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# **A Short Literature on Joint Inversion Methods in Geophysics**

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## **ABSTRACT**

Joint inversion is a geophysical approach on simultaneously inverting multiple data sets on the assumption that the sought parameters are influenced by common subsurface physical and/or hydrological property fields. Many are the data combinations among the type of data that are used for joint inversion: electrical resistivity with seismic, cross-hole electrical resistance with ground penetrating radar, magnetotelluric and seismic travelttime data, seismic with gravity and electromagnetic data, to name a few. The following literature review is structured chronologically according to the publication date of the studies.

## **INTRODUCTION**

Geophysical inverse problems are often regularized due to the presence of noise in the data and the mixed determined character of most inverse problems (Linde et. al., 2006). Additionally, inversion suffers from non-uniqueness because limited data causes the formulation of the inversion problems to be undetermined (Gunther et. al., 2016). The conventional way to reduce such non-uniqueness is to introduce external constraints such as a priori geologic information, smoothing, stochastic regularization, or inversion for a specific model type. Although tomograms obtained from a single geophysical data type can help to improve stratigraphic, geological, hydrologic or even archaeological models, ambiguity often remains.

An alternative approach for reducing model ambiguity is performing joint inversion of two or more types of collocated geophysical data to produce a single integrated model unifying the information

context included in the individual data sets (Doetsch et. al., 2010a). Nowadays, the joint inversion of several types of geophysical and hydrological data is an emerging and powerful tool for quantitative data integration. Underlying the joint inversion strategies is based on the assumption that the reconstructed parameters are influenced by a common subsurface physical and/or hydrological property field (Infante et. al., 2010).

The use of more than one geophysical dataset for geophysical inversion was first presented by Vozoff and Jupp (1975). The utilization of data from several geophysical methods can be done in three main ways: i) joint interpretation using different datasets that are separately inverted, ii) joint inversion of separate data sets for a common inversion parameter model, iii) coupled inversion, where separate datasets constrain each other based on a specific correlation equation or model (Doetsch et. al., 2012).

Joint inversions have become a popular tool in geophysics to search models of the subsurface that are in agreement with data acquired with different geophysical methods (Moorkamp et. al., 2016). Combining data sets with complementary sensitivity and resolution properties in a joint framework can mitigate problems related to inversion such as limited resolution and non-uniqueness. Models obtained from joint inversion are inherently consistent with multiple data types and can thus improve interpretation and classification as well as help to establish petrophysical relationships (Jordi et. al., 2020).

When the datasets depend on different physical parameters, joint inversion either utilizes experimental relationships between the petrophysical properties (Berge et. al., 2000; Tillman and Stocker, 2000) or the structural similarity of the geophysical models. An essential condition in the latter case is that the geophysical methods detect the same layers and targets (Hering et. al., 1995; Lines et. al., 1988). The joint inversion of two separate datasets may also assist in improving the overall resolution, creating models that are in better agreement with each other, thus assisting the interpretation (Gallardo and Meju, 2004). The combined analysis of fusion of multiple data from physical, chemical or biological observations on geological targets makes possible the discriminations and classification of subsurface materials and processes. The individual uncertainties and low resolution of some data may mislead an integrated interpretation process (Gallardo, 2007). The underlying motivation for combining two methods based on different physical properties is the possibility to decrease the inherent ambiguities of each method (Linder et. al., 2010).

For the direct parameter relationship, we need to specify a functional relation between different parameters. Such functional relation can be deduced from an empirical formula (De Stefano et. al., 2011) or derived from various logging data (Heincke et. al., 2006; Jegen et. al., 2009). The direct parameter relationships provide a strong coupling between data sets. Petrophysical links prescribe

some (often empirical) relation between the physical properties under consideration and have been shown to produce good results, if the petrophysical relationship is reliable (Jordi et. al., 2020). Though good joint inversion results can be obtained from direct parameter coupling approach it can result in spurious features when the models violate the parameter relationships (Moorkamp et. al., 2011).

Structural constraint approaches do not need a functional relation between different physical parameters and couple two or more inversion models by measuring the structural similarity between them (Zhang and Morgan, 1997; Haber and Oldenburg, 1997; Gallardo and Meju, 2003; Gunther et. al., 2006). Structural joint inversions of geophysical data acquired under static field conditions provide geometrically similar models and improve model resolution compared to individual inversions (Gallardo and Meju, 2004; Linde et. al., 2008). One key advantage of the structural approach to joint inversion is that scatterplots between different models provide less biased information about petrophysical parameters than those obtained using direct petrophysical approaches to joint inversion (Tryggvason and Linde, 2006). The goal of joint inversion with two geophysical methods is to reduce the severity of smoothing constraints by identifying common structures between the images (Gunther et. al., 2016).

The quality and resolution of time-lapse inversion results may also improve compared with static inversions as modeling and observational errors are generally smaller (Doetsch et. al., 2010b). The approach of separately inverting different time snapshots and comparing the results does not work in most cases because of the contamination of the inverted models by the data noise. Sequential time-lapse inversion over a dynamic changing environment has proved more robust resulting in general successful spatio-temporal inversion images (Day-Lewis et. al., 2002; Martinez-Pagan et. al., 2010; Karaoulis et. al., 2011a; Kim et al., 2009, 2013).

## **STATE-OF-THE-ART**

**Haber and Oldenburg (1997)** were among the first that introduced the joint inversion technique. They developed a methodology to invert two different data sets with the assumption that the underlying models have a common structure, meaning that changes occur at the same physical locations. They defined an objective function which quantifies the difference in structure between the two models and then they minimized this objective function subject to satisfy the data constraints. They tested the algorithm on synthetic and field data, where they proved that data sets are consistent with models that are quite similar. The results of their study showed that the joint inversion is superior to individual inversions.

An important study regarding the correlation among geophysical techniques is from **Meju and Gallardo (2003)** where they correlated resistivity with seismic velocity data, which was acquired

over a buried hillside. The common thread is that resistivity and velocity are both functions of porosity. After comparing the data over porous sediments and fractured granodiorite at Quorn, they concluded that there were correlation trends in the near-surface resistivity and velocity distributions. During the same year, **Gallardo and Meju (2003)** developed a 2-D joint inversion strategy incorporating a new concept of cross-gradients of electrical resistivity and seismic velocity. Their goal was to characterize heterogeneous near-surface materials. They followed two different approaches: (a) petrophysical, where they took into consideration the assumption that both techniques are related on porosity and water saturation and (b) structural, where they accepted the fact that both methods are sensing the same underlying geology. The cross-gradients criterion requires that the problem satisfies the condition  $t(m)=0$ , meaning that any spatial changes occurring in both resistivity and velocity must point in the same or opposite direction, irrespective of the amplitude. The results showed meaningful solutions and led to a better structural characterization.

**Gallardo and Meju (2004)** posit that better results can be derived from jointly estimated models. The cross-gradient function is incorporated as a constraint in a nonlinear least squares problem formulation, which is solved using the Lagrange multiplier method. They tested the technique on synthetic and field data from collocated DC resistivity and seismic refraction profiling experiments. The joint inversion results were shown to be superior to those from separate 2-D inversions of the respective data sets. Furthermore, the conformity is reached without forcing or assuming the form of the relationship between electrical resistivity and seismic velocity.

**Linde et.al. (2006)** have developed an efficient method to calculate stochastic regularization operators for given geostatistical models. They used a structural approach to jointly invert cross-hole electrical resistance and ground-penetrating radar traveltimes data in three dimensions. No petrophysical relationship between electrical conductivity and relative permittivity was assumed, but instead obtained as a by-product of the inversion. This approach made it possible to jointly invert different data sets and obtain geologically realistic models at a research site without imposing overly restrictive assumptions and allowed a lithological zonation of the site. Although the bulk properties of each zone was estimated, it did not allow them to resolve small-scale variability within the zones.

**Colombo and De Stefano (2007)** concluded that joint inversion of seismic, gravity and electromagnetic data: (a) improved both convergence speed and seismic migration quality, (b) was more stable in relation to single domain inversions, (c) could reduce the intrinsic non-uniqueness of inverse problems because the joint objective function is less subject to local minima, and finally (d) improved the resolution of non-seismic data domains.

**Moorkamp et. al. (2007)** used a genetic algorithm for joint inversion of teleseismic receiver functions and magnetotelluric data from the Slave craton. They examined the compatibility of P-wave

teleseismic receiver functions and long-period magnetotelluric observations, to infer one-dimensional lithospheric structure. They demonstrated that with joint inversion approach not only a joint model was constructed but also cases where an interface was sensed differently by both data sets, could be identified.

**Gallardo and Meju (2007)** presented a joint 2-D inversion approach for imaging collocated magnetotelluric and seismic refraction data with cross-gradient structural constraints. They applied the technique to synthetic data generated for a hypothetical complex geological model, where the results led to models with structural resemblance and improved estimates of electrical resistivity and seismic velocity. Application of the algorithm to field data sets from a test site yielded that magnetotelluric and seismic models were in excellent geometrical accordance and remarkable consistency with those obtained independently from 2-D inversion of collocated DC resistivity and seismic data.

**Gallardo (2007)** developed a generalized cross-gradient procedure that seeks multiple geometrically similar images that simplify the integration of cross-property subsurface information. He inverted P-wave, S-wave, DC resistivity and magnetic data sets recorded at a field site. The resulting geospectral image provided a natural visualization of the multiple subsurface parameters.

**Infante et. al. (2010)** presented evidence that the combination of geospectral images and geophysical signatures is a good tool to provide a natural visualization of the distribution and variations of lithological features in a test site. They used electrical resistivity and seismic data with cross-gradient joint inversion at two profiles. Geotechnical information was provided by shallow boreholes. The results showed that it is possible to evidence hindered multiparameter correlations associated with relevant geological attributes such as lithological composition and moisture content.

**Doetsch et. al. (2010a)** showed from synthetic and field vadose zone water tracer injection experiment that joint 3-D time-lapse inversion of crosshole electrical resistance tomography and ground penetrating radar traveltimes data significantly improve the imaged characteristics of the point injected plume, as well as the overall consistency between models. The joint inversion method appeared to work well for cases when one hydrological state variable controls the time-lapse response of both geophysical methods (ERT and GPR). The results showed clearly that cross-gradients joint inversions of crosshole time-lapse ERT and GPR data decreased horizontal smearing of plumes and improved the estimated centre of mass of plumes compared to individual time-lapse inversions. Same year, **Doetsch et. al. (2010b)** developed a methodology for aquifer characterization based on structural joint inversion of multiple geophysical data sets followed by clustering to form zones and subsequent inversion for zonal parameters. It was the first time that it was applied to three geophysical data types (seismic, radar and electrical resistance tomography) in three dimensions. Tests were made in synthetic and field data.

**Karaoulis et. at. (2012)** showed that joint inversion schemes reduce space-related artifacts in filtering out noise that is spatially uncorrelated. They investigated the structural cross-gradient joint inversion approach and the cross-petrophysical approach. Time-lapse approach was performed with an actively time-constrained approach. The results showed: (a) the advantage of combining structural cross-gradient or cross-petrophysical inversion approaches by using a synthetic problem, (b) the combined structural cross-gradient/actively time-constrained approach reduced the presence of artifacts with respect to individual inversion of the resistivity and seismic data sets, and (c) the combination of cross-petrophysical/ actively time-constrained approach was successful in retrieving the position of the oil/water encroachment front.

**Hamdam and Vafidis (2012)** developed joint inversion strategies to provide improved resistivity and seismic velocity images for delineating saline water zones in karstic geological formations. The idea of this approach was to quantitatively estimate the structural similarity between resistivity and seismic velocity models, using the cross product of their gradients and to achieve a unified geological model which satisfies both data sets. The proposed strategy uses a weighting factor for the cross-gradients and separate damping factors for the resistivity and seismic data, which was successfully applied on field data from the karstic region of Stilos (Crete, Greece). The independent inversion sections showed differences in the thickness of the geological layers, while the joint inversion section gave a more reasonable geological scenario.

**Haber and Gazit (2013)** studied on model fusion and joint inversion. Joint inversion was achieved by using iterative data fusion. They used four different methods for joint inversion. They suggested a new technique called Joint Total Variation to solve the problem. They experimented with the techniques on the DC resistivity problem and the borehole tomography where they showed how model fusion and joint inversion can significantly improve over existing techniques. They also explored optimization techniques, where for joint inversion they advocated the use of block coordinate descent where one model is fixed and the other is fused to it, changing the models at each sub-iteration.

**Gunther et. al. (2016)** developed a new joint inversion algorithm that could be applied to independent physical parameters. It is based on the principles of model-sided robust modeling where the algorithm uses unstructured meshes that allow the incorporation of topography and other information on the geometry of the subsurface. The synthetic study was done using refraction tomography and DC resistivity inversion. Additionally, the algorithm was applied on the Pine Creek site in Calgary (Canada), where 6m of unsaturated gravel was underlain overlying a shaly weathered sandstone bedrock. By combining the inversion results with cluster analysis, geologic features were mapped that couldn't be seen using individual inversions on their own.

**Demirci et. al. (2017)** developed a new two-dimensional joint inversion algorithm for direct current resistivity, radio-magnetotelluric and seismic refraction data based on a cross gradient approach. They also proposed a new data weighting matrix to stabilize the convergence behaviour of the joint inversion algorithms. They used synthetic data where they found that resistivity and velocity models are better than the individual inversion of each data set. Field data also were used from Bafra Plain (Turkey) to investigate saltwater intrusion. The results showed that a more unique hydrogeological scenario might be obtained, especially in highly conductive media, with the joint usage of these methods.

**Shi et. al. (2017)** presented a 3-D cross-gradient joint inversion algorithm for seismic refraction and DC resistivity data. A limited memory quasi-Newton approach was used to perform the optimization of the objective function. Tests were performed on a typical archaeological synthetic model. Their approach appeared to be more robust in conditions of noise. The algorithm was applied to the field data from an ancient city site in Hangzhou of China. Their conclusion was that the proposed algorithm can be used to jointly invert 3-D seismic and DC resistivity data to reduce the uncertainty brought by single inversion scheme.

**Hellman et. al. (2017)** proposed a structurally coupled inversion of 2-D electrical resistivity and refraction seismic data using unstructured meshes. Mean shift clustering was used to combine two images and to compare the separate and coupled inversion methodologies. The utilization of clustering analysis, such as the used Mean Shift cluster algorithm, showed improvements in automating the characterization of geological zones in the models.

**Jordi et. al. (2020)** introduced a novel scheme for calculating cross-gradient operators based on a correlation model that allowed to define the operator size by imposing physical length scales. The proposed cross-gradient operators were largely decoupled from the discretization of the modelling domain, which was particularly important for irregular meshes where cell sizes varied. They applied the technique to a synthetic resistivity tomography and ground penetrating radar 3-D cross-well experiment aiming at imaging two anomalous bodies and extracting the parameter distribution of the geostatistical background models. They also applied structural joint inversion to a couple of field data sets recorded over a karstified limestone area. By including geological a priori information via the correlation-based operators into the joint inversion, they found that P-wave velocity and electrical resistivity tomograms are in accordance with the expected subsurface geology.

## **CONCLUSIONS**

The joint inversion technique is a very useful and promising approach which can combine different geophysical data based on the assumption that the underlying models have a common structure.



Generally, there are two basic methodologies: petrophysical and structural. It is proved that the results of this data combination are superior in comparison with the individual results of each data set separately. It is also shown that joint inversion reduces space-related artefacts in filtering out noise that is spatially uncorrelated. Furthermore, one data set can improve the drawbacks of the other data set. This approach has been tested on synthetic and field data, in 2-D and 3-D dimensions.

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