



**HAL**  
open science

## Inquiry-based mathematics teaching in practice: a case of a three-phased didactical model

Morten Blomhøj, Per Øystein Haavold, Ida Friestad-Pedersen

### ► To cite this version:

Morten Blomhøj, Per Øystein Haavold, Ida Friestad-Pedersen. Inquiry-based mathematics teaching in practice: a case of a three-phased didactical model. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03760079

**HAL Id: hal-03760079**

**<https://hal.science/hal-03760079v1>**

Submitted on 24 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# **Inquiry-based mathematics teaching in practice: a case of a three-phased didactical model**

Morten Blomhøj<sup>1</sup>, Per Øystein Haavold<sup>2</sup> and Ida Friestad Pedersen<sup>3</sup>

<sup>1</sup>Aarhus University, Danish School of Education, Denmark; [blomhoej@edu.au.dk](mailto:blomhoej@edu.au.dk)

<sup>2</sup>UiT The Arctic University of Norway, Department of Education, Norway;

[per.oystein.haavold@uit.no](mailto:per.oystein.haavold@uit.no)

<sup>3</sup>UiT The Arctic University of Norway, Department of Mathematics and Statistics, Norway;

[ida.pedersen@uit.no](mailto:ida.pedersen@uit.no)

*During the latest decades, inquiry-based teaching (IBMT) has become one of the top issues at the agenda for educational politics. IBMT is seen as having a potential for enhancing the students' motivation for and appreciation of mathematics as a field of activity and as a tool for understanding the world. IBMT can be conceptualized and operationalized in different ways. In this paper, we focus on a three-phased didactical model for IBMT, which can frame the students' inquiry in and with mathematics, and support the teachers' planning and implementation of an inquiry based activity. More specifically, we present a case study of how the use of the didactical model can facilitate the implementation of IBMT, and what are the challenges that remain and need to be addressed.*

*Keywords: Inquiry based learning, professional development, case studies.*

## **Introduction**

During the latest decades, inquiry-based teaching has migrated from science education into mathematics education, and inquiry-based mathematics teaching (IBMT) has become one of the top issues at the agenda for educational politics (Artigue & Blomhøj, 2013). Inquiry-based approaches have also found their way into mathematics curricula documents in Norway as well as in other countries. IBMT is seen as having a potential for enhancing the students' motivation for and appreciation of mathematics as a field of activity and as a tool for investigating the world. In this context, at The Arctic University of Norway in Tromsø (UiT), researchers designed a four-year research and development project SUM (Sammenheng gjennom Undersøkende Matematikkundervisning) funded by The Norwegian Research Council (FINNUT) for 2017- 2021. SUM is a four-year research and developmental project, in which researchers and mathematics teachers collaborate with the overall aim of contributing to coherence in children's and students' motivation for, activities in, and learning of mathematics throughout the entire educational system. The objective of SUM is the implementation of research findings related to IBMT as a means for better coherence across transitions in the educational system. In a previous CERME paper, we reported on some preliminary findings related to how the design of the project itself affected the implementation of IBMT in educational settings (Haavold & Blomhøj, 2019). One of the main findings was that the use of a three-phased didactical model used in the project was highlighted as an important enabler for implementing IBMT in the classroom. Most teachers participating in the project had stated that the model provided them with a more structured approach for designing IBMT for their practice. The model reduced a large and complex task to several smaller less complex steps of planning. Furthermore, the teachers said that the model helped them to be aware of and maintaining their own

and their students' different roles at each phase of the model. However, the teachers also found it quite uncomfortable to give up control, and allow the students to investigate and explore more freely.

In this paper, we focus on one particular case and present and analyze an IBMT lesson with regard to how this model provides a frame for the students' work and for the teachers' planning and implementation of IBMT. More specifically, we attempt to answer the following research questions: *How can the use of a particular three-phased didactical model facilitate the implementation of IBMT, and what challenges regarding the students' learning through IBMT can be identified?*

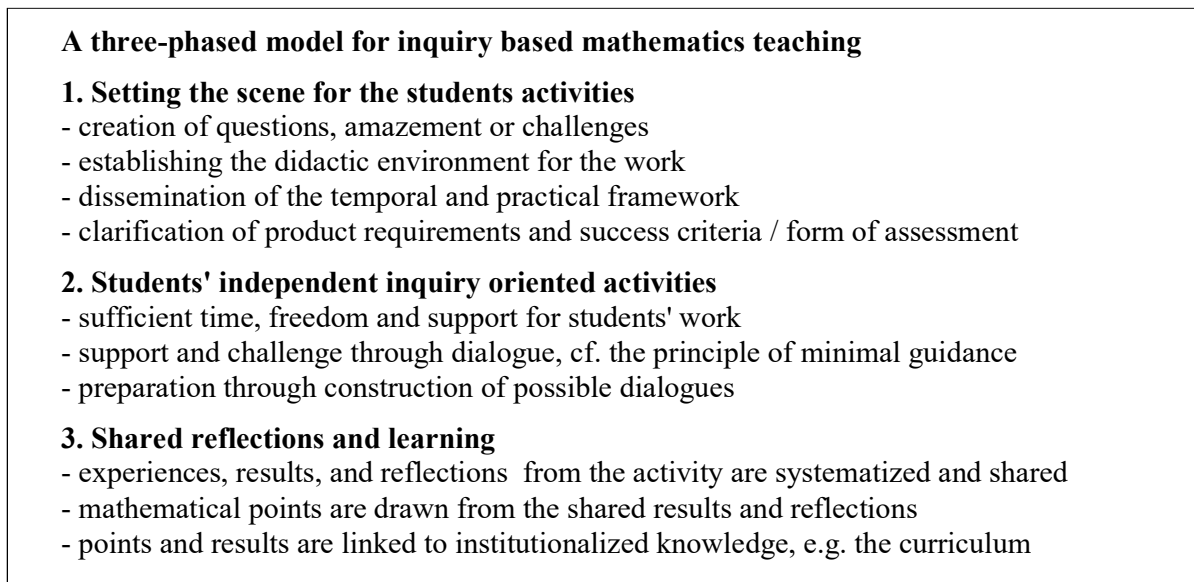
The keywords facilitate and challenges can be reframed within determinant frameworks of implementation theory, as enablers and barriers, or more generally as influential factors, that have an impact on implementation outcomes (Nilsen, 2015).

### **Inquiry based mathematics teaching**

In SUM, IBMT is conceptualized with reference to Artigue and Blomhøj (2013). Here the historical threads are retracted back to John Dewey (1859-1952), who developed an educational theory with inquiry as the main driver for humans' development of knowledge. Learning by doing has become parole for this theory, focusing on students' research-like activities. An essential point in Dewey's theory is, however, that knowledge is produced in an interplay between inquiry in problem situations and related reflections – denoted reflective inquiry (Dewey, 1938). Therefore, IBMT should engage students in relevant inquiry activities and challenge them to reflect on their experiences in order to support their learning of key mathematical concepts and methods. Artigue and Blomhøj (2013) discuss how IBMT can be understood and characterized in relation some theoretical frameworks such as problem-based learning, problem-solving, modelling, realistic mathematics education and the theory of didactical situations. Seen from the perspective of practice, IBMT has considerable overlaps with all these frameworks, and to some extent, IBMT can be seen as an overarching umbrella. From a theoretical point of view, however, the foundation of IBMT in Dewey's philosophy emphasizes the importance of anchoring the students' learning in memorable experiences and related reflections.

At seminars in the SUM project with the teachers, common factors in how IBMT can be operationalized across the different educational levels have been discussed and illustrated with concrete examples of different types of inquiry activities. Moreover, as common starting point the teachers were introduced to a three-phased didactical model for IBMT (Blomhøj, 2016, chap. 6). This model is summarized in figure 1. In the first phase, the scene should be set for the students' inquiry activities. Multiple approaches is possible, essentially the students should be motivated to investigating a phenomenon or a problem, which make sense for them. In the second phase, the focus is on the students' investigative work. Here the students should have sufficient time, freedom and support for their inquiry activities. The crucial challenge for the teacher is here to help and support just as much as needed without depriving the students the essential mathematical challenges and the related learning opportunities. In the third phase, students' experiences, results and reflections are systematized and shared in the class. The teacher can draw on the students' presentations or products and pinpoint essential elements for the class. Alternatively, the teacher can organize the results from the students' work in dialogue with the class, providing explicit and concrete anchoring to the students' work. Combinations of these formats are of course also possible. The shared experiences

and results can serve as a basis for reflections in class. It is the task of the teachers to identify key points in the students' work and to link them to content and learning objectives in the curriculum.



**Figure 1: Three-phased didactic model for IBMT**

## **Methods**

In this paper, we focus our analyses on an IBMT-lesson implemented in upper secondary school by one of the teachers (Ann) participating in the project. Ann has three years of experience teaching upper secondary and a masters' degree in mathematics education with a particular focus on secondary mathematics education. The lesson took place in a first-year upper secondary class consisting of 25 students aged 15-16 in a small to medium-sized school outside of a city in Northern Norway. Based on a survey directed at the students in the SUM project at the beginning of the school year, it seemed like the students in this class had mostly little to some experience with IBMT from their previous teaching. During interviews and in a questionnaire handed out early in the project, Ann had expressed interest for and curiosity about IBMT, but she had also said that it was difficult to find the time and the resources for designing and implementing IBMT in her classes. Furthermore, she was unsure that students' learned "what they were supposed to learn" in IBMT lessons. Ann had therefore decided, sometime during the third year of the project, that she wanted to use a so-called taxicab-geometry as a starting point for a 90 minute IBMT lesson, as it offered some "structure and clear learning targets related to proportionality and linear growth" as she put it. Prior to the lesson, Ann had been exposed to the three-phased model during the first two years of the SUM project, and had recently began using it as a tool for planning IBMT lessons in her regular practice.

Taxicab geometry is a form of non-Euclidian geometry, which can be thought of as taxicabs roaming a city with streets forming a lattice of unit-square blocks. The taxicabs can only move horizontally and vertically and U-turns are not allowed. The difference between the taxicab system and Euclidian geometry is that the usual distance function is replaced by a new metric in which the distance between two points is the sum of the absolute differences of their Cartesian coordinates, hence the distance between points in the lattice can be counted, see figure 2.

Before the lesson, Ann had prepared the following system of tasks:

1. Can you draw roundtrips that start and end at point A with lengths 8, 9, 12, and 17?
2. Can you mark all points, which have the same distance to point A and B?
3. Can you mark all points that have distance three to point A? How many points like this are there? Can you name this pattern of points?
4. Make a formula for the number of points with distance  $r$  to a specific point.
5. Make a formula for the number of points with distance less than  $r$  to a specific point.

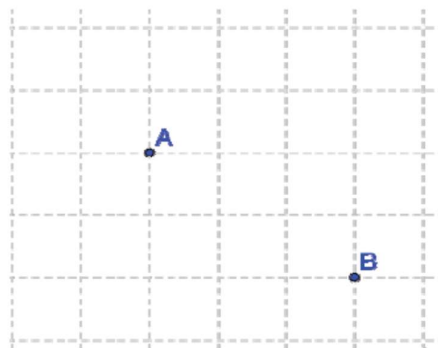


Figure 2: Points A and B with distance 5

In line with best practice regarding case studies, we used multiple data sources (Yin, 2014). We collected data from semi-structured observations of the lessons, short interviews with the teacher before and after the lesson, sound recordings of the teacher and groups of students during the lesson, and focus group interviews with students after the lesson. The main purpose of the analysis was to identify how the three-phased model provides a framework for the teachers' work in the classroom. Although there are no routine procedures for analyzing case studies, the analysis must logically piece together the available data into broader themes that capture essential aspects (Yin, 2014). Here, we attempted to identify and explain important episodes during the lesson that could help us understand both the challenges and opportunities the teacher experienced during each of the three phases of IBMT. First, we identified interesting episodes from the case based on our observational notes and sound recordings. We then cross-checked these episodes with data from our interviews before and after the lesson, in order to better understand why these episodes occurred. Finally, we discuss the potential for the students' learning, and possibly missed opportunities during these episodes.

## Results

The lesson began with Ann handing out the tasks and an explanation of the rules of the taxicab geometry to the students and telling them to sit in groups of 2-3. She then explained to the class that "this lesson is about thinking, exploring and conjecturing" and that the students had to "investigate and come up with solutions themselves". After letting the students look at the tasks and rules for a minute, she explained the rules of taxicab geometry to the whole class and she demonstrated an example of finding the distance between two points in taxicab geometry. She then told the students that they could start working on the handed out tasks. For the next five minutes, the classroom was quiet and the students did not write anything down in their own notebooks. Ann then asked the whole class to pay attention. She showed the students an example of a roundtrip of distance 8 from point A, before challenging them to find other roundtrips with distance 8, and then 9, 12, and 17.

Observations in phase 1 thus show that the students' did not immediately go to work on the tasks after Ann's short introduction and explanation. This indicates that phase 1 was not without its challenges for the teacher and students. Observation alone cannot tell us why the students sat quietly

at their desks, seemingly unwilling or unable to work on the tasks. However, interviews with the teacher and groups of students after the lesson point to at least some factors.

First, the students said that the tasks were difficult, and they were unsure of what to do. Asked why they thought the tasks were difficult, they explained that the tasks were new and unusual. One student, for instance said that; “the tasks were difficult...to understand. I did not know what to do. They were new and I have not seen this before.” The students also mentioned that the tasks lacked clear instructions, and they had to read the tasks several times before realizing that they had to solve them on their own. Second, the teacher said she expected the students to begin working on the tasks after her short introduction, and that she was surprised they sat quietly at their desks. After seeing this, Ann told us she thought the students needed a specific example to show the students how to get started. Third, we also noticed from our interviews an apparent disparity between the teacher and students regarding the purpose of the lesson. In the interviews, Ann mentioned that the lesson was built around the concepts of linear and quadratic functions and direct proportionality, as those were the key subjects in the curriculum “they were working on right now”. For instance, task 4 according to Ann, should “build a better understanding of direct proportionality and linear functions”, and in task 5 the students are challenged to develop a quadratic function. In the interviews, the students, on the other hand, expressed confusion about what the lesson was really about. As one student said, “I do not know what this has to do with the other stuff we have been working on lately”.

Based on these observations, it would seem as the students did not lack motivation or interest, but rather that their expectations caused some miscommunication at the beginning of the lesson. From the teacher’s perspective, both the purpose and intention of the tasks were clear. However, from the students’ perspective, the tasks were difficult; as they did not know how to get started on them and because they did not fully understand the learning purpose of the tasks. The issues mentioned here are not specific to this particular lesson. They are relevant for IBMT in more general. Dissemination of the temporal and practical framework and clarification of product requirements, success criteria and forms of assessment are general key issues in the first phase of IBMT. In other words, students and the teacher need to have a similar understanding of the task; what are the students expected to do, and what types of intellectual products should they produce. Brousseau and Warfield (2020) refer to this as a situation of devolution, where the students accept the challenge of an engaging and instructive mathematical situation. Similarly, Stein et al. (2008) have highlighted the importance of anticipating how students might interpret and attempt to tackle mathematical problems when implementing cognitively challenging tasks.

In the second phase, the focus is on the students’ investigative work. We observed several episodes that indicate challenges for both the teacher and the students during this phase. One issue we repeatedly noticed was that groups of students would often sit quietly, raise their hands and wait for the teacher to help them. For instance, at one point, only about five minutes into the second phase, students in seven groups raised their hands and asked the teacher for help. Only after the teacher had provided some form of guidance did the students proceed with their work. The following sequence illustrates this. Two boys raise their hands, and Ann approaches them:

Ann: How are things going? Did you get an answer to task 4?

Boy1: No, we did not understand it.

Boy2: We gave up.

Ann: No, tell me. What is it you do not understand?

The boys then show the teacher what they have done so far, and the teacher notices that they have marked points outside the intersections between the horizontal and vertical lines in the grid. The teacher asks the boys if it is possible to mark points outside the intersections.

Boy2: Aha, now I get it. The points have to be here (points to an intersection). Then it is easy. We have 4, 8, 12 points.

Boy1: I get it. We just multiply 1 by 4, then 2 by 4, then 3 by 4 and so on.

Boy2: This is brilliant! What a genius way of doing it.

Boy1: Very clever!

During the interview after the lesson, the teacher mentioned this episode with an explanation for why it occurred. According to Ann, this was an example of something she had noticed almost every time she had tried out IBMT lessons. The students are often passive during the investigative phase, and they quickly give up and ask for help from the teacher – even though they have not tried any strategies or approaches themselves. According to Ann, this is something that applies to almost all students in her class; “The students enjoy these IBMT lessons, but they give up quickly. They do not seem to want to think for themselves. They want me to tell them what to do”.

Another point worth mentioning about this episode is that the students’ solved the task by generalizing the empirical pattern of the number of points when the distance increased by one. Neither the teacher nor the students ever asked themselves why this pattern occurred. A key activity in both IBMT and mathematics in general (e.g. Barbeau & Hanna, 2008) is proving and reasoning deductively, and this was an excellent opportunity for the teacher to ask the students why this pattern appeared or if this pattern would continue when the distance became larger. The reason we bring up this point is that this was something we have noticed in several other IBMT lessons as well. Both teachers and students often seem to be satisfied with empirically based solutions. However, challenging the students’ schemas of reasoning and proof from empirical to more deductive schemas as defined by Harrel & Sowder (2007) could be seen as one of the potentials of IBMT.

In the third phase, at the end of the lesson, and after a short break, Ann told the class that they would now look at the tasks together. She asked the students if they have found any answers to the tasks, and then worked her way through each of the five tasks. For each task, she asked the students if they have an answer for her. The incorrect answers from the students were mostly sidestepped, and instead, she wrote down the correct answers from the students on the board. During this phase, Ann was very focused on connecting the students’ work to the topic of proportional growth and linear functions, as that was the topic they were currently working on. For instance, when she asked the students for solutions to task four, she stressed the importance of connecting the answers to concepts like proportionality, linear functions, and straight lines. After the lesson, Ann said in the interview, that

one of the main goals of the lesson was that the students would get a better understanding of “the relationship between tables, graphs and equations of linear functions”. Ann went on to explain that “ideally, the students’ would have been more active and more responsible for the third phase”. However, because she thought the students were a bit passive and quiet, she felt the need to make sure that the lesson ended with clear solutions to the tasks and a clear connection to the curriculum.

The interviews with the students after the lesson help us explain why the students were “passive and quiet”. As one student said, “this is just how we do it. At the end of the lesson, the teacher goes through the tasks and explain the solutions”. Another student added; “I did not quite understand the purpose of the lesson, so I prefer that teacher explain it to us”. Here we notice two things. First, it seems as if there are an expectation and habit of the lessons ending with the teacher explaining solutions and the resuming purpose of the lesson to the students. Second, it seems that the students were unsure of the purpose of the lesson in terms of the intended learning, even after the activity, and therefore they wanted the teacher to explain it to them. This observation shows that for the students, the activity constitutes a break of the didactic contract (Brousseau & Warfield, 2020).

## **Conclusion**

In this paper, we analyzed a case of IBMT with particular emphasis on the role and function of the three-phased model as a framework for the activities in the classroom. Although there are idiosyncrasies tied to the taxicab case, it illustrates several characteristics observed across many IBMT lessons in the SUM project. Thus, the points made concerning the role and function of the model in the case are to a large degree concretizations of general trends observed across many teachers, grade levels, and subjects in the SUM-project.

The taxicab lesson adhered clearly to the three-phased model. As noted by (Haavold & Blomhøj, 2019), a general observation from the SUM project is that the teachers have adopted the model as a didactical tool for planning, conducting and reflecting on their IBMT lessons. The teachers carefully separate their lessons into an introduction, an investigative phase, and a plenary summary. This might seem like an obvious and minor effect. However, in data collected at the start of the project, teachers expressed reluctance to let students investigate on their own without much guidance from the teacher (Haavold & Blomhøj, 2019). Thus, the careful use of the model is an important change of practice for many teachers. The taxicab lesson also illustrates certain challenges teachers face developing their IBMT practices. The students appeared relatively passive during the investigative phase, and according to Ann, this was something she frequently noticed when trying out IBMT lessons. In part, this may be due to the students being relatively unfamiliar with inquiry activities. Implementing IBMT lessons may necessitate renegotiating the classroom didactical contract (Brousseau & Warfield, 2020), and the teachers in the SUM project have indeed expressed views that the students need to be gradually accustomed to inquiry activities. The taxicab case illustrates this, as Ann specifically implemented a very structured IBMT lesson closely tied to the goals in the curriculum they were currently working on. Nevertheless, students’ independent work during phase two was still somewhat of a challenge. Concerning the three-phased model, it is clear that merely allocating some time for students’ independent inquiry (phase two) is not sufficient. Teachers also need to prepare for supporting students during their independent work, for example through anticipating how students may interpret the task and how they might approach solving it (cf. Stein et al., 2008).



Additionally, the taxicab lesson illustrates that phase three can be somewhat of a challenge for the teachers. Although the teacher linked the lesson and work to the curriculum, i.e. institutionalized mathematical knowledge, it is clear that the third phase lacked a systematization of and reflection on the students' solutions, as well as student engagement and active participation. Instead, the teacher mostly highlighted correct solutions without contrasting them with other solutions. The teacher mentioned in the interview that the students were passive, so she felt the need to take control and link the lesson to the curriculum. However, her aspiration to link the taxicab activity to specifically linear functions and proportionality may have contributed to hinder a productive discussion of the taxicab activity in itself and other key mathematical ideas.

To summarize, our analysis of the taxicab case illustrates how the three-phased model provides a frame for the teachers' planning and implementation of inquiry activities. In addition, it highlights the need for finding ways of supporting the teachers in planning and executing phase two and three of IBMT lessons. This plays directly into the dual nature of SUM: On the one hand, SUM as a research project aims to elucidate the extent to which teachers develop their competence for using investigative approaches in their teaching. On the other hand, SUM as a development project aims to support the teachers in developing a practice for IBMT. In that respect, the insights we gain from observing the teachers' implementation of IBMT in their practice in turn shapes how we as researchers collaborate with teachers in SUM. Hereby, SUM explores both the potentials and the limitations in developing practices of IBMT in collaboration between teachers and researchers.

## References

- Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM – Mathematics Education*, 45(6), 797–810. <https://doi.org/10.1007/s11858-013-0506-6>
- Hanna, G., & Barbeau, E. (2008). Proofs as bearers of mathematical knowledge. *ZDM – Mathematics Education*, 40(3), 345–353. <https://doi.org/10.1007/s11858-008-0080-5>
- Blomhøj, M. (2016). *Fagdidaktik i matematik* [Subject didactics in Mathematics]. Frydenlund.
- Brousseau, G., & Warfield, V. (2020). Didactic situations in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (2nd ed., 206–213). Springer. [https://doi.org/10.1007/978-3-030-15789-0\\_47](https://doi.org/10.1007/978-3-030-15789-0_47)
- Dewey, J. (1938). *Logic: The theory of inquiry*. Holt.
- Haavold, P. Ø., & Blomhøj, M. (2019). Coherence through inquiry based mathematics education. In Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Freudenthal Group & Institute; Utrecht University; ERME.
- Nilsen, P. (2015). Making sense of implementation theories, models and frameworks. *Implementation Science*, 10, Article number 53. <https://doi.org/10.1186/s13012-015-0242-0>
- Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10(4), 313–340. <https://doi.org/10.1080/10986060802229675>