



**HAL**  
open science

# Smooth Switching of Multi-LPV Control Systems Based on Youla-Kucera Parameterization

Hussam Atoui, Olivier Sename, Vicente Milanés, John Jairo Martínez Molina

► **To cite this version:**

Hussam Atoui, Olivier Sename, Vicente Milanés, John Jairo Martínez Molina. Smooth Switching of Multi-LPV Control Systems Based on Youla-Kucera Parameterization. LPVS 2022 - 5th IFAC Workshop on Linear Parameter Varying Systems, Sep 2022, Montréal, Canada. pp.19-24, 10.1016/j.ifacol.2022.11.284 . hal-03762058

**HAL Id: hal-03762058**

**<https://hal.science/hal-03762058v1>**

Submitted on 26 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

# Smooth Switching of Multi-LPV Control Systems Based on Youla-Kucera Parameterization <sup>\*</sup>

Hussam Atoui <sup>\*,\*\*</sup> Olivier Sename <sup>\*</sup> Vicente Milanés <sup>\*\*\*</sup>  
John J. Martínez <sup>\*</sup>

<sup>\*</sup> *Univ. Grenoble Alpes, CNRS, Grenoble INP, GIPSA-lab, 38000 Grenoble, France (e-mail: hussam.atoui@grenoble-inp.fr, olivier.sename@grenoble-inp.fr,*

*john-jairo.martinez-molina@grenoble-inp.fr).*

<sup>\*\*</sup> *Research Department, Renault SAS, 1 Avenue de Golf, 78280 Guyancourt, France (e-mail: hussam.atoui@renault.com)*

<sup>\*\*\*</sup> *Spanish Engineering Division, Renault España SA, Av. Madrid, 72, 47008 Valladolid, Spain (e-mail: vicente.milanés@renault.com)*

---

**Abstract:** This paper presents a smooth switching between a set of Linear Parameter-Varying (LPV) controllers that have been designed separately. The switching control strategy is based on Youla-Kucera (YK) parameterization. A generalized interpolation scheme of various LPV controllers is shown and proved for the LPV-YK concept. Such kind of parameterization is beneficial to switch or interpolate between multiple controllers without adding any constraints to the design of the local controllers and the switching signals. The proposed method is applied to an Active Magnetic Bearing (AMB) system and compared to a switched LPV controller.

*Keywords:* Linear parameter-varying system, Youla parameterization, Gain-scheduling, Linear matrix inequality

---

## 1 Introduction

Linear Parameter-Varying (LPV) control techniques are widely used in different applications as in aerospace Jiang et al. (2020) and autonomous vehicles Atoui et al. (2020a), Atoui et al. (2020b), etc. Hoffmann and Werner (2015) presents a deep LPV control review. The synthesis of an LPV control can be formulated as a Linear Matrix Inequality (LMI) optimization problem using a single Lyapunov function, either quadratic Packard (1994) or parameter-dependent Wu et al. (1996). On the other hand, a single Lyapunov function for complex models with large parameter variations may not exist, or if it exists, it could be conservative. A solution has been proposed in the literature to design multiple LPV controllers based on multiple parameter-varying Lyapunov functions, each suitable for a specific parameter subregion, and switch between them to achieve better performance Lu and Wu (2004).

### 1.1 LPV Switching Control

In 2004, switched LPV control systems have been first proposed in the pioneering work Lu and Wu (2004). The switching stability has been studied for hysteresis switching and switching with average dwell-time strategies. This methodology enhances the use of the LPV techniques in several applications, see for instance Bei Lu et al. (2006), Lescher et al. (2006), Postma and Nagamune (2012).

On the other hand, it has been stated in Lu and Wu (2004) that the switched LPV controllers may not provide a smooth transient response during switching, where aggressive performance is obtained at switching instants. Such case may lead to mechanical damage, decrease material lifetime, or signal saturation which is out of real application objectives. Following this work, challenges have been raised to smooth the control response during switching, and to relax the limitations on the switching signals. Several research studies have been involved in solving the switching smoothness. For instance, a bumpless transfer of switching controllers is proposed in Dimon et al. (2005) followed by some developments in Chen et al. (2010), and Safaei et al. (2012).

In addition, a smooth switching LPV controller has been proposed first in Hanifzadegan and Nagamune (2014). It is designed in considering adjustable interpolation functions and a higher order differential control signal. An iterative descent algorithm is applied to optimize three decision variables (the parameter-dependent Lyapunov functions, the local controllers, and the interpolation functions). It also augments the problem to two dimensional parameter regions. This concept is developed in the recent works He et al. (2018) and He et al. (2019), however, it increases the complexity and the constraints of the design of the local controllers to achieve its objectives.

---

<sup>\*</sup> Institute of Engineering Univ. Grenoble Alpes

## 1.2 LPV-YK Control

Recently, Youla-Kucera (YK) parameterization has been also used to switch between LPV controllers. It has been proposed in Atoui et al. (2021) to interpolate between two pre-defined quadratically stabilizing LPV controllers. More advanced LPV-YK control formulations have been proposed and proofed in Atoui et al. (2022). The objective is to achieve the closed-loop quadratic stability under arbitrary switching signals without requiring a constant Lyapunov function in the design of the local LPV controllers. However, the proposed theorem is still conservative since it requires quadratically stabilizing polytopic controllers.

The interest behind designing an interpolation control scheme based on YK is to parameterize a set of LPV stabilizing controllers  $K(Q)$ , where each one is parameterized by its corresponding LPV-YK parameter  $Q(\rho)$ . Notice that  $Q(\rho)$  is designed using the doubly coprime factorization Vidyasagar (2011), Atoui et al. (2021). An advantage of the YK concept is that it doesn't require of the re-design of the defined local LPV controllers. Its LMI conditions concern only the stability of the YK parameterization.

It is worth to mention some advantages of the YK parameterization to interpolate between several controllers: 1) Dimensions of the controllers could differ; 2) Interpolated controllers can be designed and tuned separately using different control techniques ( $\mathcal{H}_\infty$ , LQR, PID, ...) Tay et al. (1997); 3) It facilitates adding new controllers to an existing switching control system online as Plug&Play control theory Stoustrup (2009), without re-designing; and 4) The closed-loop stability is guaranteed under arbitrary continuous/discontinuous interpolating signals without requiring a common Lyapunov function Hespanha and Morse (2002).

## 1.3 Motivation and Contribution

Active Magnetic Bearing (AMB) system is known to be highly nonlinear, multi-variable and inherently unstable system. The nonlinearity is described in the relationship between the electro-magnetic force, current and length of the air gap. Moreover, the system (vibrational) dynamics depend on the rotational speed due to gyroscopic and electro-magnetic coupling. The AMB system has been previously controlled using the three LPV approaches: polytopic (Lu et al. (2008)), grid-based (Morsi et al. (2021)), and LFT (Lauridsen and Santos (2017)). In addition, (Lu and Wu (2004)) has implemented a switched-LPV controller on the AMB application to improve its closed-loop performance. The results have shown good performance over each parameter subregion, however, the proposed switched-LPV controller leads to aggressive transient switching.

This paper aims to design a smooth switched LPV controller based on YK parameterization which ensures the switching stability over the parameter subregions and smooth transient switching. Among all the previous works which have shown successful and smooth LPV switching as Lu and Wu (2004), Chen et al. (2010), and Hanifzadegan and Nagamune (2014), all of them require the re-design of the local LPV controllers using proposed LMIs, in addition to switching signals limitations (e.g. hysteresis switching, switching with average dwell-time, etc.). It is

worth mentioning that the re-design of all the local LPV controllers together may cause conservatism when increasing the number of subregions or parameter dimensions. The current paper proposes the YK parameterization to: 1) Simplify the design of the LPV switching control system by decreasing the complexity of the LMI conditions (no need to re-design the local LPV controllers); 2) Avoid any limitation on the switching signals without requiring constant Lyapunov function; 3) Smooth the control and state responses during the switching instants; and 4) Avoid the re-design of the switching control scheme if one needs to add or remove any of the local LPV controllers as Plug&Play.

The general contribution of this paper is to interpolate between multiple LPV controllers smoothly, each LPV controller is designed to perform well over a certain parameter sub-region. The interpolation scheme is formulated based on the LPV-YK parameterization. Notice that the proposed LPV-YK control scheme has been contributed and proved in (Atoui et al. (2022)). An interpolation scheme is drawn between multi-LPV controllers based on YK parameterization which guarantees the closed-loop stability and performance under arbitrary interpolating signals. In addition, a comparative numerical simulation is presented at the end to compare the switching smoothness between the proposed LPV-YK switching controller and the LPV switched controller proposed by Lu and Wu (2004).

## 2 Grid-based LPV-YK Control Configuration

This section represents the configuration scheme of a grid-based LPV-YK controller using the coprime factorization.

### 2.1 LPV Plant and Controllers Description

Consider a Multi-Input-Multi-Output (MIMO) LPV system  $G(\rho)$  with  $m$  inputs and  $p$  outputs:

$$G(\rho) \begin{cases} \dot{x}(t) = A(\rho(t))x(t) + B_1(\rho(t))w(t) + B_2(\rho(t))u(t) \\ z(t) = C_1(\rho(t))x(t) + D_{11}(\rho(t))w(t) + D_{12}(\rho(t))u(t) \\ y(t) = C_2(\rho(t))x(t) + D_{21}(\rho(t))w(t) + D_{22}(\rho(t))u(t) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^{n_x}$ ,  $y(t) \in \mathbb{R}^p$ ,  $u(t) \in \mathbb{R}^m$ ,  $z(t) \in \mathbb{R}^{n_z}$  are the state, output, input, controlled output vectors respectively.  $w(t) = [r \ n \ d]^T \in \mathbb{R}^{n_w}$  contains the exogenous inputs of the tracking reference  $r$ , noise  $n$  and input disturbance  $d$ . All the state-space data are continuous functions of the parameter vector  $\rho$ . Assume that  $\rho$  is in a compact set  $\mathcal{P} \subset \mathbb{R}^s$  with its parameter variation rate bounded by  $\underline{\nu}_k \leq \dot{\rho}_k \leq \bar{\nu}_k$  for  $k = 1, 2, \dots, s$ . Moreover, let us assume the following:

- $(A(\rho), B_2(\rho), C_2(\rho))$  triple is parameter-dependent stabilizable and detectable  $\forall \rho \in \mathcal{P}$ .
- $[B_2^T(\rho) \ D_{12}^T(\rho)]$  and  $[C_2(\rho) \ D_{21}(\rho)]$  have full row ranks  $\forall \rho \in \mathcal{P}$ .
- $D_{22}(\rho) = 0$ .

Suppose that the parameter set  $\mathcal{P}$  is covered by a finite number of closed subsets  $\{\mathcal{P}_i\}_{i \in Z_N}$ , where the index set  $Z_N = \{1, 2, \dots, N\}$ , and  $\mathcal{P} = \bigcup \mathcal{P}_i$ . At the boundaries between each adjacent subsets, there exist at least a single intersecting boundary or an intersecting surface.

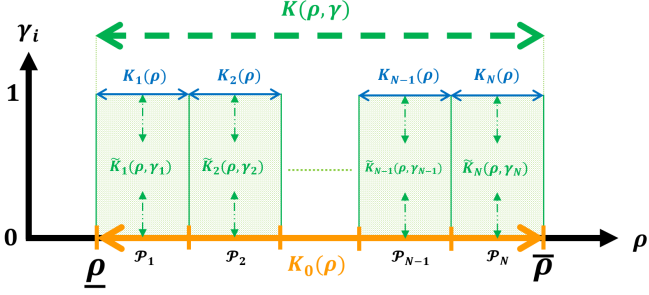


Fig. 1. LPV-YK gridded controller

Now, assume that

(A.1). *There exists an LPV output-feedback controller  $K_0(\rho)$  which exponentially stabilizes  $G(\rho)$  at the full parameter region  $\mathcal{P}_0 := \mathcal{P}$ . (following the approach in Wu et al. (1996)),*

(A.2). *Over each parameter subset  $\{\mathcal{P}_i\}_{i \in Z_N}$ , there exists an LPV controller  $K_i(\rho)$  pre-designed separately and exponentially stabilizes  $G(\rho)$  over  $\{\mathcal{P}_i\}_{i \in Z_N}$ . Each  $K_i(\rho)$  is designed to achieve a suitable performance in its corresponding parameter subregion  $\{\mathcal{P}_i\}_{i \in Z_N}$ .*

Fig. 1 shows the partitioned parameter region  $\mathcal{P}$  with intersecting boundaries. The orange solid line represents the chosen nominal LPV controller  $K_0(\rho)$ , as defined by (A.1). The blue solid lines represent the local LPV controllers  $K_i(\rho)$  ( $i \in Z_N$ ) as defined in (A.2). The overall switching controller is performed using the interpolating signal  $\gamma = [\gamma_1, \dots, \gamma_N]$  ( $\gamma_i \in [0, 1] \forall i$ ), and is represented by the LPV-YK controller  $\tilde{K}(\rho, \gamma)$ . These LPV and LPV-YK controllers are designed in the next section.

The defined LPV controllers  $K_i(\rho)$  ( $i \geq 0$ ) are described over  $\mathcal{P}_i$  as

$$K_i(\rho) : \begin{bmatrix} A_{k,i}(\rho, \dot{\rho}) & B_{k,i}(\rho) \\ C_{k,i}(\rho) & D_{k,i}(\rho) \end{bmatrix}, \quad i \in \{0, Z_N\} \quad (2)$$

where  $A_{k,i}(\rho, \dot{\rho}) \in \mathbb{R}^{n_{k,i} \times n_{k,i}}$ ,  $B_{k,i}(\rho) \in \mathbb{R}^{n_{k,i} \times m_k}$ ,  $C_{k,i}(\rho) \in \mathbb{R}^{p_k \times n_{k,i}}$  and  $D_{k,i}(\rho) \in \mathbb{R}^{p_k \times m_k}$ .

**Remark 2.1.** *At each subset  $\mathcal{P}_i$ ,  $K_i(\rho)$  may be designed using different concepts ( $\mathcal{H}_\infty$ , PID, LQR/ $\mathcal{H}_2$ , ...). As a result,  $n_{k,i}$  may differ from one controller to another.*

## 2.2 LPV Coprime Factorization

Using the YK parameterization concept, the plant model and the stabilizing LPV controllers, at each subset  $\{\mathcal{P}_i\}_{i \geq 0}$ , can be factorised using the following doubly coprime factorization:

$$\begin{aligned} G_i(\rho) &= N_i(\rho)M_i(\rho) = \tilde{M}_i(\rho)\tilde{N}_i(\rho) \\ K_i(\rho) &= U_i(\rho)V_i^{-1}(\rho) = \tilde{V}_i^{-1}(\rho)\tilde{U}_i(\rho) \end{aligned} \quad (3)$$

Notice that,  $\forall i \in \{0, Z_N\}$ , the coprime factors are computed such that  $M_i, N_i, \tilde{M}_i, \tilde{N}_i, U_i, V_i, \tilde{U}_i, \tilde{V}_i$  are stable, proper, and satisfying the following *Bezout Identity*:

$$\begin{bmatrix} \tilde{V}_i(\rho) & -\tilde{U}_i(\rho) \\ -\tilde{N}_i(\rho) & \tilde{M}_i(\rho) \end{bmatrix} \begin{bmatrix} M_i(\rho) & U_i(\rho) \\ N_i(\rho) & V_i(\rho) \end{bmatrix} = \begin{bmatrix} M_i(\rho) & U_i(\rho) \\ N_i(\rho) & V_i(\rho) \end{bmatrix} \begin{bmatrix} \tilde{V}_i(\rho) & -\tilde{U}_i(\rho) \\ -\tilde{N}_i(\rho) & \tilde{M}_i(\rho) \end{bmatrix} \\ = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (4)$$

The coprime factors are computed for every  $\rho \in \{\mathcal{P}_i\}_{i \geq 0}$  using the state-space representations written in (5)-(6).

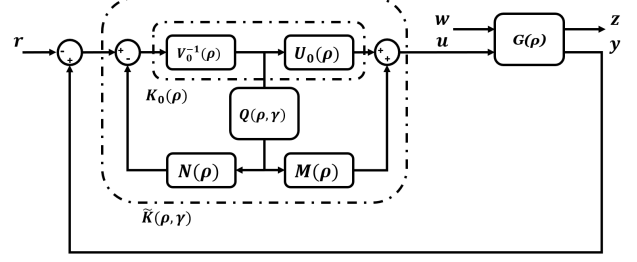


Fig. 2. YK control structure

To perform a stable coprime factorization, the closed-loops  $[A(\rho) + B_2(\rho)F_g(\rho)]$  and  $[A_{k,i}(\rho) + B_{k,i}(\rho)F_{k,i}(\rho)]$  ( $\forall i \in \{0, Z_N\}$ ) must be stable.

$$\begin{bmatrix} M_i(\rho) & U_i(\rho) \\ N_i(\rho) & V_i(\rho) \end{bmatrix} := \begin{bmatrix} A(\rho) + B_2(\rho)F_g(\rho) & 0 & B_2(\rho) & 0 \\ 0 & A_{k,i}(\rho) + B_{k,i}(\rho)F_{k,i}(\rho) & 0 & B_{k,i}(\rho) \\ F_g(\rho) & C_{k,i}(\rho) + D_{k,i}(\rho)F_{k,i}(\rho) & I & D_{k,i}(\rho) \\ C_2(\rho) & F_{k,i}(\rho) & 0 & I \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} \tilde{V}_i(\rho) & -\tilde{U}_i(\rho) \\ -\tilde{N}_i(\rho) & \tilde{M}_i(\rho) \end{bmatrix} := \begin{bmatrix} A(\rho) + B_2(\rho)D_k(\rho)C_2(\rho) & B_2(\rho)C_{k,i}(\rho) & -B_2(\rho) & B_2(\rho)D_{k,i}(\rho) \\ B_{k,i}(\rho)C_2(\rho) & A_{k,i}(\rho) & 0 & B_{k,i}(\rho) \\ F_g(\rho) - D_{k,i}(\rho)C_2(\rho) & -C_{k,i}(\rho) & I & -D_{k,i}(\rho) \\ C_2(\rho) & -F_{k,i}(\rho) & 0 & I \end{bmatrix}. \quad (6)$$

## 2.3 LPV-YK Control Structure

Fig.2 shows the structure of the LPV-YK controller  $\tilde{K}(\rho, \gamma)$  defined as:

$$\begin{aligned} \tilde{K}(\rho, \gamma) &= (U_0(\rho) + M(\rho)Q(\rho, \gamma))(V_0(\rho) + N(\rho)Q(\rho, \gamma))^{-1} \\ &= (\tilde{V}_0(\rho) + Q(\rho, \gamma)\tilde{N}(\rho))^{-1}(\tilde{U}_0(\rho) + Q(\rho, \gamma)\tilde{M}(\rho)) \end{aligned} \quad (7)$$

where,  $Q(\rho, \gamma) = \sum_{i=1}^N \gamma_i Q_i(\rho)$ , and

$$Q_i(\rho) = \tilde{U}_i(\rho)V_0(\rho) - \tilde{V}_i(\rho)U_0(\rho) \quad (8)$$

See (Atoui et al. (2022)) for more details on the stability proof.

$\gamma(\rho)$  is a vector of the parameter-dependent switching signals  $\gamma_i(\rho)$  ( $i \in Z_N$ ) that are chosen here as follows:

- if  $\gamma_i(\rho) = 0 \forall i$ ,  $\tilde{K}(\rho, \gamma) \equiv K_0(\rho)$ .
- For any  $\rho \in \mathcal{P}_m$ ,  $\gamma_m(\rho) = 1$  and  $\gamma_i(\rho) = 0 \forall i \neq m$ , which implies that  $\tilde{K}(\rho, \gamma)$  is equivalent to  $\mathcal{F}_l(J(\rho), Q_m(\rho))$  that recovers  $K_m(\rho)$ .

## 3 Model Formulation

In this section, the Active Magnetic Bearing (AMB) system is defined and reformulated as an LPV model. Then, the design steps of the grid-based LPV-YK controller are illustrated using the contributed Theorem in Atoui et al. (2022). The nonlinear gyroscopic equations of the AMB system can be represented as:

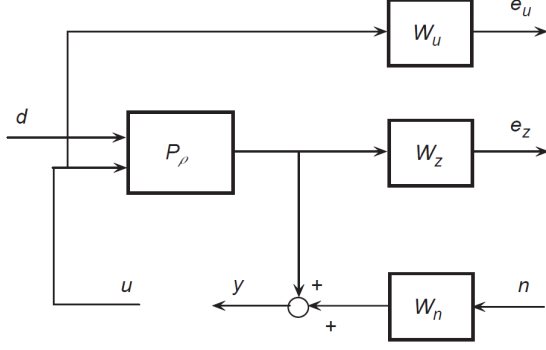


Fig. 3. Weighted open-loop interconnection Lu and Wu (2004)

$$\begin{aligned}
 l\ddot{\theta} &= -\frac{\rho J_a}{J_r} l p \dot{\psi} + \frac{1}{m} (-4c_2 l \dot{\theta} + 2c_1 \phi_{\theta} + f_{d\theta}), \\
 l\ddot{\psi} &= \frac{\rho J_a}{J_r} l \dot{\theta} + \frac{1}{m} (-4c_2 l \dot{\psi} + 2c_1 \phi_{\psi} + f_{d\psi}), \\
 \dot{\phi}_{\theta} &= \frac{1}{N} (e_{\theta} + 2d_2 l \dot{\theta} - d_1 \phi_{\theta}), \\
 \dot{\phi}_{\psi} &= \frac{1}{N} (e_{\psi} + 2d_2 l \dot{\psi} - d_1 \phi_{\psi}),
 \end{aligned} \tag{9}$$

where  $\theta$  and  $\psi$  represents the Euler angles denoting the orientation of rotor centerline.  $\phi_{\theta}$  and  $\phi_{\psi}$  denote the differential magnetic flux from electromagnetic pairs.  $J_a$ ,  $J_r$ , and  $\rho$  are the moment of inertia of the rotor in axial and radial directions, respectively.  $e_{\theta}$  and  $e_{\psi}$  represents the corresponding differences of electric voltage.  $f_{d\theta}$  and  $f_{d\psi}$  are the disturbance forces caused by gravity, modelling errors, imbalances, etc. The geometry and parameters of the AMB,  $c_1$ ,  $c_2$ ,  $d_1$ ,  $d_2$ ,  $m$  and  $l$ , can be found in Mohamed and Busch-Vishniac (1995).

### 3.1 AMB LPV Model

The nonlinear model in (9) can be written in form of a parameter-varying state-space representation as

$$\begin{aligned}
 \dot{x} &= A(\rho)x + B_1 d + B_2 u, \\
 e &= C_1 x + D_{11} d + D_{12} u, \\
 y &= C_2 x + D_{21} d + D_{22} u,
 \end{aligned} \tag{10}$$

where,

$$A(\rho) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{4c_2}{m} & 0 & 0 & -\frac{\rho J_a}{J_r} & \frac{2c_1}{m} & 0 \\ 0 & -\frac{4c_2}{m} & \frac{\rho J_a}{J_r} & 0 & 0 & \frac{2c_1}{m} \\ \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} & 0 \\ 0 & \frac{2d_2}{N} & 0 & 0 & 0 & -\frac{d_1}{N} \end{bmatrix},$$

$$B_1 = 0_{6 \times 2}, \quad B_2 = \frac{1}{N} \begin{bmatrix} 0_{4 \times 2} \\ I_2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} I_2 & 0_{2 \times 4} \\ 0_{2 \times 6} \end{bmatrix}, \quad D_{11} = 0_{4 \times 2}, \quad D_{12} = \begin{bmatrix} 0_{2 \times 2} \\ I_2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} I_2 & 0_{2 \times 4} \end{bmatrix}, \quad D_{21} = I_2, \quad D_{22} = 0_{2 \times 2}.$$

The LPV model is formulated having the rotor speed  $\rho(t)$  as the parameter-varying. The objective of designing a switching LPV control is to stabilize the AMB system over large range of rotor speeds and to attenuate the disturbance effect on the gap displacement at bearing locations, defined as  $d^T = [f_{d\theta} \ f_{d\psi}]$ .

This section aims to design a grid-based LPV-YK controller that switches between multiple LPV controllers, each suitable for a specific parameter region. Each LPV controller is designed separately achieving closed-loop exponential stability over its corresponding gridded subset.

The rotor speed is assumed to vary within the range  $\rho \in \mathcal{P} = [315, 1100] \text{ rad/s}$ , with a variation less than  $100 \text{ rad/s}^2$ . Since the system dynamics changes significantly in this speed range, it could be conservative to design a single LPV controller over the full parameter region. Thus, the parameter region is divided into two overlapped subsets  $\mathcal{P}_1 = [315 \ 720]$  and  $\mathcal{P}_2 = [700 \ 1100]$ . In this example, two methods are used to design a switching LPV controller, the first one is our proposed LPV-YK controller and the second is designed using the Theorem presented in Lu and Wu (2004).

The requirements are achieved by choosing dynamic weighting functions as  $W_z(s) = \frac{10(s+8)}{s+0.001} I_2$ ,  $W_u(s) = \frac{0.01(s+100)}{s+100000} I_2$ ,  $W_n(s) = 0.001 I_2$ , and a weighted generalized plant is obtained (see Fig. 3). The following steps are used to design  $\tilde{K}(\rho, \gamma)$  (7) for every  $\rho \in \mathcal{P}$ :

- (1) The nominal controller  $K_0(\rho)$  is designed over the full parameter region  $\mathcal{P}$  satisfying (A.1) with  $\gamma_{\infty,0} = 48.01$ . Following (A.2), the controllers  $K_1(\rho)$  and  $K_2(\rho)$  are designed separately over  $\mathcal{P}_1$  and  $\mathcal{P}_2$  achieving  $\gamma_{\infty,1} = 47.95$  and  $\gamma_{\infty,2} = 47.93$  performances, respectively. Notice that  $K_0(\rho)$  could be designed with the same mentioned weighting functions, or with more robust requirements.
- (2) The LPV state-feedback gains  $F_g(\rho)$  and  $F_{k,0}(\rho)$  are designed using the LMIs (11)-(12),

$$\begin{aligned}
 A(\rho)X_g(\rho) + X_g(\rho)A^T(\rho) + \sum_{j=1}^s \pm \{\underline{\nu}_j, \bar{\nu}_j\} \frac{\partial X_g}{\partial \rho_j} \\
 + B_2(\rho)V(\rho) + V^T(\rho)B_2^T(\rho) < 0
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 A_{k,0}(\rho)X_{k,0}(\rho) + X_{k,0}(\rho)A_{k,0}^T(\rho) + \sum_{j=1}^s \pm \{\underline{\nu}_j, \bar{\nu}_j\} \frac{\partial X_{k,0}}{\partial \rho_j} \\
 + B_{k,0}(\rho)W(\rho) + W^T(\rho)B_{k,0}^T(\rho) < 0
 \end{aligned} \tag{12}$$

where the multiple parameter-dependent Lyapunov functions at each subset are specified as affine functions of scheduling parameters. That is,

$$X_g(\rho) = X_g^0 + X_g^1 \rho, \quad X_{k,0}(\rho) = X_{k,0}^0 + X_{k,0}^1 \rho,$$

where matrices  $X_g^j$  and  $X_{k,0}^j$ ,  $j = 0, 1$  are new optimization variables to be determined. Then, compute  $F_g(\rho) = V(\rho)X_g^{-1}(\rho)$  and  $F_{k,0}(\rho) = W_{k,0}(\rho)X_{k,0}^{-1}(\rho)$ .

- (3) The state-space representations of  $M(\rho)$ ,  $N(\rho)$ ,  $U_0(\rho)$  and  $V_0(\rho)$ ,  $\tilde{U}_1(\rho)$ ,  $\tilde{V}_1(\rho)$ ,  $\tilde{U}_2(\rho)$ , and  $\tilde{V}_2(\rho)$  are computed using (5)-(6).  $Q_1(\rho)$  and  $Q_2(\rho)$  are obtained from (8).
- (4)  $Q(\rho, \gamma) = \sum_{i=1}^2 \gamma_i(\rho)Q_i(\rho)$ , where  $\gamma_i(\rho)$  is switched between  $\{0,1\}$  when  $\rho(t)$  touches the switching instants

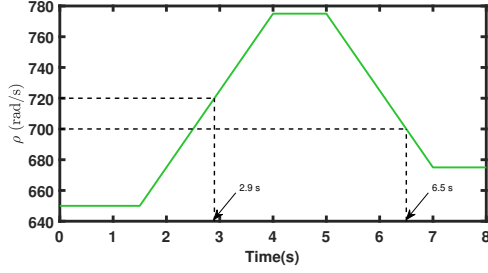


Fig. 4. Parameter-varying rotor speed  $\rho$  (rad/s)

(see Fig. 5). Then, the LPV-YK controller  $\tilde{K}(\rho, \gamma)$  is structured and implemented as shown in Fig. 2.

Then,  $\forall \rho \in \mathcal{P}$ , the state-space matrices of  $\tilde{K}(\rho, \gamma)$  are

$$\tilde{A}_k(\rho, \gamma) = \begin{bmatrix} A(\rho) + B_2(\rho)F_g(\rho) - B_2(\rho)D_q(\rho, \gamma)C_2(\rho) & -B_2(\rho)D_q(\rho, \gamma)F_{k,0}(\rho) & B_2(\rho)C_q(\rho, \gamma) \\ -B_{k,0}(\rho)C_2(\rho) & A_{k,0}(\rho) & 0 \\ -B_q(\rho)C_2(\rho) & -B_q(\rho)F_{k,0}(\rho) & A_q(\rho) \end{bmatrix}$$

$$\tilde{B}_k(\rho, \gamma) = [B_2(\rho)D_q(\rho, \gamma) \ B_{k,0}(\rho) \ B_q(\rho)]^T$$

$$\tilde{C}_k(\rho, \gamma) = [F_g(\rho) - (D_{k,0}(\rho) + D_q(\rho, \gamma))C_2(\rho) \ C_{k,0}(\rho) - D_q(\rho, \gamma)F_{k,0}(\rho) \ C_q(\rho, \gamma)]$$

$$\tilde{D}_k(\rho, \gamma) = D_{k,0}(\rho) + D_q(\rho, \gamma)$$
(13)

where

$$A_q(\rho) = \begin{bmatrix} A_{q,1} & 0 \\ 0 & A_{q,2} \end{bmatrix}, \quad B_q(\rho) = [B_{q,1} \ B_{q,2}]^T,$$

$$C_q(\rho, \gamma) = [\gamma_1 C_{q,1} \ \gamma_2 C_{q,2}], \quad D_q(\rho, \gamma) = \sum_{i=1}^2 \gamma_i D_{q,i}$$
(14)

being  $A_{q,i}$ ,  $B_{q,i}$ ,  $C_{q,i}$ , and  $D_{q,i}$  the state-space matrices of  $Q_i(\rho)$ ,  $F_g(\rho) = V(\rho)X_g^{-1}(\rho)$ , and  $F_{k,0}(\rho) = W_{k,0}(\rho)X_{k,0}^{-1}(\rho)$ .

## 5 Simulation Results

Same as in Lu and Wu (2004), the rotor speed profile  $\rho(t)$  is taken as shown in Fig. 4. The switching events happen at  $t = 2.9s$  and at  $t = 6.5s$ . The simulation is performed as in Lu and Wu (2004), where the disturbances  $f_{d\theta}$  and  $f_{d\psi}$  are injected to the system as step inputs with same magnitude but opposite signs. The comparison between the LPV switching controller Lu and Wu (2004) and the proposed LPV-YK controller  $\tilde{K}(\rho, \gamma)$  is done by observing the output responses  $x_1$  and  $x_2$  and the control inputs  $u_1$  and  $u_2$  at the switching instants.

Figs. 6a and 6b represent the states transitions that are zoomed during switching. It is observed that the switching effect is negligible for the proposed LPV-YK controller, which is not the case for the switched LPV controller in Lu and Wu (2004). Moreover, as shown in Figs. 7a and 7b, the LPV switching controller leads to high control jumps at the switching instants. However, the LPV-YK controller switches smoothly with negligible control variation.

## 6 Conclusion

This work has proposed a new YK-based switching LPV method to obtain smooth switching performance. The switching signal can be used to incorporate any ad-hoc physically-based switching, without adding any constraints or conservatism to the pre-design problem. As a result, a YK control scheme is drawn between multiple

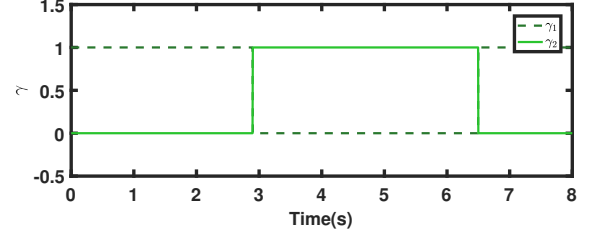
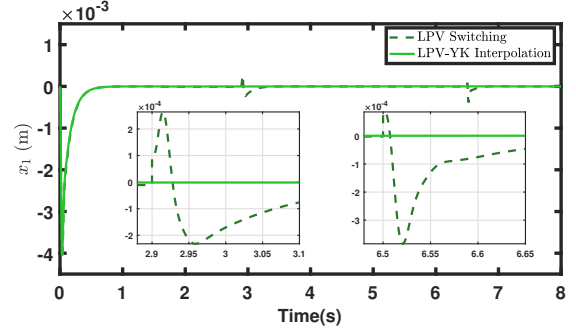
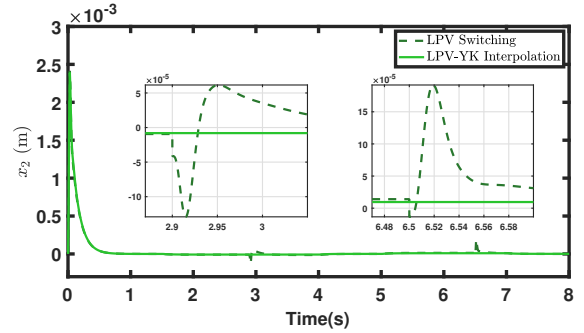


Fig. 5. Interpolating signal  $\gamma$



(a)  $x_1$  (m)



(b)  $x_2$  (m)

Fig. 6. States  $x_1$  and  $x_2$  (m)

gridded LPV controllers. An application to the AMB system is carried out to compare the proposed approach with the approaches proposed by Lu and Wu (2004) and He et al. (2019). The simulation results show the efficiency of the proposed method to perfectly handle the system with negligible effect on the control input.

As a future work, an interest appears to study the choice of  $Q$  dynamics, i.e. how to choose the best design of the state-feedback gains  $F_g$  and  $F_{k,0}$ , and how they affect the closed-loop performance.

## References

- Atoui, H., Milanés, V., Sename, O., and Martinez, J.J. (2020a). Design and experimental validation of a lateral lpv control of autonomous vehicles. In *2020 IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC)*, 1–6. doi:10.1109/ITSC45102.2020.9294459.
- Atoui, H., Sename, O., Alcalá, E., and Puig, V. (2020b). Parameter varying approach for a combined (kinematic + dynamic) model of autonomous vehicles. *IFAC-PapersOnLine*, 53, 15071–15076. doi:10.1016/j.ifacol.2020.12.2028.

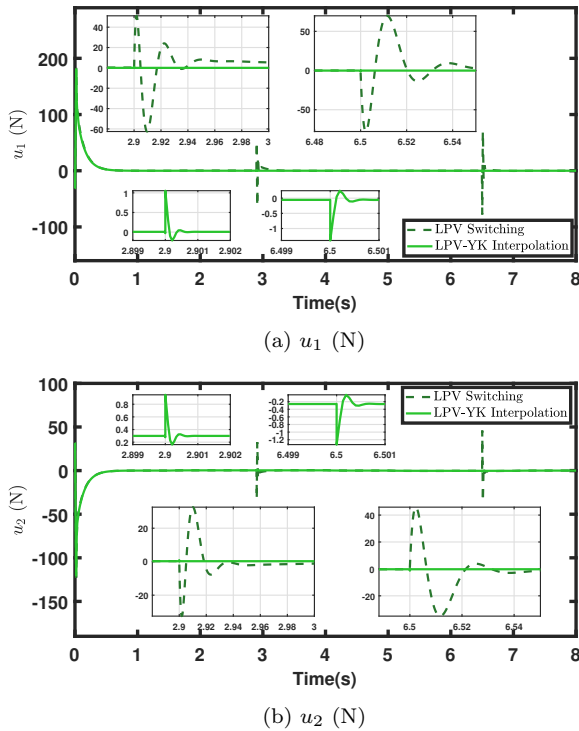


Fig. 7. Control inputs  $u_1$  and  $u_2$  (m)

- Atoui, H., Senéme, O., Milanes, V., and Martínez, J. (2022). Advanced LPV-YK Control Design with Experimental Validation on Autonomous Vehicles. URL <https://hal.univ-grenoble-alpes.fr/hal-03648497>. Working paper or preprint.
- Atoui, H., Senéme, O., Milanes, V., and Martínez, J.J. (2021). Interpolation of Multi-LPV Control Systems Based on Youla-Kucera Parameterization. URL <https://hal.archives-ouvertes.fr/hal-03196373>. Accepted to *Automatica*.
- Bei Lu, Fen Wu, and SungWan Kim (2006). Switching lpv control of an f-16 aircraft via controller state reset. *IEEE Transactions on Control Systems Technology*, 14(2), 267–277.
- Chen, P.C., Wu, S.L., and Chuang, H.S. (2010). The smooth switching control for tora system via lmis. In *IEEE ICCA 2010*, 1338–1343. doi:10.1109/ICCA.2010.5524116.
- Dimon, J., Jakob, B., and Trangbaek, K. (2005). Bumpless transfer between observer-based gain scheduled controllers. *International Journal of Control*, 78. doi:10.1080/00207170500111028.
- Hanifzadegan, M. and Nagamune, R. (2014). Smooth switching lpv controller design for lpv systems. *Automatica*, 50, 1481–1488.
- He, T., Zhu, G.G., Swei, S.S., and Su, W. (2018). Simultaneous design of smooth switching state-feedback lpv control. 3368–3373. doi:10.23919/ACC.2018.8431311.
- He, T., Zhu, G., and Swei, S. (2019). Smooth switching lpv dynamic output-feedback control. *International Journal of Control, Automation and Systems*, 18. doi:10.1007/s12555-019-0088-3.
- Hespanha, J. and Morse, A.S. (2002). Switching between stabilizing controllers. *Automatica*, 38(11), 1905–1917.
- Hoffmann, C. and Werner, H. (2015). A survey of linear parameter-varying control applications validated by experiments or high-fidelity simulations. *IEEE Transactions on Control Systems Technology*, 23(2), 416–433. doi:10.1109/TCST.2014.2327584.
- Jiang, W., Wu, K., Wang, Z., and Wang, Y. (2020). Gain-scheduled control for morphing aircraft via switching polytopic linear parameter-varying systems. *Aerospace Science and Technology*, 107, 106242. doi:https://doi.org/10.1016/j.ast.2020.106242. URL <http://www.sciencedirect.com/science/article/pii/S127096382030924X>.
- Lauridsen, J.S. and Santos, I.F. (2017). Design of robust amb controllers for rotors subjected to varying and uncertain seal forces. *European Journal of Emergency Medicine*, 4, 16–00618.
- Lescher, F., Zhao, J., and Borne, P. (2006). Switching lpv controllers for a variable speed pitch regulated wind turbine. *International Journal of Computers, Communications Control*, 1, 1334 – 1340. doi:10.1109/CESA.2006.4281844.
- Lu, B. and Wu, F. (2004). Switching lpv control designs using multiple parameter-dependent lyapunov functions. *Automatica*, 40, 1973–1980. doi:10.1016/j.automatica.2004.06.011.
- Lu, B., Choi, H., Buckner, G.D., and Tammi, K. (2008). Linear parameter-varying techniques for control of a magnetic bearing system. *Control Engineering Practice*, 16(10), 1161–1172. doi:https://doi.org/10.1016/j.conengprac.2008.01.002.
- Mohamed, A. and Busch-Vishniac, I. (1995). Imbalance compensation and automation balancing in magnetic bearing systems using the q-parameterization theory. *IEEE Transactions on Control Systems Technology*, 3(2), 202–211. doi:10.1109/87.388128.
- Morsi, A., Abbas, H.S., Ahmed, S.M., and Mohamed, A.M. (2021). Model predictive control based on linear parameter-varying models of active magnetic bearing systems. *IEEE Access*, 9, 23633–23647. doi:10.1109/ACCESS.2021.3056323.
- Packard, A. (1994). Gain scheduling via linear fractional transformations. *Systems Control Letters*, 22(2), 79–92.
- Postma, M. and Nagamune, R. (2012). Air-fuel ratio control of spark ignition engines using a switching lpv controller. *IEEE Transactions on Control Systems Technology*, 20(5), 1175–1187.
- Safaei, F.R.P., Hespanha, J., and Stewart, G. (2012). On controller initialization in multivariable switching systems. *Automatica*, 48, 3157–3165.
- Stoustrup, J. (2009). Plug&play control: Control technology towards new challenges. *European Journal of Control*, 311–330.
- Tay, T., Moore, J., and Mareels, I. (1997). *High performance control*. Springer Science & Business Media.
- Vidyasagar, M. (2011). Control system synthesis: a factorization approach, part ii. *Synthesis lectures on control and mechatronics*, 2(1), 1–227.
- Wu, F., Yang, X., Packard, A., and Becker, G. (1996). Induced l2-norm control for lpv systems with bounded parameter variation rates. *International Journal of Robust and Nonlinear Control*, 6, 983–998. doi:10.1002/(SICI)1099-1239(199611)6:9<983::AID-RNC263>3.0.CO;2-C.