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When mathematics in three acts meets mathematical modelling

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In this article, we investigate how the introduction of the pedagogical method, “mathematics in three acts” to preservice teachers influenced their mathematical modelling, while on practicum. We analyzed documentation, of groups of preservice teachers, on their experience of teaching modelling lessons. One finding was that the groups were able to use “mathematics in three acts” to produce modelling problems that fulfilled certain criteria in our selected framework, but they had difficulty finding modelling problems for the lower grades (grades 1-3). We also found that the preservice teachers appeared to emphasize the subprocess of working mathematically and devalue the subprocesses simplifying/structuring, mathematising, and validating, when guiding the pupils through a modelling process in a “mathematics in three acts” lesson.

Keywords: Mathematical modelling, “mathematics in three acts”, modelling problem, modelling process.

Introduction and literature review

Internationally, mathematical modelling was traditionally reserved for secondary schools. Only in the last two decades did researchers begin to see value in modelling for primary education (e.g., English & Watters, 2005). In Norway, modelling was introduced to primary school (grades 1-7) in 2020, when a new and revised version of the national curriculum included modelling and applications as one of several core elements. At our university, mathematical modelling was not part of the teacher education for preservice teachers for grades 1-7 until 2018. In 2018, a research project, LATAcME¹, began, which focused on mathematics teacher education for grades 1-7, including a focus on modelling. LATAcME uses an educational design research approach, that uses an analysis of experiences in design cycles, to refine or change teacher education practices in forthcoming cycles with the overall aim of improving educational practices and developing theory about those practices. According to Borromeo Ferri (2018), educational modelling courses should keep a balance between theory and practice. Consequently, at the beginning of the project, the LATAcME research-practitioner team recommended teacher educators to introduce theories about mathematical modelling in their work with preservice teachers (PSTs), and to ask the PSTs to prepare and implement a modelling lesson when on practicum.

Research by Paolucci and Wessels (2017) had shown that while PSTs were relatively proficient in identifying and presenting relevant real-world problem contexts to young children, they had difficulty with formulating problems satisfying certain design principles which characterized “good” modelling problems. Reflecting on data from the first-year design cycle (2018-19) researchers in LATAcME

¹ Learning about teaching argumentation and critical mathematics education in multilingual classrooms (LATAcME, <https://prosjekt.hvl.no/latacme/>) funded by the Research Council of Norway. The project period is 2018–2022.

observed that the modelling problems suggested by their PSTs had been based on real-world situations that were familiar to the pupils, but the modelling processes were seldom fully developed. More detailed investigations showed that the PSTs had difficulties with balancing pupils' independence when guiding the different work subprocesses (Hansen, 2021). In the next design cycle (2019-20), the research-practitioner team recommended the teacher educators introduced the PSTs to the didactical method “mathematics in three acts” (MITA) to investigate whether this method could assist PSTs in introducing modelling problems which could support pupils with an appropriate amount of guidance to work through a complete modelling process. MITA was designed by Meyer (2011) to encourage learners to pose and work on mathematical problems and it was further developed in Lomax et al (2017). The basic features of the method are illustrated in Figure 1.

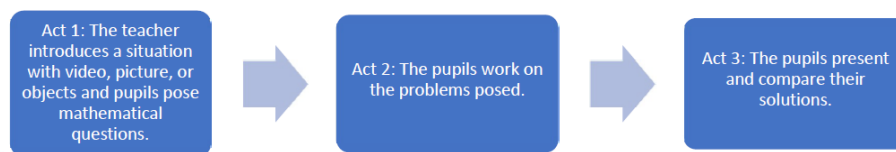


Figure 1: Mathematics in Three Acts (inspired by Meyer, 2011; Lomax et al., 2007)

However, it was by no means certain that MITA would achieve the desired effect. Dogan (2020) had found that a group of PSTs were successful in creating problems that were based on real-world contexts, but only a part of the problems could be classified as being model eliciting. To study how effective MITA was in assisting the PSTs in introducing problems that could be classified as “good” modelling problems in Act 1, we decided to evaluate the problems the PSTs adopted in their modelling lessons via MITA in terms of criteria to Dogan (2020), who proposed four criteria: reality, openness, complexity, and model eliciting. This would form the second cycle of the education design research approach for this aspect of LATAcME.

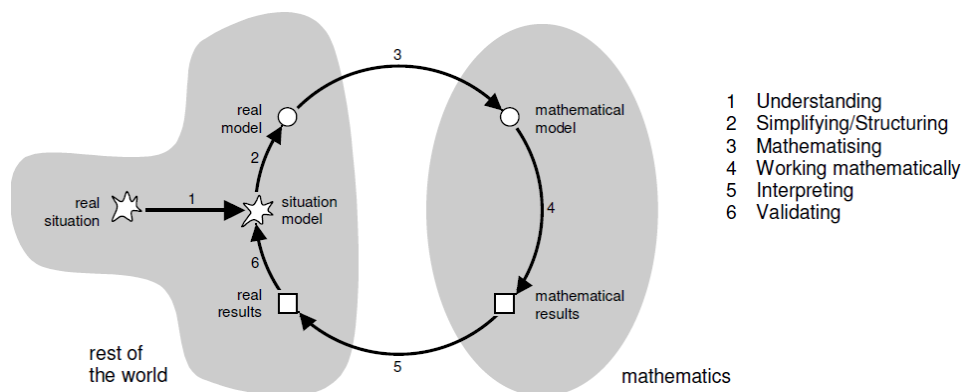


Figure 2: Modelling Cycle (Blum & Leiß, 2006)

After a modelling problem is chosen, solving it involves several subprocesses, which are not usually carried out linearly. The modelling process has been illustrated in various ways. As shown in Figure 2, Blum and Leiß (2006) emphasizes the cyclical nature of a mathematical modelling process and describes the common subprocesses, that in return require different competencies involved in solving a modelling problem (Maaß, 2007).

While MITA suggests a linear working process (see Figure 1), the modelling cycle (see Figure 2) indicates the constant movement between the sub-processes. This suggests that some of the subprocesses, such as validating, in typical modelling cyclical processes, may be easier to be overlooked when engaged in MITA. The second aim of our study was therefore to investigate to what extent subprocesses of the modelling cycle (Blum & Leiß, 2006) were present in lessons based on MITA.

Conceptual framework and research questions

Borromeo Ferri stated that in modelling lessons, “the selection and the quality of tasks for lessons are essential for mathematical understanding, for promoting students’ mathematical practices and competencies” (2018, p.41). Synthesizing earlier research, Dogan (2020) proposed four criteria to evaluate modelling problems posed by PSTs: reality, openness, complexity, and model eliciting. The reality criterion requires that the modelling problem comes from a real-world situation and aligns with the reality of the pupils (Lesh & Doerr, 2003; Maaß, 2007; Dogan, 2020). A modelling problem satisfies the reality criterion if it allows pupils to interpret the problem based on their experience and their mathematical knowledge. The openness criterion requires a modelling problem to be interpretable in multiple ways, open-ended and to allow for different solution paths (Maaß, 2007; Dogan, 2020). A complex modelling problem requires the pupils to understand the context and search for relevant data, and to be cognitively demanding to solve (Dogan, 2020; Borromeo Ferri, 2018). The model eliciting property ensures that a modelling problem should promote the modelling process, and requires the students to use mathematics to construct, describe or explain situations (Dogan, 2020; Lesh & Doerr, 2003).

Based on our twin aims for this paper, our research questions are about the MITA lessons described by groups of PSTs when reflecting on their practicum:

RQ1: Which of the four criteria (reality, openness, complexity and model eliciting), were most commonly found to be fulfilled by the modelling problems in the lessons?

RQ2: Which of the subprocesses of the modelling cycle were present in the PSTs’ descriptions of the pupils’ work processes?

Research Method

The investigation was conducted by using document analysis (Bowen, 2009). The units of analysis were 16 written assignments from groups of PSTs for 1-7 grades. The assignments were classified according to the four criteria in Dogan (2020) and the subprocesses of the modelling cycle by Blum and Leiß (2006) depicted in Figure 2.

Research Context and Participants

These PSTs were in the first semester of their second year of their teacher education. General theories on mathematical modelling and MITA were introduced to them through 3 three-hour lectures combined with literature reading. The PST-groups were then asked to plan and carry out a modelling lesson while on practicum. After practicum, the PST-groups were asked to describe and reflect on their modelling lessons. A total of 16 PST-groups with 3-5 PSTs in each group, gave permission for us to analyze their assignments. Of these groups, 15 had used the MITA structure for their modelling

lessons. One of these groups reported to have performed two different modelling lessons. Therefore, we analyzed 15 assignments which contained 16 modelling lessons.

Data analysis

To evaluate modelling problems used by the PST-groups in their modelling lessons via MITA (RQ1), we used the four criteria reality, openness, complexity, and model eliciting (Dogan, 2020; Lesh & Doerr, 2003). After having identified the modelling problems the PST-groups described in their assignments, we categorized them according to the four criteria according to the questions in Table 1 which was based on the work of Dogan (2020). In the next section, we provide an example from one of the PSTs' assignments and a detailed description of how the analysis was carried out.

Table 1: Criteria for evaluating modelling problems via MITA

Criteria	Guiding questions for each criterion
Reality	<ol style="list-style-type: none"> 1. Whether the problem sprang from real life of the pupils. 2. Whether the problem was suitable for the pupils' academic level.
Openness	Whether the problem was open for different interpretations or solving methods.
Complexity	<ol style="list-style-type: none"> 1. Whether the problem was cognitively demanding for the pupils to interpret the problem. 2. Whether the problem could make the pupils to see the need of mathematics.
Model eliciting	Whether the problem required the pupils to generate a model.

To identify the modelling subprocesses the PSTs described the pupils going through, we analyzed the documents using the characteristics of the subprocesses described by Blum and Leiß (2006) (see Figure 2).

Results

The assignments contained MITA lessons. Some included dialogues from Act 1, while others described the dialogues implicitly. We chose here to present an extract from an assignment of one of the PST-groups, where the dialogues from Act 1 were described explicitly. This was typical of the assignment data and we use it to explain how our data analysis in more detail.

Extract from an assignment

This PST group described that they had implemented a modelling session over three lessons of 45 minutes each, in a grade 4 class with 22 pupils, using the MITA structure.

The PSTs described that in Act 1 they had presented a video about global warming. The PSTs and the pupils talked about this video and tried to understand some graphs about climate change. The PSTs described that the pupil had raised many concerns, among which the PSTs had identified two interesting questions, *“Is it possible to find out how much warmer it will be when I grow up?”* and *“How many years will it take before the sea rises over the dock (Bryggen) in Bergen?”* The PSTs reported that the last question had been chosen for this modelling session.

The document then reported that in Act 2 the PST-group had started the lesson with a discussion about what would happen if the sea level continued to rise. They had afterwards asked the pupils to think about what they needed to know to answer this question. The PSTs together with the pupils had concluded that one must know the present height and how much the sea level rises per year. The PSTs then wrote *“we found that the dock height above the sea level is 720 mm”*, and *“... that the sea level rises 3.4 mm per year. Since the pupils have not learned decimal numbers, we decided to round down to 3 mm per year”*. Afterwards, they had provided the pupils with a table with two columns, where one column was a list of the years from 2016 to 2022 plus the year 2032, and the other column were the sea levels with the first three years' sea levels 43 mm, 46 mm and 49 mm. In groups, the pupils had been supposed to fill in the table, with the PSTs being available for the pupils' questions. The PSTs also mentioned that some pupils were critical about the table, by for example saying that *“It is not certain the water rises all the time”*.

In Act 3, the PST group had divided the pupils into groups of four. Having their table and calculations at hand, the pupils had been asked to answer the question “how many years will it take before the sea rises over the dock (Bryggen) in Bergen?”. After about 30 minutes of group work, the PSTs had a summary, with one of the PSTs showing a solution method with centicubes.

Analysis of the example

The problem “How many years will it take before the sea rises over the dock (Bryggen) in Bergen?” was chosen after Act 1. It is likely that Bryggen would be familiar to most pupils in the class and as such most pupils would want to find out what could happen in the future, using this as a benchmark to understand the implications of climate change. From the PSTs' description, the mathematics involved seemed to be suitable for grade 4 pupils. The problem, therefore, fulfilled the realistic criterion. The problem also fulfilled the openness criterion as it allowed several different interpretations (e.g., what does “over the dock” mean?) and different solving strategies. It also required solvers to orient themselves, simplify, find needed information, and to use mathematics to find the solution(s). Consequently, we interpreted it as being complex. As the problem also seemed to invite pupils to generate a model, the problem was also classified as model eliciting. Thus, we considered the problem to be realistic, open, complex and model eliciting.

In regard to the second research question, we identified the modelling sub-processes that were evident in what the PSTs described the pupils as doing in Act 2 and Act 3 of the modelling sequence. For the first subprocess, “understanding”, the PSTs indicated that the pupils had the opportunity to interpret the modelling problem in classroom discussions about the consequences of sea-level rise. They also had to identify and understand the information they needed to answer the problem. However, from what they described, it seemed that the PSTs took over the next subprocess “simplifying/structuring” themselves, since they provided the height of the dock and rounded down the height that the average sea level rises per year. In addition, they presented a table to represent the situation, an activity that we interpreted as part of the “mathematising” sub-process, so that the pupils just had to complete the table. By completing the table, the pupils concentrated on the subprocess “working mathematically” and implicitly the subprocess “interpreting” in the context of the real model. The subprocess of validating was not promoted by the PSTs.

General results

There were 11 problems in total (see Table 2) that satisfied all the quality criteria. Of those which did not fulfill all the criteria, most of the problems were for pupils in the first few grades of school. One modelling problem for grade 1 failed the reality criterion, and 3 modelling problems for grade 2 and 1 modelling problem for grade 3 failed the openness, complexity or/and modelling eliciting criteria.

Table 2: Number of modelling problems that fulfil each criterion

	Reality	Openness	Complexity	Model-eliciting
Number of modelling problems out of 16	15	13	12	12

Table 3: Analysis of the modelling subprocesses that were present

	Understanding	Simplifying	Mathematising	Working mathematically	Interpreting	Validating
No.	11	1	6	11	11	5

We found that where the PSTs' modelling problems did not fulfil all four criteria in our framework, there were also lack of the modelling subprocesses in the descriptions. For four of the modelling problems this was because that the complexity of the problems was too low for the grades (complexity criterion was not fulfilled). Therefore, we restricted identification of modelling subprocesses to the 11 modelling lessons in which the modelling problems satisfied all four criteria for good problems. Table 3 presents the analysis of the modelling process for these 11 modelling lessons. It shows that the sub-processes simplifying, mathematising, and validating were often absent in these lessons. In particular, the subprocess simplifying only appeared in one of the modelling lessons. In the one modelling lesson that involved all subprocesses failed to repeat subprocesses for one or more cycles, suggesting a linear rather than a cyclical approach to modelling.

Discussion

When it comes to the first research question, our result showed that 11 out of 16 lessons adopted modelling problems that satisfied all four criteria, and 15 of the problems satisfied the reality criterion (Table 2). We can compare this with the results by Dogan (2020), who found that PSTs had difficulties constructing modelling questions that fulfilled all four criteria (5 out 17 in his case fulfilled all criteria, and 12 of 17 fulfilled the reality criterion). In our analysis, we interpreted that through MITA the PSTs were able to encourage the pupils to come up with accessible modelling problems from familiar contexts. In some of the lessons, the pupils asked a wide range of questions through Act 1, which gave the PSTs some freedom to choose open, model eliciting problems with adequate complexity. This can be illustrated by an example from a PST group reported having presented the pupils with a short video of Usain Bolt running a competition. Some questions that the pupils asked were: how fast did Usain Bolt run in that competition? How many centimeters did he run? How much does he earn as a runner? How old will Usain Bolt become?

The fact that the 5 cases that failed one or more criteria in Table 1 were for lower grades (1-3 grades) is interesting. Paolucci and Wessels (2017) raised also the concern about the PSTs' capacity to create modelling problems for lower grades, especially for grade 1. One of the PST groups stated, "since there is not much theory [mathematical knowledge] available for 2nd graders, we choose to look at the mathematical curricular goals". This indicates that the PSTs thought the difficulty lay in the fact that pupils in lower grades did not have sufficient knowledge in mathematics, so that there are a limited number of mathematical topics to work with. This suggests that PSTs need more guidance from teacher educators in designing modelling problems for lower grades.

In response to the second research question, we found that the main modelling activities took place in Act 2 and Act 3. In these acts, the PSTs guided the pupils through the modelling process to solve the problems posed in Act 1. Our analysis showed that the subprocess simplifying/constructing was missing for 10 of the 11 modelling processes. In most cases the PSTs took over the modelling problems and simplified and structured them for the pupils. Only in 6 of the 11 modelling processes did the pupils need to transfer the real model to a mathematical model. Ng (2018) and Hansen (2021) pointed out similar tendencies among experienced teachers and PSTs respectively that were new beginners in teaching modelling, that is, that the teachers and PSTs tended to provide scaffolding because they perceived mathematical modelling to be challenging for their pupils.

The PSTs also did not often include the validating subprocess in the modelling cycle. Only 5 out of 11 PST groups used this sub-process. Our result showed that in the descriptions in the assignments the cyclic nature of the modelling process seemed to be absent in the lessons, whereas the subprocess "working mathematically" was always part of the process. We suspect MITA could have affected both the cyclicity of the modelling process and the focus on working mathematically. Act 3 in MITA was designed to let the pupils to compare and reflect over solution methods, and it did not suggest that pupils should go back to re-solve the problem or refine the solution to the problem. Therefore, the cyclicity was not part of the process suggested by MITA.

Conclusion

There are two main conclusions from this study. The first is that we found through MITA the PSTs were able to arrive at modelling problems that included the pupils' perspective, but that they had difficulty to find modelling problems for the lower grades (grades 1-3). The second conclusion is that even if the PSTs arrived at appropriate modelling problems together with the pupils, the corresponding modelling process through MITA did not necessarily contain all the subprocesses of the modelling cycle. In particular, the subprocesses simplifying/structuring, mathematising, and validating were often missing. This is interesting, and it asks for more attention towards how to instruct PSTs to include these subprocesses in their modelling practice in teacher education.

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