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Abstract

A systematic way to compute the finite dimensional dynamic model of the counter current heat exchanger network based on the graph theory is proposed. This contribution focuses to the serial interconnection of heat exchanger. The proposed models are build from the elementary block composed of two compartments corresponding to the hot and cold streams. The method is based on the graph based input-output representation of the convective heat flux between the heat exchangers.

Keywords: Heat exchanger network, Dynamic modelling, Graph theory

1. Introduction

Modelling and simulation are essential tools for the design of heat exchangers (HE) and Heat Exchangers Networks (HEN) (Roetzel et al. (2020)). Typical HE and HEN design approaches are primarily based on steady-state models. They are suitable for HENs retrofitting, upgrades and for estimating optimal steady-state operating points. However, dynamic performance of HEN has also to be explored in order to improve flexibility and controllability properties of HEN (Yang et al. (2021)) as well as their energy efficiency.

Roetzel et al. (Roetzel et al. (2020)) cited several mathematical models for transient analysis of heat exchangers: (i) the lumped parameter model, (ii) the distributed parameter model and (iii) the cell model. The first model is a systemic approach wherein the HE is considered as a single box; each fluid in the whole HE has an uniform temperature which is used to calculated the heat transfer through the partition wall and the energy balance involves only the inlet and outlet temperatures. The second model is an infinite dimensional model for which the HE space must be discretized (Michel and Kugi (2013)). The latter, also called lumped-distributed parameter model, consists in dividing the HE space into many elements along its length and applying the lumped method to each element (Correa and Marchetti (1987)). All the models were used for a better understanding of HE behaviour and the model building methods were discussed for small systems with simple topology like HE that are not nearly as large as district heating or heat networks. In addition, energy balances were written based on temperatures as state parameters and do not separate the convective and heat transfer parts, which makes it impractical to use them for scalable and extensible HENs.

For this purpose, we propose a systematic way to compute the finite dimensional dynamic model of a network of counter-current heat exchangers based on graph theory. This method gives the dynamic model in an iterative way and is well suited for modification of the network topology. The obtained model is given in a structure form that can be also used for control purposes.

2. Model formulation

This contribution focuses to the serial interconnection of heat exchangers. The proposed models are built from the elementary block named i composed of two well stirred compartments corresponding to the hot and cold streams as shown in Figure 1. Notations $h, \bar{h}$ stand for specific enthalpies and $Q, \bar{Q}$ for mass flows of the two streams. The indexes *in* and *out* stand for inlet and outlet flows resp. In this paper, we considered these two compartments were separated by a wall without heat accumulation for pedagogical purpose. However, this assumption could be overcomed using mean heat capacities including both the fluid and wall properties as described in Correa's paper annex (Correa and Marchetti (1987)).

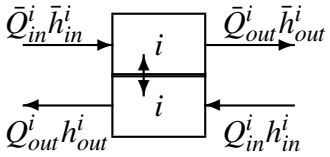

Figure 1: Elementary block i with enthalpy flows

The method is based on the graphic input-output representation of two heat exchangers in series with $[i]$ and $[j]$ elementary blocks, resp. as shown in Figure 2. The arrows are the convective inlet or outlet flows. They are also the edges of the graphs linked to the matter convection. The nodes of the matter and energy graphs are (i) the grey circles which represent the connections of the input and output convective fluxes (grey nodes without matter and energy accumulation) and (ii) each compartment where energy accumulation occurs. There are two graphs, since there are one hot and one cold streams by compartment, but they are treated simultaneously in order to implicitly take into account the heat transfer through the block partition walls (the heat transfer is represented in the block and the edge does not appear in the graph representation). The general construction of the model is based on the adjacency matrix of the convective graphs based on the material and energy balances. With this method, dynamic models of block series can be easily obtained. One of the main advantage of this approach is its modularity as the network can be extended as much boxes (representing a single or network heat exchangers) and streams as needed. So it can be used for network retrofitting or upgrade. Furthermore this modelling method gives structured matrices for which mathematical network analysis can be easily applied and used for control purposes such as dynamic energy assessment approach.

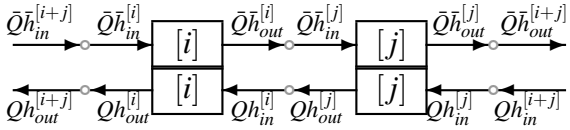

Figure 2: Two heat exchangers or two heat exchanger networks in series

The main assumptions for the modelling of blocks are the following:

(a) The compartments are perfectly stirred.

(b) The two streams remain in liquid phase.

(c) The pressure P, $\bar{P}$ in the compartments as well as the mass densities (ρ,$\bar{\rho}$), the heat capacities (c_p, $\bar{c}_p$) and the mass flow rates (Q,$\bar{Q}$) are constant.

(d) There is no heat accumulation in the block partition wall and no heat exchange with the environment.

(e) The global heat transfer coefficient λ and the volumes V, $\bar{V}$ are constant and equal for all the compartments.

With the previous assumptions, the mass balances in each block are reduced to the equalities of the inlet and outlet mass flow rates, that for brevity we denote as $\bar{Q}$ et Q:

$$0 = \bar{Q}^i_{in} - \bar{Q}^i_{out} \qquad\qquad 0 = Q^i_{in} - Q^i_{out}$$

and the enthalpies are defined as $H = \rho V h(T), \quad \bar{H} = \bar{\rho}\bar{V}\bar{h}(\bar{T})$

Proposition 1 *With assumptions (a) to (e), the energy balances in an elementary block 1 write as:*

$$\left\{\begin{array}{rcl} \frac{d}{dt}\mathbb{H}^1 & = & -\lambda(I_1 \otimes A_t)\mathbb{T}^1 - \frac{Q}{\rho V}\mathbb{A}_1\mathbb{H}^1 - \frac{\bar{Q}}{\bar{\rho}\bar{V}}\bar{\mathbb{A}}_1\mathbb{H}^1 + \mathbb{B}_1\mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\begin{bmatrix} H^1_{in} \\ \bar{H}^1_{in} \end{bmatrix} \\ F^1_{out} & = & \mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\mathbb{C}_1\mathbb{H}^1 \end{array}\right. \tag{1}$$

where $\mathbb{H}^1 = \begin{bmatrix} H^1 \\ \bar{H}^1 \end{bmatrix}$, $\mathbb{T}^1 = \begin{bmatrix} T^1 \\ \bar{T}^1 \end{bmatrix}$ *and* $\begin{bmatrix} H^1_{in} \\ \bar{H}^1_{in} \end{bmatrix}$ *are the enthalpy state, temperature and input enthalpy vectors resp.* F^1_{out} *is the ouptut enthalpy flow vector.* $\otimes$ *stands for the Kronecker product. Matrices are as follow:* $A_t = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $\mathbb{A}_1 = \Big((I_1 - S_1) \otimes A_c\Big)$, $\bar{\mathbb{A}}_1 = \Big((I_1 - S_1^T) \otimes \bar{A}_c\Big)$ *with* $I_1 = 1, S_1 = 0$, $A_c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ *and* $\bar{A}_c = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbb{V} = \begin{bmatrix} (\rho V)^{-1} & 0 \\ 0 & (\bar{\rho}\bar{V})^{-1} \end{bmatrix}$, $\mathbb{B}_1 = \begin{bmatrix} B & \bar{B} \end{bmatrix}$ *with* $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\mathbb{C}_1 = \begin{bmatrix} C \\ \bar{C} \end{bmatrix}$ *with* $C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$,

The indexes t and c stand for heat transfer through the partition wall and for convection, resp. The matrices B_c et $\bar{B}_c$ are null in this case but not for several blocks in series. The proof is trivial in this case. It suffices to write the energy balances and identify the matrices.

3. Model of two interconnected blocks

The objective of this section is to show on a simple case how the systematic modelling is constructed. For this purpose, we consider two elementary HEs composed of one block each as shown in Figure 3.

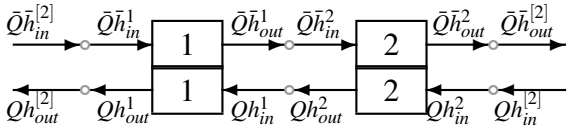

Figure 3: Two elementary HEs composed of one block each in series

Proposition 2 *With assumptions (a) to (e), the system of energy balances for 2 blocks in series writes as:*

$$\left\{\begin{array}{rcl} \frac{d}{dt}\mathbb{H}^{[2]} & = & -\lambda(I_2 \otimes A_t)\mathbb{T}^{[2]} - \frac{Q}{\rho V}\mathbb{A}_2\mathbb{H}^{[2]} - \frac{\bar{Q}}{\bar{\rho}\bar{V}}\bar{\mathbb{A}}_2\mathbb{H}^{[2]} + \mathbb{B}_2\mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\begin{bmatrix} H^{[2]}_{in} \\ \bar{H}^{[2]}_{in} \end{bmatrix} \\ F^{[2]}_{out} & = & \mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\mathbb{C}_2\mathbb{H}^{[2]} = \begin{bmatrix} Qh^{[2]}_{out} \\ \bar{Q}\bar{h}^{[2]}_{out} \end{bmatrix} \end{array}\right. \tag{2}$$

with $\mathbb{H}^{[2]^T} = \begin{bmatrix} H^1 & \bar{H}^1 & H^2 & \bar{H}^2 \end{bmatrix}$, $\mathbb{T}^{[2]^T} = \begin{bmatrix} T^1 & \bar{T}^1 & T^2 & \bar{T}^2 \end{bmatrix}$. *The matrices are given by:* $\mathbb{A}_2 = \Big((I_2 - S_2) \otimes A_c\Big)$, $\bar{\mathbb{A}}_2 = \Big((I_2 - S_2^T) \otimes \bar{A}_c\Big)$ *with* I_2 *the* 2×2 *identity matrix,* $S_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbb{B}_2 = \begin{bmatrix} 0_2 & \bar{B} \\ B & 0_2 \end{bmatrix}$, $\mathbb{C}_2 = \begin{bmatrix} C & 0_2^T \\ 0_2^T & \bar{C} \end{bmatrix}$ *with* $0_2^T = \begin{bmatrix} 0 & 0 \end{bmatrix}$.

Proof First let us consider two systems of energy balances as written in Proposition 1, the first for the block 1 and the second for the block 2. After vector and matrix concatenations, considering the global state vector $\mathbb{H}^{[2]}$ and the temperature vector $\mathbb{T}^{[2]}$ as written in Proposition 2 and denoting by $\mathbb{O}$ the zeros matrices when the size is not specified, we can immediately write:

$$\begin{cases} \frac{d\mathbb{H}^{[2]}}{dt} = & -\lambda \begin{bmatrix} (I_1 \otimes A_t) & \mathbb{O} \\ \mathbb{O} & (I_1 \otimes A_t) \end{bmatrix} \mathbb{T}^{[2]} - \frac{Q}{\rho V} \begin{bmatrix} \mathbb{A}_1 & \mathbb{O} \\ \mathbb{O} & \mathbb{A}_1 \end{bmatrix} \mathbb{H}^{[2]} - \frac{\bar{Q}}{\bar{\rho}\bar{V}} \begin{bmatrix} \bar{\mathbb{A}}_1 & \mathbb{O} \\ \mathbb{O} & \bar{\mathbb{A}}_1 \end{bmatrix} \mathbb{H}^{[2]} \\ & + \begin{bmatrix} \mathbb{B}_1 \\ 0_{2\times 2} \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^1 \\ \bar{H}_{in}^1 \end{bmatrix} + \begin{bmatrix} 0_{2\times 2} \\ \mathbb{B}_1 \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^2 \\ \bar{H}_{in}^2 \end{bmatrix} \\ F_{out}^1 = & \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} \mathbb{C}_1 & 0_{2\times 2} \end{bmatrix} \mathbb{H}^{[2]},\ F_{out}^2 = \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} 0_{2\times 2} & \mathbb{C}_1 \end{bmatrix} \mathbb{H}^{[2]} \end{cases} \tag{3}$$

Clearly the first right hand side term of (3) is equal to first right hand side term of (2). In order to write the other terms, let us consider the interconnection matrix A_d for the enthalpy flows. The matrix elements are 1 if a grey node links two enthalpy flows, 0 else. By premultiplying on both sides the flow equality equations by ρV, $\bar{\rho}\bar{V}$ for flow related to Q, $\bar{Q}$ respectively, we obtain:

$$\begin{bmatrix} QH_{in}^1 \\ \bar{Q}\bar{H}_{in}^1 \\ QH_{in}^2 \\ \bar{Q}\bar{H}_{in}^2 \\ QH_{out}^{[2]} \\ \bar{Q}\bar{H}_{out}^{[2]} \end{bmatrix} = \overbrace{\left[\begin{array}{cccc|cc} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right]}^{A_d} \begin{bmatrix} QH_{out}^1 \\ \bar{Q}\bar{H}_{out}^1 \\ QH_{out}^2 \\ \bar{Q}\bar{H}_{out}^2 \\ QH_{in}^{[2]} \\ \bar{Q}\bar{H}_{in}^{[2]} \end{bmatrix} = \left[\begin{array}{c|c} A_{d_{lu}} & A_{d_{ru}} \\ \hline A_{d_{ld}} & 0_{2\times 2} \end{array}\right] \begin{bmatrix} QH_{out}^1 \\ \bar{Q}\bar{H}_{out}^1 \\ QH_{out}^2 \\ \bar{Q}\bar{H}_{out}^2 \\ QH_{in}^{[2]} \\ \bar{Q}\bar{H}_{in}^{[2]} \end{bmatrix} \tag{4}$$

A_d is intrinsic to the series of two HEs independently on their sizes.

- Computation of the inputs of the 2 subsystems that become global input variables. We have:

$$\begin{bmatrix} QH_{in}^{[2]} \\ \bar{Q}\bar{H}_{in}^{[2]} \end{bmatrix} = A_{dru}^T \begin{bmatrix} QH_{in}^1 \\ \bar{Q}\bar{H}_{in}^1 \\ QH_{in}^2 \\ \bar{Q}\bar{H}_{in}^2 \end{bmatrix} = \begin{bmatrix} QH_{in}^2 \\ \bar{Q}\bar{H}_{in}^1 \end{bmatrix} \tag{5}$$

- Computation of the internal inputs: They are given by:

$$QH_{in}^1 = A_{dlu_1} \begin{bmatrix} QH_{out}^1 & \bar{Q}\bar{H}_{out}^1 & QH_{out}^2 & \bar{Q}\bar{H}_{out}^2 \end{bmatrix}^T \tag{6}$$

$$\bar{Q}\bar{H}_{in}^2 = A_{dlu_4} \begin{bmatrix} QH_{out}^1 & \bar{Q}\bar{H}_{out}^1 & QH_{out}^2 & \bar{Q}\bar{H}_{out}^2 \end{bmatrix}^T \tag{7}$$

where A_{dlu_1} and A_{dlu_4} are the first and fourth line of A_{dlu} respectively.

Inserting the expressions of the internal outputs with respect to global state vector, we obtain the following relations for the internal inputs:

$$\begin{bmatrix} H_{out}^1 \\ \bar{H}_{out}^1 \\ H_{out}^2 \\ \bar{H}_{out}^2 \end{bmatrix} = \overbrace{\begin{bmatrix} \mathbb{C}_1 & 0_{2\times 2} \\ 0_{2\times 2} & \mathbb{C}_1 \end{bmatrix}}^{\mathscr{C}_2} \begin{bmatrix} \mathbb{H}^1 \\ \mathbb{H}^2 \end{bmatrix} \Longrightarrow \begin{cases} QH_{in}^1 = QA_{dlu_1}\mathscr{C}_2\mathbb{H}^{[2]} \\ \bar{Q}\bar{H}_{in}^2 = \bar{Q}A_{dlu_4}\mathscr{C}_2\mathbb{H}^{[2]} \end{cases} \tag{8}$$

Let us rewrite the following terms in (3) : $\begin{bmatrix} \mathbb{B}_1 \\ 0_{2\times 2} \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^1 \\ \bar{H}_{in}^1 \end{bmatrix} + \begin{bmatrix} 0_{2\times 2} \\ \mathbb{B}_1 \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^2 \\ \bar{H}_{in}^2 \end{bmatrix}$ in order to recompose with respect to internal and global inputs. We have :

$$\begin{bmatrix} \mathbb{B}_{11} & 0_2 \\ 0_2 & \mathbb{B}_{12} \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^1 \\ \bar{H}_{in}^2 \end{bmatrix} + \begin{bmatrix} 0_2 & \mathbb{B}_{12} \\ \mathbb{B}_{11} & 0_2 \end{bmatrix} \mathbb{V} \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix} \begin{bmatrix} H_{in}^2 \\ \bar{H}_{in}^1 \end{bmatrix}$$

where $\mathbb{B}_{11}, \mathbb{B}_{12}$ are the first and second column of $\mathbb{B}_1$, that are respectively B and $\bar{B}$.

So for the global inputs, the last right hand side term of (2), we can identify that

$\mathbb{B}_2 = \begin{bmatrix} 0_2 & \mathbb{B}_{12} \\ \mathbb{B}_{11} & 0_2 \end{bmatrix} = \begin{bmatrix} 0_2 & \bar{B} \\ B & 0_2 \end{bmatrix}$.

Finally, for stream Q, let us add the convective term in (3) and the term on internal inputs and replace H_{in}^1 by the formula given in (8), we obtain:

$$-\frac{Q}{\rho V}\begin{bmatrix} \mathbb{A}_1 & \mathbb{O} \\ \mathbb{O} & \mathbb{A}_1 \end{bmatrix}\mathbb{H}^{[2]} + \frac{Q}{\rho V}\begin{bmatrix} \mathbb{B}_{11} \\ 0_2 \end{bmatrix} H_{in}^1 = -\frac{Q}{\rho V}\begin{bmatrix} \mathbb{A}_1 & \mathbb{O} \\ \mathbb{O} & \mathbb{A}_1 \end{bmatrix}\mathbb{H}^{[2]} + \frac{Q}{\rho V}\begin{bmatrix} \mathbb{B}_{11} \\ 0_2 \end{bmatrix} A_{dlu_1}\mathscr{C}_2\mathbb{H}^{[2]}.$$

It remains to show that $\begin{bmatrix} \mathbb{A}_1 & \mathbb{O} \\ \mathbb{O} & \mathbb{A}_1 \end{bmatrix} - \begin{bmatrix} \mathbb{B}_{11} \\ 0_2 \end{bmatrix} A_{dlu_1}\mathscr{C}_2 = \mathbb{A}_2$. It can be checked that $A_{dlu_1}\mathscr{C}_2$ is equal to the third line of $\mathscr{C}_2$. So $\begin{bmatrix} \mathbb{B}_{11} \\ 0_2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0_{2\times 2} & A_c \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix}$. So $\begin{bmatrix} \mathbb{A}_1 & \mathbb{O} \\ \mathbb{O} & \mathbb{A}_1 \end{bmatrix} - \begin{bmatrix} \mathbb{B}_{11} \\ 0_2 \end{bmatrix} A_{dlu_1}\mathscr{C}_2 = \Big((I_2 - S_2) \otimes A_c\Big)$.

The same computation can be made for stream $\bar{Q}$.

- Computation of the global outputs. From the equations (4) and (8), we have:

$F_{out}^{[2]} = \begin{bmatrix} QH_{out}^{[2]} \\ \bar{Q}\bar{H}_{out}^{[2]} \end{bmatrix} = A_{dld}\begin{bmatrix} QH_{out}^1 \\ \bar{Q}\bar{H}_{out}^1 \\ QH_{out}^2 \\ \bar{Q}\bar{H}_{out}^2 \end{bmatrix} = \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\overbrace{A_{dld}\,\mathscr{C}_2}^{\mathbb{C}_2}\,\mathbb{H}^{[2]}$. It is easy to check that $\mathbb{C}_2 = A_{dld}\,\mathscr{C}_2 = \begin{bmatrix} C & 0_2^T \\ 0_2^T & \bar{C} \end{bmatrix}$ This ends the proof. ■

This method can be iteratively used in order to obtain models for HE with i blocks.

4. Model of Heat Exchanger Network interconnection in series

The objective of this section is to give the model of two series interconnected HENs as shown in Figure 2. The results are given without proof.

As in the previous section, we do not present the graphic representation of matter flows since the graph is trivial and leads to equality of matter flows with two flows only: Q and $\bar{Q}$. However, as soon as mixer or splitter units will be consider this matter graph representation will be important to analyse.

First the model of a HE composed of i blocks in series is given by:

Proposition 3 *With assumptions (a) to (e), the system of energy balances for a heat exchanger composed of i blocks in series writes as:*

$$\begin{cases} \frac{d}{dt}\mathbb{H}^{[i]} &= -\lambda(I_i \otimes A_t)\mathbb{T}^{[i]} - \frac{Q}{\rho V}\mathbb{A}_i\mathbb{H}^{[i]} - \frac{\bar{Q}}{\bar{\rho}\bar{V}}\bar{\mathbb{A}}_i\mathbb{H}^{[i]} + \mathbb{B}_i\mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\begin{bmatrix} H_{in}^{[i]} \\ \bar{H}_{in}^{[i]} \end{bmatrix} \\ F_{out}^{[i]} &= \mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\mathbb{C}_i\mathbb{H}^{[i]} = \begin{bmatrix} Qh_{out}^{[i]} \\ \bar{Q}\bar{h}_{out}^{[i]} \end{bmatrix} \end{cases} \tag{9}$$

with $\mathbb{H}^{[i]}$, $\mathbb{T}^{[i]}$ are the enthalpy and temperature $2i$ vectors of the i-blocks HE resp. The matrices are given by: $\mathbb{A}_i = \Big((I_i - S_i) \otimes A_c\Big)$, $\bar{\mathbb{A}}_i = \Big((I_i - S_i^T) \otimes \bar{A}_c\Big)$ with S_i the upper shift $i \times i$ matrix and

I_i the $i \times i$ identity matrix, $\mathbb{B}_i = \begin{bmatrix} 0_{2i-2} & \bar{B} \\ B & 0_{2i-2} \end{bmatrix}$, $\mathbb{C}_i = \begin{bmatrix} C & 0_{2i-2}^T \\ 0_{2i-2}^T & \bar{C} \end{bmatrix}$ *with* 0_{2i-2} *the* $2i-2$ *zero vector.*

Now let us consider the series of the *i*-blocks HE with a *j*-blocks HE or HEN. We have for the representation of the system of energy balances:

Proposition 4 *With assumptions (a) to (e), the system of energy balances for the global system composed of two systems of the form* (9) *in series (the first of size i and the second of size j) writes:*

$$\begin{cases} \frac{d\mathbb{H}^{[i+j]}}{dt} &= -\lambda\left(I_{i+j} \otimes A_t\right)\mathbb{T}^{[i+j]} - \frac{Q}{\rho V}\mathbb{A}_{i+j}\mathbb{H}^{[i+j]} - \frac{\bar{Q}}{\bar{\rho}\bar{V}}\bar{\mathbb{A}}_{i+j}\mathbb{H}^{[i+j]} + \mathbb{B}_{i+j}\mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\begin{bmatrix} H_{in}^{[i+j]} \\ \bar{H}_{in}^{[i+j]} \end{bmatrix} \\ F_{out}^{[i+j]} &= \mathbb{V}\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\mathbb{C}_{i+j}\mathbb{H}^{[i+j]} \end{cases} \tag{10}$$

with $\mathbb{H}^{[i+j]^T} = \begin{bmatrix} \mathbb{H}^{[i]^T} & \mathbb{H}^{[j]^T} \end{bmatrix}$, $\mathbb{T}^{[i+j]^T} = \begin{bmatrix} \mathbb{T}^{[i]^T} & \mathbb{T}^{[j]^T} \end{bmatrix}$ *and the notations are the same as previously using* $i+j$ *instead of i.*

The generalization of this approach when heteregeneous blocks parameters are considered can be easily deduced. Consider the $i \times i$ heat transfer parameter diagonal matrices Λ_i with notations $\lambda_{i,k}$ for the k^{th} element. Since volume of compartment can be different for the blocks, the previous (2×2) matrix $\mathbb{V}$ has to be indexed by the number of the block: $\mathbb{V}_n$ for the n^{th} block. Let us consider the block diagonal matrix $\mathbb{W}_i$ with $\mathbb{V}_n$ on the diagonal for the n^{th} block and the matrix $W_i^T = \begin{bmatrix} \mathbb{V}_1 & \mathbb{V}_2 & \dots & \mathbb{V}_i \end{bmatrix} = \mathbb{1}_{2i}^T\mathbb{W}_i$ with $\mathbb{1}_{2i}$ the $2i$ vector of 1. The matrix $\mathbb{W}_i$ is a $(2i \times 2i)$ matrix while the matrices W_i and W_i^T have the same dimensions as $\mathbb{B}_i$ and $\mathbb{C}_i$, resp. Finally let us denote the Hadamard product as $\odot$.

Corollary 4.1 *With assumptions (a) to (d), the system of energy balances for a heat exchanger composed of i blocks in series writes as:*

$$\begin{cases} \frac{d}{dt}\mathbb{H}^{[i]} &= -\left(\Lambda_i \otimes A_t\right)\mathbb{T}^{[i]} - Q\mathbb{W}_i\mathbb{A}_i\mathbb{H}^{[i]} - \bar{Q}\mathbb{W}_i\bar{\mathbb{A}}_i\mathbb{H}^{[i]} + \left(\mathbb{B}_i \odot W_i\right)\begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\begin{bmatrix} H_{in}^{[i]} \\ \bar{H}_{in}^{[i]} \end{bmatrix} \\ F_{out}^{[i]} &= \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}\left(W_i^T \odot \mathbb{C}_i\right)\mathbb{H}^{[i]} = \begin{bmatrix} Qh_{out}^{[i]} \\ \bar{Q}\bar{h}_{out}^{[i]} \end{bmatrix} \end{cases} \tag{11}$$

5. Conclusion

The proposed systematic way of modelling series coupling of heat exchangers or heat exchanger networks has the advantage to have intrinsic matrices (with only 1 and 0 as elements) such $\mathbb{A}_i$, $\mathbb{B}_i$ and $\mathbb{C}_i$. The perspective is to extend this method to parallel interconnections in order to model collectors or distributors. The genericity of the proposed structured dynamical model can be easily implemented as a computer aided process engineering.

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