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Smooth Switching of Multi-LPV Control Systems Based on Youla-Kucera Parameterization ^{*}

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Abstract: This paper presents a smooth switching between a set of Linear Parameter-Varying (LPV) controllers that have been designed separately. The switching control strategy is based on Youla-Kucera (YK) parameterization. A generalized interpolation scheme of various LPV controllers is shown and proved for the LPV-YK concept. Such kind of parameterization is beneficial to switch or interpolate between multiple controllers without adding any constraints to the design of the local controllers and the switching signals. The proposed method is applied to an Active Magnetic Bearing (AMB) system and compared to a switched LPV controller.

Keywords: Linear parameter-varying system, Youla parameterization, Gain-scheduling, Linear matrix inequality

1 Introduction

Linear Parameter-Varying (LPV) control techniques are widely used in different applications as in aerospace Jiang et al. (2020) and autonomous vehicles Atoui et al. (2020a), Atoui et al. (2020b), etc. Hoffmann and Werner (2015) presents a deep LPV control review. The synthesis of an LPV control can be formulated as a Linear Matrix Inequality (LMI) optimization problem using a single Lyapunov function, either quadratic Packard (1994) or parameter-dependent Wu et al. (1996). On the other hand, a single Lyapunov function for complex models with large parameter variations may not exist, or if it exists, it could be conservative. A solution has been proposed in the literature to design multiple LPV controllers based on multiple parameter-varying Lyapunov functions, each suitable for a specific parameter subregion, and switch between them to achieve better performance Lu and Wu (2004).

1.1 LPV Switching Control

In 2004, switched LPV control systems have been first proposed in the pioneering work Lu and Wu (2004). The switching stability has been studied for hysteresis switching and switching with average dwell-time strategies. This methodology enhances the use of the LPV techniques in several applications, see for instance Bei Lu et al. (2006), Lescher et al. (2006), Postma and Nagamune (2012).

On the other hand, it has been stated in Lu and Wu (2004) that the switched LPV controllers may not provide a smooth transient response during switching, where aggressive performance is obtained at switching instants. Such case may lead to mechanical damage, decrease material lifetime, or signal saturation which is out of real application objectives. Following this work, challenges have been raised to smooth the control response during switching, and to relax the limitations on the switching signals. Several research studies have been involved in solving the switching smoothness. For instance, a bumpless transfer of switching controllers is proposed in Dimon et al. (2005) followed by some developments in Chen et al. (2010), and Safaei et al. (2012).

In addition, a smooth switching LPV controller has been proposed first in Hanifzadegan and Nagamune (2014). It is designed in considering adjustable interpolation functions and a higher order differential control signal. An iterative descent algorithm is applied to optimize three decision variables (the parameter-dependent Lyapunov functions, the local controllers, and the interpolation functions). It also augments the problem to two dimensional parameter regions. This concept is developed in the recent works He et al. (2018) and He et al. (2019), however, it increases the complexity and the constraints of the design of the local controllers to achieve its objectives.

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1.2 LPV-YK Control

Recently, Youla-Kucera (YK) parameterization has been also used to switch between LPV controllers. It has been proposed in Atoui et al. (2021) to interpolate between two pre-defined quadratically stabilizing LPV controllers. More advanced LPV-YK control formulations have been proposed and proofed in Atoui et al. (2022). The objective is to achieve the closed-loop quadratic stability under arbitrary switching signals without requiring a constant Lyapunov function in the design of the local LPV controllers. However, the proposed theorem is still conservative since it requires quadratically stabilizing polytopic controllers.

The interest behind designing an interpolation control scheme based on YK is to parameterize a set of LPV stabilizing controllers $K(Q)$, where each one is parameterized by its corresponding LPV-YK parameter $Q(\rho)$. Notice that $Q(\rho)$ is designed using the doubly coprime factorization Vidyasagar (2011), Atoui et al. (2021). An advantage of the YK concept is that it doesn't require of the re-design of the defined local LPV controllers. Its LMI conditions concern only the stability of the YK parameterization.

It is worth to mention some advantages of the YK parameterization to interpolate between several controllers: 1) Dimensions of the controllers could differ; 2) Interpolated controllers can be designed and tuned separately using different control techniques (\mathcal{H}_∞ , LQR, PID, ...) Tay et al. (1997); 3) It facilitates adding new controllers to an existing switching control system online as Plug&Play control theory Stoustrup (2009), without re-designing; and 4) The closed-loop stability is guaranteed under arbitrary continuous/discontinuous interpolating signals without requiring a common Lyapunov function Hespanha and Morse (2002).

1.3 Motivation and Contribution

Active Magnetic Bearing (AMB) system is known to be highly nonlinear, multi-variable and inherently unstable system. The nonlinearity is described in the relationship between the electro-magnetic force, current and length of the air gap. Moreover, the system (vibrational) dynamics depend on the rotational speed due to gyroscopic and electro-magnetic coupling. The AMB system has been previously controlled using the three LPV approaches: polytopic (Lu et al. (2008)), grid-based (Morsi et al. (2021)), and LFT (Lauridsen and Santos (2017)). In addition, (Lu and Wu (2004)) has implemented a switched-LPV controller on the AMB application to improve its closed-loop performance. The results have shown good performance over each parameter subregion, however, the proposed switched-LPV controller leads to aggressive transient switching.

This paper aims to design a smooth switched LPV controller based on YK parameterization which ensures the switching stability over the parameter subregions and smooth transient switching. Among all the previous works which have shown successful and smooth LPV switching as Lu and Wu (2004), Chen et al. (2010), and Hanifzadegan and Nagamune (2014), all of them require the re-design of the local LPV controllers using proposed LMIs, in addition to switching signals limitations (e.g. hysteresis switching, switching with average dwell-time, etc.). It is

worth mentioning that the re-design of all the local LPV controllers together may cause conservatism when increasing the number of subregions or parameter dimensions. The current paper proposes the YK parameterization to: 1) Simplify the design of the LPV switching control system by decreasing the complexity of the LMI conditions (no need to re-design the local LPV controllers); 2) Avoid any limitation on the switching signals without requiring constant Lyapunov function; 3) Smooth the control and state responses during the switching instants; and 4) Avoid the re-design of the switching control scheme if one needs to add or remove any of the local LPV controllers as Plug&Play.

The general contribution of this paper is to interpolate between multiple LPV controllers smoothly, each LPV controller is designed to perform well over a certain parameter sub-region. The interpolation scheme is formulated based on the LPV-YK parameterization. Notice that the proposed LPV-YK control scheme has been contributed and proved in (Atoui et al. (2022)). An interpolation scheme is drawn between multi-LPV controllers based on YK parameterization which guarantees the closed-loop stability and performance under arbitrary interpolating signals. In addition, a comparative numerical simulation is presented at the end to compare the switching smoothness between the proposed LPV-YK switching controller and the LPV switched controller proposed by Lu and Wu (2004).

2 Grid-based LPV-YK Control Configuration

This section represents the configuration scheme of a grid-based LPV-YK controller using the coprime factorization.

2.1 LPV Plant and Controllers Description

Consider a Multi-Input-Multi-Output (MIMO) LPV system $G(\rho)$ with m inputs and p outputs:

$$G(\rho) \begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B_1(\rho(t))w(t) + B_2(\rho(t))u(t) \\ z(t) = C_1(\rho(t))x(t) + D_{11}(\rho(t))w(t) + D_{12}(\rho(t))u(t) \\ y(t) = C_2(\rho(t))x(t) + D_{21}(\rho(t))w(t) + D_{22}(\rho(t))u(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$, $z(t) \in \mathbb{R}^{n_z}$ are the state, output, input, controlled output vectors respectively. $w(t) = [r \ n \ d]^T \in \mathbb{R}^{n_w}$ contains the exogenous inputs of the tracking reference r , noise n and input disturbance d . All the state-space data are continuous functions of the parameter vector ρ . Assume that ρ is in a compact set $\mathcal{P} \subset \mathbb{R}^s$ with its parameter variation rate bounded by $\underline{\nu}_k \leq \dot{\rho}_k \leq \bar{\nu}_k$ for $k = 1, 2, \dots, s$. Moreover, let us assume the following:

- $(A(\rho), B_2(\rho), C_2(\rho))$ triple is parameter-dependent stabilizable and detectable $\forall \rho \in \mathcal{P}$.
- $[B_2^T(\rho) \ D_{12}^T(\rho)]$ and $[C_2(\rho) \ D_{21}(\rho)]$ have full row ranks $\forall \rho \in \mathcal{P}$.
- $D_{22}(\rho) = 0$.

Suppose that the parameter set \mathcal{P} is covered by a finite number of closed subsets $\{\mathcal{P}_i\}_{i \in Z_N}$, where the index set $Z_N = \{1, 2, \dots, N\}$, and $\mathcal{P} = \bigcup \mathcal{P}_i$. At the boundaries between each adjacent subsets, there exist at least a single intersecting boundary or an intersecting surface.

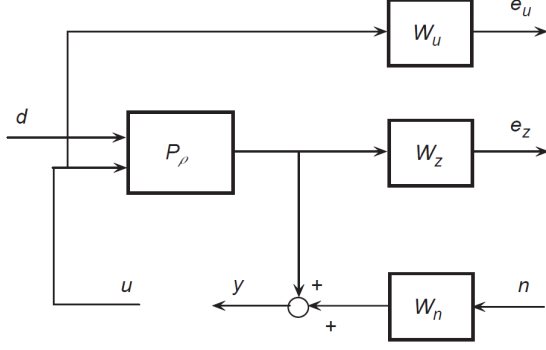


Fig. 3. Weighted open-loop interconnection Lu and Wu (2004)

$$\begin{aligned}
 l\ddot{\theta} &= -\frac{\rho J_a}{J_r} l p \dot{\psi} + \frac{1}{m} (-4c_2 l \dot{\theta} + 2c_1 \phi_\theta + f_{d\theta}), \\
 l\ddot{\psi} &= \frac{\rho J_a}{J_r} l \dot{\theta} + \frac{1}{m} (-4c_2 l \dot{\psi} + 2c_1 \phi_\psi + f_{d\psi}), \\
 \dot{\phi}_\theta &= \frac{1}{N} (e_\theta + 2d_2 l \dot{\theta} - d_1 \phi_\theta), \\
 \dot{\phi}_\psi &= \frac{1}{N} (e_\psi + 2d_2 l \dot{\psi} - d_1 \phi_\psi),
 \end{aligned} \tag{9}$$

where θ and ψ represents the Euler angles denoting the orientation of rotor centerline. ϕ_θ and ϕ_ψ denote the differential magnetic flux from electromagnetic pairs. J_a , J_r , and ρ are the moment of inertia of the rotor in axial and radial directions, respectively. e_θ and e_ψ represents the corresponding differences of electric voltage. $f_{d\theta}$ and $f_{d\psi}$ are the disturbance forces caused by gravity, modelling errors, imbalances, etc. The geometry and parameters of the AMB, c_1 , c_2 , d_1 , d_2 , m and l , can be found in Mohamed and Busch-Vishniac (1995).

3.1 AMB LPV Model

The nonlinear model in (9) can be written in form of a parameter-varying state-space representation as

$$\begin{aligned}
 \dot{x} &= A(\rho)x + B_1 d + B_2 u, \\
 e &= C_1 x + D_{11} d + D_{12} u, \\
 y &= C_2 x + D_{21} d + D_{22} u,
 \end{aligned} \tag{10}$$

where,

$$A(\rho) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{4c_2}{m} & 0 & 0 & -\frac{\rho J_a}{J_r} & \frac{2c_1}{m} & 0 \\ 0 & -\frac{4c_2}{m} & \frac{\rho J_a}{J_r} & 0 & 0 & \frac{2c_1}{m} \\ \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} & 0 \\ 0 & \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} \end{bmatrix},$$

$$B_1 = 0_{6 \times 2}, \quad B_2 = \frac{1}{N} \begin{bmatrix} 0_{4 \times 2} \\ I_2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} I_2 & 0_{2 \times 4} \\ 0_{2 \times 6} \end{bmatrix}, \quad D_{11} = 0_{4 \times 2}, \quad D_{12} = \begin{bmatrix} 0_{2 \times 2} \\ I_2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} I_2 & 0_{2 \times 4} \end{bmatrix}, \quad D_{21} = I_2, \quad D_{22} = 0_{2 \times 2}.$$

The LPV model is formulated having the rotor speed $\rho(t)$ as the parameter-varying. The objective of designing a switching LPV control is to stabilize the AMB system over large range of rotor speeds and to attenuate the disturbance effect on the gap displacement at bearing locations, defined as $d^T = [f_{d\theta} \ f_{d\psi}]$.

This section aims to design a grid-based LPV-YK controller that switches between multiple LPV controllers, each suitable for a specific parameter region. Each LPV controller is designed separately achieving closed-loop exponential stability over its corresponding gridded subset.

The rotor speed is assumed to vary within the range $\rho \in \mathcal{P} = [315, 1100] \text{ rad/s}$, with a variation less than 100 rad/s^2 . Since the system dynamics changes significantly in this speed range, it could be conservative to design a single LPV controller over the full parameter region. Thus, the parameter region is divided into two overlapped subsets $\mathcal{P}_1 = [315 \ 720]$ and $\mathcal{P}_2 = [700 \ 1100]$. In this example, two methods are used to design a switching LPV controller, the first one is our proposed LPV-YK controller and the second is designed using the Theorem presented in Lu and Wu (2004).

The requirements are achieved by choosing dynamic weighting functions as $W_z(s) = \frac{10(s+8)}{s+0.001} I_2$, $W_u(s) = \frac{0.01(s+100)}{s+100000} I_2$, $W_n(s) = 0.001 I_2$, and a weighted generalized plant is obtained (see Fig. 3). The following steps are used to design $\tilde{K}(\rho, \gamma)$ (7) for every $\rho \in \mathcal{P}$:

- (1) The nominal controller $K_0(\rho)$ is designed over the full parameter region \mathcal{P} satisfying (A.1) with $\gamma_{\infty,0} = 48.01$. Following (A.2), the controllers $K_1(\rho)$ and $K_2(\rho)$ are designed separately over \mathcal{P}_1 and \mathcal{P}_2 achieving $\gamma_{\infty,1} = 47.95$ and $\gamma_{\infty,2} = 47.93$ performances, respectively. Notice that $K_0(\rho)$ could be designed with the same mentioned weighting functions, or with more robust requirements.
- (2) The LPV state-feedback gains $F_g(\rho)$ and $F_{k,0}(\rho)$ are designed using the LMIs (11)-(12),

$$\begin{aligned}
 A(\rho)X_g(\rho) + X_g(\rho)A^T(\rho) + \sum_{j=1}^s \pm \{\underline{\nu}_j, \bar{\nu}_j\} \frac{\partial X_g}{\partial \rho_j} \\
 + B_2(\rho)V(\rho) + V^T(\rho)B_2^T(\rho) < 0
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 A_{k,0}(\rho)X_{k,0}(\rho) + X_{k,0}(\rho)A_{k,0}^T(\rho) + \sum_{j=1}^s \pm \{\underline{\nu}_j, \bar{\nu}_j\} \frac{\partial X_{k,0}}{\partial \rho_j} \\
 + B_{k,0}(\rho)W(\rho) + W^T(\rho)B_{k,0}^T(\rho) < 0
 \end{aligned} \tag{12}$$

where the multiple parameter-dependent Lyapunov functions at each subset are specified as affine functions of scheduling parameters. That is,

$$X_g(\rho) = X_g^0 + X_g^1 \rho, \quad X_{k,0}(\rho) = X_{k,0}^0 + X_{k,0}^1 \rho,$$

where matrices X_g^j and $X_{k,0}^j$, $j = 0, 1$ are new optimization variables to be determined. Then, compute $F_g(\rho) = V(\rho)X_g^{-1}(\rho)$ and $F_{k,0}(\rho) = W_{k,0}(\rho)X_{k,0}^{-1}(\rho)$.

- (3) The state-space representations of $M(\rho)$, $N(\rho)$, $U_0(\rho)$ and $V_0(\rho)$, $\tilde{U}_1(\rho)$, $\tilde{V}_1(\rho)$, $\tilde{U}_2(\rho)$, and $\tilde{V}_2(\rho)$ are computed using (5)-(6). $Q_1(\rho)$ and $Q_2(\rho)$ are obtained from (8).
- (4) $Q(\rho, \gamma) = \sum_{i=1}^2 \gamma_i(\rho)Q_i(\rho)$, where $\gamma_i(\rho)$ is switched between $\{0,1\}$ when $\rho(t)$ touches the switching instants

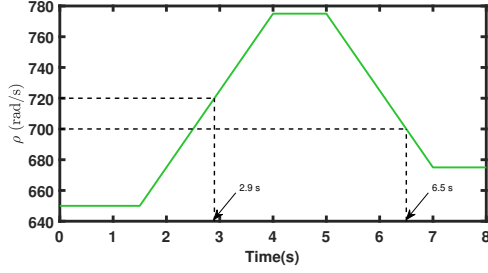


Fig. 4. Parameter-varying rotor speed ρ (rad/s)

(see Fig. 5). Then, the LPV-YK controller $\tilde{K}(\rho, \gamma)$ is structured and implemented as shown in Fig. 2.

Then, $\forall \rho \in \mathcal{P}$, the state-space matrices of $\tilde{K}(\rho, \gamma)$ are

$$\tilde{A}_k(\rho, \gamma) = \begin{bmatrix} A(\rho) + B_2(\rho)F_g(\rho) - B_2(\rho)D_q(\rho, \gamma)C_2(\rho) & -B_2(\rho)D_q(\rho, \gamma)F_{k,0}(\rho) & B_2(\rho)C_q(\rho, \gamma) \\ -B_{k,0}(\rho)C_2(\rho) & A_{k,0}(\rho) & 0 \\ -B_q(\rho)C_2(\rho) & -B_q(\rho)F_{k,0}(\rho) & A_q(\rho) \end{bmatrix}$$

$$\tilde{B}_k(\rho, \gamma) = [B_2(\rho)D_q(\rho, \gamma) \ B_{k,0}(\rho) \ B_q(\rho)]^T$$

$$\tilde{C}_k(\rho, \gamma) = [F_g(\rho) - (D_{k,0}(\rho) + D_q(\rho, \gamma))C_2(\rho) \ C_{k,0}(\rho) - D_q(\rho, \gamma)F_{k,0}(\rho) \ C_q(\rho, \gamma)]$$

$$\tilde{D}_k(\rho, \gamma) = D_{k,0}(\rho) + D_q(\rho, \gamma)$$
(13)

where

$$A_q(\rho) = \begin{bmatrix} A_{q,1} & 0 \\ 0 & A_{q,2} \end{bmatrix}, \quad B_q(\rho) = [B_{q,1} \ B_{q,2}]^T,$$

$$C_q(\rho, \gamma) = [\gamma_1 C_{q,1} \ \gamma_2 C_{q,2}], \quad D_q(\rho, \gamma) = \sum_{i=1}^2 \gamma_i D_{q,i}$$
(14)

being $A_{q,i}$, $B_{q,i}$, $C_{q,i}$, and $D_{q,i}$ the state-space matrices of $Q_i(\rho)$, $F_g(\rho) = V(\rho)X_g^{-1}(\rho)$, and $F_{k,0}(\rho) = W_{k,0}(\rho)X_{k,0}^{-1}(\rho)$.

5 Simulation Results

Same as in Lu and Wu (2004), the rotor speed profile $\rho(t)$ is taken as shown in Fig. 4. The switching events happen at $t = 2.9s$ and at $t = 6.5s$. The simulation is performed as in Lu and Wu (2004), where the disturbances $f_{d\theta}$ and $f_{d\psi}$ are injected to the system as step inputs with same magnitude but opposite signs. The comparison between the LPV switching controller Lu and Wu (2004) and the proposed LPV-YK controller $\tilde{K}(\rho, \gamma)$ is done by observing the output responses x_1 and x_2 and the control inputs u_1 and u_2 at the switching instants.

Figs. 6a and 6b represent the states transitions that are zoomed during switching. It is observed that the switching effect is negligible for the proposed LPV-YK controller, which is not the case for the switched LPV controller in Lu and Wu (2004). Moreover, as shown in Figs. 7a and 7b, the LPV switching controller leads to high control jumps at the switching instants. However, the LPV-YK controller switches smoothly with negligible control variation.

6 Conclusion

This work has proposed a new YK-based switching LPV method to obtain smooth switching performance. The switching signal can be used to incorporate any ad-hoc physically-based switching, without adding any constraints or conservatism to the pre-design problem. As a result, a YK control scheme is drawn between multiple

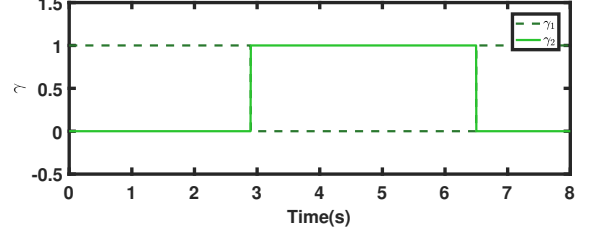
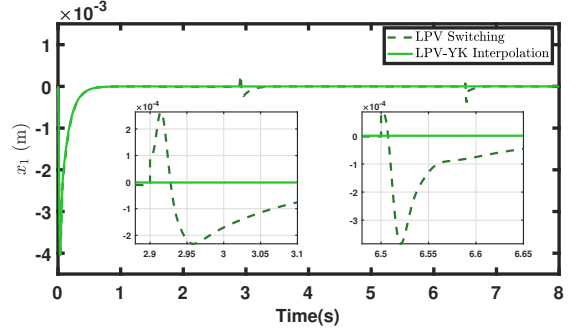
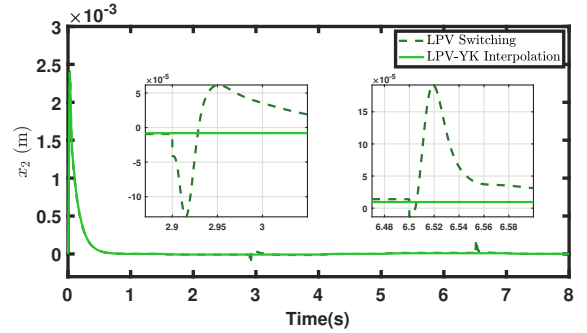


Fig. 5. Interpolating signal γ



(a) x_1 (m)



(b) x_2 (m)

Fig. 6. States x_1 and x_2 (m)

gridded LPV controllers. An application to the AMB system is carried out to compare the proposed approach with the approaches proposed by Lu and Wu (2004) and He et al. (2019). The simulation results show the efficiency of the proposed method to perfectly handle the system with negligible effect on the control input.

As a future work, an interest appears to study the choice of Q dynamics, i.e. how to choose the best design of the state-feedback gains F_g and $F_{k,0}$, and how they affect the closed-loop performance.

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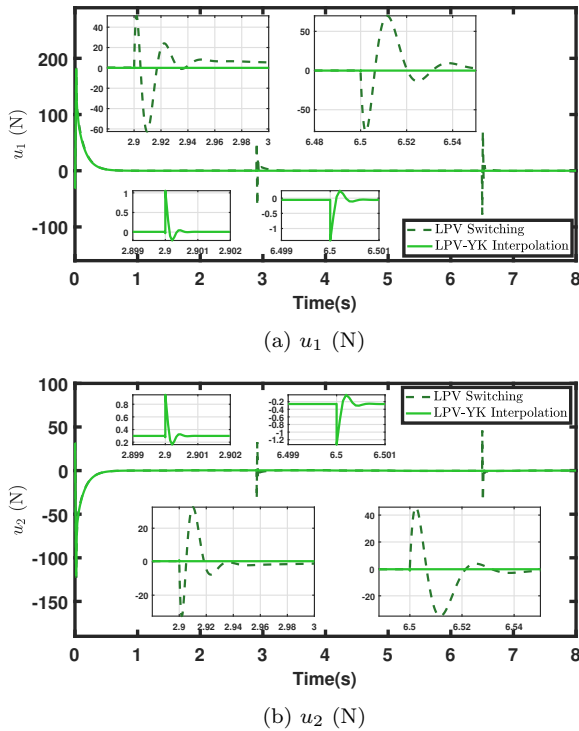


Fig. 7. Control inputs u_1 and u_2 (m)

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