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## Semi-inclusive lepton flavor universality ratio in $b \rightarrow s\ell^+\ell^-$ transitions

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We construct a semi-inclusive lepton flavor universality (LFU) ratio,  $R_\Sigma$ , to test  $\mu/e$  universality in  $b \rightarrow s\ell^+\ell^-$  transitions at  $e^+e^-$   $B$ -meson factories. Combining different decay channels, this observable maximizes the sensitivity to possible LFU violations of short-distance origin, yet preserves a clean theoretical interpretation in the case of a deviation from its Standard Model prediction,  $R_\Sigma^{\text{SM}} = 1$ .

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### I. INTRODUCTION

Within the Standard Model (SM) all the lepton Yukawa couplings are small compared to the SM gauge couplings, giving rise to an approximate accidental symmetry known as lepton flavor universality (LFU) (see e.g., Ref. [1]). In the last few years precise LFU tests have been performed by the LHCb experiment in rare  $B$ -meson decays. More precisely,  $\mu/e$  universality has been tested via measurements of the exclusive ratios [2]

$$R_M = \frac{\Gamma[B \rightarrow M\mu^+\mu^-]}{\Gamma[B \rightarrow Me^+e^-]}, \quad (1)$$

in specific dilepton invariant mass intervals, and for different final state mesons ( $M = K^+, K_S, K^{*0}, K^{*+}$ ).

Within the SM,  $R_M^{\text{SM}} = 1$  up to corrections due to phase space and QED, which do not exceed 1% [3–5] for the observables considered so far.<sup>1</sup> The experimental results reported by LHCb are all below this figure [6–8], and if combined, even in the most conservative way [9], they provide strong evidence of physics beyond the SM.<sup>2</sup> Given the potential groundbreaking impact of this result, it is extremely important to confirm it under different experimental conditions.

<sup>1</sup>The only exception is  $R_{K^*}$  for  $2m_\mu \leq m_{\ell\ell} \leq 1$  GeV, whose SM prediction is  $0.906 \pm 0.028$  [3].

<sup>2</sup>The evidence of new physics in  $b \rightarrow s\ell\ell$  transitions is further reinforced by global analyses of these processes, also taking into account the “anomalies” reported in branching ratios and angular distribution measurements (see e.g., [10,11] and references therein).

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An ideal setup for completely independent tests of  $\mu/e$  universality in rare  $B$  decays is provided by experiments performed at  $e^+e^-$   $B$ -meson factories, such as Belle-II [12]. These benefit from a much cleaner environment, compared to experiments at hadron colliders, such as LHCb. The only serious drawback, at least in the short term, is the limited statistics. To overcome this limitation, we propose testing the same short-distance dynamics via a semi-inclusive LFU ratio,

$$R_\Sigma = \frac{\sum_{H_s} \Gamma[B \rightarrow H_s\mu^+\mu^-]}{\sum_{H_s} \Gamma[B \rightarrow H_se^+e^-]}. \quad (2)$$

Here  $B$  stands for  $B$  mesons of any charge (i.e.,  $B^\pm$  and  $B^0$ ), while  $H_s$  denotes a series of well-defined exclusive final states, as well as an appropriate kinematical projection to maximize the statistics and, at the same time, retain a clean sensitivity to possible LFU effects of short-distance origin.

The main ideas behind the construction of  $R_\Sigma$  can be listed as follows:

- (i) A possible violation of  $\mu/e$  universality of short-distance origin, i.e., a violation attributed to a local  $b \rightarrow s\ell^+\ell^-$  interaction, should manifest in any exclusive  $B \rightarrow H_s\ell^+\ell^-$  decay. We can therefore combine many exclusive channels to increase the statistics. The key point to address is how to combine the different channels, taking into account the uncertainties due to unknown hadronic matrix elements.
- (ii) In principle, the simplest solution would be to consider a fully inclusive final state  $|X_s\rangle$ , of strangeness  $|S| = 1$ . However, this is quite challenging from the experimental point of view, requiring an independent (opposite-side)  $B$ -meson tag that usually implies a low efficiency. On the other hand, as pointed out in [13], LFU ratios have a rather constrained structure that allows us to combine them

even in the absence of a complete description of the underlying hadronic dynamics. Following the approach of Ref. [13], we can therefore limit the combination only to the specific sum of final states which have an easy (self-tagged) signature. The only strict requirement is to select the same combination of hadrons, in the same kinematical range, for both  $\ell = e$  and  $\ell = \mu$ .

- (iii) Following the above prescription, within the SM we expect  $R_{\Sigma}^{\text{SM}} = 1$  up to QED corrections. It is less obvious how to interpret the result if  $R_{\Sigma}^{\text{SM}} \neq 1$ . To this purpose, the key observation is that in a large fraction of the phase space, the  $b \rightarrow s\ell^+\ell^-$  SM amplitude is dominated by the product of a left-handed hadronic current times an (almost) left-handed leptonic current [14]. Hence the LFU ratios project out the left-handed component of the possible non-SM amplitude. The only exception is the region of narrow charmonia resonances and the low- $m_{\ell\ell}$  region, which are dominated by lepton-universal contributions. Cutting out the latter with appropriate kinematical cuts, we can build a semi-inclusive ratio that maximizes the sensitivity and allows for a clean theoretical interpretation.

Taking into account the above considerations, we proceed with the detailed definition of  $R_{\Sigma}$ .

## II. DEFINITION OF $R_{\Sigma}$

### A. Hadronic state and kinematical range

The hadronic states we propose to analyze together are composed of an odd number of kaons and an arbitrary number of pions. The set can be limited to charged pions and kaons only, but could also include a few neutral states (which notoriously have smaller detection efficiencies). We can generically denote the set as

$$|H_s\rangle \in \{(2n+1)|K\rangle + m|\pi\rangle\}, \quad n, m \in \mathbb{N}. \quad (3)$$

As anticipated, it is essential to ensure the same hadronic composition (i.e., the same  $n$  and  $m$ , and the same number of  $\pi^0$  and  $K_S$ ) for both  $\ell = e$  and  $\ell = \mu$ . On the other hand, neutral and charged  $B$ -meson decays can be combined in the semi-inclusive sum. Given the significant phase-space suppression of many-body channels, the analysis can be limited to at most three mesons in the final state, hence ( $m \leq 2, n = 0$ ) and ( $m = 0, n = 1$ ). However, one of the key features of the approach we propose is that there is no need to be fully inclusive; hence even a smaller subset would provide valuable results.

In order to define the kinematical range for the dilepton invariant mass, two requirements need to be fulfilled: (i) avoiding the region of the narrow charmonia,  $J/\Psi$  and  $\Psi(2S)$ , which would dilute a possible LFU-violating effect of short-distance origin, and (ii) performing

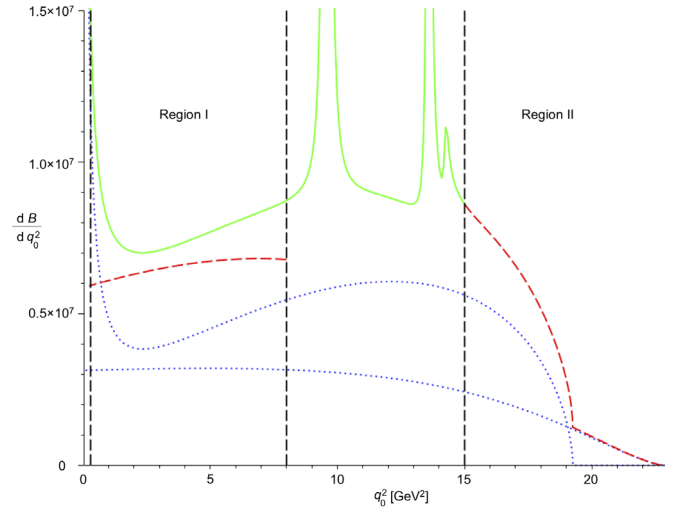


FIG. 1. Semi-inclusive dilepton spectrum obtained by summing  $\mathcal{B}(B \rightarrow K e^+ e^-)$  and  $\mathcal{B}(B \rightarrow K^* e^+ e^-)$ . The (green) solid line corresponds to the complete sum, including an estimate of long-distance contributions. The (blue) dotted lines indicate the short-distance contributions for the two separate modes. The (red) dashed line indicates the sum, after applying the longitudinal projection on the  $B \rightarrow K^* e^+ e^-$  mode. The vertical dashed lines denote the two regions for the evaluation of  $R_{\Sigma}$ .

kinematical cuts that do not induce LFU-violating effects of QED origin (i.e.,  $\alpha_{\text{em}} \log m_{\ell}$  corrections).

As demonstrated in Ref. [4], the dangerous QED collinear logs are avoided if the dilepton range is defined in terms of the collinear-safe variable

$$q_0^2 = (p_B - p_H)^2, \quad (4)$$

where  $p_H$  is the sum of all hadronic momenta. Contrary to experiments performed at hadron colliders, the variable  $q_0^2$ , which coincides with  $m_{\ell\ell}^2$  only in the limit of negligible final-state radiation, is accessible at  $e^+e^-B$  factories. Defining cuts in  $m_{\ell\ell}^2$ , rather than in  $q_0^2$ , is the main reason why the estimates of the inclusive ratios presented in [15,16] (including QED corrections) are significantly different for electrons and muons.

We recall that we are interested only in the LFU ratio and not in a precise description of the absolute decay probability in terms of short-distance dynamics. Hence we can afford to include a small (universal) long-distance contamination due to resonance tails in  $R_{\Sigma}$ . Keeping this in mind, we can extend the low- $q_0^2$  window up to  $8 \text{ GeV}^2$ , which is safely below the  $J/\Psi$  peak, and define the high- $q_0^2$  window starting from  $15 \text{ GeV}^2$ . In order to avoid the Dalitz decays ( $P \rightarrow \ell^+ \ell^- \gamma$ ) of light mesons, it is also useful to set a lower cut  $q_0^2 \geq q_{\text{min}}^2 = 0.3 \text{ GeV}^2 \approx m_{\eta}^2$ . Summarizing, as shown in Fig. 1, we propose to define  $R_{\Sigma}$  by integrating over the following two  $q_0^2$  windows:

$$\begin{aligned} \text{Region I: } & 0.3 \text{ GeV}^2 \leq q_0^2 \leq 8 \text{ GeV}^2, \\ \text{Region II: } & q_0^2 \geq 15 \text{ GeV}^2. \end{aligned} \quad (5)$$

The  $R_\Sigma$  defined in Eq. (2), with  $|H_s\rangle$  in (3) and  $q_0^2$  in (5), is a good variable to test LFU: It satisfies

$$R_\Sigma^{\text{SM}} = 1.00 \pm 0.01, \quad (6)$$

where the error is due to subleading QED corrections and is dominated by short-distance dynamics. However, its interpretation if  $R_\Sigma^{\text{exp}} \neq 1$  would not be very clean, the main problem being the photon-pole contribution in region I: a lepton-universal amplitude which necessarily dilutes a possible LFU effect. The precise estimate of this dilution requires the knowledge of the hadronic matrix elements of the dipole operator  $Q_7$  (see the Appendix), which are unknown for multimeson final states. The effect can be suppressed by further reducing the  $q_0^2$  range, and it can be partially dealt with by treating the unknown hadronic matrix elements as nuisance parameters [13]. However, as we discuss below, a more efficient strategy in the case of  $B$ -factory experiments is to get rid of the photon-pole contribution with a simple angular projection.

### B. Angular projection in the low- $q_0^2$ region

In order to define the angular projection which allows us to get rid of the photon-pole contribution, it is worth first discussing the allowed values of the angular momentum of the hadronic system ( $J_{\text{had}}$ ) and the helicity structure of the decay amplitude.

- (a) *Allowed values of  $J_{\text{had}}$ .* The matrix elements of the  $b \rightarrow s\ell^+\ell^-$  effective Lagrangian (see the Appendix), which are nonvanishing at the tree level in  $B \rightarrow H_s\ell^+\ell^-$  decays, can generally be decomposed as

$$\langle H_s | \mathcal{J}_{\text{had}}^\mu | B \rangle \times \mathcal{J}_{\text{lept}}^\mu. \quad (7)$$

Since the  $B$  meson has vanishing angular momentum ( $J^P = 0^-$ ) and  $\mathcal{J}_{\text{had}}^\mu$  transforms as a Lorentz vector, this implies that  $J_{\text{had}} = 0$  or 1. In principle, higher  $J_{\text{had}}$  can be generated by truly nonlocal contributions of the four-quark operators (via multipole expansion); however, these effects are extremely suppressed. For all practical purposes we can restrict our attention to  $J_{\text{had}} = 0$  and  $J_{\text{had}} = 1$ .

- (b) *Helicity structure of the decay amplitude.* The photon-pole contribution in a given  $B \rightarrow H_s\ell^+\ell^-$  decay is present if the  $B \rightarrow H_s\gamma$  transition, with an on-shell photon, is allowed. If  $J_{\text{had}} = 0$ , or if  $J_{\text{had}} = 1$  and is longitudinally polarized ( $J_{\text{had}} = 1^0$ ), helicity conservation forbids the  $B \rightarrow H_s\gamma$  decay. Isolating the  $J_{\text{had}} = 0$  and  $J_{\text{had}} = 1^0$  partial waves in the decay rate of  $B \rightarrow X_s\ell\ell$  would thus permit us to neglect the photon-pole contribution.

- (c) *Definition of the projection.* The helicity of the dilepton system (and, correspondingly, of the hadronic one) can be identified experimentally via the angle  $\theta_\ell$ , defined as the angle between the lepton and the  $B$  direction of flight in the  $q_0$  rest frame.<sup>3</sup> Neglecting lepton masses, the  $B \rightarrow H_s\ell^+\ell^-$  double-differential decay rate can be decomposed as [17]

$$\begin{aligned} \frac{d^2\Gamma}{dq_0^2 d\cos\theta_\ell} &= \mathcal{A}_{[J=0,1^0]}^{(1)} \times \sin^2\theta_\ell + \mathcal{A}_{[J=1^\pm]}^{(2)} \times \cos\theta_\ell \\ &+ \mathcal{A}_{[J=1^\pm]}^{(3)} \times (1 + \cos^2\theta_\ell). \end{aligned} \quad (8)$$

As explicitly indicated, the photon-pole contribution can show up only in the  $\mathcal{A}^{(2)}$  and  $\mathcal{A}^{(3)}$  coefficients. We can easily get rid of these terms with a nontrivial integration over  $x \equiv \cos\theta_\ell$  by means of the following projection operator:

$$\hat{P} \left[ \frac{d^2\Gamma}{dq_0^2 dx} \right] = \int_{-1}^{+1} dx (2 - 5x^2) \frac{d^2\Gamma}{dq_0^2 dx}. \quad (9)$$

The projector is such that

$$\hat{P}(1 + x^2) = \hat{P}(x) = 0, \quad (10)$$

$$\hat{P}(1 - x^2) = \frac{4}{3} = \int_{-1}^{+1} dx (1 - x^2). \quad (11)$$

The normalization of the projector in Eq. (11) is such that  $\hat{P}$  acts like the identity operator on the  $J_{\text{had}} = 0$  and  $J_{\text{had}} = 1^0$  components of the decay rate.

A similar projection procedure has been introduced in Ref. [15], in the context of fully inclusive  $B \rightarrow X_s\ell^+\ell^-$  decays, assuming the SM effective Lagrangian. We stress that the decomposition in Eq. (8), and the projection operator in Eq. (9), holds for any final state  $|H_s\rangle$  and also in the presence of (local) new-physics contributions.

We further stress that there is no need to operate with  $\hat{P}$  either in the case of a single kaon in the final state ( $|H_s\rangle = |K^\pm\rangle$  or  $|K_S\rangle$ ), which necessarily has  $J_{\text{had}} = 0$ , or in the high- $q_0^2$  region (region II), where the photon-pole contribution is already strongly suppressed by the value of  $q_0^2$ . The longitudinal projector  $\hat{P}$  has to be applied only on multimeson final states in region I. As shown in Fig. 1, where we illustrate the impact of the projection on the exclusive decay  $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ , the loss of statistics due to the projection is quite limited (below 20%).

<sup>3</sup>The  $q_0$  rest frame coincides with the dilepton rest frame in the limit of negligible final-state radiation [4].

### III. $R_\Sigma$ BEYOND THE SM

Following Ref. [13], the explicit expression of  $d\hat{\Gamma}_{H_s}^\ell/dq_0^2$  in terms of Wilson coefficients in generic extensions of the SM can be written as

$$\frac{d\hat{\Gamma}_{H_s}^\ell}{dq_0^2} = f_{H_s}^\ell(q^2) \{ (|C_L^\ell|^2 + |C_L^{\ell'}|^2 + \text{Re}[\eta_H^0(q^2)C_L^{\ell*}C_L^{\ell'}]) + (L \rightarrow R) + O(C_7) \}, \quad (12)$$

where the  $C_i$  are defined in the Appendix. The  $O(C_7)$  terms indicate contributions of the dipole operator that are not enhanced by the photon pole or that arise at the high- $q_0^2$  region where the longitudinal projector is not applied. Given the smallness of  $|C_7|$  in the SM [18], the experimental bounds on nonstandard  $b \rightarrow s$  dipole transitions [19], and given that the dipole amplitude is lepton-flavor universal, they can be safely neglected.

In the (well-justified) limit of neglecting the  $O(C_7)$  terms,  $R_\Sigma$  assumes the following simple form:

$$R_\Sigma = \frac{\{|C_L^\mu|^2 + |C_L^{\mu'}|^2 + \text{Re}[\langle \eta_\Sigma^0 \rangle C_L^{\mu*} C_L^{\mu'}]\} + \{L \rightarrow R\}}{\{|C_L^e|^2 + |C_L^{e'}|^2 + \text{Re}[\langle \eta_\Sigma^0 \rangle C_L^{e*} C_L^{e'}]\} + \{L \rightarrow R\}}. \quad (13)$$

The expression depends on a single combination of hadronic parameters,  $\langle \eta_\Sigma^0 \rangle$ , which controls the relative weight of vector and axial currents in the semi-inclusive sum (averaged over the various hadronic states and over the different  $q_0^2$  regions). As noted in Ref. [13], the positivity of the decay rate implies  $|\langle \eta_\Sigma^0 \rangle| \leq 2$ . In the exclusive  $B \rightarrow K$  transition, where the final state has a well-defined parity, this bound is saturated and  $\langle \eta_K^0 \rangle = 2$ . On the other hand,  $|\langle \eta_\Sigma^0 \rangle| \approx 0$  if the sum over  $H_s$  is sufficiently inclusive (leading to an average over final states with different parity).

To check how close we get to a sufficiently inclusive sum, considering only a few hadronic states, we have analyzed numerically the case where the  $|H_s\rangle$  set is limited to  $|K\rangle$  and  $|K^*\rangle$  (whose hadronic form factors are known). The semi-inclusive dilepton spectrum thus obtained is shown in Fig. 1. The numerical analysis has been performed by employing the  $B \rightarrow K$  and  $B \rightarrow K^*$  form factors from Refs. [20,21]. For illustrative purposes, in Fig. 1 we also include the effect of the narrow charmonia states in the  $B \rightarrow K\ell^+\ell^-$  case, following the procedure developed in [22].

Applying the projection operator only in the  $B \rightarrow K^*\ell^+\ell^-$  case, and only in region I, we estimate  $R_\Sigma$ , allowing for nonuniversal contributions to the Wilson coefficients. Defining

$$\Delta C_i = C_i^\mu - C_i^e, \quad (14)$$

and expanding for small  $|\Delta C_i|$ , we can write

$$R_\Sigma - 1 = \kappa_L \Delta C_L + \kappa_R \Delta C_R + \kappa_{L'} \Delta C_L' + \kappa_{R'} \Delta C_R' + O(\Delta C_i^2). \quad (15)$$

The numerical coefficients obtained with the procedure outlined above are

$$\kappa_L = 0.25 \pm 0.02, \quad \kappa_R = -0.02 \pm 0.03, \quad (16)$$

$$\kappa_{L'} = -0.03 \pm 0.02, \quad |\kappa_{R'}| < 0.01. \quad (17)$$

The errors are dominated by the uncertainty on the SM Wilson coefficients. In particular, we include a conservative 20% error on the value of  $C_9^{\text{SM}}$ , to account for the uncertainties associated with nonlocal contributions from four-fermion operators (see e.g., [23–25]). For completeness, we note that the results for various  $\kappa$  coefficients vary within the errors if estimated separately in one of the two  $q^2$  regions defined above.

The smallness of the  $\kappa'_{L,R}$  coefficients indicates that we are already very close to the inclusive limit (i.e.,  $\langle \eta_\Sigma^0 \rangle \approx 0$ ) even when we consider only  $|K\rangle$  and  $|K^*\rangle$  states. We thus conclude that  $R_\Sigma$  is a clean and sensitive probe of a single combination of LFU-violating Wilson coefficients:

$$\Delta C_L = (C_9^\mu - C_9^e) - (C_{10}^\mu - C_{10}^e). \quad (18)$$

Interestingly enough, a nonvanishing  $\Delta C_L$  is usually advocated, both for phenomenological and model-building considerations, as the origin of the violations of universality so far observed in  $b \rightarrow s\ell^+\ell^-$  transitions (see e.g., [10,26–28]).

### IV. CONCLUSIONS

The semi-inclusive LFU ratio  $R_\Sigma$ , defined by Eqs. (2), (3), and (5), and possibly improved by the angular projection discussed in Sec. II B, could allow for very clean testing of LFU even with limited statistics. The semi-inclusive transitions we propose to analyze, summing events in the two  $q_0^2$  regions, and taking into account the angular projection, correspond to an effective branching ratio  $\mathcal{B}_{\text{eff}} \approx 2 \times 10^{-6}$  (the precise value depends on how many exclusive channels will be included). Given that both charged and neutral  $B$ -meson decays can be combined, this corresponds to about 30 times the branching ratio relevant to  $R_{K^+}$  in the low- $m_{\ell\ell}$  region (as defined in the LHCb analysis [6]). Even if the overall efficiency for  $R_\Sigma$  is not as high as the one for  $R_K$  or  $R_{K^*}$ , we should expect a significant increase in the signal yield.

The Belle-II experiment has already observed the  $B \rightarrow K^*\ell^+\ell^-$  transition [29], and the statistics collected since then has more than doubled. With such statistics,  $R_\Sigma$  could possibly be measured with a O(10%) error, providing an interesting, nontrivial, independent test of  $\mu/e$  universality in  $b \rightarrow s\ell^+\ell^-$  transitions.

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APPENDIX:  $b \rightarrow s\ell^+\ell^-$  EFFECTIVE LAGRANGIAN

At energies below the electroweak scale, short-distance effects of both the SM and heavy new physics can be described by contact interactions among the light SM fields. The effective Lagrangian relevant for the  $b \rightarrow s\ell^+\ell^-$  transitions that we are interested in is

$$\mathcal{L}_{\text{eff}}^{b \rightarrow s\ell\ell} = -2\sqrt{2}G_F \frac{\alpha_e}{4\pi} V_{ts}^* V_{tb} \sum_i C_i \mathcal{O}_i + \text{H.c.} \quad (\text{A1})$$

where

$$\begin{aligned} \mathcal{O}_7 &= \frac{m_b}{e} (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, & \mathcal{O}'_7 &= \frac{m_b}{e} (\bar{s}_R \sigma_{\mu\nu} b_L) F^{\mu\nu}, \\ \mathcal{O}_9^\ell &= (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell), & \mathcal{O}'_{10} &= (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell), \\ \mathcal{O}'_9 &= (\bar{s}_R \gamma_\mu b_R) (\bar{\ell} \gamma^\mu \ell), & \mathcal{O}''_{10} &= (\bar{s}_R \gamma_\mu b_R) (\bar{\ell} \gamma^\mu \gamma_5 \ell). \end{aligned} \quad (\text{A2})$$

We do not consider scalar operators because their chirally suppressed contributions to  $b \rightarrow s\ell^+\ell^-$  amplitudes are

irrelevant for the observables considered in the ratio  $R_{\Sigma}$  defined in Eq. (2). Moreover, as discussed in the text, the nonlocal lepton-universal effects of four-quark operators are taken into account via an effective shift (and corresponding uncertainty) in the SM value of  $C_9$ .

Taking advantage of the (almost) left-handed structure of SM lepton currents, new physics effects can be best distinguished by rewriting the operator basis in terms of chirally projected operators for the leptons

$$\begin{aligned} C_L^\ell &= C_9^\ell - C_{10}^\ell, & C_L^{\ell'} &= C_9^{\ell'} - C_{10}^{\ell'}, \\ C_R^\ell &= C_9^\ell + C_{10}^\ell, & C_R^{\ell'} &= C_9^{\ell'} + C_{10}^{\ell'}, \end{aligned} \quad (\text{A3})$$

with the additional advantage that, in rates, the  $L, R$  interference is suppressed by the small lepton masses. The decay amplitude of a generic  $B \rightarrow H_s \ell^+ \ell^-$  process is decomposed as

$$\mathcal{A}^\ell(B \rightarrow H_s \ell^+ \ell^-) \propto (\mathcal{M}_{H_s, L}^\alpha)^\ell (\mathcal{J}_\alpha^L)^\ell + L \leftrightarrow R \quad (\text{A4})$$

where  $(\mathcal{J}_\alpha^X)^\ell = \bar{\ell} \gamma_\alpha \ell_X$ , with  $X = L, R$ , and

$$(\mathcal{M}_{H_s, X}^\alpha)^\ell = C_X^\ell \mathcal{J}_{H_s}^\alpha + C_X^{\ell'} \mathcal{J}_{H_s}^{\alpha'} + C_7 \mathcal{J}_{H_s}^{\alpha'} \quad (\text{A5})$$

with

$$\begin{aligned} \mathcal{J}_{H_s}^\alpha &= \langle H_s | (\bar{s}_L \gamma^\alpha b_L) | B \rangle, & \mathcal{J}_{H_s}^{\alpha'} &= \langle H_s | (\bar{s}_R \gamma^\alpha b_R) | B \rangle, \\ \mathcal{J}_{H_s}^{\alpha'} &\propto \frac{q_\beta}{q^2} \langle H_s | (\bar{s}_L \sigma^{\alpha\beta} b_R) | B \rangle, \end{aligned} \quad (\text{A6})$$

where  $q$  is the four-momentum of the lepton pair.

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