Instructive errors of Bouguer, Lambert, and Arago in the first determinations of angular reflectances on flat surfaces: discussion

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Abstract

The first photometric measurements performed in the 18th century were based on brightness matching between two illuminated surfaces. Bouguer and Lambert proposed first methods in 1760 to measure the angular reflectance of a flat surface, and Arago proposed a third one in the mid-19th century. These pioneering experiments provided rather good estimates of the values that we can predict or measure much more accurately today, attended that the human visual system was the only available light detector at that time. We show that the errors made in their measurements does not only come from experimental uncertainties but mainly on an incomplete knowledge of the physical properties of light, leading to incorrect assumptions in their models. The main errors are i) the fact that light is totally reflected at grazing incidence, ii) the glass plates they used were not perfectly clear, and iii) light is partially polarized after transmission across the surface. By highlighting the impact of these three errors, we can better understand the state of knowledge in optics at that time, and question our current practices in radiometric measurements and calculations.

1. Introduction

In 1729, in his *Essai d'optique* [1], Pierre Bouguer described the first rational method for measuring light intensity. The main principle consisted in matching the apparent amount of light emanating from two juxtaposed samples seen simultaneously. The intensity was measured relatively to a reference light source such as a candle. The principle of visual brightness matching constituted the basis of photometry until the use of photoelectric detectors in the 20th century [2,3]. Bouguer used his method to measure the transmittance of transparent media at normal incidence: glass plates, water, atmospheric air. For the reflectance of flat surfaces, the measurement was more difficult because the optical axis is modified. Pierre Bouguer presented the first reflectance measurements at different angles of incidence for a water surface and a glass plate in his *Traité d'optique* published posthumously in 1760 [4]. Since then and until the early 20th century, scientists proposed several improvements of this visual brightness matching method for the determination of angular reflectances.

With the knowledge we have today on the angular reflectance of plane interfaces between air and dielectric materials, we are able to identify some errors in the measurements made by these early experimenters or in the models that they used. Some errors seem to be due to the surface of the samples. For example, Bouguer found that the surface of a liquid was not perfectly flat but convex, which decreased the intensity of the reflected light. For glass plates, the scattering of light at the surface or in the volume may have disturbed the measurements. Another source of error relies on physiological considerations. However, Bouguer showed that two illuminated areas became visually indistinguishable when the difference of brightness was smaller than 1/64, i.e., about 1.5%. This threshold will be later called *just-noticeable difference* in psychophysics [4,5]. But Bouguer did not reach this level of precision. In his historical overview, Xiang Chen shows very well how these measurements of the surface reflectivity by visual matching went in one century from a poor quality to an accuracy close to those obtained by photodetectors [6]. In 1800, William Herschel improved a photometer proposed by Bouguer [7]. He replaced the candle by a lamp inside a dark lantern to avoid the diffusion of the light by the objects surrounding the instrument. On the other hand, Herschel noted the difficulty to adjust the illuminances when the eye was exposed to very close brightness during a long time. To remedy this problem, he added devices to his photometer to make constant adjustments while maintaining continuous observation. Other improvements in photometric devices were made to limit eye fatigue and make visual matching more accurate. Thus, high contrasts for the observation of illuminated areas were avoided [8], and brightness comparison was made on adjacent surfaces [9]. The photometers were also perfected in order to ensure a very good parallelism between the reflected light beam to be measured and the reference light beam. The most elaborate device was undoubtedly that of Rayleigh in 1886 [10]. His improvements allowed him to measure reflectances at normal incidence through a glass interface with an accuracy between 1 and 2% only. Current methods for measuring regular reflectance are detailed in Ref. [11]. Today, national metrology institutes can achieve uncertainties of the order of 0.3% at best.

In 1809, Malus demonstrated the polarization of light by reflection [12]. The understanding of the phenomenon by Fresnel allowed him to establish the relations between surface reflectance and incidence angle, now called Fresnel equations [13]. Fresnel relied on Arago's measurements to validate his equations [14]. In 1831, Potter used the discrepancies between his measurements and the theory to question Fresnel's approach [8]. At the end of the nineteenth century, the Fresnel equations were widely approved and used for example by Rayleigh to calibrate his measurements [10].

Note that the spectral properties of light were not considered at that time. The experimenters used white light sources, either natural or artificial such as candles. Diffusing screens were used to obtain uniform illumination.

In this paper, we focus on the measurements of the precursors, i.e., Bouguer, Lambert and Arago. They proposed partially indirect determinations of angular reflectances. We will not discuss their potential experimental errors but their erroneous physical assumptions. It is interesting to highlight them because these simplifying assumptions can still be made today without always being aware of the uncertainty generated, as discussed in a last Section.

2. Bouguer's extrapolation error

The *Traité d'optique* [4] by Bouguer, consisting of three books, took up a large part of his *Essai* [1]. However, Book 2, devoted to reflection by polished or diffuse surfaces, was completely original. Measurements on flat surfaces were presented in Section 2 of Book 2 while the experimental setup was described in Book 1 on the measurement of light in more general terms.

One of the devices consisted in a light source illuminating two screens placed symmetrically with respect to the studied surface (see Figure S1 in supplementary materials). The comparison is made between the brightness of a screen seen directly and that of the reflection of the other screen by the surface. The light source is moved until the two luminosities match with each other. The reflectance is then the ratio between the squared distances between the source and the screens according to the inverse square law. This device was later improved by Herschel [7].

Bouguer probably used another device for the reflectivity measurements on a water or glass surface, as reported below. Two adjustable openings in a partition allow to compare on a screen the light coming directly from the reference source and the one reflected by the studied surface (see Figure S2). The reflectance corresponds this time to the ratio between the areas of the two openings. Bouguer sometimes operated in two steps, by first measuring the reflectivity of a mercury surface and then comparing it to the light reflected from the surface under study.

Rather than making measurements at each incidence angle that would show variations related to measurement uncertainties, Bouguer preferred to translate the monotonic angular variation of the reflectance by fitting his measurements by a parametric curve of the following form:

$$R(\theta) \approx A + B(1-\cos\theta)^3 + C(1-\cos\theta)^6 \tag{1}$$

Function $(1-cos\theta)$, called *versine*, was considered as an important trigonometric function. Bouguer determined the three parameters using three measurements at 0°, 65° and 90°. Bouguer did not use decimal numbers and therefore manipulated only fractions of integers. Here are the measurements obtained for the water surface:

$$R(0°) = \frac{1}{55} = A \tag{2}$$

$$R(65°) = \frac{97}{100} = A + B\left[1-\cos(65°)\right]^3 + C\left[1-\cos(65°)\right]^6 \tag{3}$$

$$R(90°) = \frac{3}{4} = A + B + C \tag{4}$$

Bouguer probably used a trigonometric table for the *versine* value at 65°. He determined the following parameter values:

$$A = \frac{1}{55};\quad B = \frac{1}{3};\quad C = \frac{2}{5} \tag{5}$$

We reproduce in Table 1 the values as written in Bouguer's book and complete them with the numerical values of the approximation function as calculated by a computer. There are some rounding errors, unavoidable because of the calculating means of the time. Let us point out a more important calculation or recopy error for 75° (0.211 instead of 0.220). Moreover, in his determination of the parameters, Bouguer made a slight approximation since $A + B + C = 124/175 \approx 0.752$ and not $3/4$. More fundamentally, Bouguer made the mistake of constructing his curve from a value of reflectance at 90°, which is not measurable by definition. In practice, Bouguer had to

make this measurement at an angle between 85° and 89°. This extrapolation error strongly underestimates the reflectances at grazing angles. For example, Bouguer gave the predicted value of 0.61 at 87.5°, much lower than the 0.74 given by Fresnel's equations. Shifting the estimated reflectance value from 90° to the real angle of incidence probably around 87 or 88° would have significantly improved Bouguer's predictions. He did not do so perhaps because he thought he had reached the maximum reflectivity or perhaps more prosaically because the calculations generated would have been more tedious. In 1831, Potter used these erroneous results of Bouguer to assert the invalidity of Fresnel's equations predicting a total reflectance at 90 °, while "*it has been known since Bouguer's time that no known substance reflects at any incidence, all the light falling upon it*" [8].

Table 1. Angular reflectance of a water surface as written in Bouguer's book and recalculated with Eq. **(1)**.

Angle of incidence (°)	Values from Bouguer	Values from Eq. (1)
0*	**0.018**	0.018
10	0.018	0.018
20	0.018	0.018
30	0.019	0.019
40	0.022	0.023
50	0.034	0.034
60	0.065	0.066
65*	**0.097**	0.097
70	0.145	0.146
72.5	0.178	0.179
75	0.211	0.220
76.5	0.244	0.249
77.5	0.271	0.271
80	0.333	0.334
82.5	0.409	0.410
85	0.501	0.503
87.5	0.614	0.616
88	0.639	0.641
88.5	0.669	0.667
89	0.692	0.694
89.5	0.721	0.722
90*	**0.750**	0.752

***Measured values.**

Bouguer carried out similar study with a glass surface but with less details. He obtained the following function:

$$R(\theta) \approx \frac{1}{40} + \frac{7}{10}(1-\cos\theta)^3 - \frac{1}{40}(1-\cos\theta)^6 \tag{6}$$

If he had made the measurements at the same angles as with the water surface, he would have obtained:

$$R(0°) = 0.025; \quad R(65°) = 0.157; \quad R(90°) = 0.700 \tag{7}$$

These low values represent the reflectance on a single glass interface. Bouguer gave no details on how the reflection from the back side of the glass plate was not accounted for in his measurements.

Figures 1.a and 1.b compare the angular reflectance curves provided by Bouguer and the one computed according to the Fresnel formulas for a water and glass surface, of respective refractive indices 1.33 and 1.5, under unpolarized light. If, for water, the results are consistent with the Fresnel equations except for the grazing angles, higher differences are observed for the glass surface. At 65°, the deviation of Bouguer's measurement (0.157) from the Fresnel value (0.114) is significant (38% relative error).

Lambert determined the angular reflectance of a glass plate, presented in the next section. From these results, he developed a computation rule giving an empirical, approximate expression for the angular reflectance of a single interface between air and glass, given by the following formula:

$$\log_{10}\left[1-R(\theta)\right] \approx -\frac{\alpha}{\cos^2\theta} \tag{8}$$

He determined parameter $\alpha = 0.0087214$ that most closely matched his experiments, and gave tabulated reflectance values for angles varying from 0° to 80° in steps of 10°. The curve is shown as a blue dashed line in Figure 1.b. It reproduces well the increase of the reflectance with the angle of incidence, but with a striking overestimation at the grazing angles. In contrast with Bouguer's results, the value of the approximated function is equal to unity when the angle of incidence tends to 90°. But this is a point on which Lambert does not comment.

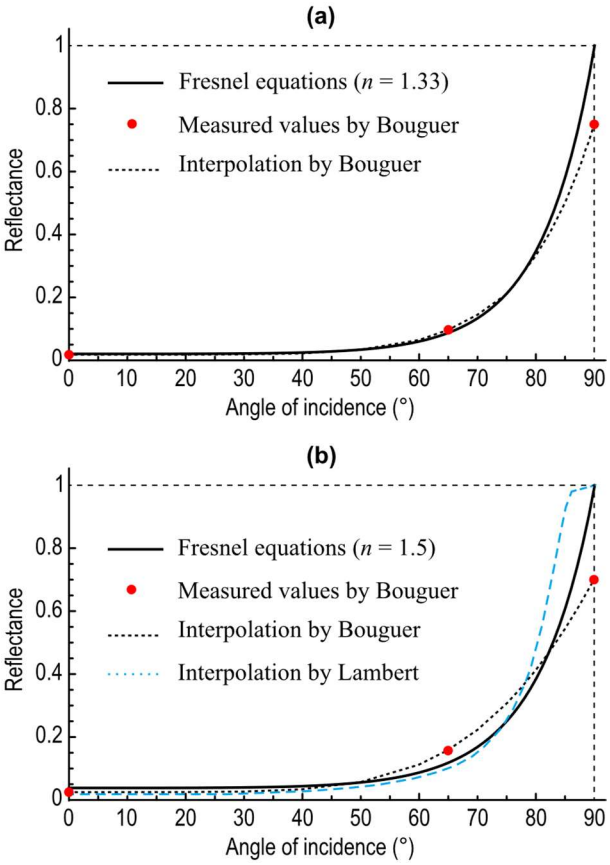

Figure 1. Angular reflectance for unpolarized light computed according to Fresnel's equations (solid back lines) and provided by Bouguer (the measured values are represented by red dots) (a) for a water surface of refractive index 1.33, (b) for a glass surface of refractive index 1.5 (Lambert interpolation is plotted in blue dashed line).

3. Lambert's untested assumptions

In 1760, Lambert published *Photometria* [15] in which he established the main laws of photometry. Lambert read Bouguer's *Essai d'optique*, which he quoted explicitly. He was not able to read Bouguer's *Traité d'optique*, but knew of its forthcoming publication. The first translation of *Photometria*, from Latin to German, was published only in 1892 [16], and only partially. In particular, Part 2 dealing with the determination of reflectances and transmittances of glass plates was lacking. This would partly explain why the results reported below are relatively unknown.

To establish the angular variation of the reflectance of a glass plate, Lambert's method is much more indirect than Bouguer's one. Lambert studied the reflectance and transmittance of a stack of identical glass plates considering the multiple reflections between the plates. Assuming perfectly clear glass, he expressed the angular reflectance $R_n(\theta)$ and transmittance $T_n(\theta)$ of a stack of n plates knowing those of one plate, $R_1(\theta)$ and $T_1(\theta)$:

$$R_n(\theta) = \frac{nR_1(\theta)}{1+(n-1)R_1(\theta)} \tag{9}$$

$$T_n(\theta) = \frac{T_1(\theta)}{n-(n-1)T_1(\theta)} \tag{10}$$

Lambert applied these relations to natural sunlight or sky light, or to artificial light from a candle. He made a mistake that he could not have been aware of: these relations are valid at oblique incidence only for a *p*- or *s*-polarized beam, and not for the unpolarized light that he used. This is shown by the graphic in Figure 2 where are plotted the angular reflectances $R_n(\theta)$ of 1 to 10 perfectly clear glass plates of optical index 1.5 as functions of the angle of incidence. The graphic compares the reflectances predicted by correctly taking into account the partial polarization by transmission through and reflection on each plate, or by assuming that light remains unpolarized after reflection or transmission events. The difference between the two approaches is maximal at the Brewster

angle, around 56° for an optical index of 1.5, angle at which the light reflected by any interface within the stack is linearly polarized (*s* component).

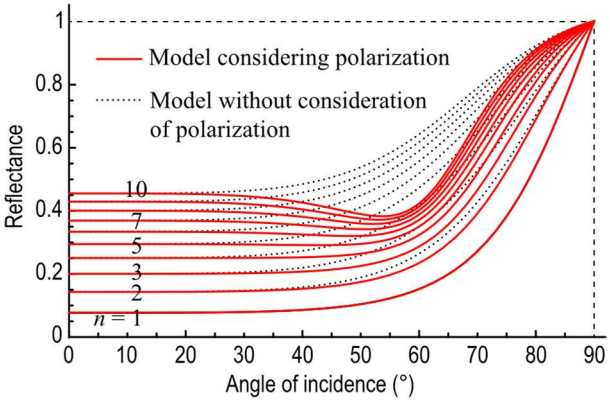

Figure 2. Reflectance of one perfectly clear glass plate (optical index 1.5) and piles of 2 to 10 identical plates, as predicted by taking into account the partial polarization by transmission through and reflection on each plate (red solid lines), or by assuming that light remains unpolarized after reflection or transmission events (dashed lines).

Lambert designed an experimental setup where a pile of glass plates was placed vertically on a horizontal white sheet of paper (see Figure S3). On this sheet was drawn a line in black ink and the device was illuminated equally on both sides of the pile. An observer could compare the images of the black line in reflection and transmission modes. He could determine the angle of incidence for which the two images of the line had the same lightness. At this angle θ_n of incidence on a pile of n plates, the following equality is satisfied:

$$R_n(\theta_n) = T_n(\theta_n) \tag{11}$$

For n perfectly clear plates, reflectance and transmittance are then equal to 1/2. From the previous relations, one can deduce the reflectance $R_1(\theta_n)$ of one clear plate at this specific angle θ_n:

$$R_1(\theta_n) = \frac{1}{1+n} \tag{12}$$

Once again, this relation should be valid only for a *p*- or *s*-polarized beam, but Lambert applied it to non-polarized beam.

Table 2. Angles θ_n determined by Lambert from Eq. (11).

Number *n* of plates	Angle of incidence θ_n (°)	$R_1(\theta_n)$
1	75.5	0.500
2	68	0.333
3	63	0.250
4	59	0.200
5	55	0.167
6	51	0.143
7	47	0.125
8	43	0.111
9	39.5	0.100

Reflectance $R_1(\theta_n)$ from one plate is deduced from Eq. (12).

Lambert determined the angles θ_n experimentally and deduced the reflectances $R_1(\theta_n)$ at these angles for one glass plate. He succeeded for n values from 1 to 9 (cf. Table 2) and noted that beyond this value, the determination became too difficult. The analysis of the results obtained by Lambert shows that the glass plate was far from being clear. We can obtain a good fit of the angles θ_n given by Lambert by considering a transmittance at normal incidence of 78% instead of 92% for a perfectly clear glass. Figure 3 shows the angular reflectance and

transmittance calculated with this absorptance. The angles verifying Eq. (11) correspond well to those given by Lambert for *n* from 1 to 6. Beyond, the differences are more important because reflectance and transmittance have very close values on a large angular range. For *n* larger than 7, there is no more angle of incidence for which reflectance and transmittance are strictly equal.

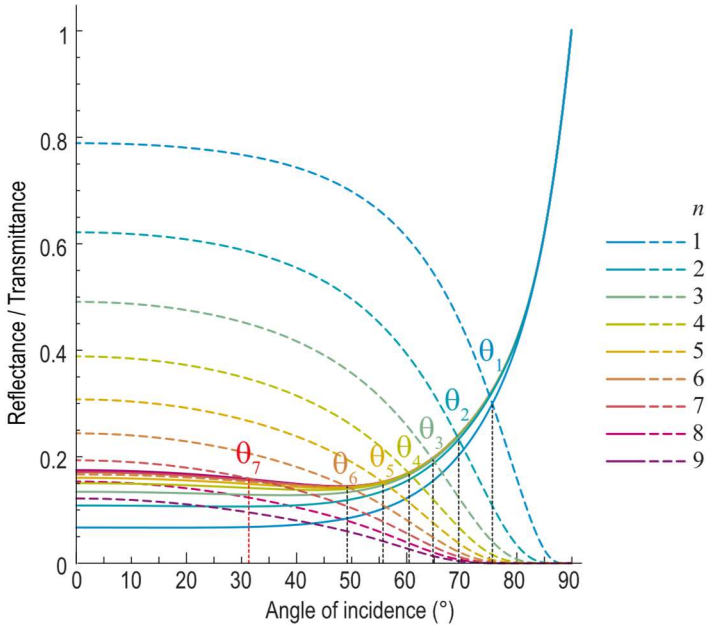

Figure 3. Reflectance (solid lines) and transmittance (dashed lines) of piles of identical plates being absorbing (optical index 1.5). The number of plates is indicated in the insert, and the angle θ_n, at which the reflectance and the transmittance of a pile of *n* plates are equal, are also shown.

The assumption of a perfectly clear glass which would imply $R_n(\theta_n) = T_n(\theta_n) = 1/2$ is not respected at all. The simulation shows that equality is obtained at significantly lower values. For example, for one glass plate, $R_1(\theta_1) = T_1(\theta_1) \approx 0.35$ with $\theta_1 = 75.5°$. This induces a significant overestimation of the reflectance of a glass plate predicted by Lambert's formula as a function of the angle of incidence, as shown in Figure 4. This overestimation is even greater when compared with the reflectance calculated by correctly accounting for the glass absorptance.

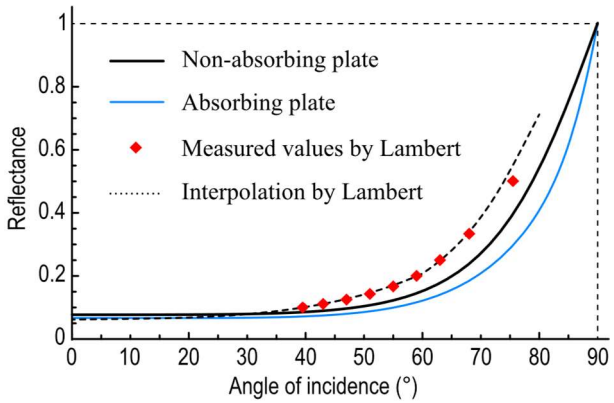

Figure 4. Reflectance for unpolarized light according to Fresnel's equations for a glass plate of refractive index 1.5 assuming perfectly clear glass (solid black line) or accounting for its intrinsic absorptance (solid blue line). Values given by Lambert (red dots for data deduced from measurements and dotted line for interpolated data).

Lambert developed various experimental and computational approaches to confirm his results. He gave tabulated values resulting from different interpolations for angles varying from 40 to 75° in steps of 5°, and from 0 to 80° in steps of 10°, plotted in dashed line in Figure 4.

In 1815, Arago developed the same type of measurement as Lambert [14]. He determined the angles of incidence for which the reflectance and transmittance are equal, for 1, 2 and 4 identical glass plates. The results he obtained show that the glass was much clearer than that used by Lambert. In 1821, Fresnel established his equations giving the reflectance of a surface at any incidence angle by considering *s*- and *p*-polarized lights [13]. To validate his theory, he relied on the experimental results obtained by Arago for 1 glass plate (2 interfaces) and for 2 glass plates (4 interfaces). His theory, taking into account the multiple reflections of light between the interfaces, showed that Arago's experimental determinations gave higher values of about 1% for one plate and 6% for two plates. These slight overestimations may be due in part to the failure to take into account the small absorptance of the glass.

Fresnel thought of using Bouguer's table because it gave him simpler cases (only 1 interface) and for a wider range of angles of incidence. He gave up on Arago's advice that Bouguer's data seemed very inaccurate (see Figure 1.b for the case of a glass surface) [13].

Since it is easier to make a direct measurement in transmission mode, the assumption of non-absorbance of the material is used to deduce the reflectance. This is what Rood proposed in 1870 for a glass plate at normal incidence [17]. Rayleigh criticized this approach that did not allow to expect good accuracy. This encouraged him to develop his sophisticated device of direct reflectance measurement [10].

4. When Arago forgot the partial polarization of light by transmission

Photometry and polarimetry were among the main fields of research invested by François Arago. He was familiar with the work of Bouguer and Lambert. He participated in the pioneering work on the polarization of light since the discovery of polarization by reflection by Malus in 1809 [12].

He designed a device that he called "photometer", based on the same principle as Lambert's but improved [18] (see Figure S4). It was in practice a measurement of the angle for which the images of an object in reflection and transmission by and through a glass plate matched. Like Lambert, he made the same assumption of a perfectly clear glass and could determine the incidence angle such that $R(\theta) = T(\theta) = 1/2$. To measure the reflectances at other angles, Arago placed a polarizing component in the eyepiece of the photometer in order to divide by two the intensity of either the transmitted light or the reflected light. Thus, the matching of the perceived lights allowed to obtain the incidence angles for which $R(\theta) = 2T(\theta)$ or $T(\theta) = 2R(\theta)$. By superimposing two polarizing components, he obtained the angles for which $R(\theta) = 4T(\theta)$ and $T(\theta) = 4R(\theta)$. He deduced the reflectance values using the conservation relation $R(\theta) + T(\theta) = 1$. Let us note an error for the most grazing angle corresponding to the case $R(\theta) = 4T(\theta)$ which induces to a reflectance of 4/5 but that Arago indicated for a value of 3/4. Arago's device also allowed him to compare the reflected intensity by the glass plate with the direct intensity, not transmitted through the plate, divided by 4 by the superposition of the two polarizing components. In other words, in this configuration $R(\theta) = 0.25$. Logically, Arago should have performed the symmetrical operation allowing him to obtain the angle for which $T(\theta) = 0.25$, i.e., $R(\theta) = 0.75$. He did not make this measurement probably because he thought he already had this value due to the mistake mentioned above.

Table 3. Angular reflectance values by Arago.

Angle of Incidence (°)	Values from Arago	Configuration
85.5	0.8000*	$R(\theta) = 4T(\theta)$
83.0	0.6667	$R(\theta) = 2T(\theta)$
78.9	0.5000	$R(\theta) = T(\theta)$
72.7	0.3333	$R(\theta) = T(\theta)/2$
68.4	0.2500	$R(\theta) = 1/4$
63.4	0.2000	$R(\theta) = T(\theta)/4$
53.4	0.1595	Indirect measurements from $T(\theta) = 1 - R(\theta)$
43.6	0.1423	
33.7	0.1280	
19.6	0.1028	
0.6	0.0810	

*Arago indicated a wrong value of 0.7500

These measurements are represented by the red points in the graphic of Figure 5. The agreement with the curve deduced from the Fresnel formulas for a perfectly clear glass plate is better than with Lambert's measurements. This can be explained by Arago's more elaborate and precise experimental setup and by the use of a clearer glass that the one used by Lambert. The method set up by Arago allowed him to reach the angular domain [63°,86°], shifted towards the grazing angles compared to that of the Lambert method [39°,76°]. To reach angles closer to the normal, Arago adapted his device and described the protocol in a new chapter [19]. He attenuated the

transmitted light beam by placing across its path an identical glass plate tilted by an angle θ. His device then allowed him to equalize the two images under an angle θ'. The operation could be formalized as follows:

$$T(\theta)T(\theta') = R(\theta') \tag{13}$$

The trick of this method was that θ' was part of the angular range previously explored by Arago. $R(\theta')$ and $T(\theta') = 1 - R(\theta')$ were therefore known. $R(\theta) = 1 - T(\theta)$ could then be deduced. These results are given in Table 3 and represented with the blue dots in Figure 5. The deviation from the theoretical curve is much larger than for the red dots. This is because the measurement is less direct. Deducting a low reflectance value (between 8 and 16%) from a high transmittance induces an important uncertainty. Moreover, the assumption of a perfectly transparent glass leads to an overestimation of the reflectance which can be non-negligible even if the absorption is low.

Arago made another mistake by not taking into account the partial polarization of light by transmission. This error is astonishing on the part of Arago because he studied a lot the polarization property of light and even designed a polarimeter allowing to measure the degree of polarization typically after transmission through a glass plate at oblique incidence. Arago succeeded in combining photometry and polarimetry, but forgot about polarization here, perhaps because photometric measurement was the most important objective. The correct equation would be:

$$\frac{1}{2}\left[T_s(\theta)T_s(\theta') + T_p(\theta)T_p(\theta')\right] = R(\theta') \tag{14}$$

where the directions of polarization *s* and *p* are specified in subscript while the absence of index concerns a quantity for non-polarized light. This would require determining the transmittances for the *s*- and *p*-polarizations for both θ and θ'. This is not possible with the Arago photometer alone. A solution combining the use of the Arago polarimeter might have been possible but would probably have be very tedious. For an angle θ close to the normal, the transmitted light remains almost unpolarized, and the two previous equations are practically equivalent. This could explain why the error is smaller for θ near 0°.

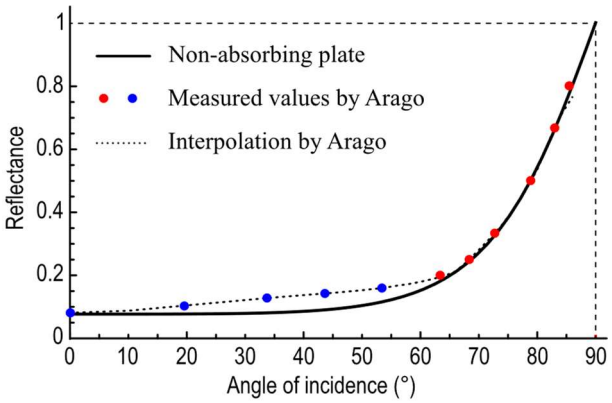

Figure 5. Reflectance for unpolarized light according to Fresnel's equations for a glass plate of refractive index 1.5 assuming perfectly clear glass (solid black line). Values given by Arago (red and blue dots for data deduced from measurements and dotted line for interpolated data).

Arago asked his assistants to interpolate the experimental reflectance data, which they did scrupulously in steps of 4° for an incidence of 0° to 54° and then in steps of 2° to 86° (dashed line in Figure 5). The erroneous tabulated values at very grazing angles were not corrected during this interpolation work. Moreover, Arago did not venture to extrapolate the reflectance at 90°. The work by Fresnel, that he knew well, would have shown him that light is totally reflected at 90°. Almost a century after Bouguer, this result was still not obvious.

In presenting his experimental results, Arago assured that the comparison with a theoretical approach would be the subject of a future chapter, that has never been written. The theoretical calculation giving the reflectance of a glass plate taking correctly into account the polarization of the light, the multiple reflections between the two interfaces, and a possible absorption of the glass is formalized by Stokes about 10 years later [20].

5. From yesterday's errors to today's approximations

From the beginning of the 20th century, the use of photoelectric detectors allowed more direct and accurate measurements of reflectance. The indirect approaches of the 18th and 19th centuries were no longer of interest and were soon forgotten. In the domain of metrology, the focus has then shifted to the optical characterization of a broad range of surfaces, from strong diffusers (e.g., perfect white diffuser [21]) to very specular reflector like highly reflective multilayer mirrors [22]. Moreover, the complete and accurate measurement of light scattering from non-strictly specular surfaces is a current objective for national metrology institutes [23].

Of course, today's researchers in optics no longer make the mistakes of their predecessors of the past centuries when the systems they are studying remain reasonably complex. However, describing the optical properties of complex objects often requires some approximations. For example, for lighting simulation, most computer programs like DIALux or RELUX assign a single reflectance or transmittance value to a given surface, regardless of the wavelength, angle of incidence or polarization state of the incident light. Thus, all surfaces are assumed to be Lambertian and the angular variations of reflections and transmissions are not taken into account.

For animated movies, where more photorealistic rendering coupled with real-time processing is required, approximated surface reflection models are often used. These models are described as "physically plausible" when, in particular, the conservation of energy is respected [24]. This means verifying that the albedo of a surface is less than unity for all incidences, even if this condition is met empirically. When numerical simulations take too long time, it is still common to use an approach similar to Bouguer's approach, i.e., to fit the measured or simulated data with a parametric function. Specifically, Christophe Schlick, a researcher in computer graphics, proposed an approximation of the Fresnel angular reflectance of air-matter interfaces with unpolarized light in order to reduce the computation time for image synthesis [25]. Interestingly, the proposed approximation, widely used in the 1990s, displays some similarity with that of Bouguer (see Equation (1)):

$$R(\theta) \approx R(0°) + \left[1 - R(0°)\right](1 - \cos\theta)^5 \tag{15}$$

The reflectance value at normal incidence, $R(0°)$, is given by the Fresnel equations if the refractive index of the material is known, while $R(90°)$ is 1 as predicted by the Fresnel equations.

In some application domains, it happens that approximations valid in a certain type of material are used for other types of materials which may look similar at first sight, without checking their validity effectively. Often used to simplify the calculations, these assumptions can induce a non-negligible bias in the predictions. Likewise, as the most used natural or artificial lights are not polarized, the partial polarization of light after a reflection, a transmission, or a scattering process is often ignored, without checking the error thus made. As soon as multiple reflections occur, the error may become significant. Figure 2 illustrates this with a pile of glass plates (e.g., a double or triple glazing), a situation where taking into account the polarization is rather easy because the reflecting interfaces are all parallel. As soon as the reflectors are not parallel, for example in a V-cavity or at the corner of a veranda, rigorous models taking into account the polarization of light along the multiple reflection process is more complex because the successive incidence planes, thereby the determination of the *s* and *p* components, differ at each reflection. This is probably the reason why most models in computer graphics assume that the light remains unpolarized along the multiple reflection process. But the error thus made, quantified in [26], can be big at some incidence angles. Errors can be even more dramatic when the incident light is polarized. Figure 6 shows an example rendered with spectral simulation done by United Visual Researchers [27,28], involving ubiquitous flat panel displays known to emit polarized light. Only by taking into account the polarization (Fig. 6.b), it is possible to render the iridescence effects in the reflected image of the screen, as seen on a picture of the real scene (Fig. 6.a).

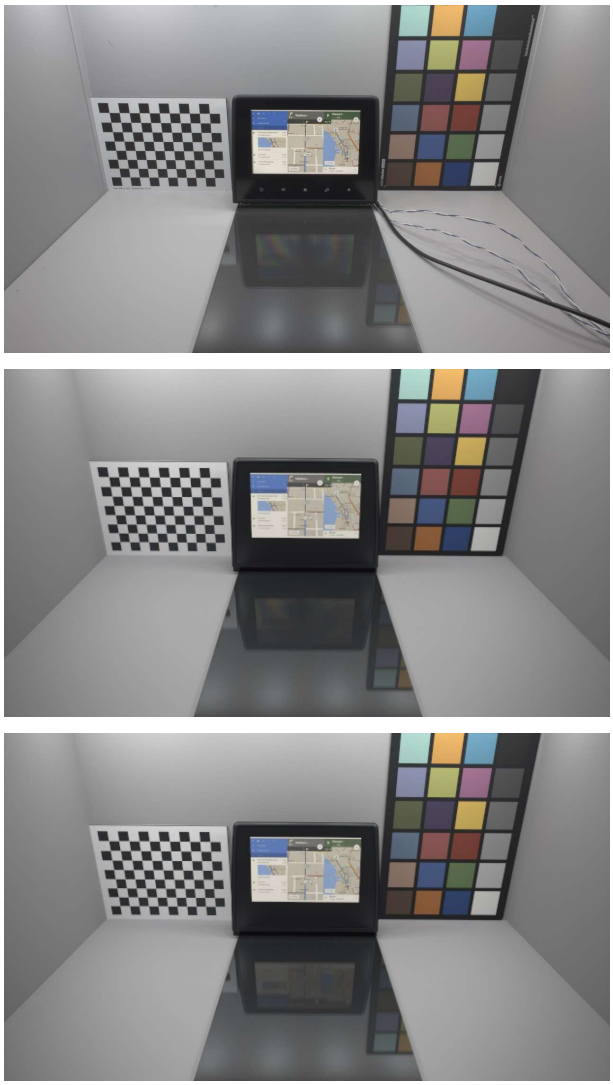

Figure 6. (a) Reference photography (Canon 5D), (b) spectral simulation considering the polarization, (c) spectral simulation without taking polarization into account. Courtesy of United Visual Researchers©.

In many studies, these simplifying assumptions are more or less explicitly expressed, and the uncertainties that they generate not always easy to estimate. Moreover, the study of error propagation is important to understand how a small uncertainty on experimental data can generate a much larger uncertainty on a quantity determined by a predictive model calibrated from these data [29].

6. Conclusion

It has been forgotten, so much so that it would seem unthinkable today to do photometry without an opto-electronic sensor, but the measurement of light quantities and the determination of the optical properties of materials was done for a long time with the naked eye. The method of photometric measurement established by Bouguer was based on the principle of brightness matching. Measuring the angular variation of reflectances was not easy. Looking back, the indirect methods set up by Bouguer, Lambert and Arago appear to be clever. Although difficult to estimate, one can imagine that the experimental uncertainties of these precursor methods were quite large. But the errors made by these scientists, that we have highlighted in this paper, are much more important. It would be absurd to judge these works in an anachronistic way. However, we can try to "understand" these errors. How could Bouguer have imagined that a light beam falling onto a surface at a perfectly grazing angle would be totally reflected whereas this effect was inaccessible to measurement? By choosing the clearest glass plates available, how could Lambert have thought that not taking into account the weak absorptance of the glass could have led to so significant errors? If Arago's method for determining the reflectance of a glass plate at near normal incidence

is incorrect, it has at least the merit of being clever. It is difficult to imagine obtaining much better measurements for these incidences by keeping the principle of his photometer.

These errors also echo our current practices in radiometry. They are no longer errors, of course, but simplifying assumptions, not always explicitly formulated. Despite the progress in numerical computing power, these simplifications are still necessary to model the light scattered by complex macroscopic structures. The impact of these assumptions on the simulation uncertainty remains a subject to be developed.

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Data availability. Data underlying the results presented in this paper are available in Refs. [4,15,18].

Supplemental document. See Supplement 1 for supporting content.

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