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How knowledge about ideal-typical modelling processes affects phase transitions in individual modelling routes

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Working on mathematical modelling tasks is usually challenging for students: several studies have shown, that students do not knowingly and consciously apply solution strategies when working on modelling tasks. In an empirical study, we investigate whether and to what extent knowledge about ideal-typical modelling processes has an influence on phase transitions in individual modelling routes. The individual acquires this knowledge in the form of an instruction that contains information about the modelling process, e.g. in form of a modelling cycle and a solution plan. In this article, the phase transitions of students who received an instruction about modelling processes are compared to those of students without such an instruction. The data for the study were collected, presented, and analysed using the Modelling-Activity-Interaction-Tool (MAI-Tool), which is based on quantitative methods to capture and analyse structures of modelling processes.

Keywords: Individual modelling routes, knowledge about ideal-typical modelling processes, phase transitions, MAI-Tool.

Introduction

Studies have already shown that knowledge about mathematical modelling has a positive influence on the modelling process (Stillman & Galbraith, 1998). This knowledge is often provided to students in the form of a solution plan (Beckschulte, 2019). Schneider et al. (2021) found out that individuals who received knowledge about the modelling process engaged intensely with mathematics and rarely deviated from the solution process.

In an empirical study, we compare individuals who have received knowledge about ideal-typical modelling processes (referred to as *the instructed*) with those who have not (*the non-instructed*). Knowledge about ideal-typical modelling processes is provided to students as an instruction in the form of the five-step modelling cycle of Kaiser and Stender (2013) (cf. Figure 1) and the corresponding solution plan of Beckschulte (2019).

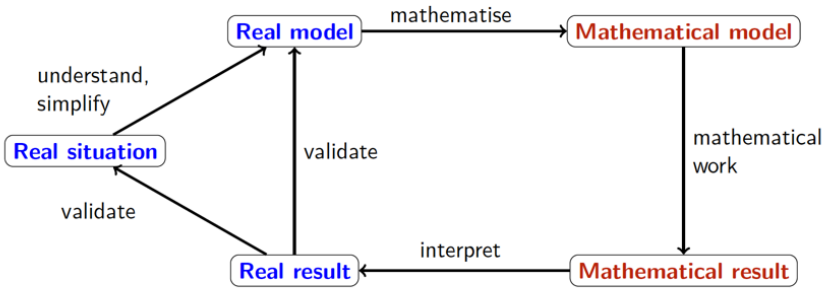

Figure 1: The modelling cycle of Kaiser and Stender (2013)

In addition, the solution process of a modelling problem is discussed using the problem of colouring the map of Germany (Leuders, 2007). The instruction is based on the fundamentals of Vorhölter and Kaiser (2016) and took about 20 minutes.

The aim of our research is to analyse how knowledge about ideal-typical modelling processes affects the structure of individual solution processes. Not only the phases, but also the phase transitions provide information about the influence of an instruction on the modelling process: especially with regard to the phases that are outside the modelling cycle (*summary solution process* and *miscellaneous*), the phase transitions can be used to show which phases occur before or after these phases. This particular view of individual modelling routes requires a tool that can represent structures of modelling processes and algorithmically evaluate them as numerical data: the MAI-Tool. In this article, we examine individual modelling routes for differences in the relative frequency of phase transitions, i.e., we investigate whether the relative number of the two cohorts differ significantly from each other.

Theoretical framework

Analysis of individual modelling routes

Methods for the representation and analysis of individual modelling processes already exist and have been applied in empirical studies. The modelling cycle is not passed linearly and there is a switching back and forth between the phases. This is what Borromeo Ferri (2007) calls an individual modelling route: Successive phases are connected by numerical arrows in the modelling cycle. Thus, the modelling cycle is not only suitable for representation, but also serves as an analysis tool for individual modelling processes.

The Modelling-Activity-Diagram (MAD) is another concept for representing and analyzing modelling processes (Ärlebäck and Albarracin, 2019): in a linear representation, activities of an individual are shown over time. The activities describe what is done in the phases of the modelling cycle.

The MAI-Tool is a newly developed tool for capturing, representing, and evaluating modelling processes (Ruzika & Schneider, 2019; Schneider et al., 2021): the tool is based on observable interactions within the group as they work on a modelling task. Following qualitative methodology, interaction units are entered into the MAI-Tool with the following information: interacting person(s) and content of the interaction. The content is assigned to a phase of the modelling cycle. A timestamp is automatically assigned to each interaction unit so that the duration is also stored. Individual modelling routes are automatically extracted from the group process. The evaluation is quantitative, as modelling processes are described with numerical data. Since the data are available digitally, they are evaluated by algorithms, so that an objective evaluation is given. This results in the advantage that the numerical data can be applied in a statistical test. The individual modelling routes are displayed graphically in the tool – depending on the focus of the evaluation.

Knowledge of mathematical modelling

Solving mathematical modelling problems is challenging for students (Blum, 2015): empirical studies have found that students rarely consciously follow a strategy and do not know how to proceed when

they encounter difficulties (Kaiser et al., 2015). Therefore, it is important to provide students with strategies that can assist them in the solving process. In the form of a solution plan that includes the modelling cycle, the phases as well as their transitions are described in detail using activities. A solution plan belongs to the general strategy aids, as no specific help is given for the task (Borromeo Ferri, 2006). By describing the entire solution process in the solution plan, knowledge about the ideal-typical course of a modelling process can be acquired: "[the] solution plan is not meant as a schema that has to be used by the students' but as an aid for difficulties that may occur in the course of the solution process" (Blum & Borromeo Ferri, 2009, p. 55).

Schneider et al. (2021) showed in an empirical study that knowledge about ideal-typical modelling processes affects the structure of individual modelling routes: individual modelling routes of the instructed were compared with those of the non-instructed for differences in the relative number and relative duration of phases in the modelling process. In particular, the phases of the *mathematical world* (*mathematical model* and *mathematical solution*) occur more frequently and for longer for the instructed compared to the non-instructed students. In addition, two phenomena were discovered that have not been considered before: non-instructed students summarise their solution process more often and longer. Moreover, they digress more often from solving the modelling task. On the basis of these analysis, they concluded that the non-instructed students lack knowledge about ideal-typical modelling processes: since they do not know how to solve a modelling task in a structured way, they are not interested in an improved solution and are satisfied with their first solution found.

Classification of phase transitions in the modelling process

To investigate differences in individual modelling routes between the instructed and the non-instructed students with respect to phase transitions, we assign the possible phase transitions to different categories.

In a first step, we divide the phases into two phase types. Phases within the ideal-typical modelling cycle (INPHASE) are *real problem*, *real model*, *mathematical model*, *mathematical solution*, *real solution*, and *validating*. Phases outside the ideal-typical modelling cycle (OUTPHASE) are not part of the modelling cycle: *summary solution process* and *miscellaneous*.

Phase transitions are characterized by their start and end phases. We distinguish between typical and atypical transitions. Typical transitions include adjacent phase transitions (APT). An APT corresponds to an ideal-typical transition in the modelling cycle with INPHASES. There is a total of three atypical transitions that do not correspond to an ideal-typical transition in the modelling cycle: The adjacent backwards phase transition (ABT) is a phase transition in which the start and end phases are reversed compared to APT. ABT are not typical phase transitions because they do not follow one another in an ideal-typical manner as in the modelling cycle. The ABT as well as at the jumps (JUMPS) consist of INPHASES: JUMPS are neither APT nor ABT. The third atypical phase transition includes at least one OUTPHASE and is called phase transition outside the modelling cycle (PTOUT). Start and/or end phase of the phase transition is an OUTPHASE. An illustration of the phases can be found in Figure 2.

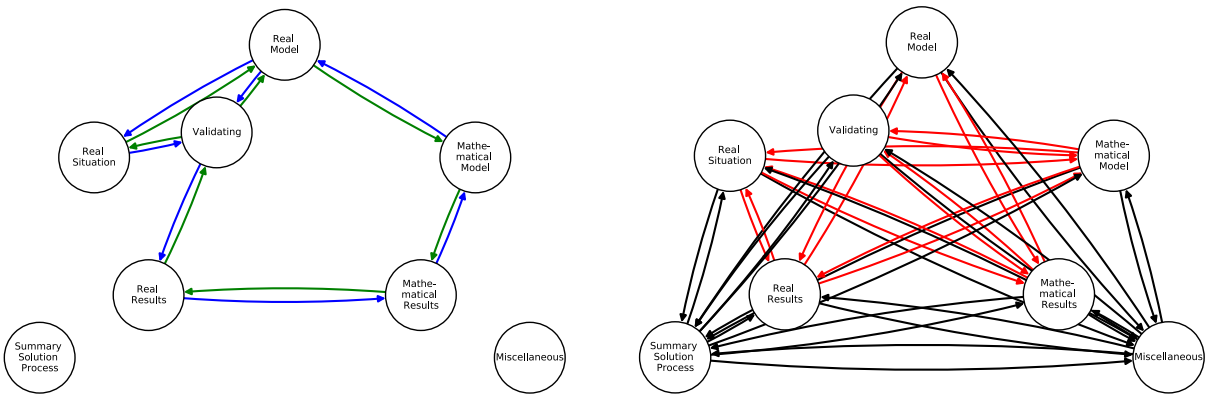

Figure 2: An overview of the phase transitions: APT (green), ABT (blue), JUMPS (red) and PTOUT (black) – illustrated on the modelling cycle

Research question and hypotheses

Based on the theoretical framework as well as on the classification of phase transitions in modelling processes, we formulate the central research question in this article:

To what extent does knowledge about ideal-typical modelling processes affect phase transitions in individual modelling routes, in particular: for which phase transitions does the relative number of individuals with instruction significantly differ from those without instruction?

Following the results of Schneider et al. (2021), we formulate hypotheses that are tested for significant differences in the phase transitions between the instructed and the non-instructed students. The instructed students engage with the mathematical world more frequently and more often during the solution process compared to the non-instructed students. From this, we derive the following hypothesis:

(H1) The instructed students have a significantly higher relative number of the APT *mathematical model* to *mathematical solution* (MM → MS) than the non-instructed students.

In particular, Schneider et al. (2021) highlighted the long duration and frequent occurrence of OUTPHASES for the non-instructed students. We hypothesize that this also affects phase transitions, especially PTOUT. Since the non-instructed students have not engaged as intensively with the *mathematical world*, it can be conjectured that they often switch from the *real model* to OUTPHASES. The following hypotheses emerge:

For the instructed students, the relative number

(H2) of the PTOUT,

(H3) of the PTOUT *summary solution process* to *miscellaneous* (SSP → MISC),

(H4) of the PTOUT *miscellaneous* to *summary solution process* (MISC → SSP),

(H5) of the PTOUT *real model* to *miscellaneous* (RM → MISC) and

(H6) of the PTOUT *real model* to *summary solution process* (RM → SSP)

is significantly lower than for the non-instructed students.

Methodological framework

The study uses a mixed-methods design, with data collection being qualitative and data analysis being quantitative. Tenth grade students from different Gymnasien (German Grammar School) were selected for the study. Only those who had indicated in a questionnaire that they had no previous experience in modelling and had not acquired knowledge about modelling were included in the study. Thus, we excluded the influence of previous experience in modelling. Working in groups of five, they complete the "filling-up" task in 30 minutes (Blum & Leiß, 2006). The sample includes 40 students (20 instructed, 20 non-instructed). The group work was videographed.

Interactions were recorded using the MAI-Tool: Grounded Theory (Strauss & Corbin, 1996) was used to qualitatively code the interaction units. Each interaction unit contained information on who interacted (with whom), which phase of the modelling process could be assigned an when the interaction began and ended. To guarantee reliability in the coding, two people coded the data. The interrater reliability was calculated using Cohen's Kappa (Cohen, 1960) and is $\kappa = [.72, .90]$ for each individual. Phase transitions are evaluated by absolute and relative frequencies in the MAI-Tool. The relative frequencies of both cohorts are analysed for significant differences using the Mann-Whitney U test. In contrast to t-tests, this non-parametric test requires fewer prerequisites. The significance level is set at $\alpha=0.05$. The effect size δ is calculated according to Fritz et al. (2012) and interpreted according to Cohen (1988). The Mann-Whitney U test and the calculation of the effect size was done using the software R. Because multiple hypotheses are tested from one data set, analyses are corrected using the Benjamini-Hochberg method (Benjamini & Hochberg, 1995).

Results of the study

The hypotheses are tested using the Mann-Whitney-U-test. The results are shown in Table 1. All hypotheses are accepted because the *p*-values for the respective variables are always below the significance level of $\alpha=0.05$. Thus, significant differences between the two cohorts can be measured for each variable. The effect size is at least weak for each hypothesis; this underlines that an effect is measured in each case.

Table 1: Results of the Mann-Whitney-U-test for group 0 (non-instructed) and group 1 (instructed) with the mean values (mv), standard deviation (sd), *p*-values and effect size δ (rounded to 3 decimal positions)

variable (corresponding hypothesis)	mv cohort 1	sd cohort 1	mv cohort 0	sd cohort 0	*p*-value	effect size δ
relative number MM → MS (H1)	0.024	0.027	0.006	0.014	0.006	0.351
relative number PTOUT (H2)	0.213	0.166	0.411	0.262	0.006	0.349
relative number	0	0	0.042	0.051	<0.001	0.53

SSP → MISC (H3)						
relative number MISC → SSP (H4)	0.001	0.004	0.025	0.045	0.032	0.259
relative number RM → MISC (H5)	0.027	0.036	0.08	0.089	0.007	0.343
relative number RM → SSP (H6)	0.01	0.012	0.002	0.006	0.007	0.343

Interpretation of the results

An overview of the phase transitions in the solution process of both cohorts can be found in Figure 3: the thicker an edge, the higher the relative number of the respective phase transition. In particular, the significant differences shown by the statistical test are evident in this figure.

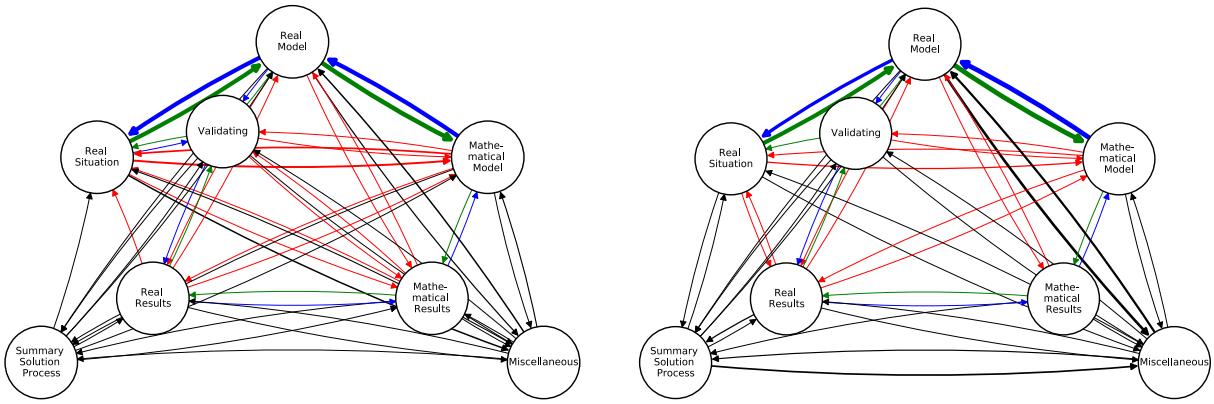

Figure 3: Overview of the phase transitions of the instructed (left) and the non-instructed (right)

The APT *mathematical model* to *mathematical solution* occurs often for the instructed in the solving process compared to the non-instructed students: when creating (and improving) the *mathematical model*, an attempt is made to translate reality into mathematics in the best possible way. From the instruction, students know that a *mathematical model* as well as its solution with mathematical concepts is relevant to solve the modelling task. In contrast, the non-instructed students have problems working mathematically: besides forming the *mathematical model*, there are also difficulties in applying mathematical concepts to solve the problem. Above all, solving the problem is a challenge for them, which is why the switch from *mathematical model* to *mathematical solution* rarely occurs.

Especially in the OUTPHASES the influence of the instruction becomes clear: the non-instructed students often switch between the OUTPHASES, which is reflected in the high relative number of PTOUT. Thus, they often stay outside the modelling cycle, i.e. also outside the processing of the modelling task. The instructed students proceed in the solution process more goal-oriented and follow

the phases of the modelling cycle, which they learned in the instruction. Therefore, they deviate less often from the solution process when working on the modelling task. Since the non-instructed students lack knowledge about modelling processes, they move away from the problem (OUTPHASES) and/or summarize their previous solution. By switching between the two OUTPHASES, they again gather new ideas on how to proceed in the solution process. The frequent switches from *real model* to an OUTPHASE and vice versa show that the non-instructed students have problems forming the *mathematical model*. Either they switch to topics that have nothing to do with the processing of the task or they recapitulate their solution process. This can be related to the phase transition *mathematical model* to *mathematical solution*: the influence of an instruction mainly affects those phase transitions that are related to the *mathematical world*. Without knowledge about the formation of the *mathematical model*, individual modelling routes are less structured and often outside the ideal-typical modelling cycle.

Further research

We have shown that individual modelling routes of the instructed and non-instructed students differ with respect to the relative number of certain phase transitions. In the next step, we build on these results: using the MAI-Tool, we will examine individual modelling routes with respect to the sequence of phases. For this we will develop a concept to describe them more precisely on the basis of patterns.

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