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Design of a didactic activity based on the Hazen-Williams model for engineering education

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The research presented is focused on engineering education, framed in the Anthropological Theory of Didactics (ATD), whose objective is to design a didactic activity of mathematical modelling. For this purpose, didactic engineering is considered. The starting point is the analysis of a civil engineers' workplace. A praxeology based on the Hazen-Williams model to design pipelines is identified using notions of hydraulics, topography, and mathematics. A didactic transposition on this praxeology is performed. A mathematical modelling activity is designed and implemented with students in the fluid mechanics' course. The central task is to determine the diameter of the pipe and ensure a water flow rate of 50lts/s. The handling of the mathematical model requires a qualitative analysis of the variables at stake and to relate knowledge of various kinds.

Keywords: Mathematical models, higher education, engineering education.

Introduction

Linking the mathematics that engineers use at work with the mathematics they study at university is a social demand, highlighted by Pollak (1988). Indeed, some research has been conducted in the workplace to identify the mathematics being used (e.g., Frejd & Bergsten, 2016; Gainsburg, 2007). Their results show the fundamental role of mathematical models, the management of which requires other knowledge, such as computational, practical, and engineering, and particularly, from experience, which is fundamental in decision-making, what Gainsburg (2007) calls the engineer's judgement. These types of mathematics can occur in training through didactic proposals that are inspired or based on the analysis of mathematics in the workplace, as suggested by Frejd and Bergsten (2016). In this line, this research was carried out within the framework of the Anthropological Theory of Didactics (ATD), in proposing to relate the workplace and specialised training as a first step for designing didactic proposals for the mathematics classroom. We mainly raise two research questions: What kind of activities of engineer's workplace can be transposed to engineering courses through mathematical modelling teaching proposals? Furthermore, what institutional conditions make it possible to integrate these proposals into engineering courses?

Some elements from ATD

The ATD proposed by Chevallard (1999; 2019) defines a model for analysing human activity in its institutional dimension. The praxeology $[T, \tau, \theta, \Theta]$ is a minimal unit of analysis of human activity. Its four components are the task type (T), the technique (τ), the technology (θ), and the theory (Θ). The 'task' refers to what is to be done; the 'technique' is how it is to be done; the 'technology' is a discourse that produces, justifies, and explains the 'technique'; while the 'theory' produces, justifies, and explains the 'technology'. Doing mathematics is a human activity closest to mathematical modelling because doing mathematics in this frame consists of and acting (produce, teach, uses) on

mathematical models, as stated in Barquero et al. (2019). Institutions offer resources and conditions that allow their subjects to develop specific activities and establish restrictions. There are different types of institutions, and a subject may belong to several institutions, occupying various positions (e.g., teacher, student, parent, citizen). According to Chevallard (1999), praxeologies can circulate between institutions, undergoing, in effect, transpositive processes, i.e., transformations. To analyse this phenomenon in the case of the training of future engineers, Romo-Vázquez (2009) classified institutions according to their relationship with knowledge into three types: production, teaching and use. Production (or research) institutions are those that produce praxeologies, such as disciplines (e.g., mathematics, hydraulics); teaching institutions are in charge of transmitting praxeologies (e.g., school mathematics, school hydraulics) and using institutions are those in which praxeologies are used (e.g., industry, workplace). This classification is made considering the primary vocation of each institution. Still, it does not mean that praxeologies are not created or taught in the workplace or that no praxeologies are taught in the disciplines. However, when a disciplinary praxeology is taught, it undergoes a didactic transposition. It means that it is transformed to become an object of teaching. Thus, mathematical praxeologies become school mathematical praxeologies (Chevallard, 1991). The didactic transposition process is illustrated by Bosch & Gascón (2006, p. 56) as follows (figure 1):

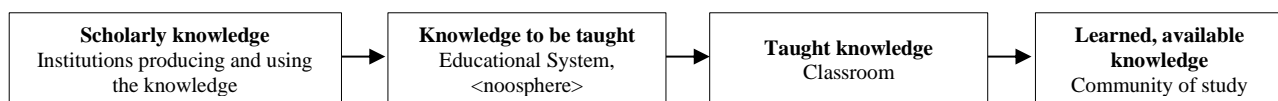


Figure 1: The didactic transposition process

In the case of training of future engineers, specific didactic transpositions can be performed. For example, transposing mathematical modelling praxeologies from the workplace to mathematics or engineering education, as suggested by some research (e.g., Frejd & Bergsten, 2016; Romo et al., 2017), as represented in figure 2.

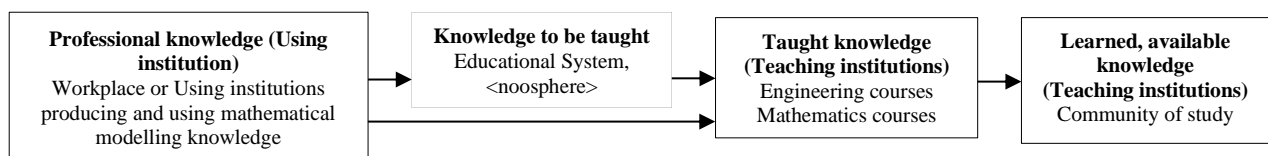


Figure 2: Transposing mathematical praxeologies from the workplace to teaching institutions

Performing this kind of transposition from Workplace to Engineering education demands identifying a local¹ mathematical modelling praxeology in a specific workplace of engineers W : $[T^e, \tau^{em}, \theta^{em}, \Theta^e] \leftarrow P_w$. Here, T^e is an engineering type task and the technique τ^{em} to perform this type of task has mathematical and engineering elements. The technology θ^{em} that justifies the technique is a mathematical model used in engineering, whereas the theory is from engineering. Praxeology P_w is then transformed into a school mathematical modelling praxeology P_s that can be constructed in engineering teaching (ET) as illustrated in figure 3.

¹ Praxeology with the same logos block for different practical blocks. It corresponds metaphorically to a modelling topic composed of various sub-topics.

$$\begin{bmatrix} T^e & \tau^{em} & \theta^{em} & \Theta^e \\ T^{se} & \tau^{sem} & \theta^{sem} & \Theta^{sem} \end{bmatrix} \leftarrow \begin{matrix} W \\ ET \end{matrix} \curvearrowright$$

Figure 3: Scheme of transposition workplace praxeology to teaching praxeology

The process of transposition demands considering the institutional conditions: workplace and teaching, and the way of the original praxeology can be transformed to live in the teaching institutions. Likewise, P_w is conceived like an *epistemological model reference* in the sense proposed by Barquero, Bosch and Gascón (2019). Thereby, P_w must be a local modelling praxeology.

Methodology: Didactic engineering

Didactic engineering constitutes a solid research methodology (Artigue, 2020), which allows the design of tasks. Its four phases are preliminary analysis, activity design and a priori analysis, experimentation, and a posteriori analysis. Based on this, we design and analyse a didactic activity of mathematical modelling for engineering education, as illustrated below.

Phase 1. Preliminary analysis: Civil engineers' mathematical modelling praxeology of hydraulics.

This analysis focused on characterising a mathematical modelling praxeology of hydraulics used by civil engineers to design pipelines in their workplaces. It was carried out jointly by a mathematics education researcher and a civil engineer with 29 years of experience, who is also a university professor (first author) of mathematics and engineering courses and a PhD student in mathematics education. Analysing his professional practice is a difficult task because some knowledge is no longer recognised, then a researcher in mathematics education questioned and asked for explanations. Civil engineers' mathematical modelling praxeology of hydraulics is represented as $[T^h, \tau^{hm}, \theta^{hm}, \Theta^h] \leftarrow P_w$, where h represents hydraulics and m mathematics. Thus, task type is T^h : Design a pipeline to transport water between two points taking advantage of the effect of gravity. Technique τ^{hm} and technology θ^{hm} : Step 1. Recognise the project's requirements and the minimum initial data to calculate the difference in level between the supply and distribution reservoirs and the total length of the pipeline path. Step 2. Determine the whole length of the pipeline path (L) using an established reference system (see figure 4). Step 3. Determine the hydraulic head of the system (ΔH), calculating the difference between supply and distribution reservoirs. ΔH is always greater than zero since the level of the distribution reservoir is lower than that of the supply reservoir, ensuring that the effect of gravity generates the flow of the fluid (figure 4).

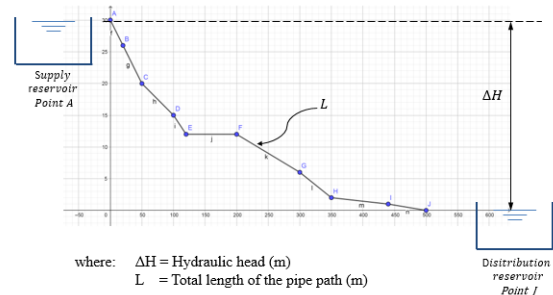


Figure 4: Schematic illustration of the pipeline path

Step 4. Select the material of manufacture of the pipe, considering the roughness of materials employing the Hazen-Williams coefficients (C)². A material with high roughness will have a lower coefficient and a lower flow rate for the same pipe diameter. In contrast, the lower roughness of the material will have a higher coefficient and a higher

² See for example Alegret & Martínez (2019, p. 45).

flow rate. Step 5. Determine the flow rate (Q) using the Hazen-Williams mathematical model: $Q = 0.2785C D^{2.63} S^{0.54}$ where: Q = Flow rate at the pipe (m^3/sec); C = Hazen-Williams coefficient (dimensionless); D = Diameter of the pipe in meters (m) and S = hydraulic head loss per length of pipe (dimensionless factor) obtained with the following formula: $S = \Delta H/L$ where: ΔH = hydraulic head, expressed in meters, L = Total length of the pipe path interconnecting both reservoirs. The applicable limits of this formula are debatable; in the third edition of Hydraulic tables by Williams & Hazen (1933), the pipe diameters to be used were limited to $0.05 \text{ m (2 in.)} < D < 1.85 \text{ m. (6 ft.)}$. Furthermore, it is recommended to be used for flow velocities values below $10 \text{ ft./sec (3.05 m/s)}$ and is valid only for water flowing at ordinary temperatures ($5 \text{ }^\circ\text{C} - 25 \text{ }^\circ\text{C}$) (Alegret & Martínez, 2019). It is worth mentioning that, in the case of not complying with the described parameters, the mathematical model could give results different from reality. Step 6. Verify that the flow rate (Q) obtained is the closest (approaching from a higher value) to the project's required flow rate (Q_p); Step 7. Check the water velocity (v) at the pipeline, which should be less than $10 \text{ ft./sec (3.048 m/sec)}$. Step 8. Report the pipeline design results. To perform these eight steps requires that engineers properly use the Hazen – Williams model, recognising the mathematical relationship of the variables involved and how they can be satisfied from experimental conditions. Sometimes it is necessary to adjust data and, above all, to contrast the results obtained by the model with the existing pipe diameters on the market. The theory Θ^e is hydraulic engineering.

Phase 2. Didactic transposition of the civil engineers' mathematical modelling P_w , design of the didactic activity and a priori analysis. A didactic transposition was performed on the P_w , obtaining a school praxeology P_s . The task type is the same; the technique is organised in P_s through three stages using subtasks $T_{n.m}$. The technology in both praxeologies is the Hazen-Williams mathematical model, and the theory is hydraulic. However, in school praxeology, the fluid mechanics's course is also involved, see figure 5.

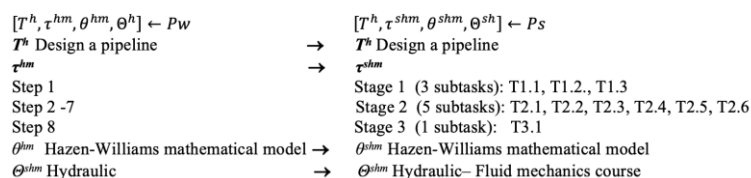


Figure 5: Scheme of didactical transposition from P_w to P_s

Thus, the school praxeology is described in this way. T^h : Design a pipeline to transport water between two points taking advantage of the effect of gravity. The technique has three stages. In stage 1, the requirements and initial data of the project are recognised through three subtasks. Firstly, T1.1. Identify the required flow rate (Q_p) at the distribution reservoir. This is the project's main datum, for example, 50 lts/sec ; T1.2. Analyse the topographic data (see Table 1), draw up a scheme and visualise the general conditions of the system to be designed. This is a first approach to the establishment of the reference system: location of the supply point as an initial point of the system, establishment of scales to use and labels to identify the implicit elements in the system. Finally, T1.3. Drawing up the topographic profile for recognising the natural terrain and identifying key points for the design of the pipeline. Civil engineers usually use the software like AutoCAD, favouring parameter manipulation for better visualisation and detailed analysis. Students can use GeoGebra, which provides an elevation

view and elements necessary in a simplified two-dimension drawing to apply the Hazen-Williams model (see figure 6). Stage 2: designing a pipeline to transport water between two points by taking advantage of the effect of gravity involves five subtasks. T2.1. Determination of the pipeline's total length is important for calculating the hydraulic head loss per length of the pipe. The lengths of the pipe sections are calculated from point to point where the changes of direction in the path occur and add together. Students can perform these calculations in GeoGebra. T2.2. Calculation of the hydraulic head, a dominant factor for the natural movement of the fluid through the pipe. This is the difference in level between the point of supply and the point of distribution calculated in meters. T2.3. Selection of the type of pipe considering three main conditions: 1) installation needs, 2) cost of the pipe, and 3) the related Hazen-Williams coefficient, depending on the material of manufacture. The students do not have information on conditions 1 and 2, so they should focus on the efficiency of the material for fluid conduction, evaluating the Hazen-Williams coefficient. T2.4. Determine the flow rate. *This is the central task* and consists of the application of the Hazen-Williams mathematical model. Although their use is not identified in professional praxeology, it is considered that students could propose an initial value of the pipe Diameter using the continuity equation $A = Q/v$, where v is the limiting velocity of the water in the pipe, Q , the flow rate for which the design is made and A , the cross-sectional area of the pipe. Considering the formula of the area of a circle, it is feasible to obtain an initial datum for the Diameter. For the students to perform this same procedure, avoiding an initial random proposal of Diameter, they will be provided with a table that favours the qualitative analysis of relationships between variables. T2.5. and T2.6. Verification of compliance with limitations or restrictions of the method, guaranteeing the correct operation of the mathematical model to obtain results following reality. 1) Pipe Diameter limited to $0.05 \text{ m} (2 \text{ inches}) < D < 1.85 \text{ m} (6 \text{ ft.})$; 2) Design flow rate greater than or equal to that required in the project [$Q \geq Q_p$]; 3) Fluid velocity in the system must be less than 10 ft. per second (3.05 m/s). To verify it, the formula $v = Q/A$ is used again. Suppose any of the restrictions are not met. In that case, students are expected to propose another diameter, develop the procedure, checking whether all the restrictions are met again, i.e. an iterative process is generated until the pipe Diameter that meets the restrictions of the method is found. As a means of verification (immediate feedback) and approaching the professional reality, students can use “Epanet”, specific software for the design of piping systems in which the Hazen-Williams mathematical model is encapsulated or implicit. Stage 3) Report the pipeline designed results. A T3.1 subtask is proposed to elaborate a report of the pipeline designed results.

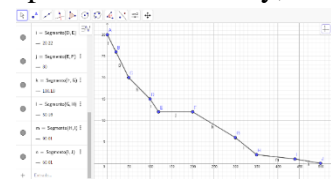


Figure 6: Topographic profile in GeoGebra

The objective of the didactic activity is to allow students to construct or reconstruct the school praxeology Ps in the classroom. For this purpose, a situation similar to those faced in workplace practice is proposed: “A team of engineers has to connect two reservoirs for the supply of water for domestic use in some town, whose flow rate average required is 50 lts/sec, according to the results of previous studies carried out by specialists. The topographic study carried out on the path of the pipeline that will connect these reservoirs yields the following data on the obligatory points (Table 1), to carry out the least amount of excavation possible.”

Table 1: Topographic data of pipeline

Punto	Coordenadas		Punto	Coordenadas	
	X	Y		X	Y
A (supply)	0	30	F	200	12
B	20	26	G	300	6
C	50	20	H	350	2
D	100	15	I	440	1
E	120	12	J(distribution)	500	0

Phase 3. Experimentation. The study was developed at the Universidad Cristóbal Colón, Veracruz, Mexico, in engineering education. The didactical activity was implemented in a fluid mechanics course. The participants were 37 students of the fourth semester, from majors of Industrial Engineering and Petroleum Engineering.

Phase 4. A posteriori analysis. The analysis was performed on the basis of the students' worksheets and the teacher's notes on this activity. Three teams were chosen to analyse their praxeologies and, mainly, how they used the mathematical model, chose the Diameter and justified their solution to the project. The criteria for selecting the teams were the clarity and coherence of their reports and the students' commitment to the development of the activity (according to the teacher's notes).

Some results

This section presents a first analysis of the praxeologies developed by three teams of students, particularly in three subtasks: the determination of the Diameter and flow rate (T2.4) and the verification of compliance with limitations or restrictions of the method (T2.5 and T2.6), which were vital in the development of the didactic activity. When the students were faced with subtask T2.4, they had calculated the value of S and identified the material for their pipe and the Hazen-Williams coefficient C . So, they still had to determine the value of the Diameter D and Q (flow rate) by using the Hazen-Williams model: $Q = 0.2785CD^{2.63}S^{0.54}$. The students had no previous experience selecting pipe diameters; then the teacher provided them with a table with different diameter values. So that they could evaluate it in the mathematical model and, depending on the results of the flow calculation, continue with the proposal of diameters until they found the one that would provide the closest flow to that required by the project. Thus, we tried to avoid using the mathematical model as a formula and favour qualitative analysis and the relationship between the variables at stake. The students determined the Diameter and verified that it was as close as possible (approaching from a higher value) to the flow rate required by the project ($Qp = 50\text{Lts/s}$). (See Tables 2a, 2b, and 2c, corresponding to samples of work achieved by the teams 1, 2, and 3, in that order).³

Table 2a: Work by team 1

Diameter (in)	Diameter (mm)	Diameter (m)	Flow rate (m ³ /sec)	Flow rate (Lt/sec)
5.7	144.78	0.14478	0.049066	49.0661151
5.8	147.32	0.14732	0.051363	51.3625321
5.9	149.86	0.14986	0.053724	53.7244023

Table 2b: Work by team 2

Diameter (in)	Diameter (mm)	Diameter (m)	Flow rate (m ³ /sec)	Flow rate (Lt/sec)
3 3/4	95.25	0.09525	0.0543	54.3
3 1/2	88.9	0.0889	0.0453	45.3
4	101.6	0.1016	0.0644	64.4

Table 2c: Work by team 3

Diameter (in)	Diameter (mm)	Diameter (m)	Flow rate (m ³ /sec)	Flow rate (Lt/sec)
5.75	146.05	0.1463	0.050431	50.4331434
5.744	145.8976	0.1459	0.050033	50.0334106
7	177.8	0.1778	0.5053	50.53

³ These tables were made by the authors, considering some rows of the original tables. The green highlighted was also made by the authors.

Team 1 selected a Diameter of 5.8 inches; it is observed that for this selection, they started evaluating diameters from the table they were provided (2, 3, 5) until they found values close to the required project's flow rate, then by 0.1-inch diameter approximations (5.7, 5.8, 5.9). They found the closest value, approaching from a higher value to the project flow rate (see table 2a). *Team 2* selected a diameter of 3¾ inches. This team presents errors in the operation of the mathematical model, it obtains higher flow rates using smaller diameters; the process of selecting the pipe diameter is very similar to that carried out by team 1, with the difference that they use approximations of 1/4 of an inch. (see table 2b). *Team 3* selected a diameter of 5.744 inches. It can be seen that they used the same procedure for diameter selection as teams 1 and 2; in this case, the approximation of the diameter proposals is 0.001 inches, trying to obtain the value of the project flow exactly equal to that required (see table 2c). The mathematical model worked as a technology that allowed them to control the technique, not considering whether the Diameter obtained corresponds to existing pipes in the market. This knowledge is constructed in the engineer's workplace. Moreover, we could ask students to verify the pipe's existence in a new redesign of the modelling activity. Thus, they would have another element to validate the Diameter obtained with the Hazen-Williams model. Concerning the verification of the obtained flow rate to be the most adjusted (approaching from higher values), it was observed that in all cases the students selected the Diameter that provided the closest magnitude to the required project flow rate, always considering values higher than this one. T2.5 Regarding the review of the water velocity at the pipeline using the continuity equation: $v = Q/A$ (where A is the cross-sectional area of the pipe). It was observed that some teams did not perform it. Where this was done, they only indicated that the parameter was not met. However, they did not make any proposal to establish a design that meets this requirement.

As an extra activity, and as a possibility for future research, it was proposed to the students to work with more specialised software –used in the workplace–, which allows them to check their results and receive immediate feedback, favouring reflection on the work carried out, proposals for improvement and connection with reality. Modelling the system in Epanet allows inserting geometric data of the system (obtained from the topographic profile in GeoGebra, figure 6), characteristics of the pipe (diameter, length, material of manufacture), accessories and equipment (e.g., valves, pumps) with the corresponding characteristics obtained from the manufacturer. The software also allows an immediate validation of the results by consulting the design parameters; these are flow rate and fluid velocity. This can be seen on the screen by moving the pointer to the desired element (figures 7 and 8).

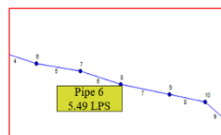


Figure 7: Epanet fluid flow rate

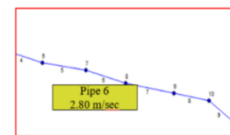


Figure 8: Epanet fluid

Conclusion

This research shows an avenue to design mathematical modelling activities for the training of engineers that connect using and teaching institutions. Didactic engineering allows performing a didactical transposition on the workplace's mathematical modelling praxeology. In this case, the analysis of the Civil engineers' mathematical modelling praxeology of hydraulics is developed by a subject of a using institution and a teaching institution. He analysed his workplace activity, identified a mathematical modelling praxeology, transposed it, generated a didactic activity and implemented

it. His professional experience as an engineer and as a teacher was fundamental in all stages of didactic engineering. The students have no difficulties carrying out this activity, but their lack of experience limits the proposed solutions: expensive materials, non-existing pipe diameters, lack of practical verifications. The use of GeoGebra allows an approach to mathematics courses in the first semesters. In contrast, the possibility of using Epanet's software allows an approach to professional practice, guiding the student in the consideration and analysis of parameters and the relationship between them for the generation of the solution with elements very close to reality. From the teacher's perspective, the software used is complementary since GeoGebra provides relevant geometric data for modelling the project in Epanet. It is considered that this didactic activity can also be adapted to be implemented in a mathematics course, and even more, it can be proposed to be developed by students from different semesters with different backgrounds and experiences.

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