patches. Thus wave numbers are related by

$$k_V = \gamma k_U \quad (\text{A11})$$

such that, in the units with  $c = 1$ ,

$$\frac{\omega_V}{k_V} = \frac{\omega_U}{k_U} = 1 \quad (\text{A12})$$

and this is equivalent to two regions  $U, V \in \mathcal{K}$  having a *diffeomorphic mapping* between them

$$N_{UV} : U \rightarrow V \quad (\text{A13})$$

$$N_{UV} : \exp(ik_U r + i\omega_U t) \rightarrow \exp(ik_V r + i\omega_V t) \quad (\text{A14})$$

The diffeomorphic map  $N_{UV}$  leads to Lorentz transformations between  $\Sigma$  and  $\Sigma'$ .

## 4. Result

Local analysis relates the length and time *scales* of two arbitrary inertial reference frames which is practically captured by proper length and proper time in special relativity. The analysis in this section does not relate arbitrary length and time *measurements* between two reference frames, that is already achieved by Lorentz transformations rather the relation between length and time *scales* is discussed within the framework of Kaal theory. This is used for generalization in large scale analysis.

### Appendix B: Kinetic Energy

A moving source must have higher KED in its neighborhood that sustains its motion as against KED bounded by static source mass. This incremental KED in its neighborhood due to virtue of its motion must then correspond with kinetic energy as it is conventionally understood. To determine this incremental KED, results from special relativity or local analysis (Appendix A) are used .

**Rotations.** In Schwarzschild conditions, KED in the neighborhood of a source mass  $M$  is determined to be

$$(\kappa_r)_{Schwarzschild} = \left(1 - \frac{2GM}{c^2 r}\right)^{-\frac{1}{2}} \kappa_{flat} \quad (B1)$$

as in Eq. (29). Say the radius of source is  $R_M$  and its constant angular velocity  $\omega_M$ . Surface of the rotating source mass presents one boundary condition i.e. KED in infinitesimal volume element at the surface is dragged by linear velocity  $\omega_M R_M \sin\theta$  in the direction of rotation. Consequently, KED  $(\kappa_r)_{rot}$  on the surface and thus in the neighborhood of rotating source must increase by the factor  $\gamma$  as

$$(\kappa_r)_{rot} = \gamma(\kappa_r)_{Schwarzschild} \quad (B2)$$

$$\gamma = \left(1 - \frac{(\omega_M R_M \sin\theta)^2}{c^2}\right)^{-\frac{1}{2}} \quad (B3)$$

This implies angular velocity of infinitesimal volume element of KED falls as radial distance increases while the linear velocity remains almost constant (here it is implied Kaal fluid has negligible viscosity). Now, kinetic energy can be described as total incremental KED in the neighborhood Kaal structure such that

$$Kinetic\ Energy = \iiint_{R_M}^r (\gamma - 1) \Delta\kappa_r dVol \quad (B4)$$

where  $\Delta\kappa_r = (\kappa_r)_{Schwarzschild} - \kappa_{flat}$ , the bump from flat conditions due to mass. Because LHS is generally known quantity based on conventional understanding, the unknown is the upper limit  $r$  of the integral on RHS. The amount of KED a source mass binds in its neighborhood is dragged just as a submerged rotating sphere drags fluid in its neighborhood (or equivalently rotation of Kaal fluid sustains rotation of the source mass).

**Translation.** Following the discussion for rotations, the case for translation is very similar with the exception of no angular dependence, as all elements on the surface of the source mass move with same linear velocity. This makes translations less interesting. Rotations are vital to observe “gravity” and hence are ubiquitous in nature. Translatory motion is equivalent to dragging a certain block of Kaal structure with velocity same as source velocity. The general equation (B4) for kinetic energy also applies for translations.

For slow moving sources  $v \ll c$ , kinetic energy is  $Mv^2/2$  and  $\gamma = 1 + v^2/2c^2$ , so (B4) can be written as

$$\frac{Mv^2}{2} = \iiint_{R_M}^r \frac{v^2}{2c^2} \Delta\kappa_r dVol \quad (B5)$$

$$Mc^2 = \iiint_{R_M}^r \Delta\kappa_r dVol \quad (B6)$$

which gives an equivalence between the known rest energy and energy bounded by mass in the present theory.

### Appendix C: Predicted vs. observed rotation curves

Plots based on MCMC model fitting using *emcee* are shown for 130 galaxies. Solid red line is the rotation curve predicted by the theory and scattered plot with error bars is actual rotation velocity from SPARC data set. The model assumes  $V_f = V_{max}$  and considers only the stellar mass ignoring other baryonic mass such as gas as described in Sec. V B. Thus if the potential is dominated by gas in certain regions of a galaxy it would be evident as deviation in the plot from observed data points. The low mass galaxies and outer regions of high mass galaxies are known to be gas dominated and so their rotation curves can exhibit slight deviation from the model. Further, in case of high mass galaxies the mass-energy distribution is not smooth and contributes to the velocity profile of the Kaal fluid. A more complex fluid dynamics model can be constructed as part of subsequent research that mimics respective rotation curves based on complex mass-energy distribution.

